Scale invariance, Stueckelberg breaking of Weyl gravity and inflation

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Based on: arXiv:1904.06596, 1812.08613, 1906.11572

- “New physics” beyond SM: new symmetry?

- SUSY @ TeV:	- nice [in] theory.... protected a hierarchy of scales. Not found @ LHC.
- scale invariance (SI);	- discrete
	- global SI. e.g.: SM with higgs $m_\phi = 0$ has scale invariance. [Bardeen 1995]
	- local SI
	- gauged local SI $=$ Weyl gauge symmetry $\rightarrow$ Weyl gravity [Weyl 1919]
	- quantum scale invariance (QSI): global, local. [Englert, Gastmans, Truffin 1976]

(1). Global scale invariance $x'_\mu = \rho\, x_\mu; \quad \phi'(\rho\, x) = \rho^{-1}\, \phi(x),$ forbids $\int d^4x\, m^2 \phi^2$

- no dim-ful coupling; all scales (M_{Planck},...) from vev's; e.g. SM with $m_h = 0$

• I show: QSI in flat spacetime protects a hierarchy of fields vev's $\phi \ll \sigma$.

(2). Local scale invariance: $g'_{\mu\nu} = \Omega(x)^2 g_{\mu\nu}, \quad \phi' = \frac{1}{\Omega(x)}\phi$

(3). Gauged local scale invariance $g'_{\mu\nu} = \Omega(x)^2 g_{\mu\nu}, \quad \phi' = \frac{1}{\Omega(x)}\phi, \quad \omega'_\mu = \omega_\mu - \partial_\mu \ln \Omega(x)$

- also called Weyl gauge symmetry. Weyl gravity/conformal geometry (1918).

- Non-metricity $\nabla_\mu\, g_{\alpha\beta} = \omega_\mu\, g_{\alpha\beta}$. Einstein critique due to ω_μ (massless)

- Not a problem if ω_μ massive/decouples ($\omega_\mu\!=\!0$) at very high scale.

• I show: Weyl gravity spontaneously broken to Einstein-Proca+cc; $m_\omega \sim M_{\text{Planck}}$
M_{Planck}=phase transition emergent scale. Weyl inflation: $0.00257 < r < 0.00303$.

(1). Global Scale Invariance – at Quantum level:

[Englert 1976, Itzykson/Zuber, Shaposhnikov 2008]

DR: $d=4-2\epsilon$: $L=\frac{1}{2}(\partial_\mu \phi)^2-\lambda\,\mu^{2\epsilon}\,\phi^4$, explicit $\not{\text{S}}$I by UV regulators, to avoid.

replace $\mu \rightarrow$ field σ, spontaneous $\langle\sigma\rangle \neq 0 \Rightarrow$ Goldstone (dilaton): $\sigma=\langle\sigma\rangle\, e^{\tau}$, $\tau \rightarrow \tau-\ln\rho$

[nothing new, recall M_{string} moduli dep]

- Action: $d=4$, $$S=\int d^4x \underbrace{\Big[(\partial_\mu\phi_j)^2-V(\phi_j)\Big]}_{\text{visible}}+\underbrace{\int d^4y\, L_h(\sigma,\partial\sigma)}_{\text{hidden}}$$ spectrum extended by σ!

- each sector SI (shift symmetry) → enhanced shift symmetry: $S_h \times S_v \Rightarrow \lambda_m \phi^2\sigma^2$: λ_m naturally small

[Fubini 1976; Volkas, Kobakhidze, Foot 2013]

$$\mu^{2\epsilon}\phi^4 \rightarrow \tilde{V} \sim (z\,\sigma)^{2\epsilon}\,\phi^4=\big(z\,\langle\sigma\rangle\big)^{2\epsilon}\Big[1+2\,\epsilon\Big(\frac{\tilde{\sigma}}{\langle\sigma\rangle}-\frac{\tilde{\sigma}^2}{2\langle\sigma\rangle^2}+\ldots\Big)+\mathcal{O}(\epsilon^2)\Big]\,\phi^4, \qquad \sigma=\langle\sigma\rangle+\tilde{\sigma}, \qquad (z:\text{dim-less})$$

$\Rightarrow$ scale inv reg=DR+dilaton with ∞-many ϵ-couplings. $\langle\sigma\rangle$:'new physics'. $\langle\phi\rangle \ll \langle\sigma\rangle$ quantum stable?

[$\sigma\sigma \rightarrow \sigma\sigma$ at 3 loops! $(\partial_\mu \ln\sigma)^4$]

- One-loop SI potential:

$$L = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(\partial_\mu\sigma)^2 - \overbrace{\frac{1}{4!}\lambda\,\phi^4}^{=V(\phi)}, \qquad V \to \tilde V = z^{2\epsilon}\,\sigma^{2\epsilon}\,V(\phi).$$

$$V_1 = -\frac{i}{2}\int\frac{d^dp}{(2\pi)^d}\ \mathrm{tr}\ \ln\left[p^2 - \tilde V_{\alpha\beta} + i\varepsilon\right] = \sum_{s=\phi,\sigma}\frac{\tilde M_s^4}{4\kappa}\ \Big[\frac{-1}{\epsilon} + \ln\frac{\tilde M_s^2}{c_0}\Big]\mu(\sigma)^{2\epsilon},$$

$$\partial^2_{\sigma\ldots}\tilde V \sim \partial^2_{\sigma\ldots}V\ldots + \epsilon\,(..); \quad \tilde M_\phi^4 = M_\phi^4 + \epsilon\ldots, \quad \tilde M_\sigma^4 \sim \epsilon^2, \qquad \kappa = (4\pi)^2$$

$$U = V + \frac{\lambda^2\phi^4}{16\kappa}\Big[\overline{\ln}\frac{\lambda\phi^2}{(z\,\sigma)^2} - \frac{1}{2}\Big] = V + \underbrace{\frac{\lambda^2\phi^4}{16\kappa}\Big[\overline{\ln}\frac{\lambda\phi^2}{(z\langle\sigma\rangle)^2} - \frac{1}{2}\Big]}_{CW} + \underbrace{\frac{\lambda^2\phi^4}{16\kappa}\Big[\frac{-\tilde\sigma}{\langle\sigma\rangle} + \frac{\tilde\sigma^2}{2\langle\sigma\rangle^2} + \ldots\Big]}_{\to 0\text{; small yet maintains SI}}$$

$\sigma = \langle\sigma\rangle + \tilde\sigma$, $\tilde\sigma$: fluctuations

$\Rightarrow$ U scale invariant due to dilaton ($\ln\sigma$).

No new poles, same beta function $\beta_\lambda^{(1)}$; $\lambda^B = \lambda Z_\lambda Z_\phi^{-2}$, $d\lambda^B/d(\ln z) = 0$, $\beta_\lambda^{(1)} = d\lambda/(d\ln z) = 3\lambda^2/\kappa$.

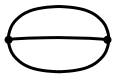
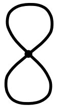
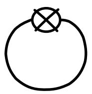
QSI with $\beta_\lambda \neq 0$, dilation current $\partial_\mu D^\mu = 0$. $\beta_\lambda = 0$ is sufficient, not necessary for QSI. CS: $dU/d(\ln z) = 0$

[Englert et al 1979, Shaposhnikov et al; Tamarit 2014]

- Two-loop SI potential: using: $\tilde V(\phi,\sigma) \sim z^{2\epsilon}\,\sigma^{2\epsilon}\,V(\phi)$ [background field method]

$$\Rightarrow \tilde V(\phi+\delta_\phi,\sigma+\delta_\sigma) = \tilde V(\phi,\sigma)+\tilde V_\alpha\delta_\alpha+\frac{1}{2}\,\tilde V_{\alpha\beta}\,\delta_\alpha\delta_\beta+\frac{1}{3!}\,\tilde V_{\alpha\beta\gamma}\,\delta_\alpha\delta_\beta\delta_\gamma+\frac{1}{4!}\,\tilde V_{\alpha\beta\gamma\rho}\,\delta_\alpha\delta_\beta\delta_\gamma\delta_\rho+\cdots \qquad \alpha,\beta=\phi,\sigma.$$

$$\tilde V_{\alpha\beta\ldots} = \partial_\alpha\partial_\beta..\tilde V$$

$V_2 = \frac{i}{12}$  $+\frac{i}{8}$  $+\frac{i}{2}$  $= \frac{i}{12}\,\tilde V_{\alpha\beta\gamma}\tilde V_{\alpha'\beta'\gamma'}\int \frac{d^dp}{(2\pi)^d}\frac{d^dq}{(2\pi)^d}\,(\tilde D_p)_{\alpha\alpha'}\,(\tilde D_q)_{\beta\beta'}\,(\tilde D_{p+q})_{\gamma\gamma'}+\cdots$

$$= (z\sigma)^{2\epsilon}\,\frac{\lambda^3\,\phi^4}{32\kappa^2}\Big\{-\frac{3}{\epsilon^2}+\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\Big\}; \qquad (\tilde D_p)_{\alpha\beta} = (D_p)_{\alpha\beta}+\epsilon\,(\ldots)_{\alpha\beta}+\epsilon^2\,(\ldots)_{\alpha\beta}$$

same poles, ϵ-shifts to propagators, vertices: $\tilde V_{\alpha\beta\gamma\ldots} = V_{\alpha\beta\gamma\ldots}+\epsilon\,(\ldots)_{\alpha\beta\gamma\ldots}+\epsilon^2\,(\ldots)_{\alpha\beta\gamma\ldots}$

[Z. Lalak, P. Olszewski,DG]
[1712.06024]

Two-loop corrected U

$$U = \frac{\lambda}{4!}\,\phi^4\,\Big\{1+\frac{3\lambda}{2\kappa}\Big(\overline{\ln}\frac{V_{\phi\phi}}{(z\sigma)^2}-\frac{1}{2}\Big)+\frac{3\lambda^2}{4\kappa^2}\Big(4+A_0-4\,\overline{\ln}\frac{V_{\phi\phi}}{(z\sigma)^2}+3\,\overline{\ln}^2\frac{V_{\phi\phi}}{(z\sigma)^2}\Big)+\frac{5\lambda^2}{\kappa^2}\frac{\phi^2}{\sigma^2}+\frac{7\lambda^2}{24\kappa^2}\frac{\phi^4}{\sigma^4}\Big\},$$

V $\qquad V^{(1)}$ $\qquad V^{(2)}$ $\qquad$ new $V^{(2,n)}$ finite
z-independent

- Two-loop: Taylor expand about $\sigma = \langle\sigma\rangle + \tilde{\sigma}$:

$$U \;=\; \frac{\lambda}{4!}\,\phi^4 \Big\{ 1 + \frac{3\lambda}{2\kappa}\Big(\overline{\ln}\frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2} - \frac{1}{2}\Big) + \frac{3\lambda^2}{4\kappa^2}\Big(4 + A_0 - 4\,\overline{\ln}\frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2} + 3\,\overline{\ln}^2\frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2}\Big) + \mathcal{O}\Big(\frac{1}{\langle\sigma\rangle}\Big)$$

- This is the "usual" CW result with $\mu = \langle\sigma\rangle$, broken SI, no dilaton present. [Cheng, I. Jack, T. Jones, S. Martin]
- new terms comparable/larger than standard two-loop terms

$$\frac{\phi^n}{\sigma^n} \sim 1., \qquad n = 1, 2; \qquad\qquad \frac{\phi}{\sigma} = \frac{\phi}{\langle\sigma\rangle}\Big(1 - \frac{\tilde{\sigma}}{\langle\sigma\rangle} + \frac{\tilde{\sigma}^2}{\langle\sigma\rangle^2} + \cdots\Big), \qquad \to 0, \text{ if } \phi \ll \sigma$$

⇒ Non-polynomial terms: - SI, vanish if $\phi \ll \sigma$, only $\log\sigma$ left;

- no $\lambda^n\phi^2\sigma^2 = \lambda^n\langle\sigma\rangle^2\phi^2 + \ldots \Rightarrow$ no tuning of higgs self-coupling λ.
- finite c-terms, cannot be seen in a scheme that breaks this symmetry.
- not forbidden by symmetry → ops mandatory, quantum generated; non-renorm
- similar operators at three-loop SI potential (computed), as counterterms.

• SM+dilaton: one-loop SI potential

[Shaposhnikov et al 2008; DG, Z. Lalak, P. Olszewski]

$$V = \frac{\lambda_\phi}{3!}(H^\dagger H)^2 + \frac{\lambda_m}{2}(H^\dagger H)\,\sigma^2 + \frac{\lambda_\sigma}{4!}\sigma^4 + \frac{4\,\lambda_6}{3}\frac{(H^\dagger H)^3}{\sigma^2} + \cdots$$

$$= \frac{\lambda_\phi}{4!}\phi^4 + \frac{\lambda_m}{4}\phi^2\,\sigma^2 + \frac{\lambda_\sigma}{4!}\lambda_\sigma\,\sigma^4 + \frac{\lambda_6}{6}\frac{\phi^6}{\sigma^2}\cdots\,;\ (\lambda_m<0),$$

$$\to \frac{\lambda_\phi}{4}\Big(\phi^2 + \frac{3\lambda_m}{\lambda_\phi}\sigma^2\Big)^2 + \text{loops.}$$

min $\langle\sigma\rangle\neq 0$:

1) $9\lambda_m^2 = \lambda_\phi\lambda_\sigma +$ loops

2) $\dfrac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = \dfrac{-3\lambda_m}{\lambda_\phi}(1+\text{loops})$

$\not{\text{S}}$I : tuning (i) or $V_{\text{min}}=0 \Rightarrow$ EWSB; $m_{\tilde\phi} \approx \lambda_\phi\langle\phi\rangle^2 = (-3)\lambda_m\langle\sigma\rangle^2 \ll \langle\sigma\rangle^2$ if $|\lambda_m| \ll \lambda_\phi$ one classical tuning

$$V^{(1)} = \text{old CW with} \quad \mu \to z\sigma$$

$$V^{(1,n)} \equiv \frac{1}{48\kappa}\Big[(-16\lambda_m\lambda_\phi - 18\lambda_m^2 + \lambda_\phi\lambda_\sigma)\,\phi^4 - \lambda_m(48\lambda_m + 25\lambda_\sigma)\phi^2\sigma^2 - 7\lambda_\sigma^2\sigma^4$$

$$+ (\lambda_\phi\lambda_m + 6\lambda_6\lambda_\sigma)\frac{\phi^6}{\sigma^2} + 8\,(4\lambda_\phi - 2\lambda_m)\,\lambda_6\frac{\phi^8}{\sigma^4} + (192\lambda_6 + 2\lambda_\phi)\lambda_6\frac{\phi^{10}}{\sigma^6} + 40\lambda_6^2\frac{\phi^{12}}{\sigma^8}\Big],$$

large σ : $S_v \times S_h$

$\to 0$ if $\phi \ll \sigma$ and $\lambda_m \to 0$.

no $\lambda_\phi^2\phi^2\sigma^2 \sim \lambda_\phi\,\phi^2\langle\sigma\rangle^2$

One-loop corrected potential $U = V + V^{(1)} + V^{(1,n)}$ gives:

$$\Delta m_{\tilde{\phi}}^2 = \frac{-\lambda_m}{\lambda_\phi}\frac{\langle\sigma\rangle^2}{16\kappa}\Big\{ 27\Big[g^4\Big(\ln\frac{g^2}{4}+\frac{1}{3}\Big) + 2\,g_2^4\Big(\ln\frac{g_2^2}{4}+\frac{1}{3}\Big) - 16\,h_t^4\Big(\ln\frac{h_t^2}{2}-\frac{1}{3}\Big)\Big] + 4\,\lambda_\phi^2\Big[5\ln\frac{\lambda_\phi^2}{12} - 8 + \ln 27\Big]\Big\} = \lambda_m\lambda_\phi\langle\sigma\rangle^2 + \cdots \ll \langle\sigma\rangle^2, \text{ but no } \quad \lambda_\phi^2\,\langle\sigma\rangle^2.$$

- $\beta_{\lambda_m} \propto \lambda_m$, so λ_m stays ultraweak, natural once chosen classically small.
- result is not a DR artefact, since all scales from fields' vev's. More scalars?

⇒ (classical) hierarchy $\langle\phi\rangle \ll \langle\sigma\rangle$ protected by quantum scale inv $S_v \times S_h$, (all orders, spont $\not{S}$I)

- Curved spacetime - Einstein gravity breaks $S_v \times S_h$ by $\big(\xi_1\phi^2 + \xi_2\sigma^2\big)R \Rightarrow \beta_{\lambda_m} = \xi_1(...) + \xi_2(...) + \lambda_m(...)$
  - hierarchy of vev's (Higgs vs dilaton)? maybe preserved, fixed point for ξ_1, ξ_2.
  - what is the fate of the dilaton? can it decouple?

(2). Local scale invariance:

[t'Hooft 1104.4543; 1410.6675, IJMP 2016]

invariance under $\hat{g}_{\mu\nu} = \Omega(x)^2 \, g_{\mu\nu}, \quad \hat{\sigma} = \frac{\sigma}{\Omega(x)} \quad (a)$

want to generate spontaneously: $L_{EH} = -\frac{1}{2}\sqrt{g}\, M_p^2\, R \;\Rightarrow\; L_1 = -\sqrt{g}\,\frac{1}{2}\Big[\frac{1}{6}\sigma^2\, R + g^{\mu\nu}\,\partial_\mu\sigma\,\partial_\nu\sigma\Big]$ (inv.)

a) - has a ghost (σ)!

b) - Fake conformal symmetry? [Jackiw, Pi 2015],

c) - Going to Einstein frame: d.o.f. number not conserved! ⇒ something is missing.... if $\langle\sigma\rangle$ somehow fixed, is this really spontaneous breaking? does not look no.

To avoid!

Next:

⇒ solution to (a),(b),(c): gauged local scale invariance: Weyl gravity: dilaton σ eaten by Weyl "photon"; hence, no ghost.

(3). Gauged local scale invariance - Weyl quadratic gravity:

invariance under $\hat{g}_{\mu\nu}(x) = \Omega(x)^2\, g_{\mu\nu}(x), \quad \hat{\sigma}(x) = \frac{\sigma(x)}{\Omega(x)}, \quad \hat{\omega}_\mu(x) = \omega_\mu(x) - \frac{1}{q}\,\partial_\mu \ln \Omega(x)^2 \quad (b)$

Weyl geometry $\tilde{\nabla}_\mu\, g_{\alpha\beta} = -q\,\omega_\mu\, g_{\alpha\beta};$ with $\tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + (q/2)\left[\delta^\rho_\mu\,\omega_\nu + \delta^\rho_\nu\,\omega_\mu - g_{\mu\nu}\,\omega^\rho\right]$ inv of (b)

$\Gamma^\rho_{\mu\nu}$ = Levi-Civita. .

$$\tilde{R} = R - 3\,q\,D_\mu\omega^\mu - \frac{3}{2}q^2\,\omega^\mu\omega_\mu. \quad \Rightarrow \quad \hat{\tilde{R}} = \frac{\tilde{R}}{\Omega^2}, \qquad \tilde{D}_\mu\sigma = (\partial_\mu - q/2\,\omega_\mu)\,\sigma \;\Rightarrow\; \hat{\tilde{D}}_\mu\hat{\sigma} = \frac{\tilde{D}_\mu\sigma}{\Omega}$$

⇒ if $\omega_\mu \to 0$: $\tilde{\Gamma}^\rho_{\mu\nu} \to \Gamma^\rho_{\mu\nu}$, Weyl geometry → Riemann geometry; $\tilde{R} \to R$, Weyl tensor $\tilde{C}_{\mu\nu\rho\sigma} \to C_{\mu\nu\rho\sigma}$

⇒ invariants of (b): $\sqrt{g}\,\tilde{R}^2, \quad \sqrt{g}\,\sigma^2\tilde{R}, \quad \sqrt{g}\,(\tilde{D}_\mu\sigma)^2, \quad \sqrt{g}\,F^2_{\mu\nu}, \quad \sqrt{g}\,\tilde{C}^2_{\mu\nu\alpha\beta}$ where $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$

no other invariants like higher dimensional ops (no scale to suppress them!) $\tilde{X}$ denotes a quantity in Weyl geometry

- Weyl gravity = Einstein gravity + c.c. + massive ω_μ [D.G. arXiv:1812.08613, 1904.06596]

$$L_0 = \sqrt{g}\,\Big\{\frac{\xi_0}{4!}\,\tilde R^2 - \frac{1}{4}\,F_{\mu\nu}^2\Big\}, \qquad \xi_0>0, \qquad \text{original Weyl action (1918)}$$

$$= \sqrt{g}\,\Big\{\frac{\xi_0}{4!}\,\big(-2\,\sigma^2\,\tilde R - \sigma^4\big) - \frac{1}{4}\,F_{\mu\nu}^2\Big\} \qquad \text{eom: } \sigma^2=-\tilde R; \quad \text{dilaton: } \ln\sigma \to \ln\sigma - \ln\Omega$$

+ Gauss-Bonnet + Weyl-tensor-squared.

- Weyl gauge transf of "gauge fixing":

$$\Omega = \frac{\xi_0\,\sigma^2}{6M_p^2} \Rightarrow \hat g_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \hat\sigma^2 = \frac{6\,M_p}{\xi_0}, \quad \hat\omega_\mu=\omega_\mu - \frac{1}{q}\partial_\mu \ln\sigma^2; \quad \text{use } \tilde R = R - 3qD_\mu\omega^\mu - \frac{3}{2}q^2\omega_\mu\omega^\mu$$

$$L_0 = \sqrt{\hat g}\,\Big\{-\frac{1}{2}\,M_p^2\,\hat R - \frac{3\,M_p^4}{2\,\xi_0} + \frac{3}{4}\,q^2\,M_p^2\,\hat\omega_\mu\,\hat\omega^\mu - \frac{1}{4}\,\hat F_{\mu\nu}^2\Big\}.$$

⇒ Einstein-Proca action for massive ω_μ which "absorbed" the dilaton $\ln\sigma$ field! mass of $\omega_\mu \propto q\,M_p$.

⇒ Stueckelberg mechanism, massless σ + ω_μ →massive ω_μ. no ghost! # dof=3 conserved!

⇒ massive ω_μ decouples; M_p: emergent scale of Weyl gauge symmetry breaking. ⇒ Weyl gravity viable

⇒ $M_p \sim \langle\sigma\rangle$ dynamically in FRW universe [arxiv:1801.07676 by G. Ross, C. Hill, P. Ferreira] see talk by Graham Ross

- Geometric Stueckelberg* mechanism: A Weyl gauge invariant action:

$$L_1 = \sqrt{g}\,\Big[-\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\,(\tilde{D}_\mu\sigma)^2 + \cdots\Big], \qquad F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$$

$$\tilde{D}_\mu\sigma \;=\; (\partial_\mu - q/2\,\omega_\mu)\,\sigma = (-q/2)\,\sigma\,\big[\omega_\mu - (1/q)\,\partial_\mu \ln\sigma^2\big].$$

"gauge fixing" transformation: $\Omega^2 = \dfrac{\sigma^2}{M^2}, \quad \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \hat{\sigma} = \dfrac{1}{\Omega}\,\sigma = M, \quad \hat{\omega}_\mu = \omega_\mu - \dfrac{1}{q}\,\partial_\mu \ln\Omega^2$

→ Proca action $L_1 = \sqrt{\hat{g}}\,\Big[-\dfrac{1}{4}\,\hat{F}_{\mu\nu}^2 + \dfrac{q^2}{8}\,M^2\hat{\omega}_\mu\hat{\omega}^\mu + \cdots\Big].$

- spontaneous breaking; ω_μ has eaten the scalar $\ln\sigma$ (note $\ln\sigma \to \ln\sigma - \ln\Omega$)
- number of d.o.f. is conserved (=3).

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[*] Baron Ernst Carl Gerlach Stueckelberg von Breidenbach zu Breidenstein und Melsbach

- Weyl gravity with matter $\qquad$ ϕ- scalar field, higgs-like, etc

$$\begin{aligned} L_2 &= \sqrt{g}\,\Big\{\frac{\xi_0}{4!}\,\tilde R^2-\frac{1}{4}\,F_{\mu\nu}^2\Big\}-\frac{\sqrt{g}}{12}\,\xi\,\phi^2\,\tilde R+\sqrt{g}\,\Big\{\frac{1}{2}(\tilde D_\mu\phi)^2-\frac{\lambda}{12}\phi^4\Big\}, \qquad \tilde R^2\to -2\sigma^2\,\tilde R-\sigma^4 \\ &= \sqrt{g}\,\Big\{-\frac{1}{4!}(\xi_0\,\sigma^2+\xi\,\phi^2)\,\tilde R-\frac{1}{4}\,F_{\mu\nu}^2+\frac{1}{2}(\tilde D_\mu\phi)^2-\frac{1}{4!}\,(\lambda\,\phi^4+\xi_0\,\sigma^4)\Big\}, \qquad \rho^2=\frac{1}{6}\,(\xi_0\sigma^2+\xi\,\phi^2) \end{aligned}$$

- Weyl gauge transformation (b): "gauge fixing": $\qquad \Omega=\frac{\rho^2}{M_p^2},\quad \hat\rho=M_p,\quad \hat\omega_\mu=\omega_\mu-\frac{1}{q}\ln\rho^2$

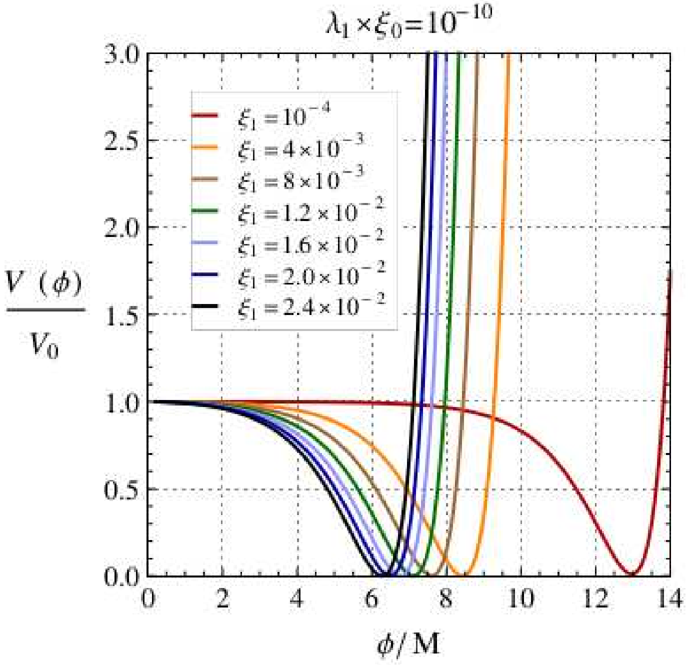
Einstein-Proca action for ω_μ :
$$L_2=\sqrt{\hat g}\,\Big\{-\frac{1}{2}\,M_p^2\,\hat R+\frac{3}{4}M_p^2\,q^2\,\hat\omega_\mu\hat\omega^\mu-\frac{1}{4}\hat F_{\mu\nu}^2+\frac{1}{2}(\hat{\tilde D}_\mu\hat\phi)^2-V\Big]\Big\},$$

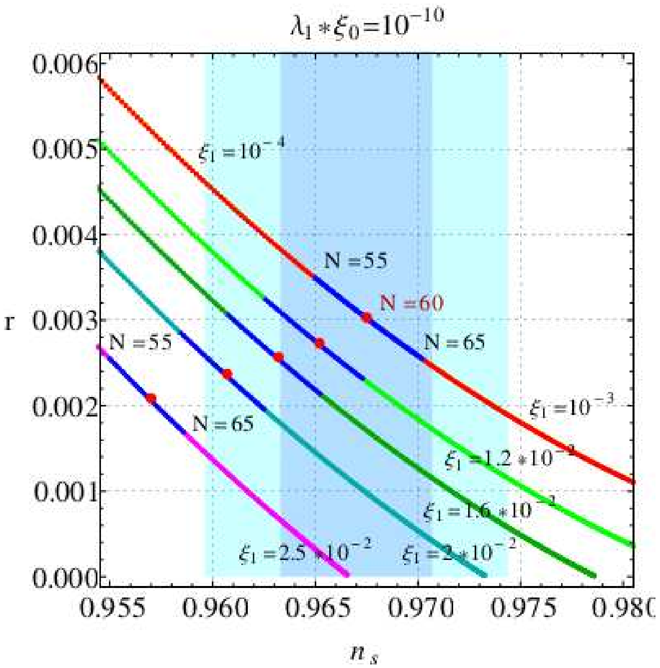
$$V=\frac{3M_p^4}{2\,\xi_0}\Big[1-\frac{\xi\hat\phi^2}{6\,M_p^2}\Big]^2+\frac{\lambda}{4!}\,\hat\phi^4+\mathcal{O}\Big(\frac{\phi^2}{M_p^2}\Big),\quad \phi\ll M_p.$$

$\Rightarrow$ No dilaton $\ln\sigma$ left (eaten by ω_μ). Gravitational higgs mechanism. Also $m_\phi^2=(-\xi/\xi_0)\,M_p^2$.

$\Rightarrow$ Fixed point for ξ? The limit $\xi\to 0\Rightarrow S_v\times S_h\Rightarrow\beta_\xi\propto\xi$? not $(\xi+1/6)$! Quantum corrections?

- Weyl inflation: $V = V_0 \left\{ \left[1 - \xi_1 \sinh^2 \frac{\phi}{M_p\sqrt{6}}\right]^2 + \lambda_1\xi_0 \sinh^4 \frac{\phi}{M_p\sqrt{6}} \right\}, \quad V_0 = \frac{3}{2}\frac{M_p^4}{\xi_0}; \quad \phi > M_p$: OK!





$$\lambda_1\xi_0 \ll \xi_1^2 \ll 1: \quad \epsilon = \frac{\xi_1^2}{3} \sinh^2 \frac{2\phi}{M_p\sqrt{6}} + \mathcal{O}(\xi_1^3); \qquad \eta = -\frac{2\xi_1}{3} \cosh \frac{2\phi}{M_p\sqrt{6}} + \mathcal{O}(\xi_1^2)$$

$$n_s = 1 + 2\eta - 6\epsilon = 1 - \frac{4}{3}\xi_1 \cosh \frac{2\phi_*}{M_p\sqrt{6}} + \mathcal{O}(\xi_1^2); \quad \Rightarrow \quad r = 3(1-n_s)^2 + \mathcal{O}(\xi_1^2)$$

$\Rightarrow 0.002567 \leq r \leq 0.00303$ if $n_s = 0.9670 \pm 0.0037$; $n_s \approx 1 - 2/\overline{N}$, $r = 12/\overline{N}^2$, $\overline{N} \approx N + 4.1$; N efolds

r midly smaller than Starobinsky, where: $\overline{N} \to N$; $r_s \approx 0.0031$, $n_s \approx 0.968$ $(N = 60)$

[D.G. arxiv:1906.11572; G. Ross, C. Hill, P, Ferreira, J. Noller 1906.03415] see next talk by Graham Ross.

- Conclusions:

I. Flat space:

$\Rightarrow$ Quantum SI can protect a classical hierarchy of: higgs vev $\ll$ dilaton vev

- All scales from fields vev's: hierarchy stability not a DR artefact

II. Curved space:

- Weyl's original gauge symmetry/gravity, viable theory; no ghost dilaton; $\#$ dof conserved.

$\Rightarrow$ Weyl gravity $=$ Einstein-Proca for ω_μ $+$ cc, after Stueckelberg, dilaton eaten by Weyl "photon" $m_\omega \sim q M_p$.

$\Rightarrow$ Weyl inflation predicts: $0.00257 \leq r \leq 0.00303$ for $N = 60$, n_s= measured 68%CL (such r values will soon be tested, LiteBIRD, CMB-S4).

BACKUP SLIDES

- Dilatation current D_μ: $$D^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)}(x^\nu \partial_\nu \phi_j + d_\phi) - x^\mu \mathcal{L}, \qquad d_\phi = (d-2)/2 \quad \text{(scalars)},$$

\- in $d = 4 - 2\epsilon$, potential $\tilde{V}$: $$\partial_\mu D^\mu = (d_\phi + 1)(\partial_\mu \phi_j) \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)} + d_\phi \, \phi_j \frac{\partial \mathcal{L}}{\partial \phi_j} - d\mathcal{L}$$
$$= d\,\tilde{V} - \frac{d-2}{2}\phi_j \frac{\partial \tilde{V}}{\partial \phi_j}, \qquad \phi_j = \phi, \sigma; \quad \text{(onshell, canonical k.t.)}$$

- in SI theory: $\tilde{V}$ homogeneous in d dim's: $\tilde{V}(\rho\, \phi_j) = \rho^{2d/(d-2)}\, \tilde{V}(\phi_j) \Rightarrow \partial_\mu D^\mu = 0$, conserved.

- in "usual" reg: $\mu =$const, no σ: $\tilde{V} = \mu^{2\epsilon} V(\phi)$, and V is scale inv in $d = 4$:

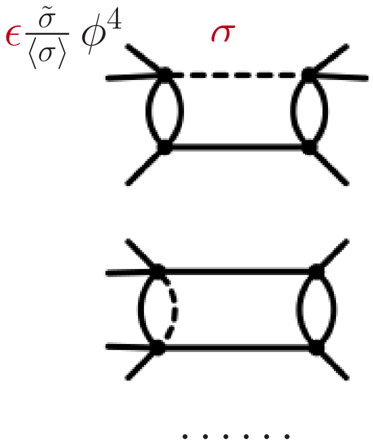
$$\partial_\mu D^\mu = 2\,\epsilon\; \mu^{2\epsilon}\, V \sim 2\,\epsilon\, \mu^{2\epsilon} \left[\lambda + \frac{\beta_\lambda}{\epsilon} + \cdots\right] \frac{\partial V}{\partial \lambda} \propto \beta_\lambda \frac{\partial V}{\partial \lambda}, \quad [=0 \text{ only if } \beta_\lambda = 0]. \quad \text{(scale anomaly)}$$

\- different field content! [Shaposhnikov et al, Tamarit 2013]

"For scale invariance, though, the situation is hopeless; any cutoff procedure necessarily involves a large mass and a large mass necessarily breaks scale invariance in a large way. This argument does not show that the occurence of anomalies is inevitable [.....]" (S. Coleman: Aspects of Symmetry, p.82, 1985).

- Three-loop SI potential: Counterterms!

[Monin 2015]



$$\delta L_3 = \frac{1}{2}\,\delta_\phi^{(3)}\,(\partial_\mu\phi)^2 - \mu^{2\epsilon}\Big\{\frac{1}{4!}\,\delta_\lambda^{(3)}\,\lambda\,\phi^4 + \frac{1}{6}\,\delta_{\lambda_6}^{(3)}\,\lambda_6\,\frac{\phi^6}{\sigma^2} + \frac{1}{8}\,\delta_{\lambda_8}^{(3)}\,\lambda_8\,\frac{\phi^8}{\sigma^4}\Big\}$$

$$\delta_{\lambda_6}^{(3)} = \frac{3}{2}\frac{\lambda^4}{\lambda_6\,\kappa^3\,\epsilon}, \qquad \delta_{\lambda_8}^{(3)} = \frac{275}{864}\frac{\lambda^4}{\lambda_8\,\kappa^3\,\epsilon.} \;\Rightarrow\; \gamma_\phi^{(3)},\ \beta_{\lambda_6},\ \beta_{\lambda_8} = ...$$

Integrate CS: Three-loop term: $U_3 = \Delta V + V^{(3)} + V^{(3,n)}$. $V^{(3)}$='old'$[\,\mu \to z\sigma\,]$; new: $V^{(3,n)}$ and ΔV

$$\Delta V = \frac{\lambda_6\,\phi^6}{\sigma^2} + \frac{\lambda_8\,\phi^8}{\sigma^4}, \qquad V^{(3)} = \frac{\lambda^4\,\phi^4}{\kappa^3}\Big\{\mathcal{Q} + \Big(\frac{97}{128} + \frac{9}{64}A_0 + \frac{\zeta[3]}{4}\Big)\,\overline{\ln}\frac{V_{\phi\phi}}{(z\sigma)^2} - \frac{31}{96}\overline{\ln}^2\frac{V_{\phi\phi}}{(z\sigma)^2} + \frac{9}{64}\overline{\ln}^3\frac{V_{\phi\phi}}{(z\sigma)^2}\Big\}$$

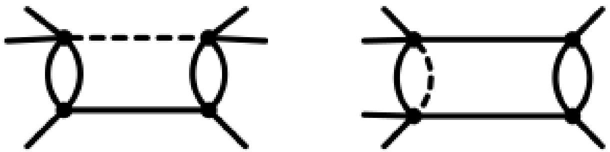
$$V^{(3,n)} = \frac{\lambda^3\,\phi^4}{2\,\kappa^3}\Big\{\Big(27\lambda - \frac{\lambda_6}{2}\Big)\frac{\phi^2}{8\,\sigma^2} + \Big(\frac{401\,\lambda}{72} - \lambda_8\Big)\frac{\phi^4}{16\,\sigma^4}\Big\}\,\overline{\ln}\frac{V_{\phi\phi}}{(z\sigma)^2}. \qquad V_{\phi\phi} \equiv \frac{\lambda}{2}\phi^2.$$

[D.G. arXiv:1712.06024]

⇒ SI effective operators always suppressed by σ ⇒. At large σ enhanced symmetry $S_v \times S_h$ restored ⇒

⇒ forbids c-terms $\lambda^2\,\phi^2\sigma^2 = \lambda^2\,\langle\sigma\rangle^2\,\phi^2 + \cdots$ No tunning of Higgs selfcoupling λ for large σ.

$\epsilon \frac{\tilde{\sigma}}{\langle\sigma\rangle}\, \phi^4$



• Three-loop SI potential

Counterterm: $\delta L_3 = \frac{1}{2}\,\delta_\phi^{(3)}\,(\partial_\mu\phi)^2 - \mu^{2\epsilon}\Big(\frac{1}{4!}\,\delta_\lambda^{(3)}\,\lambda\,\phi^4 + \frac{1}{6}\,\delta_{\lambda_6}^{(3)}\,\lambda_6\,\frac{\phi^6}{\sigma^2} + \frac{1}{8}\,\delta_{\lambda_8}^{(3)}\,\lambda_8\,\frac{\phi^8}{\sigma^4}\Big)$ [Monin 2015]

$$\delta_\phi^{(3)} = -\frac{\lambda^3}{4\kappa^3}\Big(\frac{1}{6\epsilon^2} - \frac{1}{12\epsilon}\Big), \qquad \delta_{\lambda_6}^{(3)} = \frac{3}{2}\frac{\lambda^4}{\lambda_6\,\kappa^3\,\epsilon}, \qquad \delta_{\lambda_8}^{(3)} = \frac{275}{864}\frac{\lambda^4}{\lambda_8\,\kappa^3\,\epsilon.}$$

So $\gamma_\phi^{(3)} = \lambda^3/(16\kappa^3)$. With $Z_X = 1 + \delta_X$ and $\lambda_6^B = \mu^{2\epsilon}(\sigma)\lambda_6 Z_{\lambda_6} Z_\phi^{-3} Z_\sigma$ and $(d/d\ln z)\,\lambda_6^B = 0$,

$$\beta_{\lambda_6} \;=\; \frac{\lambda^2\,\lambda_6}{2\kappa^2} + \frac{\lambda^3}{\kappa^3}\Big(9\lambda - \frac{3}{8}\lambda_6\Big), \qquad \text{(similar } \beta_{\lambda_8}).$$

Callan-Symanzik eq: $V^{(3)}$ ["usual" with $\mu\to\sigma$]$+V^{(3,n)}$ [new]: gives solution shown on previous page:

$$\frac{\partial V^{(3)}}{\partial\ln z} + \beta_\lambda^{(1)}\frac{\partial V^{(2)}}{\partial\lambda} + \beta_\lambda^{(2)}\frac{\partial V^{(1)}}{\partial\lambda} + \beta_\lambda^{(3)}\frac{\partial V}{\partial\lambda} + \gamma_\phi^{(2)}\frac{\partial V^{(1)}}{\partial\ln\phi} + \gamma_\phi^{(3)}\frac{\partial V}{\partial\ln\phi} = \mathcal{O}(\lambda_j^5).$$

$$\frac{\partial V^{(3,n)}}{\partial\ln z} + \beta_{\lambda_j}^{(1)}\frac{\partial V^{(2,n)}}{\partial\lambda_j} + \beta_{\lambda_j}^{(3,n)}\frac{\partial V}{\partial\lambda_j} = \mathcal{O}(\lambda_j^5), \qquad \lambda_j = \lambda, \lambda_6, \lambda_8.$$

- SM+dilaton: one-loop beta functions (similar to when $\mu =$ constant, same field content):

$$\beta_{\lambda_\phi} = \frac{1}{\kappa}\left[3\left(\frac{9}{4}g_2^4+\frac{3}{4}g_1^4+\frac{3}{2}g_1^2g_2^2-12h_t^4\right)-4\lambda_\phi\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)+4\lambda_\phi^2+3\lambda_m^2+96\lambda_m\lambda_6\right]$$

$$\beta_{\lambda_m} = \frac{2\lambda_m}{\kappa}\left[\lambda_\phi+2\lambda_m+\frac{1}{2}\lambda_\sigma-\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)\right]\propto\lambda_m, \quad \Rightarrow \lambda_m \text{ stays small (f.p.).}$$

- for new couplings

$$\beta_{\lambda_6} = \frac{3\lambda_6}{\kappa}\left[6\lambda_\phi-8\lambda_m+\lambda_\sigma-2\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)\right]$$

$$\beta_{\lambda_8} = \frac{2}{\kappa}\left[2\lambda_6\left(28\lambda_6+\lambda_m\right)-4\lambda_8\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)\right]$$

$$\beta_{\lambda_{10}} = 10\left[4\lambda_6^2-\lambda_{10}\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)\right]$$

$$\beta_{\lambda_{12}} = 2\left[3\lambda_6^2-6\lambda_{12}\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)\right]$$

$\Rightarrow$ if one sets $\lambda_{6,8,10,..}=0$ at tree level, then $\beta_{\lambda_{6,8,10,12}}=0$ at one-loop, but emerge at 2-loops.

- Weyl quadratic gravity with matter $\rightarrow$ Einstein gravity + massive ω_μ [DG arXiv:1812.08613, 1904.06596]

$$L_1 = \sqrt{g}\,\Big[\frac{\xi_0}{4!}\,\tilde R^2 - \frac{1}{4}\,F_{\mu\nu}^2\Big], \quad \xi_0 > 0, \qquad \text{original Weyl action (1918)}$$

$$= \sqrt{g}\,\Big[\frac{\xi_0}{4!}\big(-2\,\sigma^2\,\tilde R - \sigma^4\big) - \frac{1}{4}\,F_{\mu\nu}^2\Big] \qquad \text{eom: } \sigma^2 = -\tilde R; \quad \text{dilaton: } \ln\sigma \rightarrow \ln\sigma - \ln\Omega$$

Use $\tilde R = R - 3q\,D_\mu\omega^\mu - 3/2\,q^2\,\omega_\mu\omega^\mu$ then, in Riemann language:

$$L_1 = \sqrt{g}\,\Big\{ -\frac{\xi_0}{2}\,\Big[\frac{1}{6}\sigma^2\,R + (\partial_\mu\sigma)^2\Big] - \frac{\xi_0}{4!}\sigma^4 + \frac{q^2}{8}\,\xi_0\,\sigma^2\,\big(\omega_\mu - 1/q\,\partial_\mu\ln\sigma^2\big)^2 - \frac{1}{4}\,F_{\mu\nu}^2\Big\}.$$

- gauge transf (b) "gauge fixing!" $\Omega = \dfrac{\xi_0\,\sigma^2}{6M_p^2} \Rightarrow \hat g_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \hat\sigma^2 = \dfrac{6\,M_p}{\xi_0}, \quad \hat\omega_\mu = \omega_\mu - \dfrac{1}{q}\partial_\mu\ln\sigma^2$

$$L_1 = \sqrt{\hat g}\,\Big\{ -\frac{1}{2}\,M_p^2\,\hat R - \frac{3\,M_p^4}{2\xi_0} + \frac{3}{4}\,q^2\,M^2\hat\omega_\mu\,\hat\omega^\mu - \frac{1}{4}\,\hat F_{\mu\nu}^2\Big\}.$$

$\Rightarrow$ Einstein-Proca action for (massive) ω_μ which has "eaten" $\ln\sigma$ field! no ghost! # dof conserved!

cons. current $\partial^\alpha(F_{\alpha\mu}\sqrt{g}) + \dfrac{1}{2}\sqrt{g}\,\xi_0\,\phi_0\,\Big[\partial_\mu - \dfrac{q}{2}\,\omega_\mu\Big]\,\phi_0 = 0$