

Gravitational wave energy budget in strongly supercooled phase transitions

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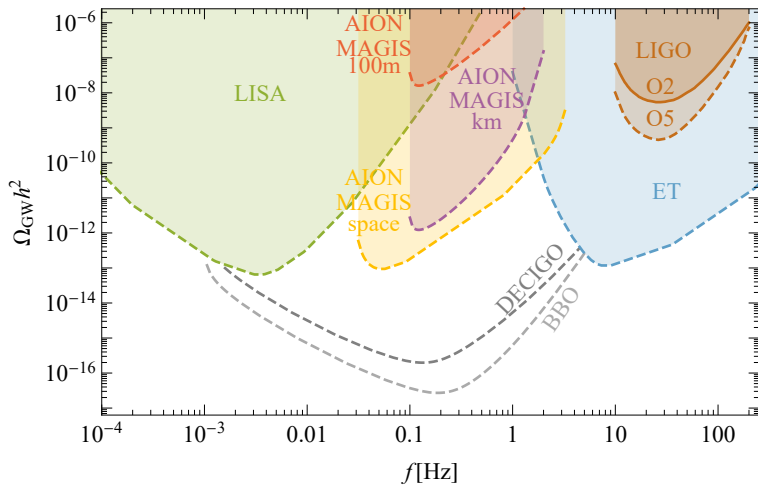


Based on:

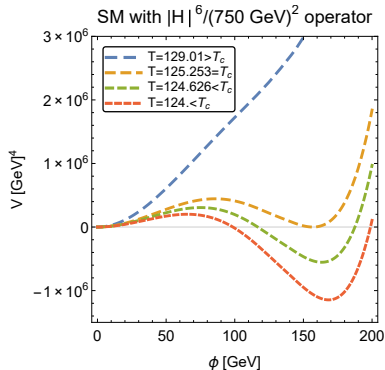
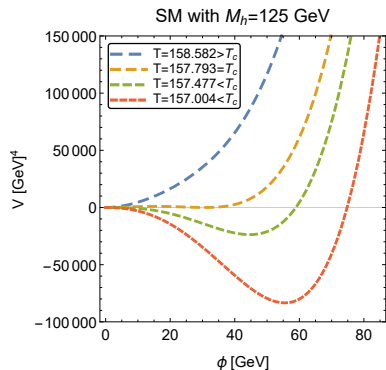
J. Ellis, ML, J. M. No, V Vaskonen arXiv:1903.09642

J. Ellis, ML, J. M. No arXiv:1809.08242

Experimental outlook



First order phase transition



If $M_h < 85 \text{ GeV}$ in SM we would have a **I order phase transition**
Kajantie, Laine, Rummukainen, Shaposhnikov 97'

phase transition dynamics

Bubble: static field configuration passing the barrier (excited through thermal fluctuations)

- decay rate

$$\Gamma(T) \approx T^4 \exp\left(-\frac{S_3(T)}{T}\right),$$

- $\mathcal{O}(3)$ symmetric action

$$S_3(T) = 4\pi \int dr r^2 \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + V(\phi, T) \right].$$

- EOM \rightarrow bubble profile

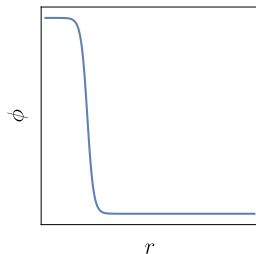
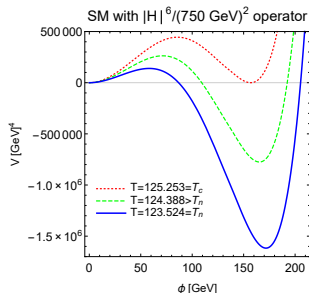
$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} - \frac{\partial V(\phi, T)}{\partial \phi} = 0,$$

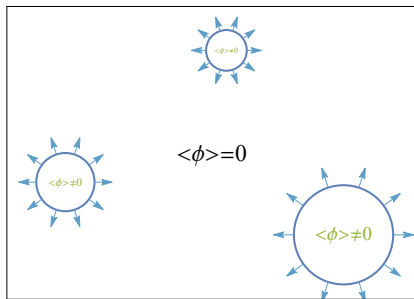
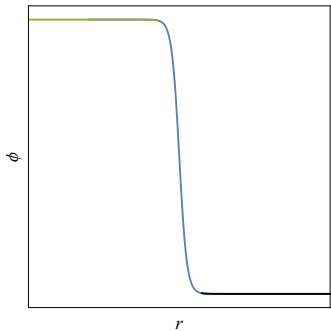
$$\phi(r \rightarrow \infty) = 0 \quad \text{and} \quad \dot{\phi}(r=0) = 0.$$

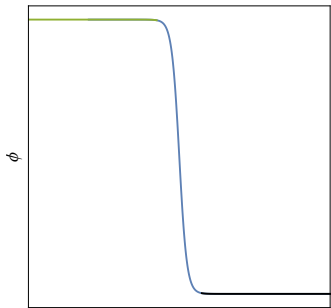
- nucleation temperature

$$N(T_n) = \int_{t_c}^{t_n} dt \frac{\Gamma(t)}{H(t)^3} = \int_{T_n}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H(T)^4} = 1$$

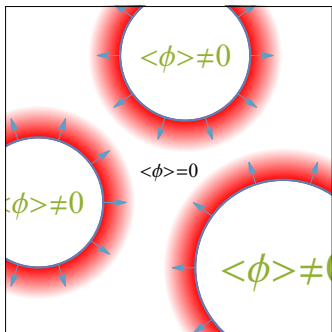
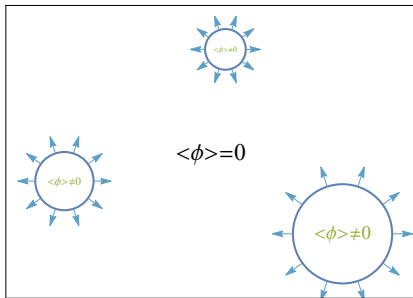
Linde '81 '83



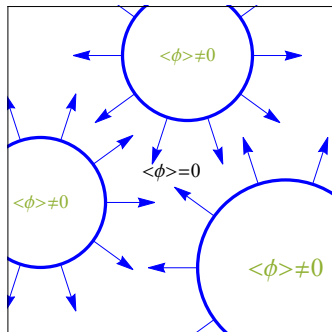




\Rightarrow



?



Gravitational waves from a PT

- Strength of the transition

$$\alpha \approx \left. \frac{\Delta V}{\rho R} \right|_{T=T_*}, \quad \Delta V = V_f - V_t$$

- Characteristic scale

$$HR_* = (8\pi)^{\frac{1}{3}} \left(\frac{\beta}{H} \right)^{-1} = H_* N_b^{-\frac{1}{3}} = H_* \left(\int dt' \left(\frac{a(t')}{a(t)} \right)^3 \Gamma(t') P(t') \right)^{-\frac{1}{3}}$$

- Signals are produced by three main mechanisms:

- collisions of bubble walls: $\Omega_{\text{col}} \propto \left(\kappa_{\text{col}} \frac{\alpha}{\alpha+1} \right)^2 (HR_*)^2$
Kamionkowski '93, Huber '08, Hindmarsh '18,
- sound waves: $\Omega_{\text{sw}} \propto \left(\kappa_{\text{sw}} \frac{\alpha}{\alpha+1} \right)^2 (HR_*) (H\tau_{\text{sw}})$
Hindmarsh '13 '15 '17 Ellis '18
- turbulence $\Omega_{\text{turb}} \propto \left(\kappa_{\text{sw}} \frac{\alpha}{\alpha+1} \right)^{\frac{2}{3}} (HR_*) (1 - H\tau_{\text{sw}})$
Caprini '09 Ellis '19
- Sound wave period lasts $H\tau_{\text{sw}} \equiv \min \left[1, \frac{HR_*}{U_f} \right]$
- The frequency of the signal changes as $f \propto \frac{T_*}{HR_*}$

- Energy of the bubble

$$\mathcal{E} = 4\pi\gamma\sigma R^2 - \frac{4\pi}{3}R^3 p, \quad \gamma = \frac{1}{\sqrt{1 - \dot{R}^2}}$$

- Vacuum pressure on the wall
Coleman '73

$$p_0 = \Delta V$$

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- Leading order plasma contribution
Bodeker '09 Caprini '09

$$p_1 = \Delta V - \Delta P_{\text{LO}} \approx \Delta V - \frac{\Delta m^2 T^2}{24},$$

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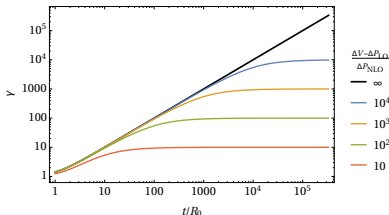
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- Leading order plasma contribution
Bodeker '09 Caprini '09

$$p_1 = \Delta V - \Delta P_{\text{LO}} \approx \Delta V - \frac{\Delta m^2 T^2}{24},$$

- Next-To-Leading order plasma contribution
Bodeker '17

$$p = \Delta V - \Delta P_{\text{LO}} - \gamma \Delta P_{\text{NLO}} \approx \Delta V - \frac{\Delta m^2 T^2}{24} - \gamma g^2 \Delta m_V T^3.$$



- γ factor at which the bubble stops accelerating and the value it would reach if we neglected ΔP_{NLO}

$$\gamma_{\text{eq}} \equiv \frac{\Delta V - \Delta P_{\text{LO}}}{\Delta P_{\text{NLO}}}, \quad \gamma_* \equiv \frac{2 R_*}{3 R_0},$$

- Finally the efficiency factors read

$$\kappa_{\text{col}} = \frac{E_{\text{wall}}}{E_V} = \begin{cases} \frac{\gamma_{\text{eq}}}{\gamma_*} \left[1 - \frac{\Delta P_{\text{LO}}}{\Delta V} \left(\frac{\gamma_{\text{eq}}}{\gamma_*} \right)^2 \right], & \gamma_* > \gamma_{\text{eq}} \\ 1 - \frac{\Delta P_{\text{LO}}}{\Delta V}, & \gamma_* \leq \gamma_{\text{eq}}, \end{cases}$$

$$\kappa_{\text{sw}} = \frac{\alpha_{\text{eff}}}{\alpha} \frac{\alpha_{\text{eff}}}{0.73 + 0.083\sqrt{\alpha_{\text{eff}}} + \alpha_{\text{eff}}}, \quad \text{with } \alpha_{\text{eff}} = \alpha(1 - \kappa_{\text{col}}).$$

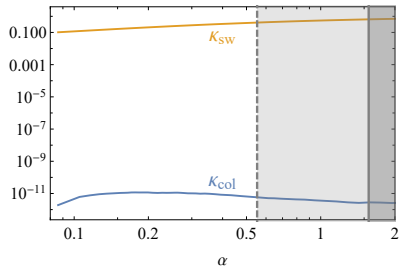
Particle physics examples

- Two models with very different potential cosmological evolution

$$H = \frac{1}{3M_{\text{pl}}^2} (\rho_R + \Delta V) = H_R + \frac{\Delta V}{3M_{\text{pl}}^2}$$

Standard Model + $|H|^6$ operator

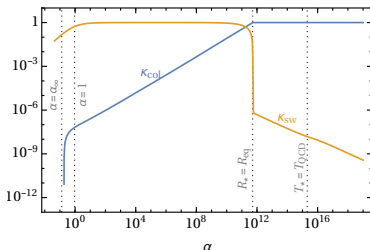
$$V(\phi) \simeq m^2 \phi^2 + \lambda \phi^4 + \frac{\phi^6}{\Lambda^2}$$

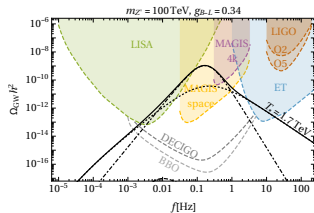
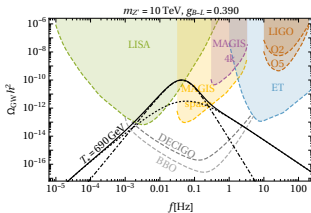
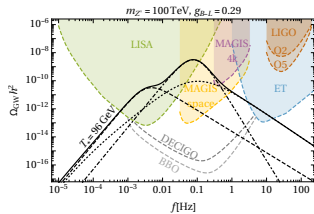
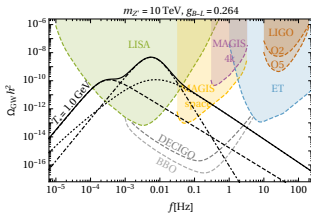
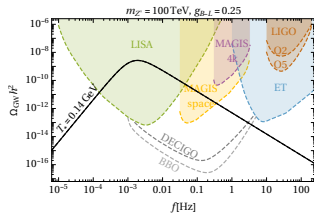
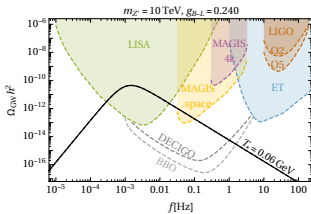


$U(1)_{B-L}$ extension of SM

$$V(\varphi) \simeq \frac{3g_{B-L}^4 \varphi^4}{4\pi^2} \left[\log\left(\frac{\varphi^2}{v_\varphi^2}\right) - \frac{1}{2} \right] + g_{B-L}^2 T^2 \varphi^2$$

$$m_{Z'} = 2g_{B-L} v_\varphi$$





Plasma related GW sources

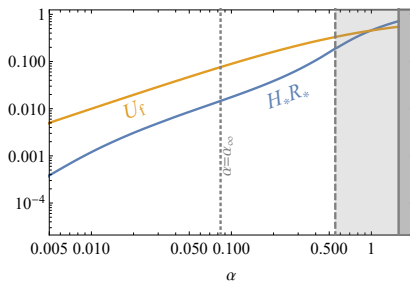
- Sound wave period last a fraction of the Hubble time

$$H\tau_{\text{sw}} \equiv \min \left[1, \frac{HR_*}{U_f} \right]$$

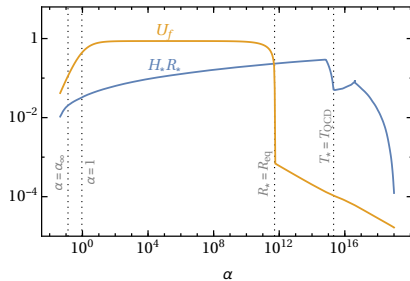
- Root-mean-square four-velocity of the plasma

$$U_f \approx \sqrt{\frac{3}{4} \frac{\kappa_{\text{sw}} \alpha}{1 + \alpha}} \xrightarrow{v_w \approx 1} \frac{\sqrt{3}\alpha}{2(1 + \alpha)\sqrt{0.73 + 0.083\sqrt{\alpha} + \alpha}}.$$

Standard Model + $|H|^6$ operator



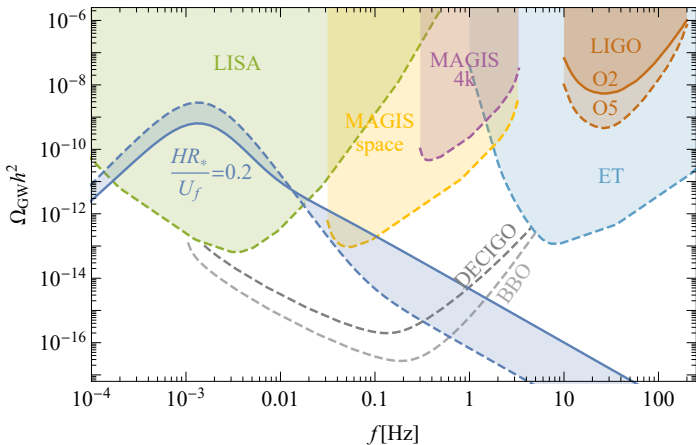
$U(1)_{B-L}$ extension of SM



- Sound wave spectrum reduction and earlier onset of turbulence

$$\Omega_{\text{sw}} \propto H\tau_{\text{sw}} = \frac{HR_*}{U_f}, \quad \Omega_{\text{turb}} \propto 1 - H\tau_{\text{sw}} = 1 - \frac{HR_*}{U_f}$$

$$T_* = 100 \text{ GeV}, \quad \alpha = 1, \quad HR_* = 10^{-1}$$

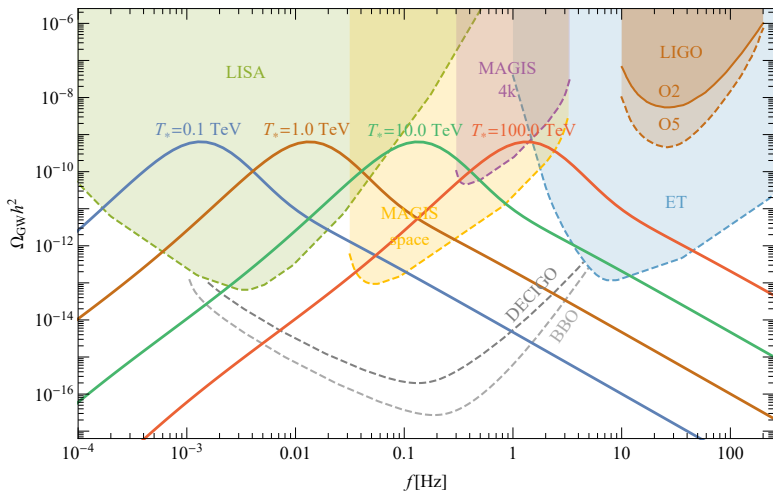


Conclusions

- Observable bubble collision GW signal requires significant supercooling. We derived the efficiency factor quantifying this requirement precisely.
- Sound wave period generically last less than a Hubble time. This leads to a much waker sound wave sourced GW signal and potentially a significant increase in the signal sourced by turbulence.

- The frequency of the signal changes as $f \propto \frac{T_*}{HR_*}$

$$\alpha=1, HR_*=10^{-1}$$



Power-law integrated sensitivity

$$\Omega_{\text{GW}}^{\text{noise}} = \frac{2\pi}{3} \frac{f^3 S_h}{H_0^2}, \quad \text{SNR} = \sqrt{\tau \int df \left(\frac{\Omega_{\text{GW}}^{\text{signal}}}{\Omega_{\text{GW}}^{\text{noise}}} \right)^2}$$

