Cosmology of models with spontaneous scalarization: instability and a cure

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Beyond General Relativity, Beyond Cosmological Standard Model
Testable gravity modifications: neutron stars and black holes

Benchmarks for testing General Relativity:

- LIGO/Virgo
- LISA
- EHT
- GRAVITY
- ...
Vanilla scalar-tensor theory:

\[
S_E = \int d^4x \frac{\sqrt{-g}}{2\kappa} \left( -R + 2\partial_\mu \varphi \partial^\mu \varphi \right) + S_m \left[ A^2(\varphi)g_{\mu\nu}, \psi_m \right]
\]

\[\kappa = 8\pi G\]

\[A(\varphi) = e^{\frac{1}{2}\beta \varphi^2}\]

\[\beta\] is a constant, which feeds into deviations from GR

\[\varphi\] is dimensionless
Original model of scalarization by Damour-Esposito-Farèse

Essence of scalarization:

\[ \Box \varphi + \frac{\kappa}{2} \alpha(\varphi) T^m = 0 \]

\[ \alpha(\varphi) \equiv \frac{d \ln A(\varphi)}{d\varphi} = \beta \varphi \]

\[ T^m_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}} \]

- \( \varphi = 0 \) is the GR solution;
- For \( \beta < 0 \) and \( T^m > 0 \), tachyonic mass \( \Rightarrow \) other solutions?

This is indeed the case for \( \beta \lesssim -4 \) for neutron stars
Scalar-Gauss-Bonnet model of scalarization

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + f(\phi) \hat{G} \right] \]

\[ \hat{G} = R_{\mu\nu\sigma\alpha} R^{\mu\nu\sigma\alpha} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \]

\[ \Box \phi + f'(\phi) \hat{G} = 0 \]

f(phi)-GB model

\[ \Box \phi + \frac{\kappa}{2} \alpha(\phi) T^m = 0 \]

Vanilla DEF model

When \( f'(\phi_0) = 0 \) and \( f''(\phi_0) > 0 \) then black holes and neutron stars are spontaneously scalarized
Instability of the GR solution

Trivial scalar $\phi_{\text{GR}} = 0$ and background Schwarzschild metric

Perturbations: $\left(\Box + f''(\phi_0)\hat{G}\right)\delta\phi = 0$

Decomposition in spherical harmonics: $\delta\phi = \frac{u(r)}{r}e^{-i\omega t}Y_{lm}(\theta, \varphi)$

Schrödinger-like equation: $\frac{d^2u}{dr_*^2} + \omega^2 u = V_{\text{eff}}(r)u$

$V_{\text{eff}}(r) = \left(1 - \frac{rg}{r}\right)\left(\frac{rg}{r^3} - \frac{12r^2}{r^6}f''(\phi_0)\right)$
Instability of the GR solution

\[ f(\phi) = \frac{1}{8}\lambda^2 \phi^2 + ... \]

Scalarization happens for \( \lambda \gtrsim r_g \), i.e. \( \lambda \sim \frac{M_\odot}{M_P^2} \)

Similarly in DEF model:

\[ \Box \delta \varphi + \frac{\kappa}{2} \beta T^m \delta \varphi = 0 \]

For \( \beta \lesssim -4 \) the GR solution for neutron stars, \( \varphi_{GR} = 0 \), is unstable
There is a tachyonic instability in these models for GR solutions.

Is this instability present in cosmology?

Then there is a problem:
- *Unstable solutions*
- *Non-GR cosmology*
instability in cosmology

**DEF model:**

- **very unstable:** inflation
- **“stable”:** radiation
- **unstable:** matter
- **slightly unstable:** dark energy

**Scalar-GB model:**

- **very unstable:** inflation
- **stable:** radiation
- **stable:** matter
- **slightly unstable:** dark energy
Shapiro time-delay measurement: $\gamma_{\text{PPN}} = 1 \pm (2.1 \pm 2.3) \times 10^{-5}$

Can we “naturally” have a small value for the cosmological scalar field?
instability in DEF model

Matter domination:
\[ \varphi_0 \lesssim 10^{-3} \quad \Rightarrow \quad \varphi_{\text{eq}} \lesssim 10^{-10} \]

Radiation domination:
\[ \frac{\varphi_{\text{eq}}}{\varphi_i} \simeq 10 \quad \Rightarrow \quad \varphi_i \lesssim 10^{-11} \]

Inflation: Fine-tune \( \varphi = 0 \) in the beginning of inflation.

Perturbations:
\[ \langle (\delta \varphi)^2 \rangle_{k \in \{k_{\min}, k_{\max}\}} = \frac{2^{2\nu} \Gamma^2(\nu)}{2(2\nu - 3)\pi^2} \cdot \frac{H^2}{M_{\text{Pl}}^2} \cdot \left| \frac{\eta_*}{\eta} \right|^{2\nu - 3} \gg 1 \]
idea: Give large normal mass to the scalar responsible for scalarization during inflation and kill the mass after inflation ends

\[ \sim \varphi^2 \chi^2_{infl} \]

\[ V(\varphi, \chi) = g^2 \varphi^2 \chi^2 \]

We assume \( m_{\text{eff}} = g^2 \chi^2 \gg H^2 \)

For \( \chi \approx M_{Pl} \) and \( H \approx 10^{13} \) GeV, \( g^2 > 10^{-12} \)
Curing DEF model

The background:

\[ \ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi = 0 \]

\[ m^2 = g^2\chi^2 + 6\beta H^2 \]

Starting from \( \varphi \approx 1 \) by the end of inflation the field \( \varphi \) is relaxed to

\[ \varphi \lesssim 10^{-39} \]

Perturbations:

\[ \sqrt{\langle \delta\varphi^2 \rangle_{\text{unstable}}(\eta_f)} \approx 10^{-46} \]
Vanilla scalar-tensor theory:

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S_E = \int d^4 x \frac{\sqrt{-g}}{2\kappa} \left( -R + 2\partial_\mu \varphi \partial^\mu \varphi \right) + S_m \left[ A^2(\varphi) g_{\mu\nu}, \psi_m \right]
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\[
A(\varphi) = e^{\frac{1}{2} \beta \varphi^2}
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Scalar-Gauss-Bonnet theory

\[
S = \int d^4 x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + f(\phi) \hat{G} \right]
\]

\[
f(\phi) \hat{G} = \frac{1}{8} \lambda^2 \phi^2 \hat{G} + \ldots
\]

\(\lambda \sim \frac{M_\odot}{M_P^2} \sim 10^{19} \text{GeV}^{-1}\)
No Cure for scalar-Gauss-Bonnet

\[
\frac{t_{\text{inst}}}{t_{\text{inf}}} \sim \frac{1}{N\lambda H_{\text{inf}}} \sim 10^{-34}
\]

for inflation with the scale \( H_{\text{inf}} \sim 10^{13}\text{GeV} \) and \( N \sim 10^2 \) e-foldings

\[
\frac{t_{\text{inst}}}{t_0} \sim \frac{H_0}{m_{\text{eff}}} \sim \frac{1}{\lambda H_0} \sim 10^{23}
\]

Destabilisation occurs very quickly $$\Rightarrow$$ no standard cosmological inflation
Trying to cure scalar-Gauss-Bonnet scalarization model

The same idea with coupling to inflaton?

\[ \sim g^2 \chi^2 \phi^2 \]

However:

\[ g^2 \gtrsim \frac{6\lambda^2 H^4}{\chi^2} \sim 10^{53} \]

for \( \chi \sim M_P, \lambda H \sim 10^{32} \), and \( H \sim 10^{13} \) GeV

Extreme strong coupling!
Trying to cure scalar-Gauss-Bonnet scalarization model

quartic self-interaction: \[ \sim g\phi^4 \]

the scalar field is in the minimum of the effective potential during inflation

However: \[ \phi_{\text{min}} \sim 10^{64} M_P \]

Also extra potential energy: \[ \sim 10^{180} M_P^4 \]
Vanilla DEF and modern scalar-Gauss-Bonnet models of scalarization gives interesting modifications of gravity, leading to different from GR neutron stars or/and black hole solutions.

Both models do contain dangerous instabilities during inflation.

It is possible to `naturally’ cure the original DEF model.

It seems however that the scalar-Gauss-Bonnet model cannot be cured.