Primordial-Black-Holes-as-CDM Scenario and Gravitational Waves

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Based on
cosmic spacetime diagram

Inflation
\( H_{\text{inf}} \sim 10^{-5}M_{\text{Pl}} \)

\[ a/k_0 \]

\[ a/10 \]

\[ a_{\text{ini}} \]

\[ a_{\text{end}} \]

\[ a(1\text{GeV}) \]

\[ a_{\text{eq}} \]

\[ a_0 \]

\[ N = \log a \]

\[ -120 \left( = \ln \frac{H_{\text{inf}}}{H_0} \right) \]

\[ \sim -60 \]

\[ \sim -30 \]

\[ \sim -10 \]

\[ 0 \]
There are some constraints on small scales, but quite weak.
A peak in the primordial curvature perturbation, which leaves horizon at $a^*$, and frozen after inflation.

$$k_* = Ha_*$$

### PBH formation

$$N = \log a$$

Rad.dom. $1/H_r \sim a^2$

$1/H_0$
The peak re-enters horizon during radiation era. If the amplitude $> O(0.1)$, PBH will form.
fraction $\beta$ that turns into PBHs

for Gaussian probability distribution

- When $\sigma_M << \delta_c$, $\beta$ can be approximated by exponential:

$$\beta \approx \sqrt{\frac{2}{\pi}} \frac{\sigma_M}{\delta_c} \exp \left(- \frac{\delta_c^2}{2\sigma_M^2} \right)$$
Non-Gaussianity can increase ($f_{NL}>0$) or decrease ($f_{NL}<0$) the PBH abundances, substantially if $\sigma(M_H) \ll 1$.

Young & Byrnes, 1307.4995
Formation of PBHs
Curvature perturbation to PBH

➢ gradient expansion/separate universe approach

\[ 6H^2(t,x) + R^{(3)}(t,x) = 16\pi G \rho(t,x) + \cdots \]

Hamiltonian constraint (Friedmann eq.)

\[ R^{(3)} \approx - \frac{4}{a^2} \nabla^2 \mathcal{R}_c \approx \frac{8\pi G}{3} \delta \rho_c \]

\[ \frac{\delta \rho_c}{\rho} \approx \mathcal{R}_c \quad \text{at} \quad \frac{k^2}{a^2} = H^2 \]

Young, Byrnes & MS '14

➢ If \( R^{(3)} \sim H^2 \quad (\Leftrightarrow \delta \rho_c / \rho \sim 1) \), it collapses to form BH

\[ M_{\text{PBH}} \sim \rho H^{-3} \sim 10^5 M_\odot \left( \frac{t}{1\text{s}} \right) \sim 20 M_\odot \left( \frac{k}{1\text{pc}^{-1}} \right)^{-2} \]

➢ Spins of PBHs are expected to be very small

formation of a closed universe

\[ R^{(3)} \approx 0 \]

\[ H^{-1} = a/k \]
Scalaron $\phi$ becomes massive at the end of the 1st stage.

Field $\chi$ plays the role of inflaton at the 2nd stage.
sharp peak

can produce nearly monochromatic PBH mass spectrum
Constraints on PBH mass spectrum

$M_{\text{PBH}} \ [M_\odot]$
disappears due to finite source size effect
Katz et al. 1807.11495

Wave Effect makes it impossible to constrain PBH on small scales.

Niikura et al. 1701.02151v3
PBHs as CMD

\[ f(M) \equiv \frac{\Omega_{PBH}}{\Omega_{DM}} \propto \exp \left[ -\frac{O(0.1)}{P(k)} \right] \]

a sharp peak in \( P(k) \)

\[ f(M) \]

\[ M_{PBH} [M_\odot] \]

monochromatic PBH mass fcn
GWs from Large Scalar Curvature Perturbation
GWs can capture PBHs!

- Background GWs at LISA band
- PBHs = CDM with $M_{\text{PBH}} \sim 10^{21}$g generates GWs with $f \sim 10^{-3}$ Hz
- LIGO-Virgo : 10 - 1000 Hz
Induced GWs

- The equation of motion for the tensor perturbation with source

\[ h''_k + 2\mathcal{H}h'_k + k^2h_k = \mathcal{S}(k, \eta) \sim \int d^3l \ l_i l_j \Phi_1(\eta)\Phi_{k-1}(\eta) \]

- where the source term is (Ananda et al. gr-qc/0612013)

\[
\mathcal{S}(k, \eta) = 36 \int \frac{d^3l}{(2\pi)^{3/2}} \frac{l^2}{\sqrt{2}} \sin^2 \theta \begin{pmatrix} \cos 2\varphi \\ \sin 2\varphi \end{pmatrix} \Phi_1 \Phi_{k-1} \\
\times \begin{bmatrix} j_0(ux) j_0(vx) \hspace{1cm} - 2 \frac{j_1(ux) j_0(vx)}{ux} \hspace{1cm} - 2 \frac{j_0(ux) j_1(vx)}{vx} \hspace{1cm} + 3 \frac{j_1(ux) j_1(vx)}{uvx^2} \end{bmatrix}.
\]

- This equation can be solved by the Green function method.
The quantity we want to calculate is

\[ \Omega_{GW}(k) \equiv \frac{1}{12} \left( \frac{k}{Ha} \right)^2 \frac{k^3}{\pi^2} \langle h_k(\eta)h_k(\eta) \rangle. \]

This gives an estimate: \( \Omega_{GW} \sim \langle hh \rangle \sim \langle SS \rangle \sim \langle \Phi\Phi\Phi\Phi \rangle \sim \mathcal{P}_\Phi^2 \)

It seems too naive to believe that \( \Phi \) stays Gaussian when it becomes very large on small scales.

As a first step, consider local non-Gaussianity

(Komatsu & Spergel astro-ph/0005036)

\[ \mathcal{R}(x) = \mathcal{R}_g(x) + F_{NL} \left[ \mathcal{R}_g^2(x) - \langle \mathcal{R}_g^2(x) \rangle \right]. \]
• The 2pt of $\Phi$ is

$$\langle \Phi_k \Phi_p \rangle = (2\pi)^3 \delta^{(3)}(k + p) \frac{4}{9} \left( P_\mathcal{R}(k) + 2F_{\text{NL}}^2 \int d^3 l \ P_\mathcal{R}(|k - l|)P_\mathcal{R}(l) \right).$$

• And for the power spectrum of the primordial curvature perturbation, we assume a narrow peak at around $k^\ast$.

$$P_\mathcal{R}(k) = \frac{\mathcal{A}_\mathcal{R}}{(2\pi)^{3/2}2\sigma k^\ast} \exp \left( -\frac{(k - k^\ast)^2}{2\sigma^2} \right).$$

• Narrow means $\sigma << k^\ast$. This is for simplicity.
• Up: $F_{NL} > 0$, and we fix the PBH abundance to be 1.

• Down: $F_{NL} < 0$, and we fix the peak amplitude to be $A_R = 10^{-2}$

• Gray curve: LISA

• Frequency: PBH window $< -$ LISA band

• Coincidence, but fortunate for our universe.
Slope: \( \lesssim k^3 \)

\[ \Omega_{GW} h^2 \]

\[ k^3 \]

\[ f / \text{Hz} \]
\( \Omega_{GW} h^2 \)

\( 10^{-7} \)

\( 10^{-10} \)

\( 10^{-13} \)

\( f / \text{Hz} \)

\( 0.0001 \)

\( 0.001 \)

\( 0.01 \)

Slope: \( \lesssim k^3 \)

Slope: \( k^3 \)

Single Peak

Multiple Peaks

\( F_{NL} = 0 \)

\( F_{NL} = 10 \)

\( F_{NL} = 20 \)

\( F_{NL} = 50 \)
Slope: $\lesssim k^3$

$F_{NL}=0$

$F_{NL}=10$

$F_{NL}=20$

$F_{NL}=50$

Single Peak

Cutoff: $2k_*$

Multiple Peaks

Cutoff: $4k_*$
PBH abundance at the formation: $\beta(M)$
$f_{GW} = 3 \times 10^{-2}$ Hz

$F_{NL} < 0$
Too much GWs

PBH as all Dark Matter

\( f_{GW} = 3 \times 10^{-3} \text{ Hz} \)

\( f_{GW} = 3 \times 10^{-2} \text{ Hz} \)

\( A_R = 10^{-2} \)

\( A_R = 10^{-3} \)

\( A_R = 10^{-4} \)
\( f_{GW} = 3 \times 10^{-2} \, \text{Hz} \)

\( f_{GW} = 3 \times 10^{-3} \, \text{Hz} \)

\( A_R = 10^{-2} \)

\( A_R = 10^{-3} \)

\( A_R = 10^{-4} \)
\[ f_{GW} = 3 \times 10^{-3} \text{ Hz} \]  
\[ f_{GW} = 3 \times 10^{-2} \text{ Hz} \]  
\[ A_R = 10^{-2} \]  
\[ A_R = 10^{-3} \]  
\[ A_R = 10^{-4} \]  

PBH as all Dark Matter
$f_{GW} = 3 \times 10^{-3}$ Hz

$A_R = 10^{-2}$

$A_R = 10^{-3}$

$A_R = 10^{-4}$

PBH as all Dark Matter

LISA bound
If PBH serves as all DM, the induced GWs must be detectable by LISA, independent of $A_R$ or $F_{NL}$.
Summary

• GWs induced by non-Gaussian scalar perturbations:
  \( k^3 \) - slope, multiple peaks, cutoff

• If PBHs = CDM, induced GWs must be detectable by LISA, indep of \( f_{NL} \), no matter how small \( \mathcal{A}_R \) is.

• Conversely if LISA doesn’t detect the induced GWs, it constrains the PBH abundances on mass range \( 10^{19} \text{g} \) to \( 10^{22} \text{g} \) where no other experiment can explore.