Kerr Primordial Black Holes

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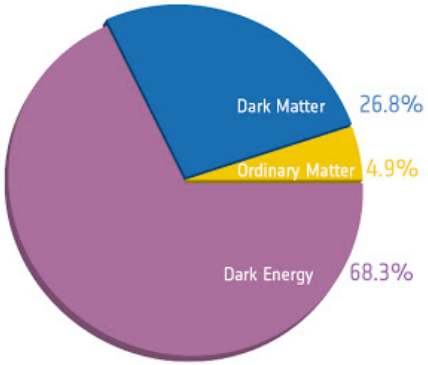
In collaboration with Alexandre Arbey & Joseph Silk

July 7th 2019

Beyond General Relativity, Beyond Cosmological Standard Model
University of Warsaw

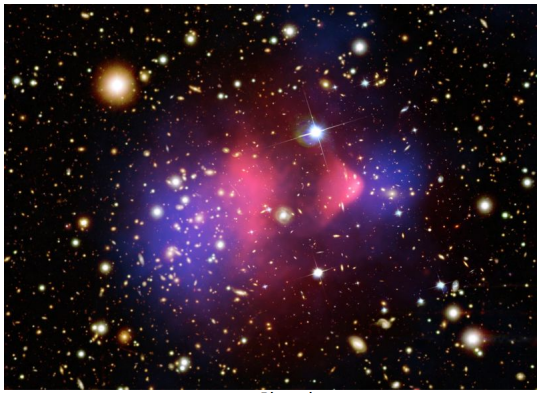
Dark Matter

Cosmic Microwave Background



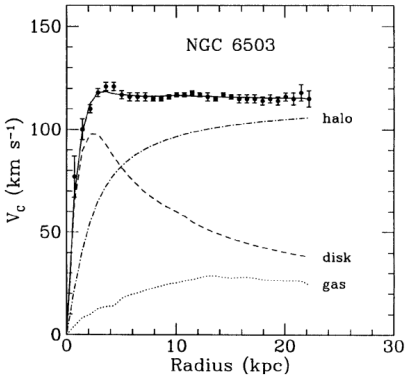
source: Planck Collaboration

Galaxy clusters collision



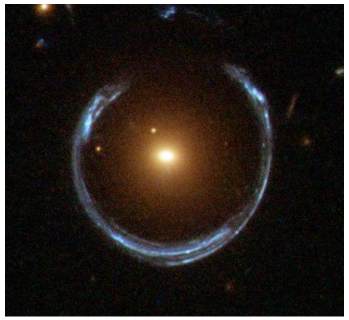
source: Chandra

Galaxy rotation curves



Freese [arXiv:1701.01840]

Gravitational lensing



source: Hubble

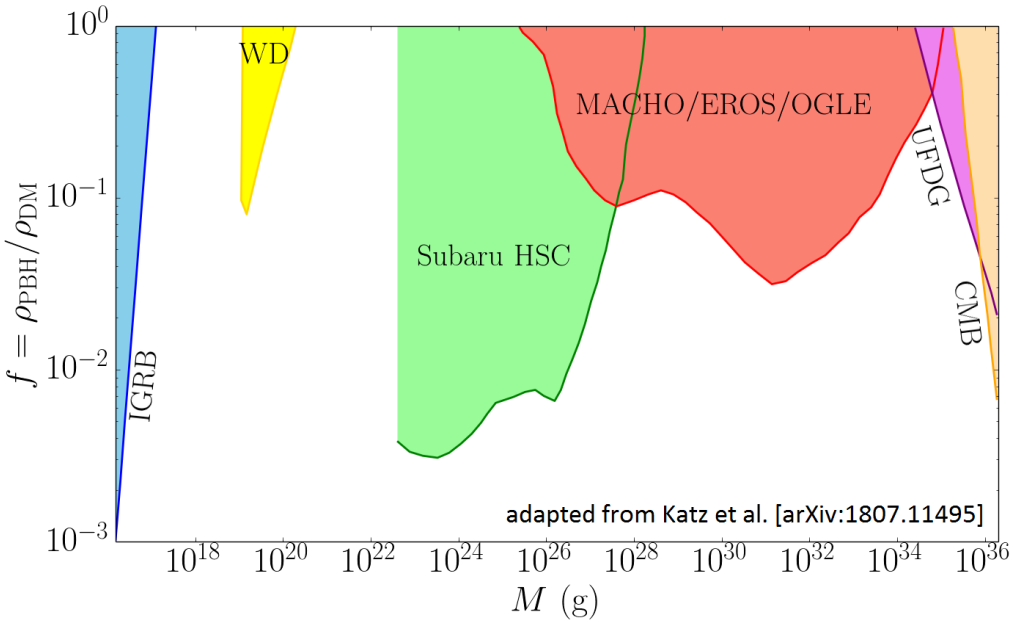
Observations of large scale structures, galaxies and cosmology show that 90% of matter is dark

Primordial Black Holes

A plausible DM candidate

- no Standard Model / General Relativity extension
- dynamically cold
- BH existence proven
- mass ranges still available for BHs to represent all of DM

PBH Hawking radiation, lensing and dynamical constraints



Primordial Black Holes

Multiple inflationary origins

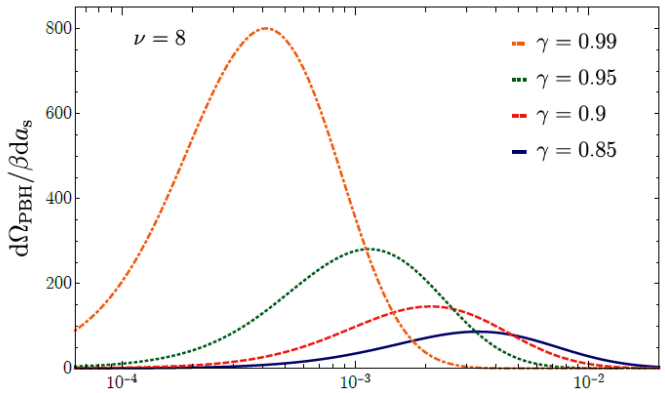
- collapse of large primordial overdensities
- phase transitions
- collapse of cosmic strings or domain walls

Spin predictions

$$a^* \equiv J/M^2$$

Standard inflationary model

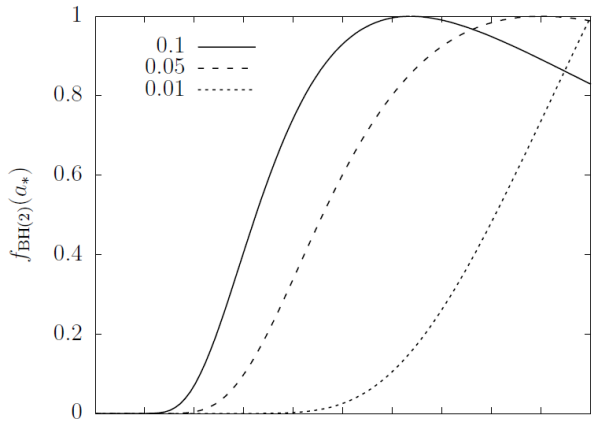
$\Longrightarrow$ low spin



De Luca et al. [arXiv:1903.01179]

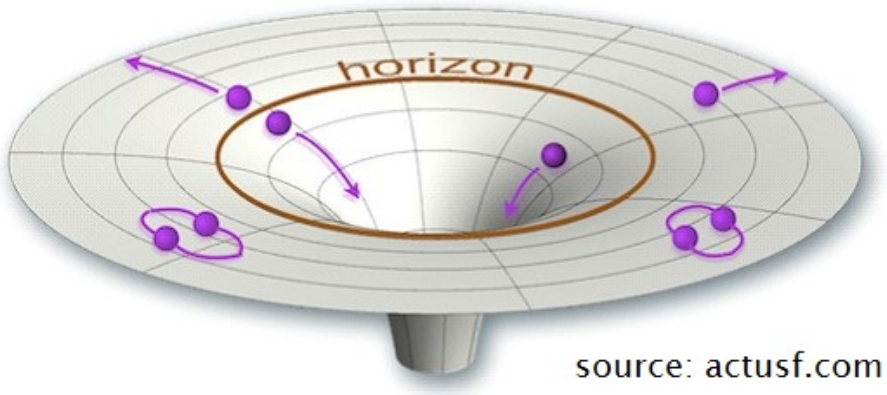
Transient matter domination

$\Longrightarrow$ high spin



Harada et al. [arXiv:1707.03595]

BH Hawking radiation



Fundamental equation for Kerr BHs

Rate of emission of Standard Model particles i at energy E by a BH of mass M and spin parameter a^*:

$$Q_i \equiv \frac{\mathrm{d}^2 N_i}{\mathrm{d}t\mathrm{d}E} = \frac{1}{2\pi} \sum_{\mathrm{dof.}} \frac{\Gamma_i(M, E, a^*)}{e^{E/T(M,a^*)} \pm 1}$$

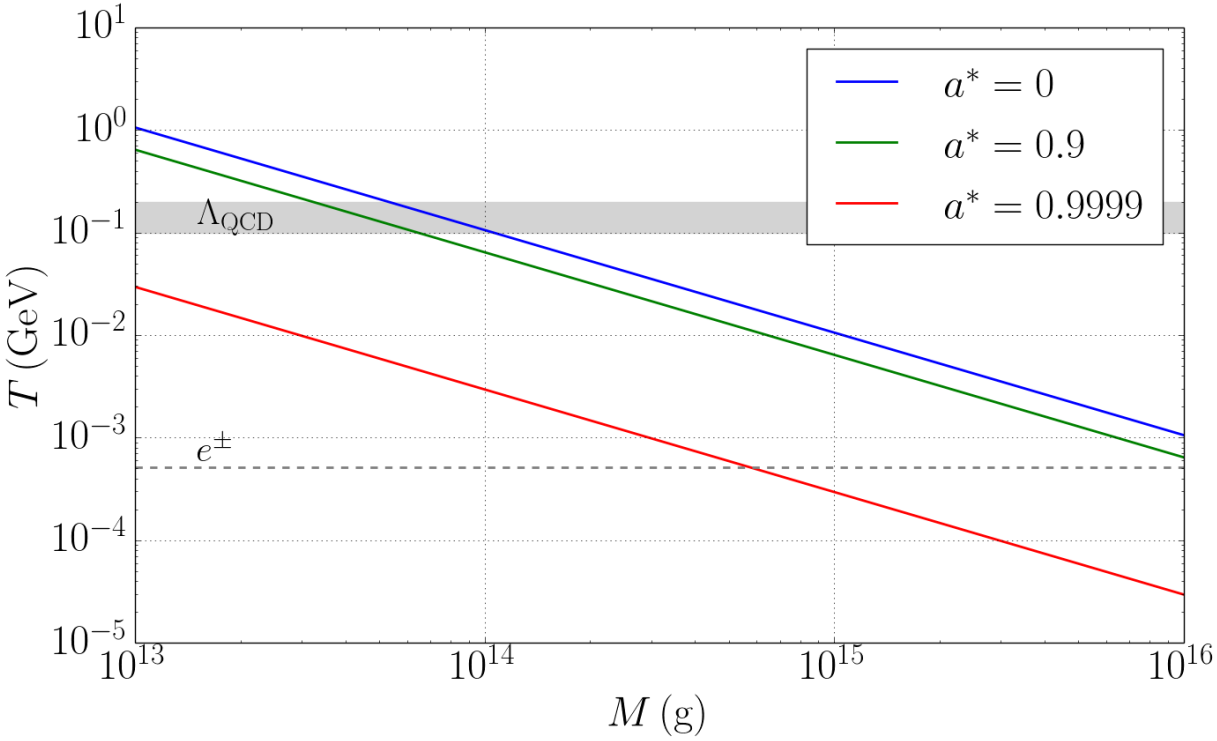
Γ_i is the greybody factor $\iff$ transmission coefficient (deviation from Planck's blackbody law)

Reduced temperature

Hawking temperature for Kerr BHs

$$T(M, a^*) = \frac{1}{4\pi M}\left(\frac{\sqrt{1-(a^*)^2}}{1+\sqrt{1-(a^*)^2}}\right) \xrightarrow[a^*=0]{\text{Schwarzschild}} \frac{1}{8\pi M}$$

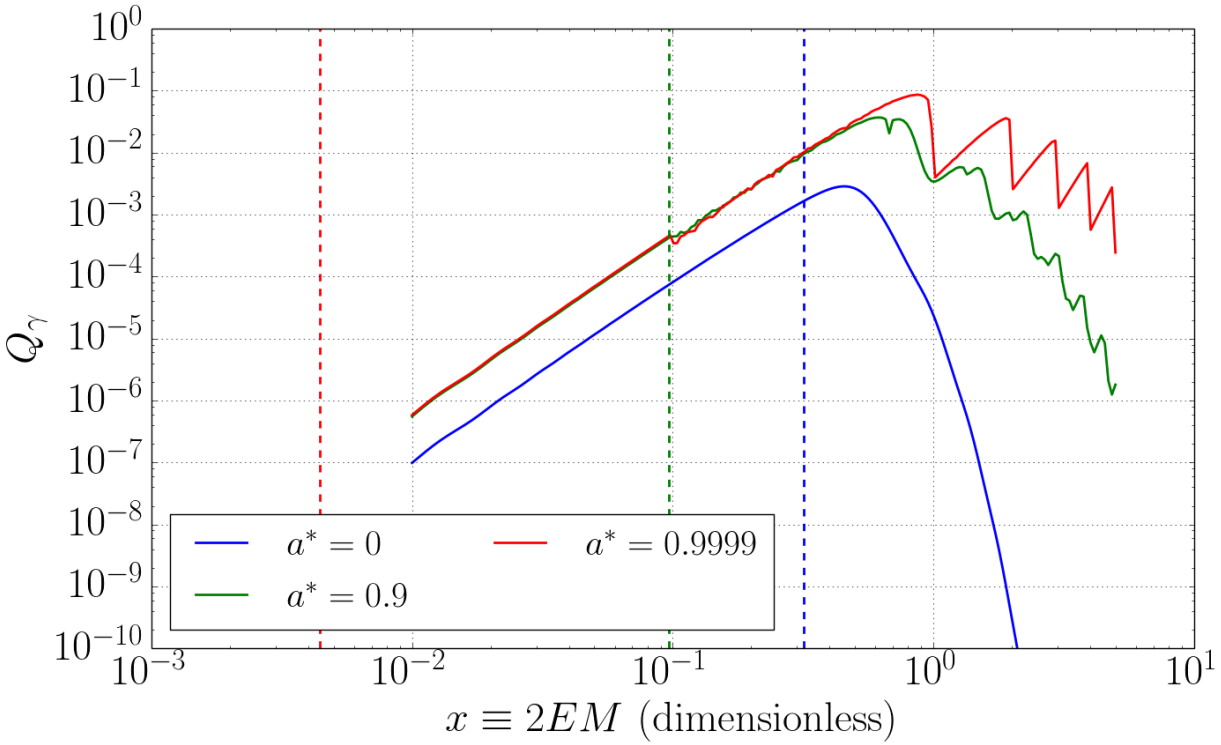
Comparison with the $e^{\pm}$ rest mass and the QCD scale Λ_{QCD}



Enhanced emission

BH-particle spin coupling $\implies$ superradiance effects (see e.g. Chandrasekhar & Detweiler papers in the 1970s)
The Hawking radiation is enhanced for high-spin particles.

Example of spin 1 massless emissivity (photon)
Dotted lines = Hawking temperature



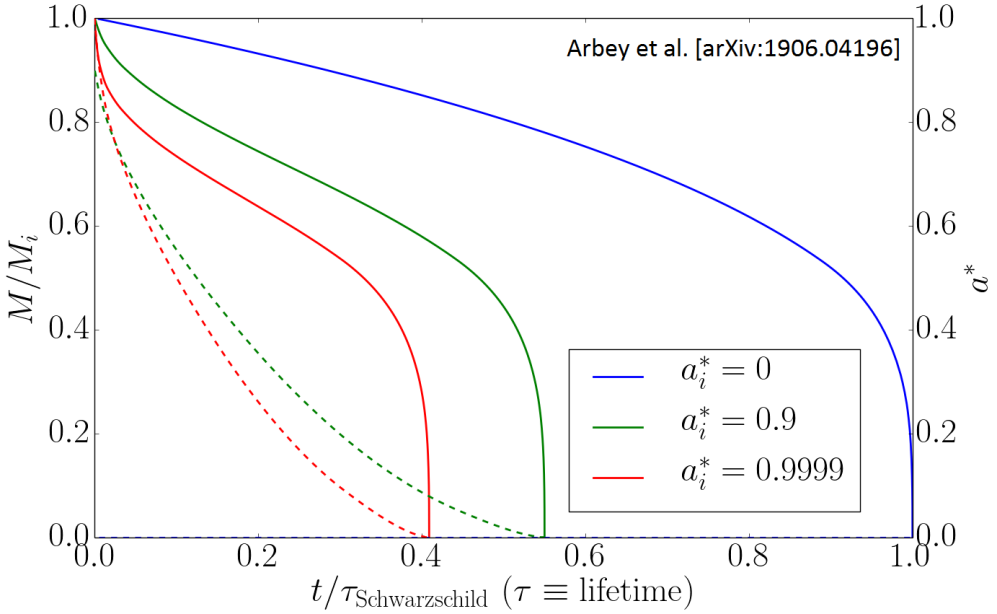
Reduced lifetime

Evolution equations

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{f(M,a^*)}{M^2} \qquad f \sim \int_E ener. \times emiss.$$

$$\frac{\mathrm{d}a^*}{\mathrm{d}t} = \frac{a^*(2f(M,a^*) - g(M,a^*))}{M^3} \qquad g \sim \int_E ang.\ mom. \times emiss.$$

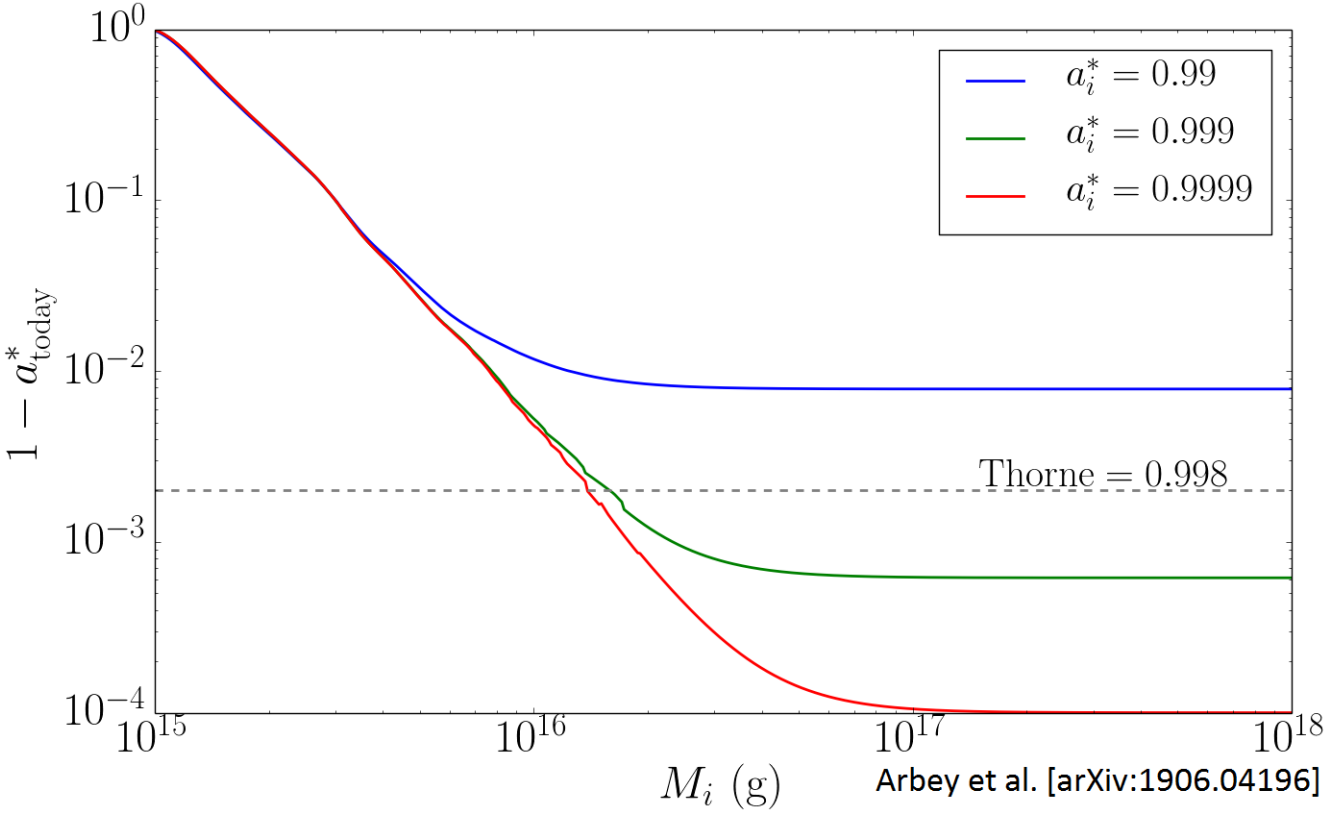
BH mass (solid) and spin (dotted) evolution



Extremal spin today?

Could high spin BHs exist today? Can we get over Thorne's limit (disk accretion on rotating BHs)?
→ Yes, with sufficiently massive and extremal PBHs

PBH final spin as a function of its initial mass



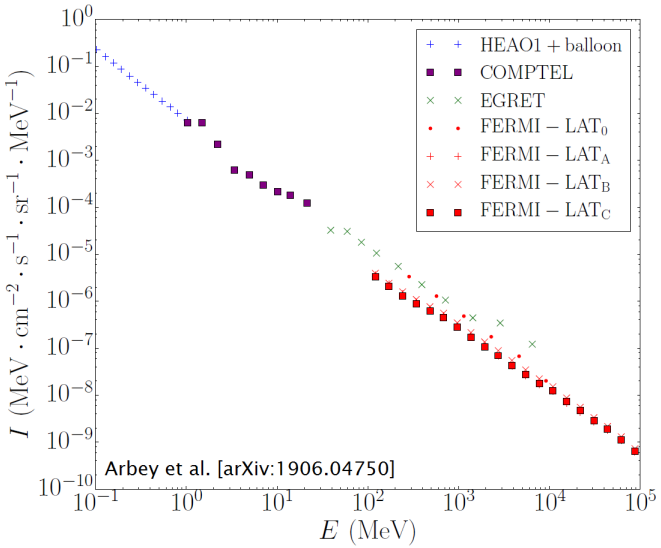
Arbey et al. [arXiv:1906.04196]

Isotropic gamma ray background constraints

Origin

Diffuse background +

- Active galactic nuclei
- Gamma ray bursts
- DM annihilation/decay?
- Hawking radiation?



Flux estimation for BHs

Arbey *et al.* [arXiv:1906.04750]

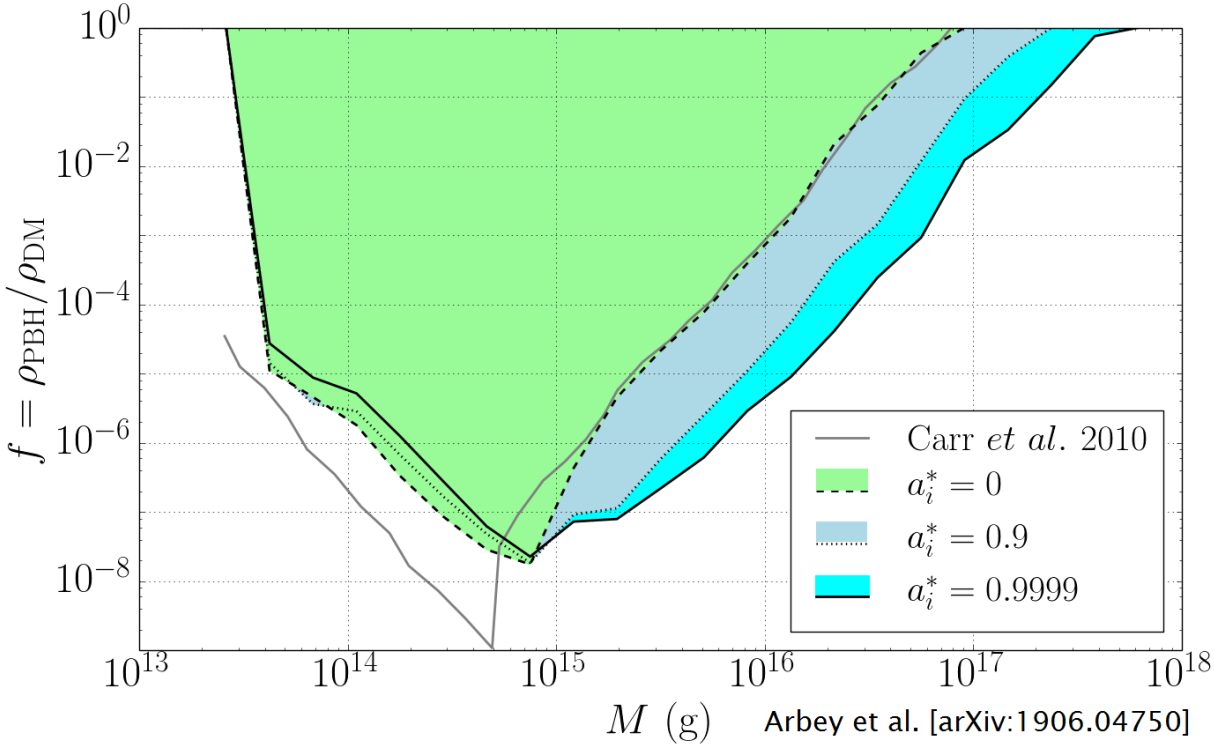
$$I \approx \frac{1}{4\pi} E \int_{t_{\rm CMB}}^{t_{\rm today}} (1+z(t)) \times \int_M \left[\frac{{\rm d}n}{{\rm d}M} \frac{{\rm d}^2 N}{{\rm d}t{\rm d}E} (M, (1+z(t))E)\, {\rm d}M \right] {\rm d}t$$

Extension to Kerr PBHs

Main spin effects

- enhanced luminosity $\Longrightarrow$ stronger constraint
- reduced temperature $\Longrightarrow$ reduced emission energy $\Longrightarrow$ weaker constraint

Monochromatic constraint from the IGRB



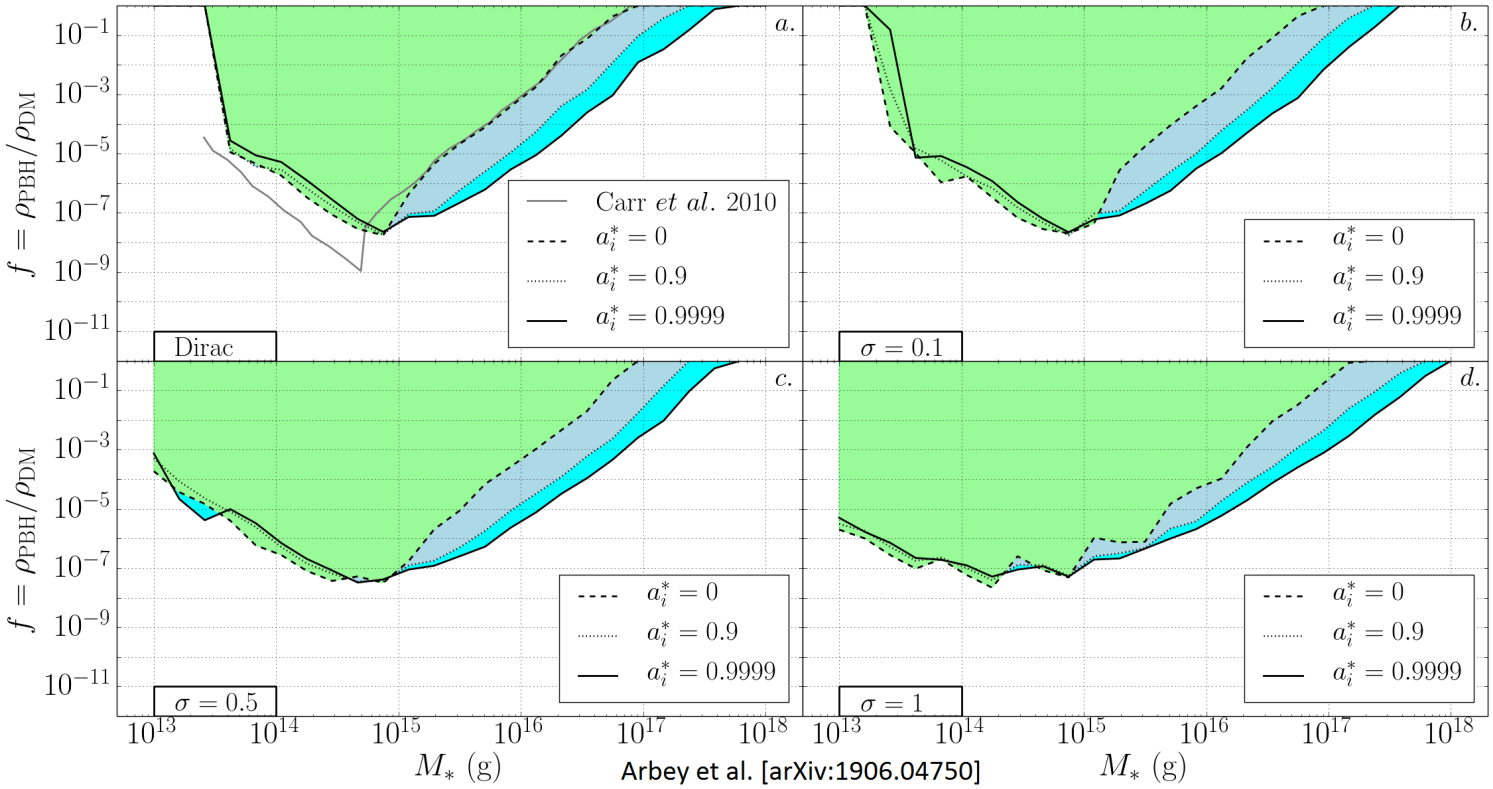
Arbey et al. [arXiv:1906.04750]

Extension to broad mass functions

Main width effects

$$M\mathrm{d}n/\mathrm{d}M \propto \exp(-\ln(M/M_*)^2/2\sigma^2)$$

- broadening of the energy spectrum $\Longrightarrow$ stronger constraint
- broadening of the mass distribution $\Longrightarrow$ greater DM total density $\Longrightarrow$ weaker constraint



Arbey et al. [arXiv:1906.04750]

BlackHawk

A public C code computing the Hawking radiation:

- Schwarzschild & Kerr PBHs
- primary & secondary spectra of Standard Model particles
- extended mass functions
- time evolution of the PBHs

Download: `http://blackhawk.hepforge.org`
Manual: arXiv:1905.04268

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- Manual
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BlackHawk

By Alexandre Arbey and Jérémy Auffinger

Calculation of the Hawking evaporation spectra of any black hole distribution

BlackHawk is a public C program for calculating the Hawking evaporation spectra of any black hole distribution. This program enables the users to compute the primary and secondary spectra of stable or long-lived particles generated by Hawking radiation of the distribution of black holes, and to study their evolution in time.

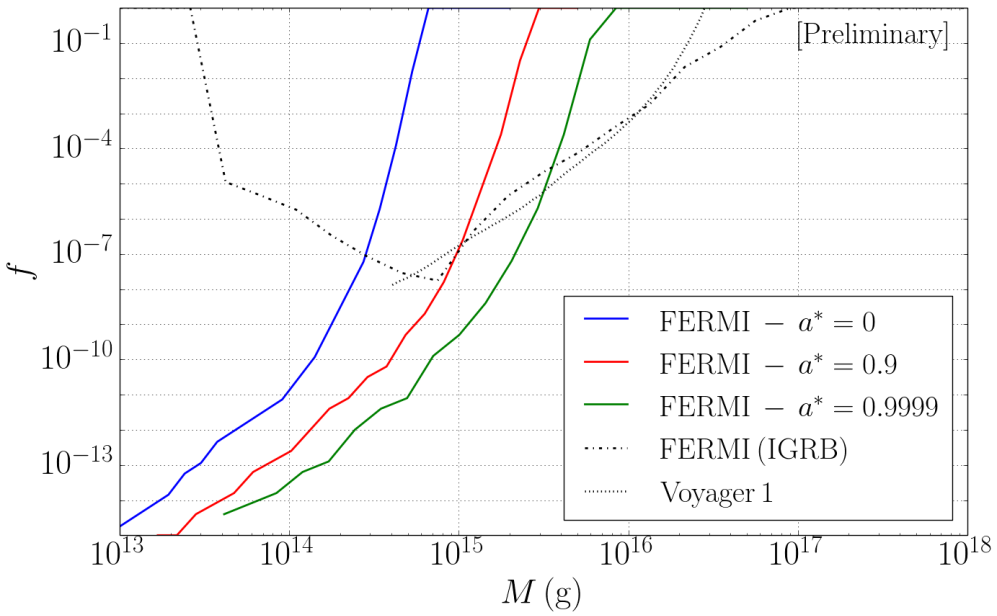
If you use BlackHawk to publish a paper, please cite:
A. Arbey and J. Auffinger, arXiv:1905.04268 [gr-qc]

For any comment, question or bug report please contact us.

On-going work

- Big Bang Nucleosynthesis (see e.g. Sedel'nikov 1996, Kohri 2000)
- galactic gamma & X-rays (see e.g. Ballestros *et al.* [arXiv:1906.10113])
- galactic positrons (see e.g. Boudaud & Cirelli [arXiv:1807.03075], DeRocco & Graham [arXiv:1906.07740], Laha [arXiv:1906.09994])

Dwarf Spheroidal galaxies (dSph) gamma ray constraints



Conclusion

Main results

- New public code `BlackHawk` to compute the Hawking radiation
- Study of the evolution of Kerr PBHs and constraints from IGRB
- Extension to more realistic broad PBH mass functions

Perspectives

- Closing the remaining PBH mass windows for all DM into PBHs?
- Primordial BH / Astrophysical BH discrimination using GW events? [arXiv:1905.13019]
- Other constraints...

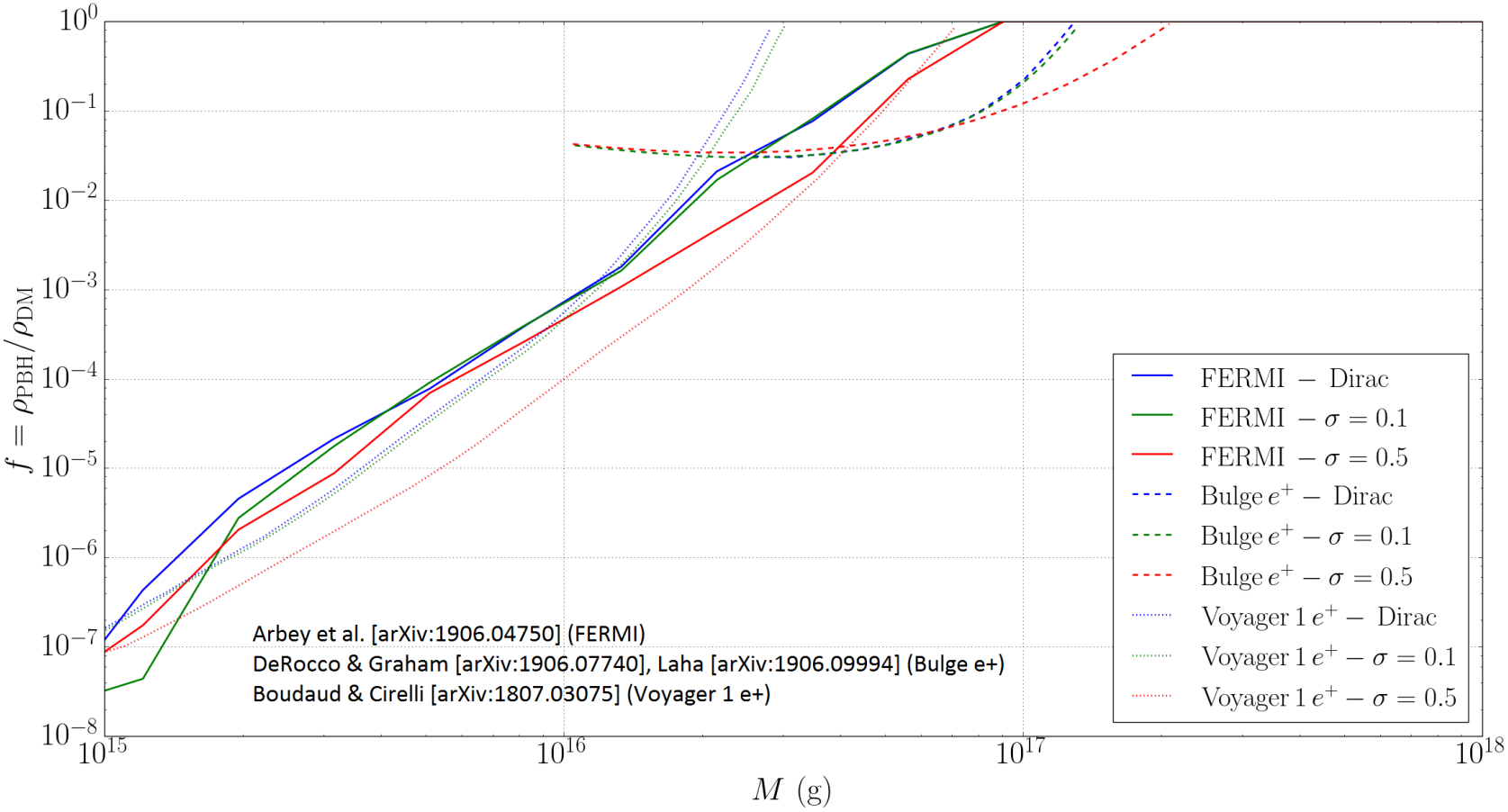
Thank you!

Publications:

- `BlackHawk`: `http://blackhawk.hepforge.org` [1905.04268]
- Any extremal black holes are primordial [1906.04196]
- Constraining primordial black hole masses with the isotropic gamma ray background [1906.04750]

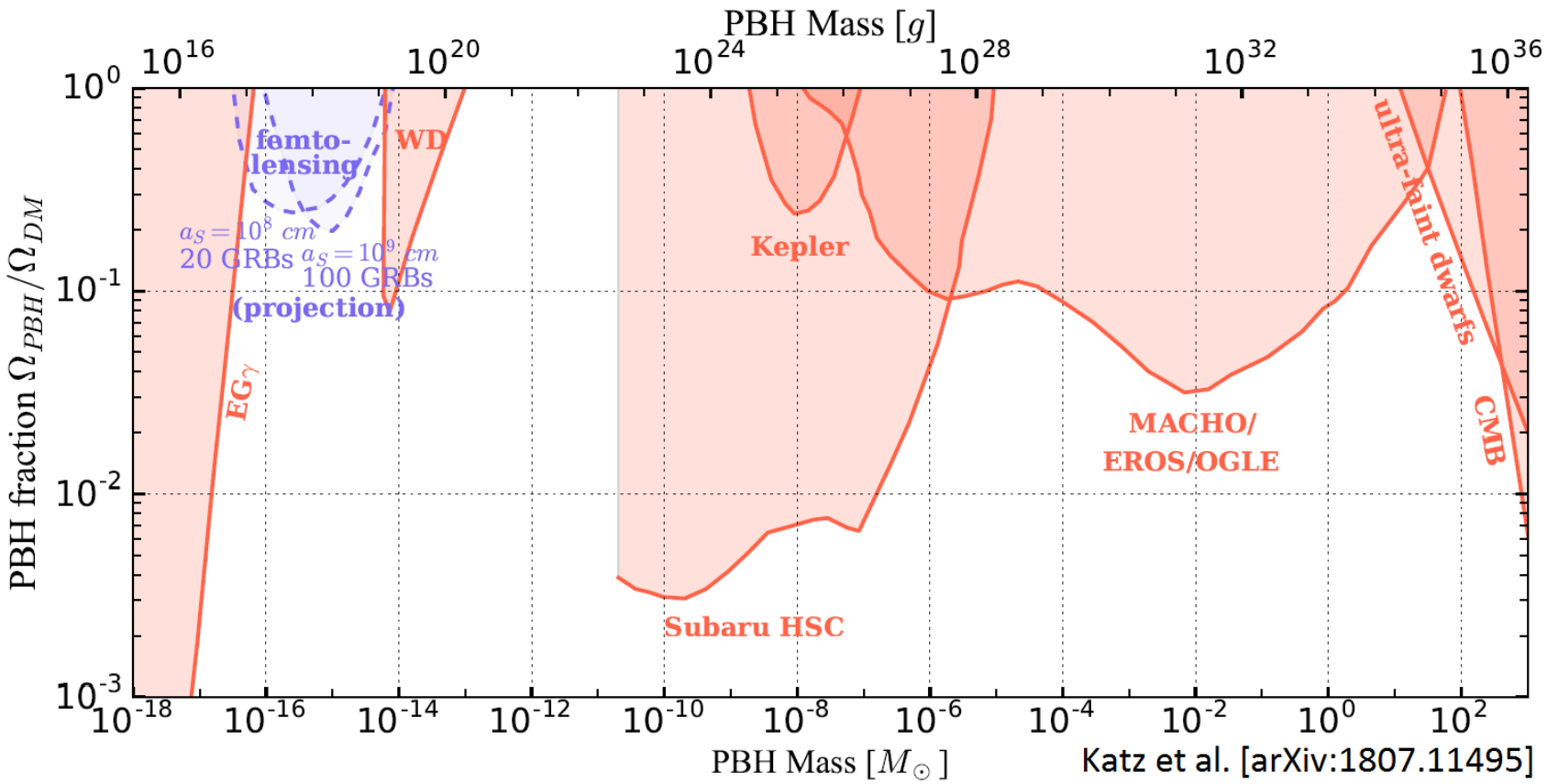
Other constraints

Comparison with recent e^+ constraints



Other constraints

Monochromatic constraints with femtolensing prospects



Katz et al. [arXiv:1807.11495]

Kerr Hawking radiation equations

Kerr metric

$$\mathrm{d}s^2 = \left(1 - \frac{2Mr}{\Sigma^2}\right)\mathrm{d}t^2 + \frac{4a^* M^2 r \sin(\theta)^2}{\Sigma^2}\mathrm{d}t\mathrm{d}\phi - \frac{\Sigma^2}{\Delta}\mathrm{d}r^2$$
$$- \Sigma^2 \mathrm{d}\theta^2 - \left(r^2 + (a^*)^2 M^2 + \frac{2(a^*)^2 M^3 r \sin(\theta)^2}{\Sigma^2}\right)\sin(\theta)^2 \mathrm{d}\phi^2$$

$\Sigma \equiv r^2 + (a^*)^2 M^2 \cos(\theta)^2$ and $\Delta \equiv r^2 - 2Mr + (a^*)^2 M^2$

Equations of motion in free space

$$\text{Dirac: } (i\partial\!\!\!/ - \mu)\psi = 0 (\text{fermions})$$

$$\text{Proca: } (\Box + \mu^2)\phi = 0 (\text{bosons})$$

$\mu =$ rest mass

Kerr Hawking radiation equations

Teukolsky radial equation

$R = \text{radial part of } \psi/\phi$

$$\frac{1}{\Delta^s}\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta^{s+1}\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \left(\frac{K^2 + 2is(r-M)K}{\Delta} - 4isEr - \lambda_{slm} - \mu^2 r^2\right)R = 0$$

$K \equiv (r^2 + a^2)E + am$, $s =$ spin, $l =$ angular momentum and m $=$ projection

Transformation into a Schödinger equation

Change $R \longrightarrow Z$ and $r \longrightarrow r^*$ (generalized Eddington - Finkelstein coordinate) (Chandrasekhar & Detweiler 1970s)

$$\frac{\mathrm{d}^2 Z}{\mathrm{d}r^{*2}} + (E^2 - V(r^*))Z = 0 \tag{1}$$

Solved with purely outgoing solution $Z \underset{r^* \to -\infty}{\longrightarrow} e^{-iEr^*}$

Transmission coefficient $\Gamma \equiv |Z_{\text{out}}^{+\infty}/Z_{\text{out}}^{\text{hor.}}|^2$

Kerr Hawking radiation equations

Chandrasekhar potentials

$$V_0(r) = \frac{\Delta}{\rho^4}\left(\lambda_{0\,lm} + \frac{\Delta + 2r(r-M)}{\rho^2} - \frac{3r^2\Delta}{\rho^4}\right)$$

$$V_{1/2,\pm}(r) = (\lambda_{1/2\,lm}+1)\frac{\Delta}{\rho^4} \mp \frac{\sqrt{(\lambda_{1/2,l,m}+1)\Delta}}{\rho^4}\left((r-M) - \frac{2r\Delta}{\rho^2}\right)$$

$$V_{1,\pm}(r) = \frac{\Delta}{\rho^4}\left((\lambda_{1\,lm}+2) - \alpha^2\frac{\Delta}{\rho^4} \mp i\alpha\rho^2\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\Delta}{\rho^4}\right)\right)$$

$$V_2(r) = \frac{\Delta}{\rho^8}\left(q - \frac{\rho^2}{(q-\beta\Delta)^2}\left((q-\beta\Delta)\left(\rho^2\Delta q'' - 2\rho^2 q - 2r(q'\Delta - q\Delta')\right)\right.\right.$$
$$\left.\left. +\rho^2(\kappa\rho^2 - q' + \beta\Delta')(q'\Delta - q\Delta')\right)\right)$$

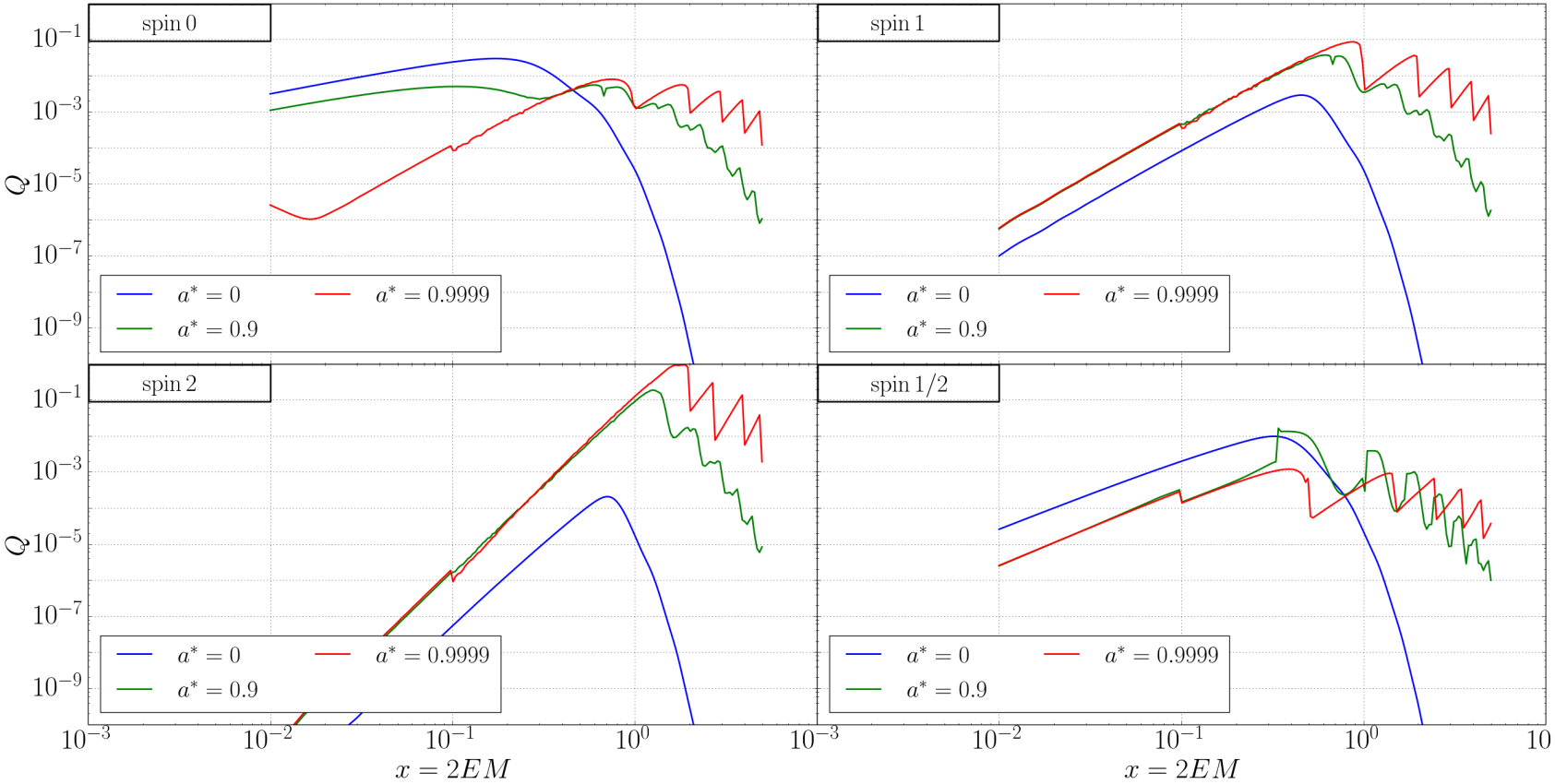
$\rho^2 \equiv r^2 + \alpha^2$ and $\alpha^2 \equiv a^2 + am/E$

$$q(r) = \nu\rho^4 + 3\rho^2(r^2 - a^2) - 3r^2\Delta$$
$$q'(r) = r\left((4\nu+6)\rho^2 - 6(r^2 - 3Mr + 2a^2)\right)$$
$$q''(r) = (4\nu+6)\rho^2 + 8\nu r^2 - 6r^2 + 36Mr - 12a^2$$

$$\beta_\pm = \pm 3\alpha^2$$

$$\kappa_\pm = \pm\sqrt{36M^2 - 2\nu(\alpha^2(5\nu+6) - 12a^2) + 2\beta\nu(\nu+2)}$$

All spins luminosities



Evolution parameters

Page parameters (Page 1976)

$$f(M,a^*) \equiv -M^2 \frac{\mathrm{d}M}{\mathrm{d}t} = M^2 \int_0^{+\infty} \sum_{\text{dof.}} \frac{E}{2\pi} \frac{\Gamma(E,M,a^*)}{e^{E'/T} \pm 1} \mathrm{d}E$$

$$g(M,a^*) \equiv -\frac{M}{a^*} \frac{\mathrm{d}J}{\mathrm{d}t} = \frac{M}{a^*} \int_0^{+\infty} \sum_{\text{dof.}} \frac{m}{2\pi} \frac{\Gamma(E,M,a^*)}{e^{E'/T} \pm 1} \mathrm{d}E$$

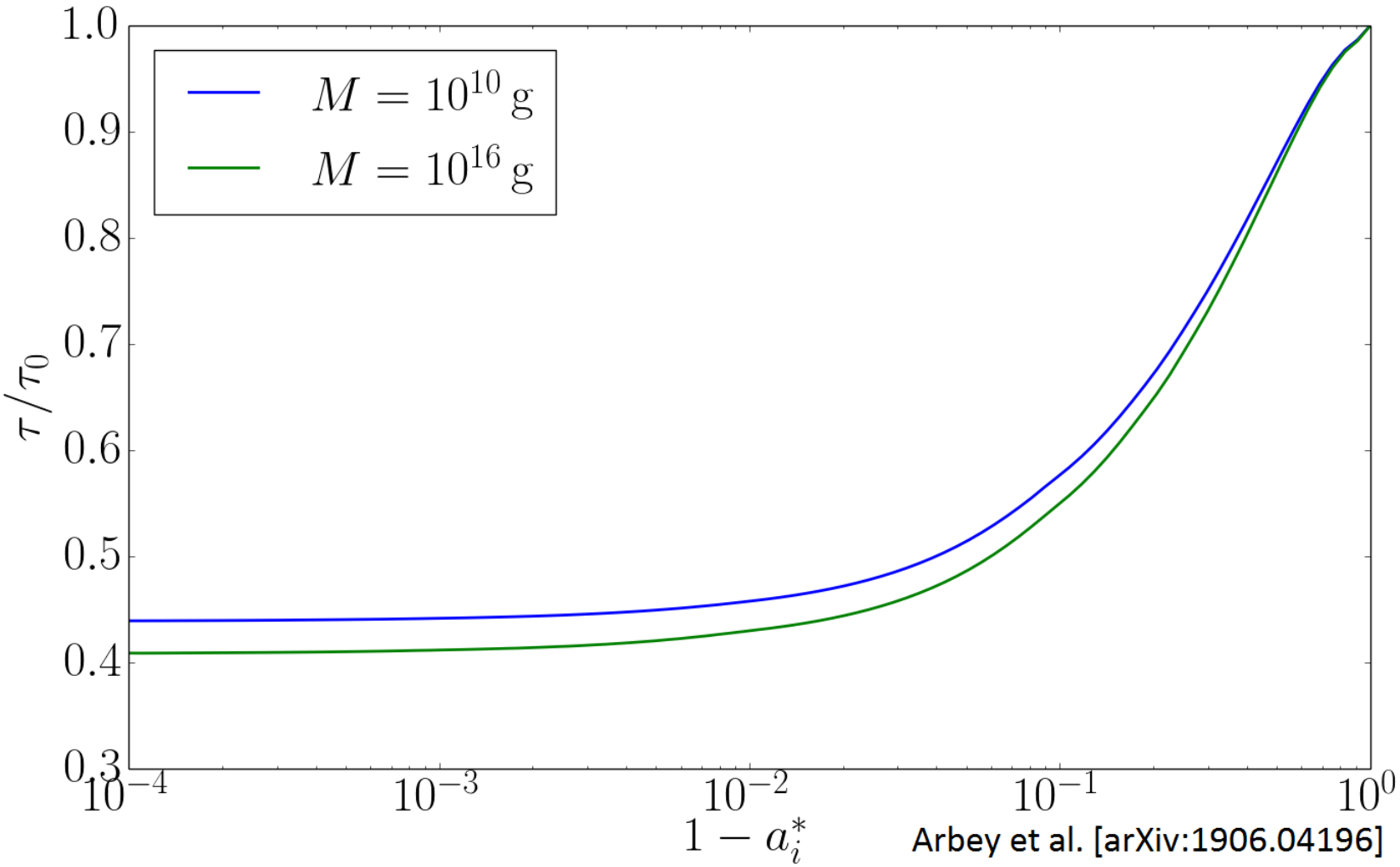
Evolution equations (Page 1976)

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{f(M,a^*)}{M^2}$$

$$\frac{\mathrm{d}a^*}{\mathrm{d}t} = \frac{a^*(2f(M,a^*) - g(M,a^*))}{M^3}$$

Reduced lifetime

Diminution of the BH lifetime τ with increasing initial spin a_i^*, compared to the Schwarzschild case (τ_0)



Arbey et al. [arXiv:1906.04196]

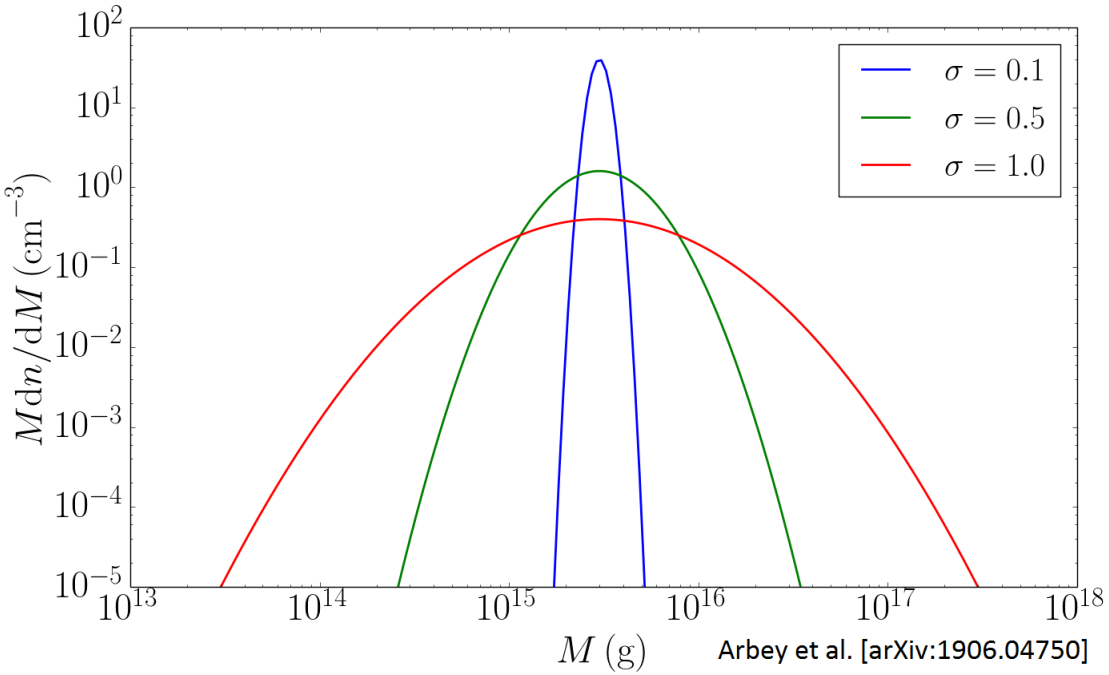
Log-normal distribution

Definition

$$\frac{\mathrm{d}n}{\mathrm{d}M} = \frac{A}{\sqrt{2\pi}\sigma M} \exp\left(-\frac{(\log(M/M_*))^2}{2\sigma^2}\right)$$

$M^* =$ central mass, $\sigma =$ width (dimensionless)

Log-normal distributions (normalized to unity, $M^* = 3 \times 10^{15}$ g)



Arbey et al. [arXiv:1906.04750]