Dark Inflation

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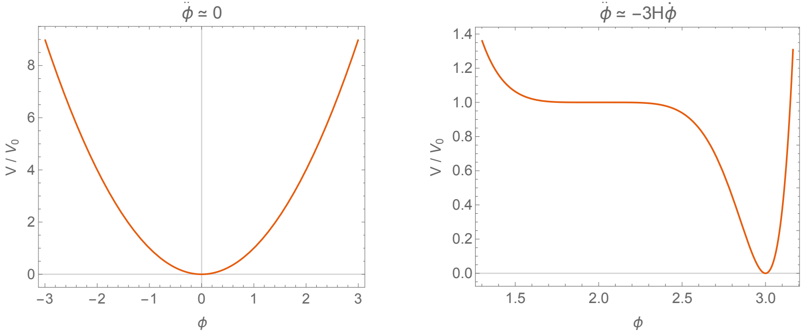
Beyond 2019
(with Vahid Kamali and Mohammad Reza Setare)

Slow-roll and beyond

The EOM of the homogeneous scalar field in the FRW background

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0 \tag{1}$$



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Constant-roll inflation: Taking $\beta \in (0, 1)$. Can be solved analytically!

$$\dot{H} = H_\phi \dot{\phi} = -\frac{1}{2}\dot{\phi}^2 \;\Rightarrow\; \dot{\phi} = -2H_\phi \tag{3}$$

We put it into (2) and we get $H(\phi)$ and in consequence $V(\phi)$

Constant roll

EOM of the inflaton now takes the form of

$$H_{\phi\phi} = \frac{3\beta}{2M_p^2} H\,, \tag{4}$$

which gives

$$H = C_1 \exp\left(\pm\sqrt{\frac{3\beta}{2}}\phi\right) + C_2 \exp\left(\pm\sqrt{\frac{3\beta}{2}}\phi\right)\,. \tag{5}$$

This gives the following potential

$$V = 3(1-\beta)C_1^2 e^{\sqrt{6\beta}\phi} + 6(\beta+1)C_1 C_2 + 3(1-\beta)2^2 e^{-\sqrt{6\beta}\phi} \tag{6}$$

For $\beta = 0$ one finds $V = 3(C_1 + C_2)^2 = const$. This is this part of the potential, where one finds the strongest deviation from the slow-roll

Constant roll using e-folds

The same results may be obtained in a different way. From (2) one finds

$$\dot{\phi} = \dot{\phi}_0 a^{-3\beta} = \dot{\phi}_0 e^{-3\beta N}\,, \tag{7}$$

where N is the number of e-folds. Then, from

$$\dot{H} = \frac{1}{2}\frac{d}{dN}(H^2) = -\frac{1}{2}\dot{\phi}_0^2 e^{-6\beta N}\,. \tag{8}$$

Simple differential equation, which gives

$$3H^2 = \frac{\dot{\phi}_0^2}{2\beta} e^{-6\beta N} + V_0\,, \tag{9}$$

$$V = V_0 + \frac{1-\beta}{2\beta}\dot{\phi}_0^2 e^{-6\beta N}\,. \tag{10}$$

This shows that β cannot be bigger than 1. Otherwise the field would have go uphill

Evolution of primordial inhomogeneities

Slow-roll inflation: we have 2 parts of curvature perturbations, but only one of them can survive the horizon crossing

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Thus, for $\beta > 1/2$ one obtains the growth of the B_k part. May be used to generate primordial black holes, but requires lots of fine-tuning of the potential

Warm inflation

One expects the inflaton to be coupled to other fields, which introduces a dissipation term in the EOMs of inflaton and radiation (relativistic particles)

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_\phi = \ddot{\phi} + 3H(1 + Q)\dot{\phi} + V_\phi = 0\,, \qquad (13)$$

$$\dot{\rho}_r + 4H\rho_r = \dot{\phi}^2\Gamma = 3HQ\dot{\phi}^2\,, \qquad (14)$$

where $Q = \Gamma/H$. In regular (cold) inflation $Q \ll 1$ and the dissipation term becomes important after inflation, during reheating.

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For $\Gamma \gg H$ (which is $Q \gg 1$)one finds in the slow-roll approximation

$$3H(1 + Q)\dot{\phi} + V_\phi \simeq 0\,, \qquad \rho_r \simeq \frac{3}{4}Q\dot{\phi}^2 \quad (15)$$

Hot inflationary Universe!

Radiation in warm constant-roll

We want to go with warm inflation beyond the slow-roll approximation. Assuming $\ddot{\phi} = -3\beta H\dot{\phi}$ and $Q = const$ one finds

$$\rho_r = \rho_{r0} a^{-4} + \frac{3Q\dot{\phi}_0^2}{2(2-3\beta)} a^{-6\beta} \,. \tag{16}$$

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For $\beta > 0$ the radiation energy density is exponentially suppressed and therefore $T \to 0$. Constant-roll warm inflation is actually is actually cold! The difference is, that now you can massively increase the cosmic friction term by taking $Q \gg 1$.

Potential in warm constant-roll

We re-do the calculations just like in the cold inflation, which gives

$$V = V_0 + \frac{\dot{\phi}_0^2}{2\beta}(1 + Q - \beta)a^{-6\beta} \, . \tag{17}$$

One can obtain analytical solutions for $\phi(N)$, $V(\phi)$ etc. Allows much bigger β! The energy density of the potential is decreasing for $\beta < 1 + Q$.

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ϵ can now decrease even faster! In the extreme case of $\beta = 1 + Q$

$$\epsilon \propto \frac{\dot{\phi}^2}{V_0} \propto a^{-6(1+Q)}\,, \tag{18}$$

The super-horizon perturbations may grow faster as well

$$\zeta_k \simeq A_k + B_k a^{3(2\beta-1)} = A_k + B_k a^{3(1+2Q)}\,. \tag{19}$$

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- Constant-roll inflation $\rightarrow$ generalization of the slow-roll paradigm
- Growing super-horizon modes, mechanism for primordial BH production
- Constant-roll warm inflation - surprisingly cold!
- Big cosmic friction term $\Rightarrow$ stronger suppression of ϵ $\Rightarrow$ faster growth of primordial inhomogeneities
- May decrease the fine-tunning of inflationary potentials designed to produce PBH