Scalar-tensor theories beyond Riemannian

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KA and K. Shimada, 1806.02589, 1904.10175

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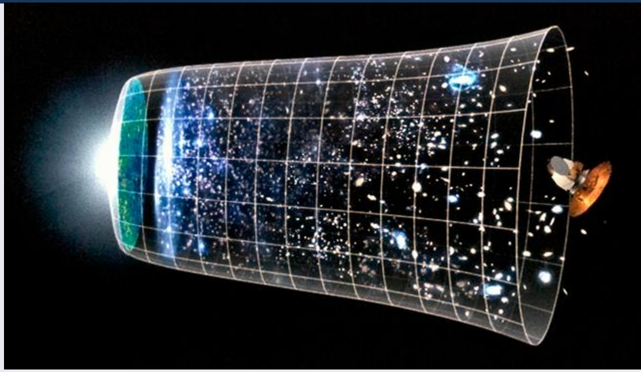
Introduction

Cosmic puzzles?

→ Einstein gravity would be modified.
but how?
We do not know fundamental theory of gravity…



A systematic way: EFT approach

Suppose that GR is somehow modified by a scalar (=Inflaton, DE, or DM…) and consider general theories as far as possible within validity of EFT.

$$\mathcal{L}_{ST} = \mathcal{L}_{ST}(g, R, \phi, \nabla\phi, \nabla\nabla\phi)$$

We should include all possible terms unless prohibited by some reason.

Some reason? → We usually use ghost-freeness as a criteria.

Why ghost-free?

If a theory is fundamental: must be ghost-free.
If a theory is just an EFT: a heavy ghost can exist. $m_{\rm ghost} \gg \Lambda_{\rm cut-off}$

Interesting phenomena are often obtained by $\nabla_\mu \nabla_\nu \phi$ of $\mathcal{L}_{ST}$

However, domination of higher derivatives → Ostrogradsky ghost at this scale

We should tune $\nabla_\mu \nabla_\nu \phi$ terms to get interesting theory while avoiding ghost.

- ✓ **Horndeski theory**: most general with at most second derivative EOM
  Horndeski, 1974
- ✓ **Beyond Horndeski**: higher than second but degenerate
  e.g. Langlois and Noui 2016
- ✓ **U-degenerate**: not degenerate in general but degenerate in a gauge
  De Felice, et al. 2018

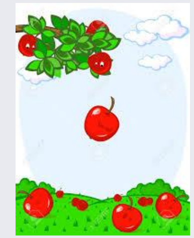
Why ghost-free? Is that just ad-hoc tuning? or any fundamental reason?

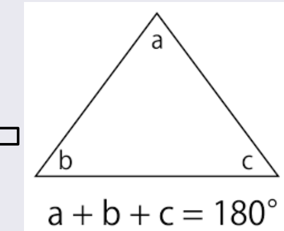
Introduction again

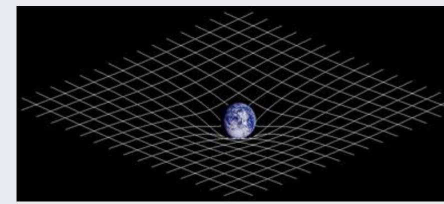
- Gravity beyond Newton and Einstein

$$\Delta\Phi = 4\pi G\rho \to G_{\mu\nu} = 8\pi G T_{\mu\nu} \to \ ???$$

We are tring to find beyond Einstein!!







- Spacetime beyond Euclidian and Riemannian

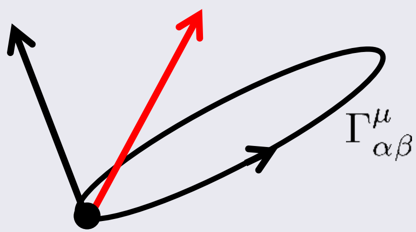
$$\eta_{\mu\nu} \to g_{\mu\nu}(x) \to \ ???$$

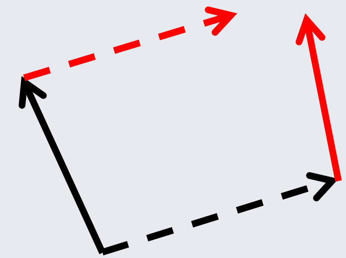
Why not considering beyond Riemannian as well?

Indeed, if we go beyond Riemannian,
the ghost-freeness can be seen by the existence of a local symmetry!

Curvature, Torsion, and Non-metricity

Curvature and torsion are defined without any knowledge of the metric.





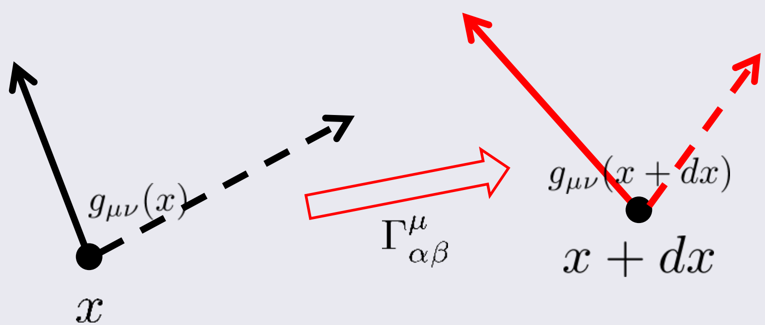
Curvature: path dependence

$$\overset{\Gamma}{R}{}^{\mu}{}_{\nu\alpha\beta}(\Gamma) := 2\partial_{[\alpha}\Gamma^{\mu}_{\nu|\beta]} + \Gamma^{\mu}_{\sigma[\alpha}\Gamma^{\sigma}_{\nu|\beta]}$$

Torsion: non-closed parallelogram

$$T^{\mu}{}_{\alpha\beta} := 2\Gamma^{\mu}_{[\beta\alpha]}$$

Length and angle are not conserved under the parallel transport



$$A_{\mathrm{PT}}\cdot B_{\mathrm{PT}} - A\cdot B = -\,Q_{\alpha\mu\nu}A^{\mu}B^{\nu}dx^{\alpha}$$

$$Q_{\mu}{}^{\alpha\beta} := \overset{\Gamma}{\nabla}_{\mu}g^{\alpha\beta}$$

Non-metricity tensor

What we usually postulate?

The metric and the connection are independent in the first place.

Riemannian geometry assumes torsionless and metric compatibility

$$T^{\alpha}{}_{\mu\nu} = 0, \quad Q_{\mu}{}^{\alpha\beta} = 0$$

These kill all degrees of freedom of connection.

$$\Gamma^{\mu}_{\alpha\beta} = \left\{ {}^{\mu}_{\alpha\beta} \right\} - \frac{1}{2}(T^{\mu}{}_{\alpha\beta} - T_{\beta}{}^{\mu}{}_{\alpha} + T_{\alpha\beta}{}^{\mu}) - \frac{1}{2}(Q^{\mu}{}_{\alpha\beta} - Q_{\beta}{}^{\mu}{}_{\alpha} - Q_{\alpha\beta}{}^{\mu})$$

$$\left\{ {}^{\mu}_{\alpha\beta} \right\} := \frac{1}{2} g^{\mu\nu}(\partial_{\alpha} g_{\beta\nu} + \partial_{\beta} g_{\alpha\nu} - \partial_{\nu} g_{\alpha\beta})$$

Cf. Euclidian geometry also assumes no curvature $\overset{\Gamma}{R}{}^{\mu}{}_{\nu\rho\sigma} = 0$

These are not necessary to be assumed a priori in gravitational theories.

Indeed, Riemannian description can be obtained as an effective theory just like Euclidian description (Minkowski sp.) is obtained.

Theories of gravity beyond Riemannian

Theories of gravity in Riemannian

$$[g_{\alpha\beta}] = 0, \quad [\Gamma^{\alpha}_{\beta\gamma}] = 1$$

$$\mathcal{L}_g = \mathcal{L}_g(g, \partial g, \partial^2 g) = \mathcal{L}_g(g, R)$$

w/ Γ: defined by general connection
w/o Γ: defined by Levi-Civita

$$R \sim \partial^2 g + \cdots, \overset{\Gamma}{R} \sim \partial\Gamma + \cdots$$

Theories of gravity in metric-affine

$$\begin{aligned}\mathcal{L}_G &= \mathcal{L}_G(g, \partial g, \partial^2 g, \Gamma, \partial\Gamma) \\ &= \mathcal{L}_G(g, \overset{\Gamma}{R}, T, Q, \overset{\Gamma}{\nabla} T, \overset{\Gamma}{\nabla} Q) \\ &= \mathcal{L}_G(g, R, T, Q, \nabla T, \nabla Q)\end{aligned}$$

$$\Gamma^{\mu}_{\alpha\beta} = \left\{ {\mu \atop \alpha\beta} \right\} - \frac{1}{2}(T^{\mu}{}_{\alpha\beta} - T_{\beta}{}^{\mu}{}_{\alpha} + T_{\alpha\beta}{}^{\mu}) - \frac{1}{2}(Q^{\mu}{}_{\alpha\beta} - Q_{\beta}{}^{\mu}{}_{\alpha} - Q_{\alpha\beta}{}^{\mu})$$

metric + additional tensors ($T^{\alpha}{}_{\mu\nu}$ and $Q_{\mu}{}^{\alpha\beta}$)

Generally have non-vanishing mass terms

Theories of gravity beyond Riemannian

Theories of gravity in Riemannian

$$[g_{\alpha\beta}] = 0, \quad [\Gamma^{\alpha}_{\beta\gamma}] = 1$$

$$\mathcal{L}_g = \mathcal{L}_g(g, \partial g, \partial^2 g) = \mathcal{L}_g(g, R)$$

w/ Γ: defined by general connection

Theories of gravity in metric-affine

$$\mathcal{L}_G = \mathcal{L}_G(g, \partial g, \partial^2 g, \Gamma, \partial\Gamma)$$

$$= \mathcal{L}_G(g, \overset{\Gamma}{R}, T, Q, \overset{\Gamma}{\nabla} T, \overset{\Gamma}{\nabla} Q)$$

$$= \mathcal{L}_G(g, R, T, Q, \nabla T, \nabla Q) \simeq \mathcal{L}_G(g, R)$$

Taking limit of infinite masses

$$\Rightarrow \quad T^{\alpha}{}_{\mu\nu} \simeq 0, \quad Q_{\mu}{}^{\alpha\beta} \simeq 0$$

Cf. $M_{pl} \to \infty$ yields $R_{\mu\nu\rho\sigma} \simeq 0$

)

$$-\frac{1}{2}(Q^{\mu}{}_{\alpha\beta} - Q_{\beta}{}^{\mu}{}_{\alpha} - Q_{\alpha\beta}{}^{\mu})$$

metric + additional tensors ($T^{\alpha}{}_{\mu\nu}$ and $Q_{\mu}{}^{\alpha\beta}$)

Generally have non-vanishing mass terms

Ghost-freeness from symmetry?

Let us suppose that EH action is somehow modified by a scalar in IR regime.

= connection is not dynamical

$$\mathcal{L}_{ST}(g,\Gamma,\phi)=\frac{M_{\rm pl}^2}{2}\overset{\Gamma}{R}+\mathcal{L}_\phi(g,\phi,\overset{\Gamma}{\nabla}_\mu\phi,\overset{\Gamma}{\nabla}_\mu\overset{\Gamma}{\nabla}_\nu\phi)$$

"Scalar-tensor theories" in metric-affine formalism

$(T^\alpha{}_{\mu\nu}\neq 0, \quad Q_\mu{}^{\alpha\beta}\neq 0)$

The connection can be integrated out. Then, we get

$$\mathcal{L}_{ST}(g,\phi)=\frac{M_{\rm pl}^2}{2}R+\mathcal{L}'_\phi(g,\phi,\nabla_\mu\phi,\nabla_\mu\nabla_\nu\phi)$$

Scalar-tensor theories in metric formalism

We usually tune $\mathcal{L}'_\phi$ to avoid the Ostrogradsky ghost.

However, a local symmetry of connection can give ghost-freeness.

"Conformal" symmetries

- Weyl transformation

$$g_{\mu\nu} \to g'_{\mu\nu} = e^{2\Omega(x)} g_{\mu\nu}$$

Angle does not change.

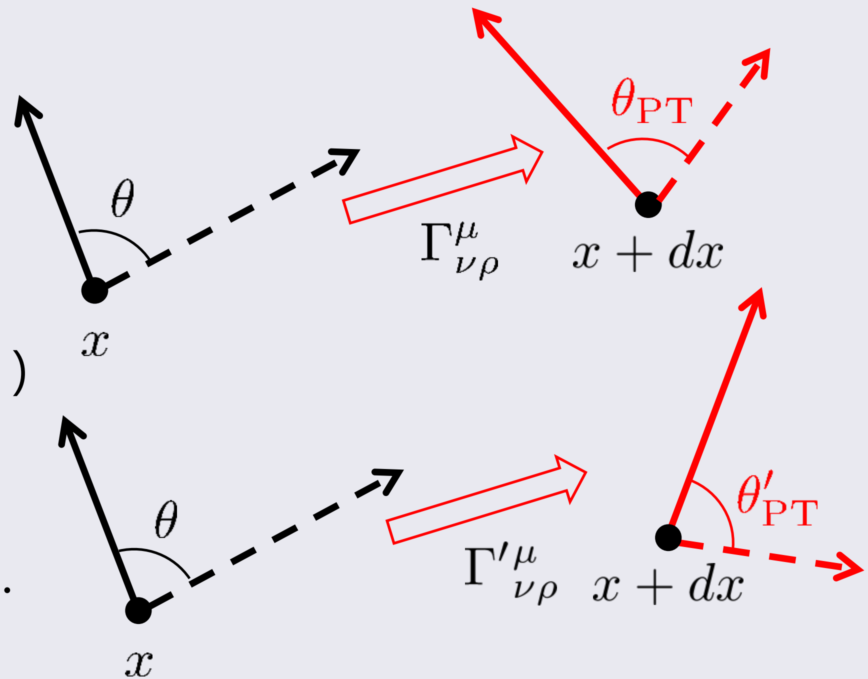
- Projective transformation (PT)

(defined only if ${T^\alpha}_{\mu\nu} \neq 0, {Q_\mu}^{\alpha\beta} \neq 0$)

$$\Gamma^\mu_{\nu\rho} \to \Gamma'^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \delta^\mu_\nu U_\rho(x)$$

Change of angle does not change.

$$\theta - \theta_{\rm PT} = \theta' - \theta'_{\rm PT}$$



This transformation is somehow suggestive...

EH action and standard matter action are invariant under PT.

Ghost-freeness from projective invariance?

$$\mathcal{L}_{ST}(g,\Gamma,\phi) = \frac{M_{\rm pl}^2}{2}\overset{\Gamma}{R} + \mathcal{L}_\phi(g,\phi,\overset{\Gamma}{\nabla}_\mu\phi,\overset{\Gamma}{\nabla}_\mu\overset{\Gamma}{\nabla}_\nu\phi)$$

Projective invariant

$$\overset{\Gamma}{\nabla}_\mu\phi = \partial_\mu\phi\,, \quad \overset{\Gamma}{\nabla}_\mu\overset{\Gamma}{\nabla}_\nu\phi = \partial_\mu\partial_\nu\phi - \Gamma^\alpha_{\nu\mu}\partial_\alpha\phi$$

The 00-component is problematic…

$$\overset{\Gamma}{\nabla}_\mu\overset{\Gamma}{\nabla}_\nu\phi = \pounds_n A_* n_\mu n_\nu + \cdots$$

$$A_* = n^\mu\partial_\mu\phi = -\dot{\phi}/N + \cdots$$

n^μ is the normal vector

The projective invariance of $\mathcal{L}_\phi$ is now invariance under

$$\overset{\Gamma}{\nabla}_\mu\overset{\Gamma}{\nabla}_\nu\phi \to \overset{\Gamma}{\nabla}_\mu\overset{\Gamma}{\nabla}_\nu\phi - U_\mu\partial_\nu\phi$$

In the gauge $\phi=\phi(t)$, i.e., $\partial_\mu\phi \propto n_\mu \Rightarrow \overset{\Gamma}{\nabla}_\mu\overset{\Gamma}{\nabla}_\nu\phi \to \overset{\Gamma}{\nabla}_\mu\overset{\Gamma}{\nabla}_\nu\phi - A_* U_\mu n_\nu$

Since U_μ is an arbitrary vector, the 00-component is just a gauge mode!

Ghost-freeness from projective invariance?

$$\mathcal{L}_{ST}(g,\Gamma,\phi) = \underbrace{\frac{M_{\rm pl}^2}{2}\overset{\Gamma}{R}}_{} + \underbrace{\mathcal{L}_\phi(g,\phi,\overset{\Gamma}{\nabla}_\mu\phi,\overset{\Gamma}{\nabla}_\mu\overset{\Gamma}{\nabla}_\nu\phi)}_{}$$

Projective invariant

The ghostly part is absorbed by the projective gauge mode!
→ The theory is definitely free from the Ostrogradsky ghost.

Since we have ignored kinetic terms of the connection ($T^\alpha{}_{\mu\nu}$ and $Q_\mu{}^{\alpha\beta}$), the connection can be integrated out.

$$\mathcal{L}_{ST}(g,\Gamma(g,\phi),\phi) = \frac{M_{\rm pl}^2}{2}R + \mathcal{L}'_\phi(g,\phi,\nabla_\mu\phi,\nabla_\mu\nabla_\nu\phi)$$

We cannot see the projective invariance any more…
However, we can still see the Ostrogradsky ghost-freeness of the theory.

Generalization

We find more general ghost-free projective invariant Lagrangian

$$\mathcal{L} = f_1 \overset{\Gamma}{R} + f_2 \overset{\Gamma}{G}{}^{\mu\nu} \overset{\Gamma}{\nabla}_\mu \phi \overset{\Gamma}{\nabla}_\nu \phi + f_3 \overset{\Gamma}{G}{}^{\mu\alpha\nu\beta} \overset{\Gamma}{\nabla}_\mu \phi \overset{\Gamma}{\nabla}_\nu \phi \overset{\Gamma}{\nabla}_\alpha \overset{\Gamma}{\nabla}_\beta \phi + f_4 \overset{\Gamma}{R}_{\mu\nu} \overset{\Gamma}{\nabla}{}^\mu \phi \overset{\Gamma}{\nabla}{}^\nu \phi + \mathcal{L}_\phi$$

where $\overset{\Gamma}{G}{}^{\mu\nu\alpha\beta} := \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} \overset{\Gamma}{R}_{\rho\sigma\gamma\delta}$ dual Riemann tensor

$\overset{\Gamma}{G}{}^{\mu\nu} := \overset{\Gamma}{G}{}^{\mu\alpha\nu}{}_\alpha$ Einstein tensor

$f_1 = f_1(g, \phi, \overset{\Gamma}{\nabla}_\mu \phi, \overset{\Gamma}{\nabla}_\mu \overset{\Gamma}{\nabla}_\nu \phi)$ and so on: projective invariant

This is still not the most general PI theory...

$$\mathcal{L}_{\mathrm{PI}} = \mathcal{L}_\phi + F^{\mu\nu\rho\sigma} \overset{\Gamma}{R}_{\mu\nu\rho\sigma} + O((\overset{\Gamma}{R}{}^\mu{}_{\nu\rho\sigma})^2)$$

$F^{\mu\nu\rho\sigma} = F^{\mu\nu\rho\sigma}(g, \phi, \overset{\Gamma}{\nabla} \phi, \overset{\Gamma}{\nabla} \overset{\Gamma}{\nabla} \phi)$: PI with $g_{\mu\nu} F^{\mu\nu\rho\sigma} = 0$

However, most general PI theory= ghost free, up to quadratic order

Hidden structure of ghost-free theories

Up to quadratic order in the connection (Here, $f_1 = f_1(\phi, (\partial\phi)^2)$ and so on)

✓ The most general ST theory = The (quadratic) U-degenerate theory

De Felice, et al. 2018

$$\mathcal{L}(g,\Gamma,\phi) = f_1 \overset{\Gamma}{R} + f_2 \overset{\Gamma}{G}{}^{\mu\nu} \overset{\Gamma}{\nabla}_\mu\phi \overset{\Gamma}{\nabla}_\nu\phi + f_4 \overset{\Gamma}{R}_{\mu\nu} \overset{\Gamma}{\nabla}{}^\mu\phi \overset{\Gamma}{\nabla}{}^\nu\phi + F_2 + F_3\mathcal{L}_3^{\mathrm{gal}\Gamma} + F_4\mathcal{L}_4^{\mathrm{gal}\Gamma}$$
$$+ C_1 \epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'}{}_\sigma \overset{\Gamma}{\nabla}_\mu\phi \overset{\Gamma}{\nabla}_{\mu'}\phi \overset{\Gamma}{\nabla}_\nu \overset{\Gamma}{\nabla}_{\nu'}\phi \overset{\Gamma}{\nabla}_{[\rho} \overset{\Gamma}{\nabla}_{\rho']}\phi + C_2(\mathcal{L}_3^{\mathrm{gal}\Gamma})^2$$
$$+ C_3(g^{\mu\beta}g^{\nu\delta}g^{\alpha\gamma} - g^{\mu\nu}g^{\alpha\gamma}g^{\beta\delta}) \overset{\Gamma}{\nabla}_\mu\phi \overset{\Gamma}{\nabla}_\nu\phi \overset{\Gamma}{\nabla}_\alpha \overset{\Gamma}{\nabla}_\beta\phi \overset{\Gamma}{\nabla}_\gamma \overset{\Gamma}{\nabla}_\delta\phi$$

$$\mathcal{L}_n^{\mathrm{gal}\Gamma} = \epsilon\epsilon(\overset{\Gamma}{\nabla}\phi)^2(\overset{\Gamma}{\nabla}\overset{\Gamma}{\nabla}\phi)^{n-2}$$

The degeneracy conditions are satisfied only in the unitary gauge.

✓ Projective invariance + Galileon type combinations = (quadratic) DHOST

Langlois and Noui 2016

$$\mathcal{L}(g,\Gamma,\phi) = f_1 \overset{\Gamma}{R} + f_2 \overset{\Gamma}{G}{}^{\mu\nu} \overset{\Gamma}{\nabla}_\mu\phi \overset{\Gamma}{\nabla}_\nu\phi + F_2 + F_3\mathcal{L}_3^{\mathrm{gal}\Gamma} + F_4\mathcal{L}_4^{\mathrm{gal}\Gamma}$$

The degeneracy conditions are satisfied in any gauge.

✓ **Theories beyond quadratic order = new theories are found**

(and can be further extended, perhaps)

Simple and interesting model?

- General building blocks: $g_{\mu\nu}, \overset{\Gamma}{R}{}^{\mu}{}_{\nu\rho\sigma}(\Gamma), \phi, \overset{\Gamma}{\nabla}_{\mu}\phi, \overset{\Gamma}{\nabla}_{\mu}\overset{\Gamma}{\nabla}_{\nu}\phi$

  w/o non-minimal coupling: no ghost due to projective invariance

$$\mathcal{L}(g,\Gamma,\phi) = \frac{M_{\rm pl}^2}{2}\overset{\Gamma}{R} + \mathcal{L}_{\phi}(g,\phi,\overset{\Gamma}{\nabla}_{\mu}\phi,\overset{\Gamma}{\nabla}_{\mu}\overset{\Gamma}{\nabla}_{\nu}\phi) + O((\overset{\Gamma}{R}{}^{\mu}{}_{\nu\rho\sigma})^2)$$

  w/ non-minimal coupling: would-be ghost-free (no counter example now)

- **Minimal** building blocks: $g_{\mu\nu}, \overset{\Gamma}{R} := g^{\alpha\beta}\overset{\Gamma}{R}{}^{\mu}{}_{\alpha\mu\beta}(\Gamma), X := (\partial\phi)^2$

$$\mathcal{L}(g,\overset{\Gamma}{R},X) = P(X) + f(X)\overset{\Gamma}{R} + O(\overset{\Gamma}{R}{}^2)$$

Generalized k-essence?

$$\Leftrightarrow \quad \mathcal{L} = P(X) + f(X)R(g) + \frac{6f_X^2}{f}\phi^{\alpha}\phi^{\beta}\phi_{\alpha\gamma}\phi^{\gamma}_{\beta}$$

$\phi_\mu = \nabla_\mu\phi, \ \phi_{\mu\nu} = \nabla_\mu\nabla_\nu\phi$

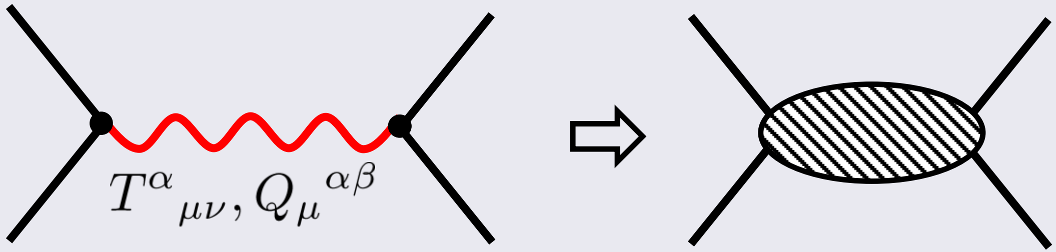
The counter term is obtained by integrating out Γ KA and K. Shimada, 2018. 06.

A safe model from constraints on speed of GW and graviton decay!

Creminelli et al, 2018. 09.

Summary

- Gravity beyond Riemannian = adding new heavy particles

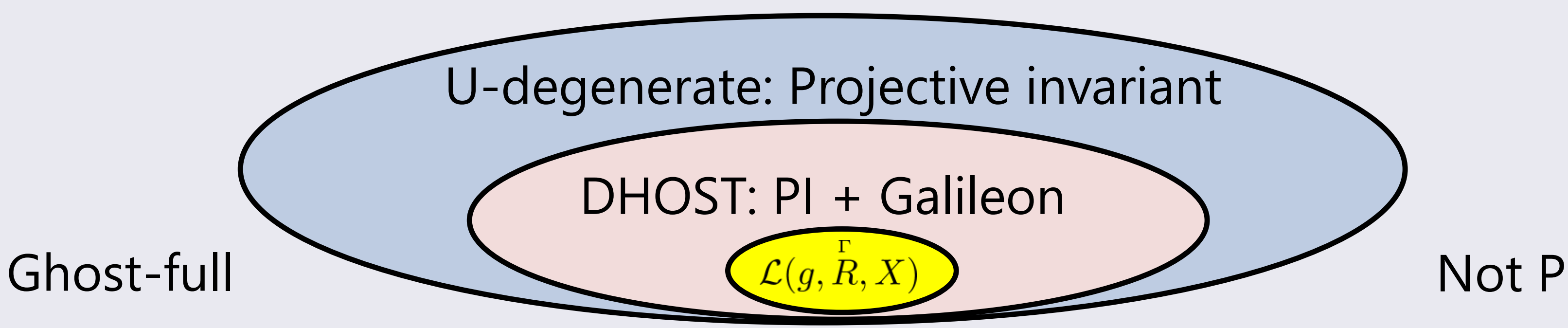


(In UV, we can indeed see massive spin-2,1,0 particles. w/S. Mukohyama, in prep.)

- Projective invariance provides a ghost-free structure in the unitary gauge.

$$\Gamma^{\mu}_{\nu\rho} \to \Gamma'^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \delta^{\mu}_{\nu} U_{\rho}(x)$$

→ Complicated ghost-free theories would be meaningful in EFT sense.



Hidden structure of ghost-free theories

If we explicitly integrate the connection out…

$$\mathcal{L}(g,\Gamma,\phi)=f_1\overset{\Gamma}{R}+f_2\overset{\Gamma}{G}{}^{\mu\nu}\overset{\Gamma}{\nabla}_\mu\phi\overset{\Gamma}{\nabla}_\nu\phi+F_2+F_3\mathcal{L}_3^{\mathrm{gal}\Gamma}+F_4\mathcal{L}_4^{\mathrm{gal}\Gamma}$$

$$\mathcal{L}_n^{\mathrm{gal}\Gamma}=\epsilon\epsilon(\overset{\Gamma}{\nabla}\phi)^2(\overset{\Gamma}{\nabla}\overset{\Gamma}{\nabla}\phi)^{n-2}$$

$$\mathcal{L}(g,\Gamma(g,\phi),\phi)=fR(g)+P+Q_1g^{\mu\nu}\phi_{\mu\nu}+Q_2\phi^\mu\phi_{\mu\nu}\phi^\nu+C^{\mu\nu,\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma}$$

$$C^{\mu\nu,\rho\sigma}=\alpha_1g^{\rho(\mu}g^{\nu)\sigma}+\alpha_2g^{\mu\nu}g^{\rho\sigma}+\frac{1}{2}\alpha_3(\phi^\mu\phi^\nu g^{\rho\sigma}+\phi^\rho\phi^\sigma g^{\mu\nu})$$
$$+\frac{1}{2}\alpha_4(\phi^\rho\phi^{(\mu}g^{\nu)\sigma}+\phi^\sigma\phi^{(\mu}g^{\nu)\rho})+\alpha_5\phi^\mu\phi^\nu\phi^\rho\phi^\sigma$$

$$\phi_\mu=\nabla_\mu\phi,\ \phi_{\mu\nu}=\nabla_\mu\nabla_\nu\phi$$

$$f=f_1-\frac{1}{2}f_2X\,,\quad P=F_2+\frac{3X(f_{1\phi}-F_3X)^2}{2f_1-f_2X+2F_4X^2}\,,$$

$$Q_1=-2f_\phi+\frac{4f_1(f_{1\phi}-F_3X)}{2f_1-f_2X+2F_4X^2}\,,\quad Q_2=\frac{2f_\phi}{X}-\frac{4(f_1-3f_{1X})(f_{1\phi}-F_3X)}{X(2f_1-f_2X+2F_4X^2)}\,,$$

$$\alpha_1=-\alpha_2=-\frac{f_2}{2}-\frac{f_1(f_2-2F_4X)}{2f_1-f_2X+2F_4X^2}\,,\quad \alpha_3=2f_{2X}+\frac{4f_1F_4+(4f_{1X}-f_2)(f_2-2F_4X)}{2f_1-f_2X+2F_4X^2}\,,$$

$$X:=g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$\alpha_4=-2f_{2X}+2f_1^{-1}f_{1X}(3f_{1X}-f_2)+f_1^{-2}f_{1X}X(f_{1X}f_2-4f_1f_{2X})+\frac{f_2^2-4f_1F_4-2f_2F_4X}{2f_1-f_2X+2F_4X^2}\,,$$

$$\alpha_5=-f_1^{-2}f_{1X}(f_{1X}f_2-4f_1f_{2X})+\frac{2f_{1X}\{4f_1F_4+(3f_{1X}-f_2)(f_2-2F_4X)\}}{f_1(2f_1-f_2X+2F_4X^2)}\,,$$

Too complicated… but the original one is just generalized Galileon