Possible resolution of a spacetime singularity with field transformations

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in collaboration with

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G.Domenech, C.Wetterich (Heidelberg) + MS [on BH, to appear soon]
Beyond singularity
Take away message

We resolve a curvature singularity by means of "field transformations" = by changing a "frame" we live in (not by means of quantum gravity)

Einstein frame = $R + \phi$

conformal frame = $\Omega R + \phi$

Singularity
We resolve a curvature singularity by means of "field transformations" = by changing a "frame" we live in (not by means of quantum gravity)

Einstein frame = $R + \phi$

conformal frame = $\Omega R + \phi$

No singularity in the new frame !! ⇒ beyond singularity
contents today

✓ brief introduction  (singularity, field tr., etc)

✓ case 1 ~ bianchi-1 universe    [hide a singularity]

✓ case 2 ~ FLRW universe     [resolve a singularity]

✓ summary
contents today

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✓ summary
Spacetime singularity

✓ Spacetime (curvature) singularities may exist
  - after gravitational collapse (center of BH)
  - the beginning/end of the universe

✓ divergence of curvature tensors \((R, R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}, W...W..., \text{etc})\)
  \(\Rightarrow\) divergence of energy scale (action)
  \(\Rightarrow\) breakdown of classical gravity
  \(\Rightarrow\) need quantum gravity (in general)

✓ Is there any singularity which can be resolved even in the framework of classical gravity? \(\Rightarrow\text{field transformations}\)??
metric transformations

1. **conformal** (Weyl) transformation

\[ \tilde{g}_{\mu \nu} = \Omega^2 g_{\mu \nu} \]

\[ \Omega : \text{a function of scalar/vector fields} \]

→ cause a change of volume factor \( \sqrt{-g} \to \Omega^4 \sqrt{-g} \)

2. **disformal** transformation \( g_{\mu \nu} \)

\[ \tilde{g}_{\mu \nu} = g_{\mu \nu} + \Gamma \partial_\mu \phi \partial_\nu \phi \]

\[ = g_{\mu \nu} + \Gamma A_\mu A_\nu \]

\( \Gamma : \text{a function of scalar/vector fields} \)

→ ?? in general [change of time in cosmology]

+AN [1505.00174, 1507.05390]

(in the absence of matter, these are **null transformations**)

see also talks by Emir & Alex
Rough idea

✓ Let us focus on a cosmic initial singularity

Einstein frame

\[ ds^2 = a^2(\tau) \eta_{\mu\nu} dx^\mu dx^\nu \]

Singularity at \( a = 0 \)

\[ ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \]

Nothing at \( a = 0 \)

✓ The geometry can be OK.. but matter will be singular

⇔ **hide** the metric singularity **behind the singular matter**

If the action is divergent, can we **trust the EOM & solution** ?

⇒ **eliminate** the singularity **imposing the regularity of action**
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✓ summary
hairly inflation

✓ inflation with anisotropic hair

\[ \mathcal{L} = \frac{1}{2} R - \frac{1}{2} (\nabla \phi)^2 - V_0 e^{\lambda \phi} - \frac{1}{4} e^{2\rho \phi} F_{\mu\nu}^2 \]

✓ homogeneous background

- Bianchi (= anisotropic) universe

\[ ds^2 = -dt^2 + t^{2\alpha} \left[ t^{-4\beta} dx^2 + t^{2\beta} (dy^2 + dz^2) \right] \]

\[ R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \propto 1/t^4 \]

\[ W_{\alpha\beta\gamma\delta} W^{\alpha\beta\gamma\delta} \propto 1/t^4 \]

- non-trivial BG vector field:

\[ A_\mu = (0, A_x(t), 0, 0) \]

see also

AN+ [1411.5489]
(power-law) solution

✓ solution:

\[
\phi = -(2/\lambda) \log t
\]

(singular @ t=0)

\[
A_x \propto t^{-1+3\alpha}
\]

(singular @ t=0)

\[
\alpha = \frac{4}{3\lambda(\lambda + 2\rho)} + \frac{\rho}{\lambda} + \frac{1}{6}
\]

isotropic

\[
\beta = -\frac{4}{3\lambda(\lambda + 2\rho)} + \frac{1}{3}
\]

anisotropic

✓ Note that

\[
A_\mu dx^\mu = A_x dx
\]

\[
dx = \left(1/A_x\right) A_\mu dx^\mu
\]
finding a tr. to flat

✓ remember \( A_\mu dx^\mu = A_x dx \)

\[
ds^2 = -dt^2 + t^{2\alpha} \left[ t^{-4\beta} dx^2 + t^{2\beta} (dy^2 + dz^2) \right]
\]

\[
= -dt^2 + t^{2(\alpha+\beta)} (dx^2 + dy^2 + dz^2) + t^{2\alpha} (t^{-4\beta} - t^{2\beta}) dx^2
\]

\[
= t^{2(\alpha+\beta)} (-d\tau^2 + dx^2 + dy^2 + dz^2) + t^{2\alpha} (t^{-4\beta} - t^{2\beta}) dx^2
\]

\[
= t^{2\alpha+2\beta} \left[ \eta_{\mu\nu} + \frac{t^{-4\beta} - 1}{A_x^2(t)} A_\mu A_\nu \right] dx^\mu dx^\nu
\]

\[
ds^2 = \Omega^2 \left( g_{\mu\nu} + \Gamma A_\mu A_\nu \right) dx^\mu dx^\nu
\]

\[
Y = A_\mu A^\mu \propto t^{-2+4(\alpha+\beta)}
\]
Short summary so far

✓ There is no curvature singularity in the new frame (Minkowski)

✓ Matter fields are divergent in general @ t=0 (φ~Log[t])

✓ Weyl$^2$ is not invariant under disformal tr. ↔ conformal tr.

✓ Singular nature of the metric can be “hidden” in the singular matter fields (so far)

Einstein frame  →  conformal frame
- curvature **singularity**   →   - flat spacetime
- **singular matter**
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✓ summary
simplifying the model

✓ Let us simplify the model by **isotropizing** the universe:

\[
\mathcal{L} = \frac{1}{2} R - \frac{1}{2} (\nabla \phi)^2 - V_0 e^{\lambda \phi} - \frac{1}{4} e^{2\rho \phi} F^2_{\mu \nu} \]

\[\beta = A_x = 0\]

✓ simplified isotropic solution:

\[
\phi = -(2/\lambda) \log t \quad \alpha = 2/\lambda^2 \quad V_0 = \alpha(-1 + 3\alpha)
\]

(singular @ t=0)

\[a(t) = t^\alpha\]

✓ curvature invariants:

\[
R \propto 1/t^2 \quad R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \propto 1/t^4
\]

Curvature singularity

@ t=0
regularity of action

✓ The value of action will be invariant under any field tr. 
  ⇔ singular action cannot be regularised by field tr.

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} (\nabla \phi)^2 - V_0 e^{\lambda \phi} \right] \]

\[ = \int d\tau d^3x t^{2(2\alpha-1)} \alpha (-1 + 3\alpha) \]

\[ = \int d\tau d^3x 1/4 \]

choose \( \alpha = 1/2 \) !!

✓ The value of total action is regular (constant)
  though \( \phi \) & curvature are singular @ \( t=0 \) (\( \phi \sim \log[t] \), \( R \sim 1/t^2 \))
  ⇒ action can be rewritten in terms of regular variables ?
rewriting action

✓ Regular variable:

\[ \phi = -2 \log(\tau/2) \quad \Leftrightarrow \quad \Omega^2 = e^{-\phi} = (\tau/2)^2 \]

\[ a(t) = t^\alpha = t^{1/2} = \tau/2 = \Omega \quad \Leftrightarrow \quad g \rightarrow \Omega^2 \; g \]

✓ New (regular) action in terms of regular variables:

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{4} e^{2\phi} \right] \quad (\alpha=1/2 \iff \lambda=2) \]

\[ = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \Omega^2 R - 3\Omega \Box \Omega - 2(\nabla \Omega)^2 - \frac{1}{4} \right] \quad (\phi = -\log \Omega^2) \]

[beyond GR + \Omega]
solution in the new frame

✓ EOM in the new frame:
\[
\begin{align*}
\frac{1}{2} \Omega^2 G_{\mu\nu} &= \Omega \nabla_\mu \nabla_\nu \Omega - \frac{1}{2} \left[ \Omega \Box \Omega + (\nabla \Omega)^2 + \frac{1}{4} \right] g_{\mu\nu} \\
\Box \Omega - \frac{1}{2} \Omega R &= 0
\end{align*}
\]

✓ trivial regular solution:
\[
g_{\mu\nu} = \eta_{\mu\nu} \quad & \quad \Omega = c_1 + c_2 \tau \rightarrow \tau/2 \quad \text{manifestly regular}
\]

✓ value of action:
\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \Omega^2 R - 3\Omega \Box \Omega - 2(\nabla \Omega)^2 - \frac{1}{4} \right] \rightarrow \int d^4x \frac{1}{4}
\]

⇒ we have successfully resolved the singularity
Summary & discussion

✓ There is a class of singularities which can be hidden by field transformations

✓ There are a class of singularities which can be resolved by field transformations

✓ The regularity of the action is a necessary condition to resolve a singularity

✓ Complete classification of singularities will be necessary

✓ How realistic our approach is ?? (include fluctuations, etc)
Dziękuję Ci
Thank you