Cosmological Implications of Electroweak Vacuum Instability

Arttu Rajantie
Based on:

- Herranen, Markkanen, Nurmi & AR, PRL113 (2014) 211102
- AR & Stopyra, PRD95 (2017) 025008
- AR & Stopyra, PRD97 (2018) 025012
- Figueroa, AR & Torrenti, PRD98 (2018) 023532
Hypothetical phenomenon: Brexiton decouples from other fields and turns into a slowly decaying spectator.
Brexiton

- Decays due to interaction with the real world
All renormalisable terms allowed by symmetries in Minkowski space

- 19 parameters – all have been measured
- Can be extrapolated all the way to Planck scale

For central experimental values $M_H = 125.18$ GeV and $M_t = 173.1$ GeV
- $\lambda$ becomes negative at $\mu_\Lambda \approx 9.9 \times 10^9$ GeV
- Minimum value $\lambda_{\text{min}} \approx -0.015$ at $\mu_{\text{min}} \approx 2.8 \times 10^{17}$ GeV

(Buttazzo et al 2013)
Vacuum Instability

- Higgs effective potential
  \[ V(\phi) \approx \lambda (g\phi)\phi^4 \]

- Becomes negative at \( \phi > \phi_c \approx 10^{10} \text{GeV} \)

- True vacuum at Planck scale?

- Current vacuum metastable against quantum tunnelling

- Barrier at \( \phi_{\text{bar}} \approx 4.6 \times 10^{10} \text{ GeV} \), height \( V(\phi_{\text{bar}}) \approx (4.3 \times 10^9 \text{ GeV})^4 \)
  (Based on a 3-loop calculation by Bednyakov et al. 2015)
Tunneling Rate

- Bubble nucleation rate:
  - $\Gamma \sim e^{-B}$, where
  - $B = \text{“bounce” action}$ (Coleman 1977)
  - Solution of Euclidean eom

- Constant $\lambda < 0$: (Fubini 1976)
  $$\phi(r) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{r^2 + R^2}$$

- Action $B = \frac{8\pi^2}{3|\lambda|}$

- When $\lambda$ runs, $B \approx \frac{8\pi^2}{3|\lambda_{\text{min}}|} \approx 1800$
  and $\Gamma \sim \mu_{\text{min}}^4 e^{-B}$

- Depends sensitively on Higgs and top masses
Assumptions:

- Bubbles grow at the speed of light (see Stopyra’s talk) and destroy everything they hit.

⇒ There cannot have been any bubbles in our past light cone.

Diagram:
- Past Light Cone
  - conformal time $\eta$
  - today $\eta_0$
- hot Big Bang
- inflation
- comoving distance
Past Light Cone

- Probability of no bubble in the past light cone:
  \[ P(\mathcal{N} = 0) = e^{-\langle \mathcal{N} \rangle}, \]
  where \( \langle \mathcal{N} \rangle \) is the expected number of bubbles \( (d\eta = dt/\alpha) \),
  \[ \langle \mathcal{N} \rangle = \frac{4\pi}{3} \int_{\eta_0}^{\eta} d\eta \, a(\eta)^4 (\eta_0 - \eta)^3 \Gamma(\eta) \]

- Therefore, we must have \( \langle \mathcal{N} \rangle \lesssim 1 \)
- Integrate over the whole history of the Universe: inflation, reheating, hot Big Bang, and late Universe
- (For anthropists: \( \frac{d\langle \mathcal{N} \rangle}{dt} \Delta t \lesssim 1 \))
- ((For quantum immortalists: You may go. There is nothing for you in this talk.))
Number of bubbles in past lightcone: $\langle \mathcal{N} \rangle \approx 0.125 \Gamma / H_0^4$

If $\langle \mathcal{N} \rangle \ll 1$, no contradiction $\Rightarrow$ Metastability
Higgs-Curvature Coupling

- Curved spacetime:
  \[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \xi R \phi^\dagger \phi \]
  (Chernikov&Tagirov 1968)

- Symmetries allow one more renormalisable term:
  Higgs-curvature coupling \( \xi \)

- Required for renormalisability, runs with energy — Cannot be set to zero!

- Last unknown parameter in the Standard Model
Running $\tilde{\xi}$

$$\mu \frac{d\xi}{d\mu} = \left(\xi - \frac{1}{6}\right) \frac{12\lambda + 6y_t^2 - \frac{3}{2} g'^2 - \frac{9}{2} g^2}{16\pi^2}$$

- Becomes negative if $\xi_{\text{EW}} = 0$
- Conformal value $\xi = 1/6$
- RG invariant at 1 loop but not beyond
Measuring $\xi$

- Curved spacetime:
  \[ \mathcal{L} = \mathcal{L}_{SM} + \xi R \phi^\dagger \phi \]
- Ricci scalar $R$ very small today
  \[ \Rightarrow \text{Difficult to measure } \xi \]
- Colliders: Suppresses Higgs couplings (Atkins&Calmet 2012)
  - LHC Bound $|\xi| \lesssim 2.6 \times 10^{15}$
  - Future (?) ILC: $|\xi| \lesssim 4 \times 10^{14}$
- In contrast, $R$ was high in the early Universe
Late Universe Stability Bounds

- Find the gravitational instanton by solving field + Einstein equations numerically (AR&Stopyra 2016)
Hot Big Bang

- High temperature: Higher bubble nucleation rate (Espinosa et al 2008)
- If reheat temperature $T_{RH}$ is high enough, this dominates over late-time contribution
- Top mass bound (Delle Rose et al 2016):

\[
\frac{M_t}{\text{GeV}} < 0.283 \left( \frac{\alpha_s - 0.1184}{0.0007} \right) + 0.4612 \frac{M_h}{\text{GeV}} \\
+ 1.907 \log_{10} \frac{T_{RH}}{\text{GeV}} + \frac{1.2 \times 10^3}{0.323 \log_{10} \frac{T_{RH}}{\text{GeV}} + 8.738}.
\]

(Markkanen, AR, Stopyra, 2018)
Higgs Fluctuations from Inflation

- Inflation: $H \lesssim 9 \times 10^{13}$ GeV (Planck+BICEP2 2015)
- Assume light Higgs, no direct coupling to inflaton
- Equilibrium field distribution (Starobinsky&Yokoyama 1994)
  
  $$P(\phi) \propto \exp \left[ -\frac{8\pi^2}{3H^4} V(\phi) \right]$$

- Tree-level potential
  
  $$V(\phi) = \lambda(\phi^2 - v^2)^2$$

- Nearly scale-invariant fluctuations with amplitude $\phi \sim \lambda^{-1/4} H$
Higgs Fluctuations from Inflation

- Equilibrium $P(\phi) \propto \exp \left[ -\frac{8\pi^2}{3H^4} V(\phi) \right]$

- Running $\lambda$:
  Fluctuations take the Higgs over the barrier if $H \gtrsim \phi_{\text{bar}} \approx 10^{10}\text{GeV}$
  (Espinosa et al. 2008; Lebedev & Westphal 2013; Kobakhidze & Spencer-Smith 2013; Fairbairn & Hogan 2014; Hook et al. 2014)

- Does this imply $H \lesssim 10^{10}\text{GeV}$?
Effective Higgs mass term \( m^2_{\text{eff}}(t) = m^2_H + \xi R(t) \)

Ricci scalar in FRW spacetime:

\[
R = 6 \left( \frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} \right) = 3(1 - 3w)H^2
\]

- Radiation dominated: \( w = 1/3 \) \( R = 0 \)
- Matter dominated: \( w = 0 \) \( R = 3H^2 \)
- Inflation / de Sitter: \( w = -1 \) \( R = 12H^2 \)
Higgs During Inflation

- Inflation: Constant $R = 12H^2$
- Effective mass term
  $$m_{\text{eff}}^2 = m_H^2 + \xi R = m_H^2 + 12\xi H^2$$
- Tree level: (Espinosa et al 2008)
  - $\xi > 0$: Increases barrier height
    Makes the low-energy vacuum more stable
  - $\xi < 0$: Decreases barrier height
    Makes the low energy vacuum less stable
- $H$ contributes to loop corrections:
  For $H \gg \phi$, $V(\phi) \approx \lambda(H)\phi^4 \Rightarrow \text{No barrier!}$ (HMNR 2014)
Effective Potential in Curved Spacetime

- One-loop computation in de Sitter:

\[
V^{\text{eff}}_{\text{SM}}(\phi(\mu)) = -\frac{1}{2}m^2(\mu)\phi^2(\mu) + \frac{\xi(\mu)}{2}R\phi^2(\mu) + \frac{\lambda(\mu)}{4}\phi^4(\mu) + V_\Lambda(\mu) - 12\kappa(\mu)H^2 + \alpha(\mu)H^4 \\
+ \frac{1}{64\pi^2} \sum_{i=1}^{31} \left\{ n_i M_i^4(\mu) \left[ \log \left( \frac{|M_i^2|}{\mu^2} \right) - d_i \right] + n_i' H^4 \log \left( \frac{|M_i^2(\mu)|}{\mu^2} \right) \right\}. \tag{5.3}
\]

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<td>$\zeta_{\omega_z} m_{\omega_z}^2 - 2H^2$</td>
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(MNRS 2018)
Potential in Curved Spacetime

- One-loop computation for $\xi = 0$ (in units of $\mu_{\text{inst}} \approx 6.6 \times 10^9 \text{GeV}$)
(De-)Stabilising the Potential

\[ \xi \gtrsim 0.06 \]

\[ V_{\text{max}} \sim \frac{36 \xi^2 H^4}{|\lambda|} \]

\[ \xi \lesssim 0.02 \]
If $H \gtrsim \mu_{\text{inst}} = 6.6 \times 10^9 \text{GeV}$ and there is no new physics, vacuum stability during inflation requires $\xi \gtrsim 0$
In real inflationary models, $H$ depends on time: Affects decay rate $\Gamma$ and volume of past light cone

Simplest case: Quadratic chaotic inflation

$$V(\chi) = \frac{1}{2}m^2\chi^2$$

Most bubbles produced near the end of inflation

Stability requires $\xi \gtrsim 0.044$

(Mantziris, Markkanen & AR, in progress)
Multiple coexisting solutions (AR&Stopyra, PRD 2018)

Quantum (Coleman-de Luccia) tunnelling rate $\Gamma \sim e^{-B}$ nearly constant until Hawking-Moss starts to dominate

$\Rightarrow$ Hawking-Moss is always the relevant process for the constraint
Multiple Solutions

(A&Stopyra, PRD 2018)
Reheating: Inflation \((R = 12H^2) \rightarrow \text{radiation} \ (R = 0)\)

\[ R(t) = \frac{2m^2\chi^2 - \dot{\chi}^2}{M^2_{\text{Pl}}} \]

Effective Higgs mass \(m^2_{\text{eff}} = m^2_H + \xi R\) oscillates:
- Parametric resonance (“Geometric preheating”) (Bassett&Liberati 1998, Tsujikawa et al. 1999)
- \(R\) goes negative when \(\chi \sim 0\)
  - If \(\xi > 0\), Higgs becomes tachyonic (HMNR 2015)
  - Exponential amplification

\[
\langle \phi^2 \rangle_H \sim \frac{2}{3\sqrt{3}\xi} \left( \frac{H}{2\pi} \right)^2 e^{\frac{2\sqrt{\xi}X_{\text{ini}}}{M^2_{\text{Pl}}}} \sim \frac{H^2}{\xi} e^{2\sqrt{\xi}}
\]
Vacuum Decay at the End of Inflation

Not enough growth

Instability!

Field backreaction

\( H/\Phi_{\text{bar}} \)

\( \Delta h \approx 10 \Lambda_f \)

\( \Delta h \approx 10^{2/3} \Lambda_f \)

Becomes nonlinear

(HMNR 2015)

A. Rajantie, Cosmological Implications of EW Vacuum Instability, 1 July 2019
\[ V(\chi) = \frac{1}{2} m^2 \chi^2, \quad M_{\text{top}} = 172.12 \text{ GeV} \]
Stability depends on top mass and speed of reheating

\[ M_{\text{top}} = 173.34 \text{ GeV} \]: vacuum decay before \( m_t = 100 \) if \( \xi \geq 9 \)
Constraints on $\xi$

- Minimal scenario:
  Standard Model + $m^2\chi^2$ chaotic inflation, no direct coupling to inflaton
  \[0.04 \leq \xi \leq 9\]

- 15 orders of magnitude stronger than the LHC bound
  \[|\xi| \leq 2.6 \times 10^{15}\]

- Caveats:
  - Assumes no direct coupling to inflaton (see, e.g., Ema et al. 2016, 2017)
    - Would still need $|\xi| \leq O(1)$
  - Assumes no new physics
    - Could stabilise potential altogether, or destabilise further
  - Assumes high scale inflation $H \approx 10^9$ GeV