Role of matter in modified gravity: A search for new interactions

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arXiv:1902.01391 [with Kazuya Koyama]
+ ongoing work [with Ryo Namba]

Beyond $\times$ *(GR, Cosmological Standard Model)*
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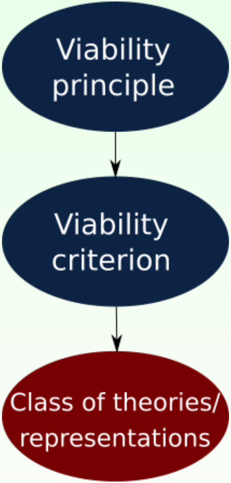
Building classes of theories



- Lack of axiomatic formulation for gravity theories. Conceptual basis? (*)
- Viability principles: Qualitative statements. (e.g. EPs, inertial frame invariance, ingredients, stability, etc.) not necessarily picking out a single theory.
- Viability criteria: A qualitative formulation of the principle that acts as a selection rule.
- Maximise the number of theories by defining some classes of theories, not directly derived from, but compatible with some viability principles. Class of theories provide a framework to evaluate experimental results.

(*) For a detailed discussion of shortcomings of formulating gravity theories, see [Sotiriou, Faraoni, Liberati '08]

Principle/Criterion mismatch
Restrictive criteria as a source for missing interactions

- Classical modified gravity built in vacuum, then Jordan frame is selected

$$S = \int \sqrt{-g}\,\mathcal{L}_{\text{gravity}} + \sqrt{-g}\,\mathcal{L}_{\text{matter}}\,.$$

- No conceptual issues. For a complete class of theory, the matter coupling should not be an issue...
- ...unless it is incomplete: a mismatch between principles and criteria

Principle: *Vacuum stability* $\Longrightarrow$ Criterion: ≤ 2 *derivatives in eom.*

- This stability criterion is too restrictive: high derivative eoms allowed if the kinetic matrix degenerate.
- Principle/criterion mismatch in Horndeski theory: not the most general scalar-tensor action with no vacuum ghost.

Degenerate extensions of Horndeski theory

- $\exists$ auxiliary field $\Leftarrow$ Beyond Horndeski, DHOST [Gleyzes et al'15; Langlois, Noui'16; Ben Achour et al'16 ...]
- Cataloguing degenerate terms $\Rightarrow$ careful survey of constraint algebra.

Representations / Choice of variables

Multiple geometries approach: another way of deriving missing interactions

- Modified gravity $\Rightarrow$ additional gravitational dof. Geometry not unique. Infinite # of representations equally valid.

Jordan frame: matter follows the geodesics of the metric

$$S = \int \sqrt{-g}\left[\frac{\phi\, R}{16\,\pi\, G} - \frac{\omega(\phi)}{2}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + \sqrt{-g}\mathcal{L}_{\text{matter}}\left(\{\psi_n\}, g_{\mu\nu}\right)$$

$$\downarrow$$

$$\tilde{g}_{\mu\nu} = \phi\, g_{\mu\nu}$$

$$\downarrow$$

Einstein frame: gravitational part same as GR+ canonical scalar

$$S = \int \sqrt{-\tilde{g}}\left[\frac{\tilde{R}}{16\,\pi\, G} - \frac{1}{2}\,\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} - \tilde{V}(\tilde{\phi})\right] + \frac{\sqrt{-\tilde{g}}}{[\phi(\tilde{\phi})]^2}\mathcal{L}_{\text{matter}}\left(\{\psi_n\}, \frac{\tilde{g}_{\mu\nu}}{\phi(\tilde{\phi})}\right)$$

- Classically, the two pictures are dynamically equivalent. Physics doesn't change (e.g. [Deruelle, Sasaki'11]), but interpretation does [Sotiriou et al'08].
- Bekenstein's nomenclature: Gravitational and physical geometries.
- Could have started with GR+scalar matter and derived BD theory.

Disformal relations

Generalising to Scalar-Tensor theories

- The most general relation in Scalar-Tensor theories with $\tilde{g} = \tilde{g}(g, \phi, \partial\phi)$

$$\tilde{g}_{\mu\nu} = C(\phi, \partial_\alpha\phi\,\partial^\alpha\phi)g_{\mu\nu} + D(\phi, \partial_\alpha\phi\,\partial^\alpha\phi)\partial_\mu\phi\,\partial_\nu\phi$$

- Rescales clocks and rulers according to the scalar field and its gradient.

C: isotropic $\qquad$ D: anisotropic

- Originally proposed to construct new gravitational theories.

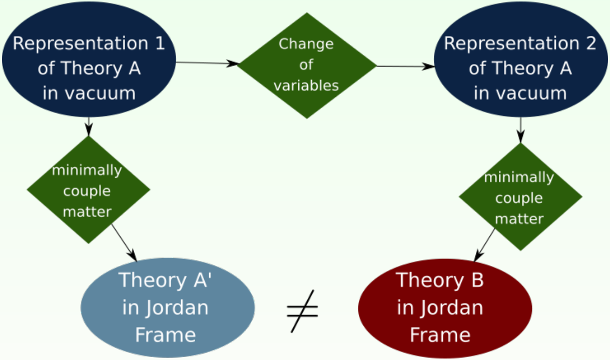
> insight into field theory. It appears likely that disformal transformations will help to supplement those insights. At the most immediate level, they provide a method for constructing novel gravitational theories based on pairs of disformally related geometries. As far as we are aware, such theories have been considered only once before [12], and the motivation there was to relate the standard in-

[Bekenstein'93]

c.f. talk by Davis

The multiple geometries approach
Constructing a new theory from an old one

If the theory class is incomplete, one may obtain a new theory starting from two representations of the same theory in vacuum.



Multiple geometries approach for S–T theories

- Start with Horndeski Lagrangian, then couple matter to a different metric

$$\sqrt{-g_H}\,\mathcal{L}_H + \sqrt{-g_J}\mathcal{L}_{\text{matter}}$$

- If $g_J = g_J(g_H, \phi, \partial\phi)$ eoms are at most 2nd order in derivatives.
- In the Jordan frame, using instead $g_H = g_H(g_J, \phi, \partial\phi)$, high derivatives emerge... [Bettoni, Liberati'13]
- ... but they are Ostrogradski-stable [Zumalacárregui, Garcia-Bellido'13]

$$\sqrt{-g_J}\;(\mathcal{L}_{\text{beyondH}} + \mathcal{L}_{\text{matter}})$$

The multiple geometries approach
A prescription for generic gravity theories

- Consider a gravity theory in vacuum, with extra fields $\{\Phi_n\}$ which contribute *(or should contribute)* to gravitational interactions.
- Write a disformal relation between two representations that depends on extra fields and their gradients:

$$\tilde{g}_{\mu\nu} = \tilde{g}_{\mu\nu}(g_{\mu\nu}, \{\Phi_n\}, \partial\{\Phi_n\})$$

- Stay agnostic about the “right choice” of metric variables, or matter coupling: introduce matter fields $\{\Psi_n\}$ coupled to another metric disformally related to the first one

$$\sqrt{-\tilde{g}}\,\mathcal{L}_{\text{gravity}}(\tilde{g}_{\mu\nu}, \{\Phi_n\}) + \sqrt{-g}\,\mathcal{L}_{\text{matter}}(g^{\mu\nu}, \{\Psi_n\})$$

- Going back to the Jordan frame will produce higher order terms, which will still be consistent extensions of the original theory, provided that one can invert the two-metric relation.
- Although one can still catalogue degenerate theories by studying the constraint algebra, multiple geometries approach provides a simple tool for generating new theories with guaranteed Ostrogradski-stability.

Application:
vector-tensor theory

(HC SVNT DRACONES)

Application: Vector-tensor theories
Why?

- Large scale ($\sim$ Mpc) magnetic fields in voids $> 10^{-15}$G. [Neronov, Vovk'10]
- Can these have primordial origin? *Primordial magnetogenesis*
- Problem: conformal symmetry of the Maxwell term in 4D. For FLRW metric $g_F^{\mu\nu} = a^2 \eta^{\mu\nu}$,

$$\sqrt{-g_F} F_{\mu\nu} F_{\alpha\beta} g_F^{\mu\alpha} g_F^{\nu\beta} = F_{\mu\nu} F_{\alpha\beta} \eta^{\mu\alpha} \eta^{\nu\beta}$$

$\Rightarrow$ Maxwell field oblivious to expansion.

- Generalised Maxwell à la Galileons? In the absence of gravity, a no-go result exists: *("equations of motion can only be linear in second derivatives of the vector field")* [Deffayet, AEG, Mukohyama, Wang'14]
- Multiple geometries approach provides a novel way of finding new $U(1)$–tensor interactions which can break the conformal symmetry, without resorting to new fields.

Beyond Einstein-Maxwell
Simple Conformal Transformation

- Starting with Einstein-Maxwell theory, we adopt the multiple geometries approach by coupling matter to a different geometry

$$S = \int d^4x \left\{ \sqrt{-\tilde{g}} \left[\frac{M_p^2}{2} R[\tilde{g}] - \frac{1}{4} F_{\mu\nu} F_{\alpha\beta} \tilde{g}^{\mu\alpha} \tilde{g}^{\nu\beta} \right] + \sqrt{-g}\, \mathcal{L}_{\text{matter}} \right\}$$

- $U(1)$ field with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.
- Simple transformation: Einstein frame metric is conformally related to the Jordan frame one

$$\tilde{g}_{\mu\nu} = C([F^2], [F^4]) g_{\mu\nu} \,,$$

where $[F^2] = F_{\mu\nu} F_{\alpha\beta} g^{\nu\alpha} g^{\mu\beta}$ and $[F^4] = F_{\mu\nu} F_{\alpha\beta} F_{\rho\sigma} F_{\eta\tau} g^{\nu\alpha} g^{\beta\rho} g^{\sigma\eta} g^{\tau\mu}$

- In the Jordan frame the action is

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2 C}{2} \left(R + \frac{3}{2} \nabla^\mu \log C \, \nabla_\mu \log C \right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{\text{matter}} \right]$$

- Has high derivative terms of the form $(\partial\partial A)^2$

[AEG, Namba, *in progress*]

Beyond Einstein-Maxwell

Equations of motion

- Vector field equations:

$$\nabla_\nu \mathscr{F}^{\mu\nu} = 0 \,, \qquad \mathscr{F}^{\mu\nu} \equiv F^{\mu\nu} + 2M_p^2 C \left(R - 3\,\frac{\nabla^2 C}{C} + \frac{3}{2}\,\frac{\nabla^\rho C\,\nabla_\rho C}{C^2} \right) \left(\frac{C_{\mathcal{F}_2}}{C} F^{\mu\nu} + 2\,\frac{C_{\mathcal{F}_4}}{C}\,(F^3)^{\mu\nu} \right)$$

- Equations contain fourth derivatives of A_μ.
- Trace of metric equations:

$$\left(R - 3\,\frac{\nabla^2 C}{C} + \frac{3}{2}\,\frac{\nabla^\rho C \nabla_\rho C}{C^2} \right) \left(\frac{C_{\mathcal{F}_2}}{C}\,[F^2] + 2\,\frac{C_{\mathcal{F}_4}}{C}\,[F^4] - \frac{1}{2} \right) = \frac{T}{2\,C\,M_p^2}$$

- Using this, the derivative order of vector eom becomes 2

$$\nabla_\nu \left[F^{\mu\nu} + \left(\frac{\frac{C_{\mathcal{F}_2}}{C} F^{\mu\nu} + 2\,\frac{C_{\mathcal{F}_4}}{C}\,(F^3)^{\mu\nu}}{\frac{C_{\mathcal{F}_2}}{C}\,[F^2] + 2\,\frac{C_{\mathcal{F}_4}}{C}\,[F^4] - \frac{1}{2}} \right) T \right] = 0$$

[AEG, Namba, *in progress*]

Application:
massive spin–2 field

Application: Massive spin–2 field theory

Massive gravity cosmology?

- dRGT potentials: Delicately tuned mass terms protected from problematic (ghosty) 6th mode at *all* scales. [de Rham, Gabadadze, Tolley'11]
- Cosmological no-go: all exact homogenous and isotropic solutions are unstable. [de Felice, AEG, Mukohyama '12]
- Approximately FLRW background is stable [d'Amico et al.'11]

c.f. talk by Mukohyama

- Do "beyond dRGT" interactions exist? If so, can they provide a healthy and interesting cosmological model?
- Uniqueness theorems do not leave much room: 6th mode reappears if:
  1. mass term is not the dRGT potential
  2. kinetic part is not the Einstein-Hilbert term [de Rham,Matas,Tolley'13]
  3. matter couples to 2 metrics simultaneously [Yamashita, De Felice, Tanaka'14]
- Loophole: Theory has cutoff above $\Lambda_3 = \left(M_p m^2\right)^{1/3}$. Interactions with characteristic scales above the cutoff would be fine.
  Example: composite matter coupling [de Rham, Heisenberg, Ribeiro'14]

Extending the disformal relation to scalar field space

$$S = \int \sqrt{-\tilde{g}}\, M_p^2 \left[R[\tilde{g}] + m^2 \mathcal{F}_{\text{dRGT}}(\sqrt{\tilde{g}^{-1} f}) \right] + \sqrt{-g}\, \mathcal{L}_{\text{matter}}$$

- $\tilde{g} \to$ dRGT frame, $g \to$ Jordan frame.
- We have 4 scalar fields ϕ^a with an internal Poincaré symmetry, since

$$f_{\mu\nu} = \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

- Poincaré symmetry $\supset$ Translation invariance. Only $\partial\phi$ dependence.
- The generalisation of disformal relation to dRGT (defined $\gamma \equiv g^{-1} f$)

$$\tilde{g}_{\mu\nu} = C([\sqrt{\gamma}^n]) g_{\mu\nu} + D([\sqrt{\gamma}^n])(g\sqrt{\gamma})_{\mu\nu} + E([\sqrt{\gamma}^n]) f_{\mu\nu} + F([\sqrt{\gamma}^n])(f\sqrt{\gamma})_{\mu\nu}$$

- Tune the unknown functions to remove all dangerous interactions below Λ_3. This requires a perturbative treatment.
- Removing interactions at scales $< \Lambda_3$ reveals no non trivial value of the derivatives of the four functions C, D, E, F up to quartic order! It is compatible with constant coefficients.

[AEG, Koyama '19]

Disformal relation for dRGT
A better suited formulation

- For constant coefficients, we get the unique transformation:

$$\tilde{g} = \frac{g}{\alpha^2}\left(\mathbb{1} + \beta\sqrt{\gamma}\right)^2$$

- This is precisely the *composite matter coupling* obtained by requiring that 1-loop matter corrections to the graviton potential do not detune the dRGT potential.
- Requiring the ghost to be pushed beyond the EFT, we found the same result and showed that it is (possibly) unique.

[AEG, Koyama '19]

Summary and Discussion

- Multiple geometries approach can generate consistent extensions of known gravity theories with extra fields.

Other applications

- Provided that the transformation is
  1. invertible;
  2. does not introduce a new dof,

  the modified matter interaction is guaranteed to be Ostrogradski stable.
- Multiscalar-tensor theories (for dark energy, inflation ...)
- A natural framework for interacting DE models with ~~WEP~~ .