Kerr Primordial Black Holes

Jérémy Auffinger
In collaboration with Alexandre Arbey & Joseph Silk

July 7th 2019

Beyond General Relativity, Beyond Cosmological Standard Model
University of Warsaw
Introduction

Kerr PBH Hawking radiation

Gamma ray constraints

BlackHawk

Conclusion
Dark Matter

Cosmic Microwave Background

Galaxy clusters collision

Galaxy rotation curves

Gravitational lensing

Observations of large scale structures, galaxies and cosmology show that 90% of matter is dark
Primordial Black Holes

A plausible DM candidate

- no Standard Model / General Relativity extension
- dynamically cold
- BH existence proven
- mass ranges still available for BHs to represent all of DM

PBH Hawking radiation, lensing and dynamical constraints

adapted from Katz et al. [arXiv:1807.11495]
Primordial Black Holes

Multiple inflationary origins

- collapse of large primordial overdensities
- phase transitions
- collapse of cosmic strings or domain walls

Spin predictions

\[ a^* \equiv \frac{J}{M^2} \]

Standard inflationary model \( \implies \) low spin

Transient matter domination \( \implies \) high spin
BH Hawking radiation

Fundamental equation for Kerr BHs

Rate of emission of Standard Model particles $i$ at energy $E$ by a BH of mass $M$ and spin parameter $a^*$:

$$ Q_i \equiv \frac{d^2 N_i}{dtdE} = \frac{1}{2\pi} \sum_{\text{dof.}} \frac{\Gamma_i(M, E, a^*)}{e^{E/T(M,a^*)} \pm 1} $$

$\Gamma_i$ is the greybody factor $\Leftrightarrow$ transmission coefficient (deviation from Planck’s blackbody law)
Reduced temperature

Hawking temperature for Kerr BHs

\[ T(M, a^*) = \frac{1}{4\pi M} \left( \frac{\sqrt{1 - (a^*)^2}}{1 + \sqrt{1 - (a^*)^2}} \right) \]

Schwarzschild \( a^* \rightarrow 0 \)

\[ T \xrightarrow{a^* = 0} \frac{1}{8\pi M} \]

Comparison with the \( e^\pm \) rest mass and the QCD scale \( \Lambda_{QCD} \)

![Graph showing comparison of temperatures for different values of \( a^* \) with \( e^\pm \) rest mass and the QCD scale.](image)
Enhanced emission

BH-particle spin coupling $\rightarrow$ superradiance effects (see e.g. Chandrasekhar & Detweiler papers in the 1970s)
The Hawking radiation is enhanced for high-spin particles.

Example of spin 1 massless emissivity (photon)
Dotted lines = Hawking temperature
Reduced lifetime

Evolution equations

\[
\frac{dM}{dt} = -\frac{f(M, a^*)}{M^2} \\
\frac{da^*}{dt} = \frac{a^*(2f(M, a^*) - g(M, a^*))}{M^3}
\]

\[f \sim \int_E \text{ener.} \times \text{emiss.}\]

\[g \sim \int_E \text{ang. mom.} \times \text{emiss.}\]

BH mass (solid) and spin (dotted) evolution

Abbey et al. [arXiv:1906.04196]

\[\frac{a_i}{a_i^*} = 0 \quad \frac{a_i^*}{a_i^*} = 0.9 \quad \frac{a_i^*}{a_i^*} = 0.9999\]
Extremal spin today?

Could high spin BHs exist today? Can we get over Thorne’s limit (disk accretion on rotating BHs)?

→ Yes, with sufficiently massive and extremal PBHs
Isotropic gamma ray background constraints

Origin

Diffuse background +
- Active galactic nuclei
- Gamma ray bursts
- DM annihilation/decay?
- Hawking radiation?

Flux estimation for BHs

\[
I \approx \frac{1}{4\pi} E \int_{t_{CMB}}^{t_{today}} \left(1 + z(t)\right)
\times \int_M \left[ \frac{dn}{dM} \frac{d^2N}{dt dE} (M, (1 + z(t)) E) \right] dM \]
Extension to Kerr PBHs

Main spin effects

- enhanced luminosity $\Rightarrow$ stronger constraint
- reduced temperature $\Rightarrow$ reduced emission energy $\Rightarrow$ weaker constraint

Monochromatic constraint from the IGRB
Extension to broad mass functions

Main width effects  \( Mdn/dM \propto \exp(-\ln(M/M_*)^2/2\sigma^2) \)

- broadening of the energy spectrum \( \Rightarrow \) stronger constraint
- broadening of the mass distribution \( \Rightarrow \) greater DM total density \( \Rightarrow \) weaker constraint
BlackHawk

A public C code computing the Hawking radiation:

- Schwarzschild & Kerr PBHs
- primary & secondary spectra of Standard Model particles
- extended mass functions
- time evolution of the PBHs

Download: http://blackhawk.hepforge.org
On-going work

- Big Bang Nucleosynthesis (see e.g. Sedel’nikov 1996, Kohri 2000)
- galactic gamma & X-rays (see e.g. Ballestros et al. [arXiv:1906.10113])
- galactic positrons (see e.g. Boudaud & Cirelli [arXiv:1807.03075], DeRocco & Graham [arXiv:1906.07740], Laha [arXiv:1906.09994])
Conclusion

Main results

- New public code BlackHawk to compute the Hawking radiation
- Study of the evolution of Kerr PBHs and constraints from IGRB
- Extension to more realistic broad PBH mass functions

Perspectives

- Closing the remaining PBH mass windows for all DM into PBHs?
- Other constraints...
Thank you!

Publications:

- BlackHawk: http://blackhawk.hepforge.org [1905.04268]
- Any extremal black holes are primordial [1906.04196]
- Constraining primordial black hole masses with the isotropic gamma ray background [1906.04750]
Other constraints

Comparison with recent $e^+$ constraints

Arbey et al. [arXiv:1906.04750] (FERMI)
DeRocco & Graham [arXiv:1906.07740], Laha [arXiv:1906.09994] (Bulge $e^+$)
Boudaud & Cirelli [arXiv:1807.03075] (Voyager 1 $e^+$)
Other constraints

Monochromatic constraints with femtolensing prospects

PBH fraction $\Omega_{PBH}/\Omega_{DM}$ vs. PBH Mass $[M_\odot]$
Kerr Hawking radiation equations

Kerr metric

\[ ds^2 = \left(1 - \frac{2Mr}{\Sigma^2}\right) dt^2 + \frac{4a^* M^2 r \sin(\theta)^2}{\Sigma^2} dt d\phi - \frac{\Sigma^2}{\Delta} dr^2 \]
\[ - \Sigma^2 d\theta^2 - \left(r^2 + (a^*)^2 M^2 + \frac{2(a^*)^2 M^3 r \sin(\theta)^2}{\Sigma^2}\right) \sin(\theta)^2 d\phi^2 \]

\[ \Sigma \equiv r^2 + (a^*)^2 M^2 \cos(\theta)^2 \text{ and } \Delta \equiv r^2 - 2Mr + (a^*)^2 M^2 \]

Equations of motion in free space

Dirac: \( (i\partial - \mu)\psi = 0(\text{fermions}) \)

Proca: \( (\Box + \mu^2)\phi = 0(\text{bosons}) \)

\( \mu = \text{rest mass} \)
Kerr Hawking radiation equations

Teukolsky radial equation

\[
R = \text{radial part of } \frac{\psi}{\phi}
\]

\[
\frac{1}{\Delta^s} \frac{d}{dr} \left( \Delta^{s+1} \frac{dR}{dr} \right) + \left( \frac{K^2 + 2is(r-M)K}{\Delta} - 4isEr - \lambda_{slm} - \mu^2 r^2 \right) R = 0
\]

\(K \equiv (r^2 + a^2)E + am, \ s = \text{spin}, \ l = \text{angular momentum and } m = \text{projection}\)

Transformation into a Schödinger equation

Change \(R \rightarrow Z\) and \(r \rightarrow r^*\) (generalized Eddington - Finkelstein coordinate) (Chandrasekhar & Detweiler 1970s)

\[
\frac{d^2 Z}{dr^{*2}} + (E^2 - V(r^*)) Z = 0 \quad (1)
\]

Solved with purely outgoing solution \(Z \rightarrow e^{-iEr^*}\) as \(r^* \rightarrow -\infty\)

Transmission coefficient \(\Gamma \equiv \left| Z_{\text{out}}^{+\infty} / Z_{\text{hor.}}^{\text{out}} \right|^2\)
Kerr Hawking radiation equations

Chandrasekhar potentials

\[ V_0(r) = \frac{\Delta}{\rho^4} \left( \lambda_{0\,lm} + \frac{\Delta + 2r(r - M)}{\rho^2} - \frac{3r^2\Delta}{\rho^4} \right) \]

\[ V_{1/2,\pm}(r) = (\lambda_{1/2\,lm} + 1) \frac{\Delta}{\rho^4} \pm \sqrt{(\lambda_{1/2\,lm} + 1)\Delta} \left( (r - M) - \frac{2r\Delta}{\rho^2} \right) \]

\[ V_{1,\pm}(r) = \frac{\Delta}{\rho^4} \left( (\lambda_{1\,lm} + 2) - \alpha^2 \frac{\Delta}{\rho^4} \mp i\alpha \rho^2 \frac{d}{dr} \left( \frac{\Delta}{\rho^4} \right) \right) \]

\[ V_2(r) = \frac{\Delta}{\rho^8} \left( q - \frac{\rho^2}{(q - \beta \Delta)^2} \left( (q - \beta \Delta) \left( \rho^2 \Delta q'' - 2\rho^2 q - 2r(q'\Delta - q\Delta') \right) \right. \right. \]
\[ \left. \left. + \rho^2 (\kappa \rho^2 - q' + \beta \Delta')(q'\Delta - q\Delta') \right) \right) \]

\[ \rho^2 \equiv r^2 + \alpha^2 \text{ and } \alpha^2 \equiv a^2 + am/E \]

\[ q(r) = \nu \rho^4 + 3\rho^2(r^2 - a^2) - 3r^2\Delta \]

\[ q'(r) = r \left( (4\nu + 6)\rho^2 - 6(r^2 - 3Mr + 2a^2) \right) \]

\[ q''(r) = (4\nu + 6)\rho^2 + 8\nu r^2 - 6r^2 + 36Mr - 12a^2 \]

\[ \beta_\pm = \pm 3\alpha^2 \]

\[ \kappa_\pm = \pm \sqrt{36M^2 - 2\nu(\alpha^2(5\nu + 6) - 12a^2) + 2\beta\nu(\nu + 2)} \]
All spins luminosities

![Graph showing luminosities for different spins and parameters.](image-url)
Evolution parameters

Page parameters (Page 1976)

\[
f(M, a^*) \equiv -M^2 \frac{dM}{dt} = M^2 \int_0^{+\infty} \sum_{\text{dof.}} \frac{E}{2\pi} \frac{\Gamma(E, M, a^*)}{e^{E'/T} \pm 1} \, dE
\]

\[
g(M, a^*) \equiv -\frac{M}{a^*} \frac{dJ}{dt} = \frac{M}{a^*} \int_0^{+\infty} \sum_{\text{dof.}} \frac{m}{2\pi} \frac{\Gamma(E, M, a^*)}{e^{E'/T} \pm 1} \, dE
\]

Evolution equations (Page 1976)

\[
\frac{dM}{dt} = -\frac{f(M, a^*)}{M^2}
\]

\[
\frac{da^*}{dt} = \frac{a^* (2f(M, a^*) - g(M, a^*))}{M^3}
\]
Reduced lifetime

Diminution of the BH lifetime $\tau$ with increasing initial spin $a_i^*$, compared to the Schwarzschild case ($\tau_0$)

$M = 10^{10} \text{ g}$

$M = 10^{16} \text{ g}$

Arbey et al. [arXiv:1906.04196]
Log-normal distribution

Definition

\[
\frac{dn}{dM} = \frac{A}{\sqrt{2\pi}\sigma M} \exp\left(-\frac{(\log(M/M_*)^2}{2\sigma^2}\right)
\]

\(M^* = \) central mass, \(\sigma = \) width (dimensionless)

Log-normal distributions (normalized to unity, \(M^* = 3 \times 10^{15}\) g)