Resolving the Strong Coupling Problems in Massive Gravity and Bigravity

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GG '17; GG, Older, Pirtskhalava, in preparation
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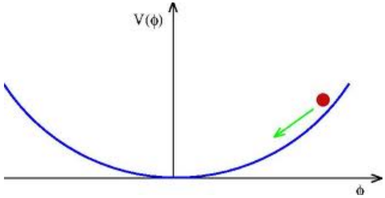
Ambitious questions in massive gravity and bigravity:

- Late-time universe (can massive gravity give dark energy?)

  Accelerated Universe without cosmological constant or scalar potentials? Tests via growth of structure

- Early universe (is inflation the only option?)

  Can bigravity give inflation or its alternatives?

- A long-standing puzzle: can these theories resolve the Cosmological Constant Problem?

Massive gauge and gravitational fields:

- Mass due to "matter" in a lab

  Plasma, superconductors, charged condensates,..

- Mass due to a relativistic Higgs vacuum

  Our universe permeated by the Higgs VEV

  Degrees of freedom of the massive gauge boson/graviton: often there are more degrees of freedom, beyond the longitudinal modes of massive gauge bosons, e.g., the ion acoustic wave in plasma, Higgs boson in a relativistic systems

  For a relativistic theory the Higgs boson needed for a weakly coupled UV completion

Minimal number of degrees of freedom:

- $SU(2)$ massive gauge fields (Vainshtein, Khriplovich '71):

$$\frac{\partial\pi^a\partial\pi^a}{(1+\pi^a\pi^a/v^2)^2}$$

the strong interaction scale is $v = m/g$

- In pure massive gravity (de Rham, GG, Tolley), the strong scale similar to DGP as derived in (Luty, Porrati, Rattazzi)

$$\Lambda_3 = (M_{\rm P} m^2)^{1/3}$$

- Vainshtein mechanism (Vainshtein 72; Deffayet, Dvali, GG, Vainshtein)

General considerations (Arkani-Hamed, Georgi, Schwartz)

The longitudinal mode gets kinetic term via mixing with a tensor:

$$\mathcal{L}_2 = (\partial h)^2 + m^2 h \partial\partial\pi + hT$$

Mass scale is irrelevant in the leading approximation:

$$\mathcal{L}_2 = (\partial \hat{h})^2 - m^4 (\partial\pi)^2 + \hat{h}T + m^2 \pi T$$

and rescale, $\pi \to \pi/m^2$; the fifth force (vDVZ discontinuity)

$$\text{Nonlinear interactions} \qquad m^4 \pi (\partial\partial\pi)^2 \to \frac{\pi(\partial\partial\pi)^2}{M_{\rm P} m^2}\,.$$

GR Extended by Mass and Potential Terms

Previous no-go statements invalid: *de Rham, GG, '10*

The Lagrangian of the theory: *de Rham, GG, Tolley, '11*

Using $g_{\mu\nu}(x)$ and 4 scalars $\phi^a(x)$, $a = 0, 1, 2, 3$, define

$$\mathcal{K}^\mu_\nu(g,\phi) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\tilde{f}_{\alpha\nu}} \qquad \tilde{f}_{\alpha\nu} \equiv \partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}$$

The Lagrangian is written using notation $tr(\mathcal{K}) \equiv [\mathcal{K}]$:

$$\mathcal{L} = M_{\rm P}^2\sqrt{g}\left(R + m^2\left(\mathcal{U}_2 + \alpha_3\,\mathcal{U}_3 + \alpha_4\,\mathcal{U}_4\right)\right)$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2] \sim det_2(\mathcal{K})$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \sim det_3(\mathcal{K})$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4] \sim det_4(\mathcal{K})$$

Cosmology of pure massive gravity: *D'Amico, de Rham, Dubovsky, GG, Pirtskhalava, Tolley, '11 Gümrükcüoglu, Lin, Mykohyama 11*

For instance, *Koyama,Niz,Tasinato*:

$$ds^2 = -d\tau^2 + e^{m\tau}(d\rho^2 + \rho^2 d\Omega^2)$$

while, ϕ^0 and ϕ^ρ, are inhomogeneous functions. Selfacceleration is a generic feature of this theory. Anisotropic solutions and fluctuations: Gümrükcüoglu, Lin, Mukohyama '12.

Some of the more complex solutions might be OK (Gümrükcüoglu, Lin, Mukohyama, De Felice, et al.), or else extensions beyond pure massive gravity are needed for cosmology.

Brief summary:

Linearized theory: 3 NG Bosons eaten up by the tensor field that becomes massive. The theory guarantees unitary 5 degrees of freedom on (nearly) Minkowski backgrounds.

Nonlinear interactions are such that there are 5 degrees of freedom on any background. However, there is no guarantee that some of these 5 degrees of freedom aren't bad on certain backgrounds for certain values of the two free parameters (instabilities, superluminalities).

The theory is not finished – further completion needed. No IR obstruction to UV completion for certain values of the parameters of the theory – C. Cheung, G. Remmen (see also de Rham, Melville, Tolley).

On curved backgrounds, e.g., AdS:

(Porrati; Kogan, Mouslopoulos, Papazoglou)

On curved backgrounds, e.g., on AdS_4, with $-\Lambda < 0$, one obtains, $-\Lambda m^2(\partial\pi)^2$. This would raise the strong scale as long as the magnitude of the cosmological constant is large, $\Lambda >> m^2$.

A way to use the above while being in Minkowski, GG
Embedding into a $D = 4 + n > 4$ dimensional massive gravity

$$-M_D^{2+n}\bar{m}^2\bar{\Lambda}\left(\partial_D\Pi(x^\mu, z^1, z^2, ..., z^n)\right)^2$$

M_D and $\bar{m}$ are Planck and graviton mass in D-dim, respectively.

The action and coupling to the matter:

$$\tilde{\mathcal{L}}_2 = (\partial \hat{h})^2 - m^4(\partial\pi)^2 - M_D^{2+n} L^n \bar{m}^2 \bar{\Lambda} (\partial\pi)^2 + \hat{h} T + m^2 \pi T$$

As long as, $M_D^{2+n} L^n \bar{m}^2 \bar{\Lambda} >> m^4$, (in $M_{\mathrm{P}} = 1$ units), all OK

Below the scale of new physics still the old theory; it has to be that

$$m_{KK} < \Lambda_3$$

Rescaling the π field:
a) would remove the vDVZ problem
b) would raise the strong scale

Example: warped massive gravity, GG '17

4D massive gravity embedded in 5D AdS massive gravity:
The 5D massive action just a generalization of the 4D action

$$S_5 = M_5^3 \int d^4x\, dz\, \sqrt{-\bar{g}} \left(\bar{R}(\bar{g}) + 2\bar{\Lambda} + 2\bar{m}^2 \mathcal{V}(\bar{\mathcal{K}}^M_N) \right)$$

where

$$\mathcal{V}(\bar{\mathcal{K}}) = \det_2(\bar{\mathcal{K}}) + \beta_3 \det_3(\bar{\mathcal{K}}) + \beta_4 \det_4(\bar{\mathcal{K}}) + \beta_5 \det_5(\bar{\mathcal{K}})$$

with the definition

$$\bar{\mathcal{K}}^A{}_B = \delta^A_B - \sqrt{\bar{g}^{AM} \bar{f}_{MB}}, \qquad \bar{f}_{MN} = \partial_M \Phi^I \partial_N \Phi^J \tilde{f}_{IJ}(\Phi)$$

and $\Phi^J(x^\mu, z)$, $(I, J = 0, 1, 2, 3, 5)$, five scalar Stückelberg fields.
(F. Hassan, R. A. Rosen, arbitrary fiducial metric, bigravity)

The total action:

$$S_{total} = S_5 + S_4 + S_{Boundary} \; ,$$

with S_5 and S_4 defined similarly in 5D and 4D respectively; $S_{Boundary}$ is the Gibbons-Hawking plus certain additional terms.

Bulk boundary connection

$$\bar{g}_{\mu\nu}(x,z)|_{z=0} = g_{\mu\nu}(x)$$

$$\delta^a_J \Phi^J(x,z)|_{z=0} = \varphi^a(x)$$

$$\delta^I_a \delta^J_b \tilde{f}_{IJ}(\Phi)|_{\Phi^z=0} = \eta_{ab}$$

Classical solutions:

The fiducial metric is assumed to be AdS (justified a posteriori)

$$ds^2_{Fid} = \tilde{f}_{IJ} d\Phi^I d\Phi^J = \frac{L^2}{(\Phi^z + L)^2}\left[\eta_{ab} d\Phi^a d\Phi^b + (d\Phi^z)^2\right]$$

Then, the physical metric has a solution ($z > 0$, $\Phi^z > 0$)

$$ds^2 = \bar{g}_{AB} dx^A dx^B = A^2(z)\left[\eta_{\mu\nu} dx^\mu dx^\nu + dz^2\right], \quad A(z) \equiv \frac{L}{z+L}$$

This could be obtained in bigravity, with a weak coupling of the massless graviton

$$\tilde{M}_5^3 \int d^5\Phi \sqrt{\tilde{f}(\Phi)}\left(R(\tilde{f}(\Phi)) + 2\bar{\Lambda}\right)$$

Linearized theory – the spectrum:

The linearized theory is continuous in the following massless limit:

$$m \to 0, \quad \bar{m} \to 0, \quad m/\bar{m} \to 0, \quad \bar{\Lambda} = fixed$$

In the above limit the spectrum consists of:
RS zero mode, KK gravitons, KK vectors and scalars

Away from the limit: the RS zero mode disappears, a resonance in the KK tower (similar to a scalar, Dubovsky, Rubakov, Tinyakov)

The strong coupling is due to the longitudinal mode of the resonance graviton; the latter gets a large kinetic term due to the background

$$-M_5^3 \bar{\Lambda} (\partial \Pi)^2, \quad \Pi = \frac{\Pi^c}{\sqrt{M_5^3 \bar{\Lambda}}} = \frac{\Pi^c}{M_5^{3/2} \bar{H}}$$

Nonlinear interactions:

Bulk generic (C. de Rham, GG, 10)

$$M_5^3 \bar{m}^2 \bar{h} \left(\left(\frac{\nabla\nabla\Pi}{\bar{m}} \right)^2 + \left(\frac{\nabla\nabla\Pi}{\bar{m}} \right)^3 + \left(\frac{\nabla\nabla\Pi}{\bar{m}} \right)^4 \right)$$

Bulk special (related to total derivatives)

$$M_5^3 \bar{m}^2 \bar{\Lambda} \left(\left(\frac{\nabla\Pi}{\bar{m}} \right) \left(\frac{\nabla\Pi}{\bar{m}} \right) \left(\frac{\nabla\nabla\Pi}{\bar{m}} \right) \ldots + \left(\frac{\nabla\Pi}{\bar{m}} \right) \left(\frac{\nabla\Pi}{\bar{m}} \right) \left(\frac{\nabla\nabla\Pi}{\bar{m}} \right)^3 \right)$$

The Π field needs to be canonically normalized

As a result, the 5D strong scale is

$$\Lambda_{5D} \simeq (M_5^{3/2}\bar{m}\bar{H})^{2/7} = \Lambda_{7/2}\left(\frac{\bar{H}}{\bar{m}}\right)^{2/7} >> \Lambda_{7/2}$$

How about the 4D strong scale?

Effective kinetic term of the longitudinal mode, captures all KK's

$$-\frac{M_5^3\bar{\Lambda}}{2}\,\pi(x)\,\sqrt{-\Box_4}\,\frac{K_1(L\sqrt{-\Box_4})}{K_2(L\sqrt{-\Box_4})}\,\pi(x) \tag{1}$$

In the low energy approximation $L\sqrt{-\Box_4} << 1$

$$L\,\frac{M_5^3\bar{\Lambda}}{2}\,\pi(x)\,\Box_4\,\pi(x) \tag{2}$$

The 4D strong scale:

$$\Lambda_* \simeq (M_5^{3/2} \bar{m} \bar{H}^{1/2})^{1/3}$$

$$\Lambda_* \sim (M_{\rm P} \bar{m} \bar{H})^{1/3} = (\Lambda_2^2 \bar{H})^{1/3}$$

For: $\bar{H} \sim 10^{16}\ GeV$, $M_5 \sim 10^{18}\ GeV$, and $\bar{m} \sim m \sim 10^{-42}\ GeV$

$$\Lambda_{5D} \sim GeV, \quad \Lambda_* \sim MeV$$

is some 19 orders of magnitude greater than $\Lambda_3 \sim 10^{-19}\ MeV$.

A 4D holographic dual description

- What is a 4D dual of a 5D AdS massive graviton?

  The 4D stress-tensor that sources the bulk 5D massive graviton acquires an anomalous dimension proportional to

$$\delta = \frac{\bar{m}^2}{\bar{H}^2}$$

  Therefore, the 4D stress-tensor cannot be conserved, yet, the dual is a CFT. No local Lagrangian description, yet all the correlation functions can be calculated from AdS/CFT. Hence, an effective 1PI action exists, Domokos, GG, '15.

- Introducing the RS brane endowed with a 4D massive graviton: as before a nonlocal CFT, has no stress tensor; the 4D massive graviton couples to the non-local CFT. This coupling generates an additional kinetic term for π due to nonlocality

Quantum generated bulk graviton mass

- Can the strong scale be raised even further?

  Inducing the bulk graviton mass by quantum corrections in the 5D AdS bulk (Porrati; Duff, Liu); new states in the 5D with special boundary conditions (Aharony, Clark, Karch; Kiritsis) .

$$AdS_5 \, U \, AdS_5 \quad ---> \quad CFT_1 \times CFT_2$$

  This appears to lead to a theory with the strong scale at $\bar{H} >>> \Lambda_3$, and could be of the order of the GUT scale (GG, Older, Pirtskhalava, in preparation, see Dan Older's talk).

- AdS/CFT interpretation: nonlocal CFT, has no stress tensor; emergent massive graviton as a bound state of the CFT

Conclusions

- Beyond Einstein theories of gravity: interesting applications to cosmology of late-time universe as well as to early universe.
- The strong coupling problem. In a classical theory the strong coupling scale can be raised by at least 19 order of magnitude.
- Graviton mass by quantum loop effects in 5D AdS raises the strong coupling scale in 4D. Union of two AdS spaces as dual of product of CFT's leading to a theory with the strong scale at $\bar{H}$, that could be as high as the GUT scale.