Stability of de Sitter spacetime against the backreaction of the infrared modes of scalar fields

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Based on:
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Non Perturbative Renormalization Group

Flow in the infrared limit

Conclusion
Quantum field theory in de Sitter spacetime

We do a semi-classical treatment with

- a classical background metric
- quantum fields as a content

It is maximally symmetric

We will consider the Expanding Poincaré patch → FLRW with constant Hubble rate $H$

- $\text{d}s^2 = -\text{d}t^2 + a^2(t)\text{d}\vec{X}^2$, $a(t) = e^{Ht}$
- Conformal time, $\text{d}\eta = \frac{\text{d}t}{a(t)}$
  
  
  $\text{d}s^2 = a^2(\eta)\left(-\text{d}\eta^2 + \text{d}\vec{X}^2\right)$. 
Gravitational effects in de Sitter: particle creation

Similar effects when one puts a quantum field with a constant background field:

- Schwinger effect: pair creation in the presence of an electric field $\vec{E}$
- Unruh-Hawking radiation: pair creation in the presence of a black hole

In both cases: the creation of pairs has potentially nontrivial backreaction on the background source.
Free scalar field

The mode function of light scalar fields $m \ll H$ in Bunch-Davies vacuum is $\phi_k(\eta) \sim H_{3/2} \left( \frac{k}{a(\eta)} \right)$.

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$|H_{3/2}(z)|^2$ vs $z$

amplified fluctuating modes
For scalar field in dS,
- Large gravitational effects in the infrared (superhorizon scales)
- Infrared modes are amplified
- Interactions cannot be treated perturbatively

A. A. Starobinsky, J. Yokoyama ’94 ; N. C. Tsamis, R. P. Woodard ’05 ; C. P. Burgess et al. ’10

It is interesting to study the **backreaction** of these infrared modes fluctuations to test whether de Sitter space is stable under their amplification and interactions.

E. Mottola ’85 ; I. Antoniadis et al. ’86 ; R. H. Brandenberger et al. ’96 ; Unruh ’98 ; A. M. Polyakov ’10, ’12
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Formulation of NPRG is done in term of the effective action

\[ e^{-i \mathcal{W}_\kappa[j, g]} = \int \mathcal{D} \phi e^{iS[\phi, g] + i \Delta S_\kappa[\phi, g] + i \int j \phi} \]

\( g_{\mu\nu} \) background metric, \( S \) action of an \( O(N) \) scalar theory.

\[ \Gamma_\kappa[\phi, g] + \Delta S_\kappa[\phi, g] = -\mathcal{W}_\kappa[j, g] + j \cdot \phi, \quad \phi = \langle \hat{\phi} \rangle \]

\[ R_\kappa(p) \]

\( \kappa^2 \)

\[ \Gamma_{\kappa \to \infty} = S \]

\[ \Gamma_{\kappa \to 0} = \Gamma \]
We want to solve the flow of $\Gamma_\kappa$: it obeys the Wetterich equation, which is IR and UV finite

$$\dot{\Gamma}_\kappa = \frac{1}{2} \text{tr} \dot{R}_\kappa (\Gamma^{(2)}_\kappa + R_\kappa)^{-1}.$$ 

C. Wetterich ’93

The physical values for $g$ and $\varphi$ are simultaneously determined at each scale $\kappa$ through

$$\frac{\delta \Gamma_\kappa}{\delta \varphi} = 0, \quad \frac{\delta \Gamma_\kappa}{\delta g_{\mu \nu}} = 0 \quad \text{or} \quad G^\kappa_{\mu \nu} = \langle T^\kappa_{\mu \nu} \rangle$$

We take constant values of $\varphi$ and de Sitter spacetime: it gives the flow of the Hubble constant
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The theory flows towards a zero dimensional theory
J. Serreau ’14 ; M. Guilleux, J.Serreau ’15

\[ e^{H^{-4} \Omega} \mathcal{W}_\kappa(j,h) = \int d^N \phi e^{-H^{-4} \Omega \left( U_{\text{in}}(\phi,h) + \frac{\kappa^2}{2} \phi^2 - j \cdot \phi \right)} \]

- It coincides with the equilibrium probability distribution in the stochastic formalism
  A. A. Starobinsky, J. Yokoyama ’94
- It is the **effective theory for the scalar field averaged over a Hubble patch** at constant values of the field

With \( U_{\text{in}}(\phi,h) = \alpha - \frac{B}{2} H^2 + \frac{\lambda}{8} \phi^4 \), \( \alpha \propto \Lambda \), \( \beta H^2 \propto R \)

\[ H^2_k = \frac{4 \alpha}{\beta} + \frac{2 \kappa^2}{\beta} \langle \phi^2 \rangle + \frac{\lambda}{2 \beta} \langle \phi^4 \rangle \]
Result for a massless self-interacting field

- Enhanced superhorizon modes draw energy from the gravitational field
- The dynamical generation of a mass screens this effect
- Finite renormalization of $H^2$: $H^2_\infty \approx H^2_{cl} + \frac{2H^4_{cl}}{\beta \Omega}$, $H^2_0 \approx H^2_{cl} + \frac{H^4_{cl}}{\beta \Omega}$

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Perturbation theory

Expansion parameter is $\frac{\lambda H^4}{\kappa^4}$: perturbation theory breaks down when $\kappa$ decreases.

\[ H_{\kappa}^2 \]

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• The gravitational mass generation screens the renormalization of the Hubble parameter
• Non minimal coupling between the scalar fields and gravitational field has a non trivial effect on the flow
• Goldstone modes do not contribute
• A full non perturbative treatment is needed as the perturbative approach breaks down

Perspectives :

• Generalize to FLRW metric
• apply the same treatment directly to the effective stochastic theory, using QFT formulation of Langevin equation