ASTROPHYSICS GROUP, DEPARTMENT OF PHYSICS AND ASTRONOMY

Is Vacuum Decay During Inflation Fatal?

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Beyond General Relativity, Beyond Cosmological Standard Model

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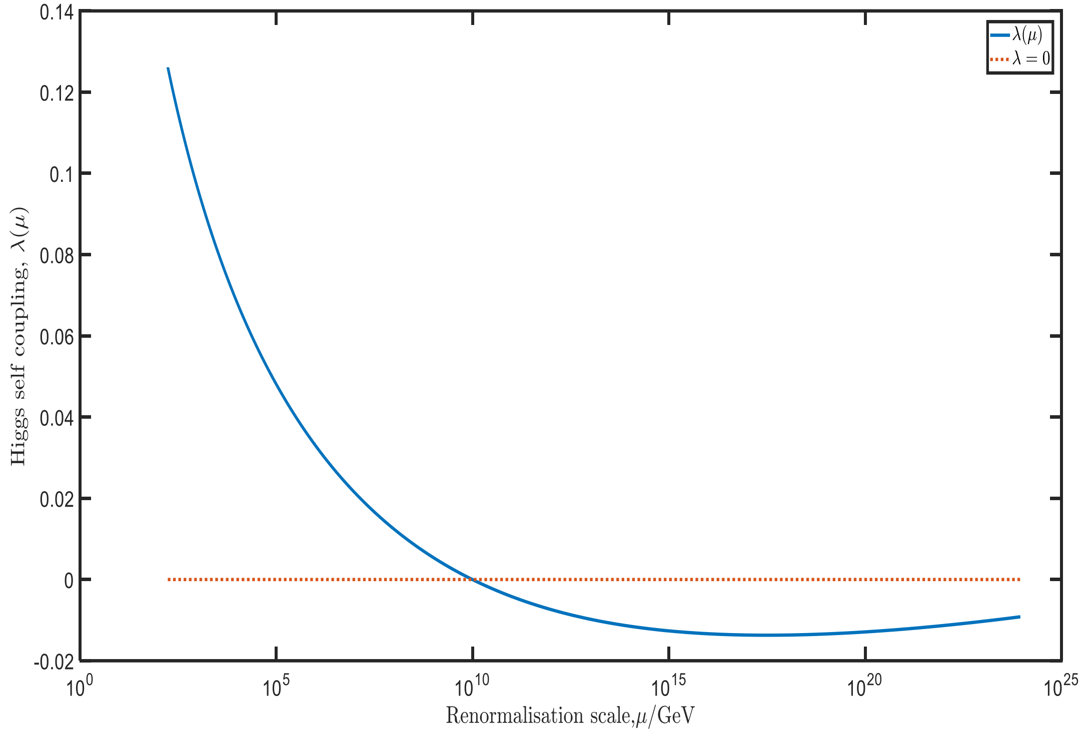
Based on material in https://spiral.imperial.ac.uk/handle/10044/1/66255
See also: https://doi.org/10.3389/fspas.2018.00040 (arXiv:1809.06923)

Introduction and Overview

- Current best estimates - $m_H = 125.10 \pm 0.14\ \text{GeV}, m_T = 173.1 \pm 0.9\ \text{GeV}$[1]
- Vacuum is metastable with these values, but long-lived.
- Flat space – bubbles expand.
- Negative energy density true vacuum – collapse to singularity.
- But what about curved space?
- Cosmological implications - see talk by Arttu Rajantie



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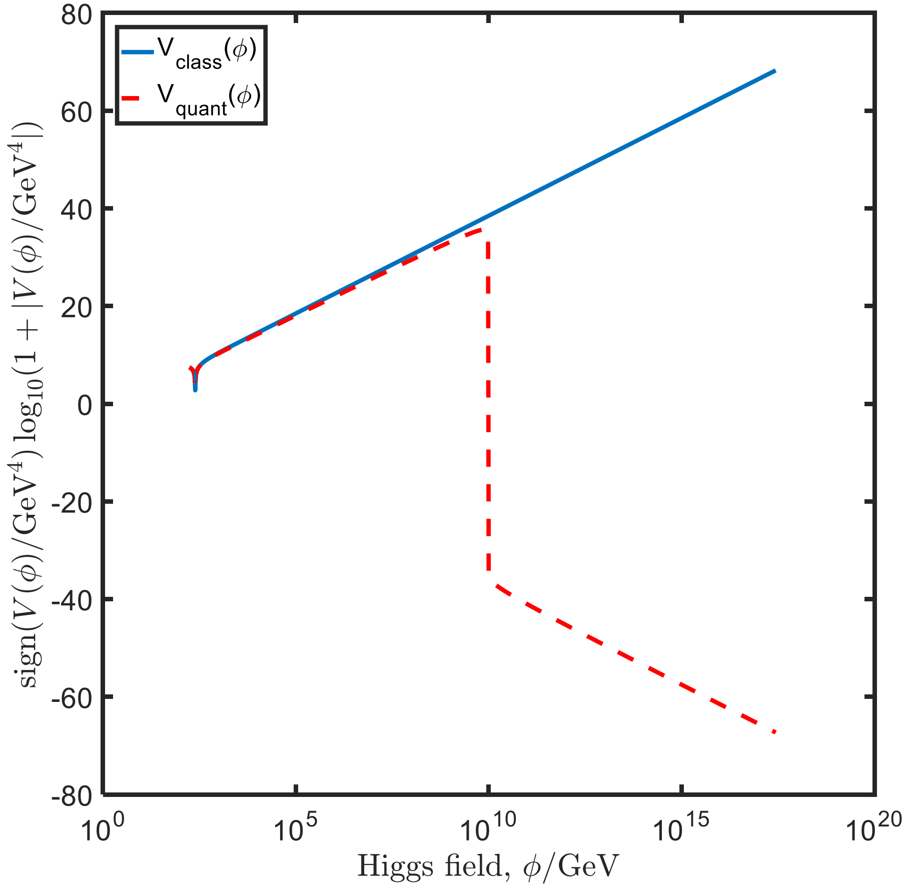

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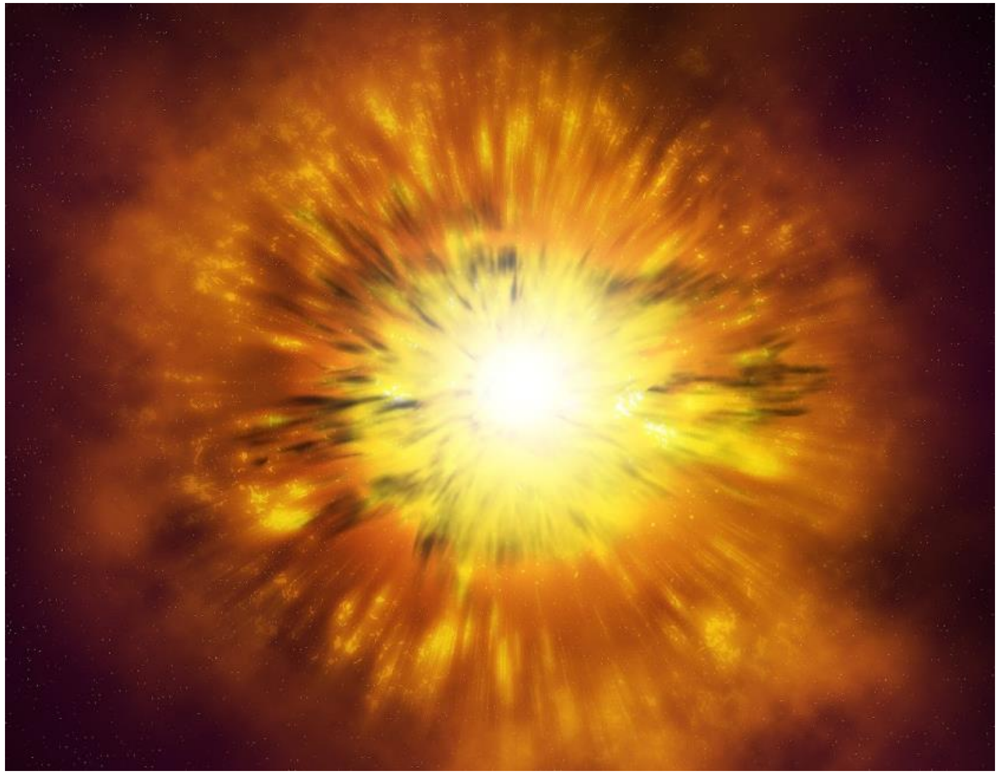
Introduction and Overview

- Situation less clear during inflation. Bubbles expand, but so does space.
- Some say anti-de-Sitter interior means bubble is inflated away.
- As we will see – not quite that simple.

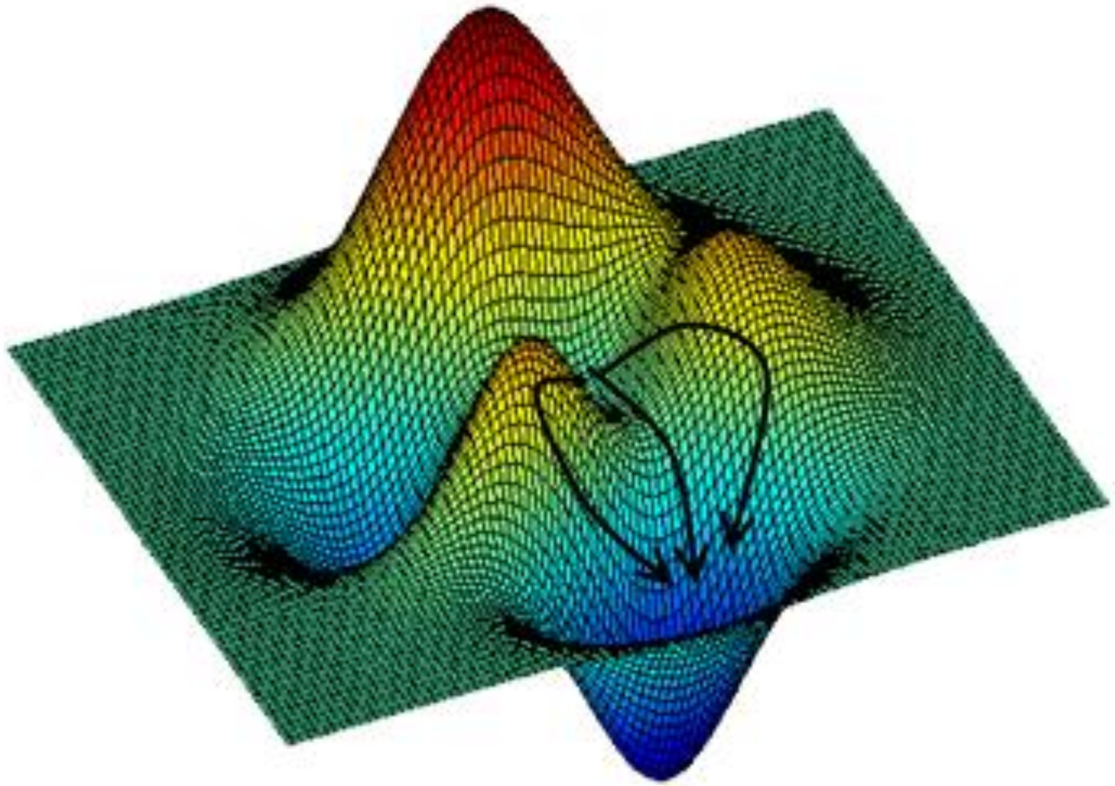


Introduction and Overview

- Review bubble expansion in flat space.
- Include gravity – formation of singularity.
- De Sitter Bubbles
- Conclusions

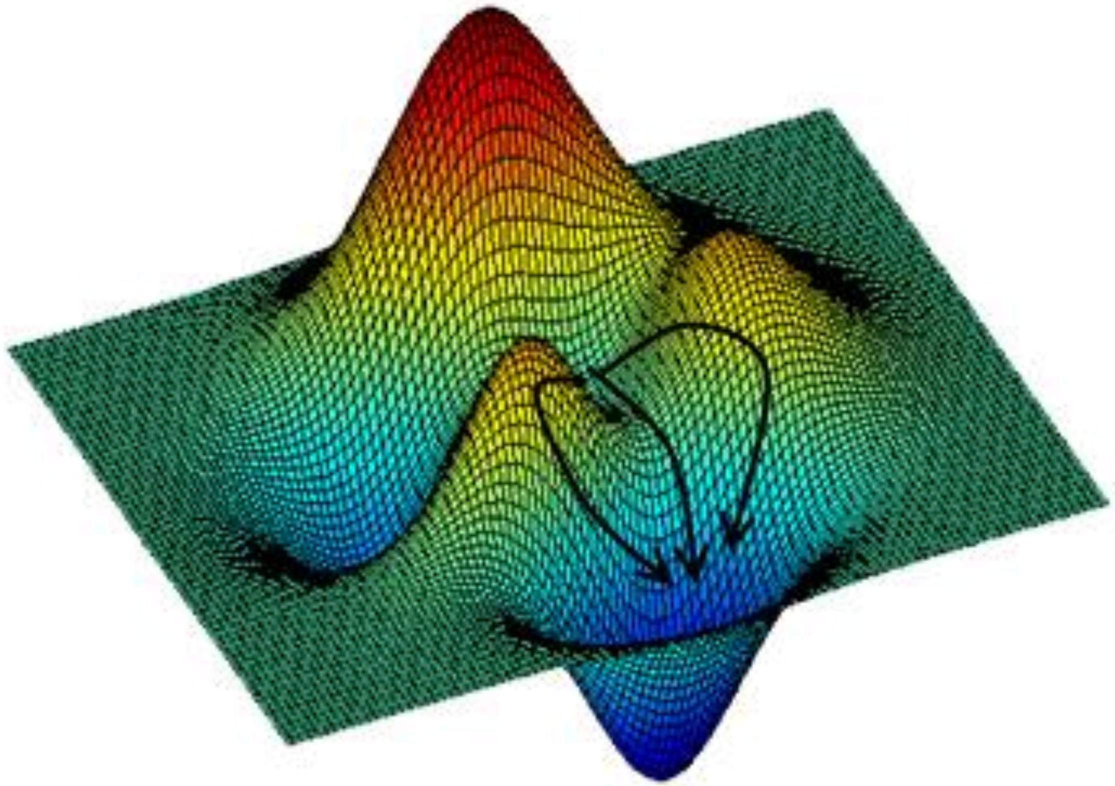
Review bubble expansion in flat space

- Compute shape and decay rate – quantum tunnelling with ∞ degrees of freedom.
- 1 d.o.f. -> WKB approx., $\Gamma \propto \exp\left(-2\int_{x_1}^{x_2} \mathrm{d}x\sqrt{2(V(x)-E)}\right)$
- ∞ d.o.f. – sum over many paths. Minimise *Euclidean* action:
- $S_E[\phi] = \int\, d^4x \left[\frac{1}{2}(\nabla\phi)^2 + V(\phi)\right]$



Review bubble expansion in flat space

- Quantum Tunnelling in QFT -> PDE problem
- $O(4)$ symmetry[2]: PDE -> ODE
- Solve $\ddot{\phi}_B + \frac{3}{\chi}\dot{\phi}_B - V'(\phi_B) = 0$, with $\Gamma \propto e^{-S_E[\phi]}$
- 1 d.o.f. – emerge at single point on other side of barrier.
- ∞ d.o.f. – emerge at a field configuration, $\phi_B(r)$ given by solution by *analytic continuation*.



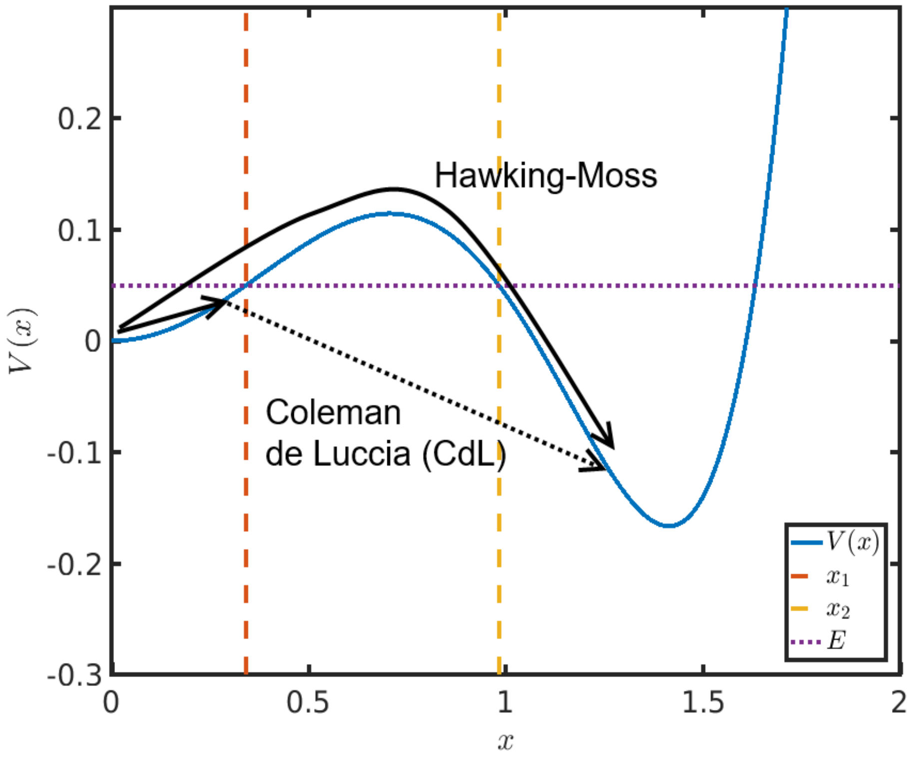
[2] S. Coleman, V. Glaser and A. Martin, Action minima among solutions to a class of euclidean scalar field equations, Communications in Mathematical Physics 58 (1978) 211.

Review bubble expansion in flat space

- Bounce $O(4)$ symmetric -> $O(3,1)$ symmetry in real space. Unique solution is $\phi(r,t) = \phi_B\left(\sqrt{r^2 - c^2t^2}\right)$.
- Position bubble with $\phi = \phi_0$ moves as $r^2 - c^2t^2 = R_0^2, R_0 = \phi_B^{-1}(\phi_0)$.
- $v = \dot{r}(t) = \frac{c^2 t}{\sqrt{R_0^2 + c^2 t^2}} \to c$
- So bubble rapidly approaches speed of light.
- But what if $r < ct$? Corresponds to patch of spacetime *inside* the bubble. Need gravity to understand it.

Gravitational Collapse of Bubble

- Need to include gravity! See [3]
- $S_E = \int d^4x\sqrt{|g|}\left[\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + V - \frac{M_P^2}{2}R\right],$
- $\Gamma \propto e^{-S_E}$
- $\ddot{\phi} + \frac{3\dot{a}}{a}\dot{\phi} - V'(\phi) = 0$
- $\dot{a}^2 = 1 - \frac{a^2}{3M_P^2}\left(-\frac{\dot{\phi}^2}{2} + V(\phi)\right)$
- $ds^2 = d\chi^2 + a^2(\chi)[d\psi^2 + \sin^2\psi\, d\Omega_2^2]$
- How to continue this back?



[3] S. Coleman and F. De Luccia, Gravitational effects on and of vacuum decay, Phys. Rev. D 21 (1980) 3305.

Gravitational Collapse of Bubble

- Good explanation given by [4]. In flat space, simply do $t = -i\tau$.
- In curved space, define $f(\chi)$ by $f' = \frac{f}{a}, f(0) = 0$ and transform as:
- $\left.\begin{array}{c}\tilde{r} = f(\chi)\sin\psi \\ \tilde{\tau} = f(\chi)\cos\psi\end{array}\right\} \Rightarrow ds^2 = \frac{a^2(\chi)}{f^2(\chi)}(d\tilde{\tau}^2 + d\tilde{r}^2 + \tilde{r}^2\, d\Omega_2^2)$
- Conformally flat, so now do $\tilde{t} = -i\tilde{\tau}$, (equivalently $\psi_+ = i(\psi - \pi/2)$)
- $\left.\begin{array}{c}\tilde{r} = f(\chi)\cosh\psi_+ \\ \tilde{t} = f(\chi)\sinh\psi_+\end{array}\right\} \Rightarrow ds^2 = \frac{a^2(\chi)}{f^2(\chi)}(-d\tilde{t}^2 + d\tilde{r}^2 + \tilde{r}^2\, d\Omega_2^2)$

[4] P. Burda, R. Gregory and I. G. Moss, The fate of the higgs vacuum, Journal of High Energy Physics 2016 (2016) 25.

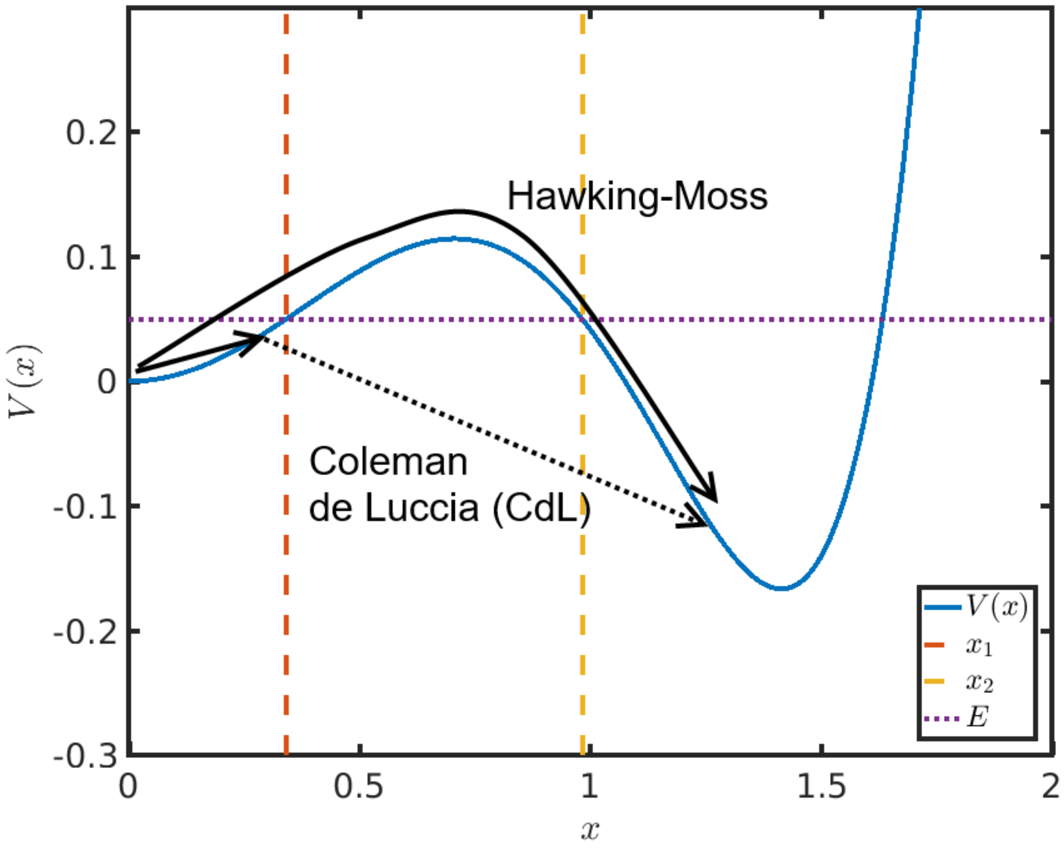
Gravitational Collapse of Bubble

- $\Rightarrow \tilde{r}^2 - \tilde{t}^2 = f^2(\chi)$ (c.f. flat space, with $f(\chi) = \chi$)
- So solution is $\phi(\tilde{r}, \tilde{t}) = \phi_B\left(f^{-1}\left(\sqrt{\tilde{r}^2 - \tilde{t}^2}\right)\right)$ (c.f. flat space $\phi_B\left(\sqrt{\tilde{r}^2 - \tilde{t}^2}\right)$)
- But $\tilde{r}^2 - \tilde{t}^2 = f^2(\chi)$ clearly only works for $\tilde{r} > \tilde{t}$, ie, outside the lightcone of nucleated bubble.
- Burda et al. [3] showed you can get the $\tilde{r} < \tilde{t}$ portion with:
- $\left.\begin{matrix} \tilde{r} = f(\chi)\sinh\psi_- \\ \tilde{t} = f(\chi)\cosh\psi_- \end{matrix}\right\} \Rightarrow ds^2 = -d\chi^2 + a^2(\chi)[d\psi_-^2 + \sinh^2\psi_- \, d\Omega_2^2]$

[4] P. Burda, R. Gregory and I. G. Moss, The fate of the higgs vacuum, Journal of High Energy Physics 2016 (2016) 25.

Gravitational Collapse of Bubble

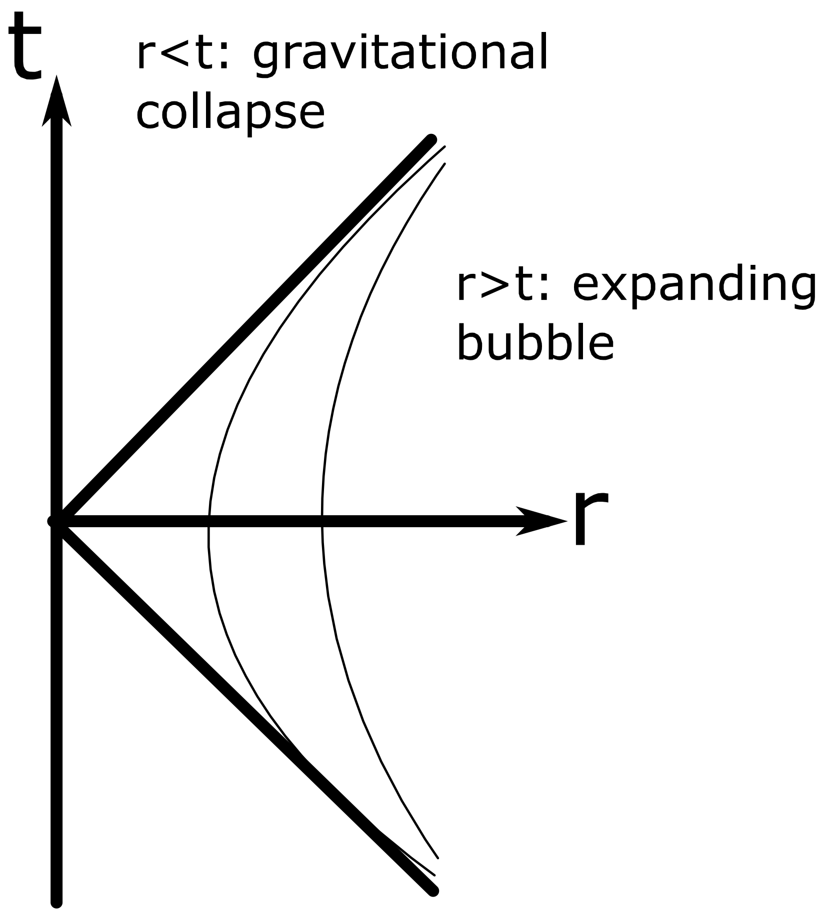
- Same equation of motion as bounce, but *non-inverted* potential.
- $\ddot{a} = -\frac{a}{3M_P^2}\underbrace{\left(\dot{\phi}^2 - V(\phi)\right)}_{>0\ !!!}$
- $a = 0$, *big-crunch* singularity guaranteed in finite time. Same conclusion as [3].



[3] S. Coleman and F. De Luccia, Gravitational effects on and of vacuum decay, Phys. Rev. D 21 (1980) 3305.

Gravitational Collapse of Bubble

- So what happens next?
- $r > t$ describes the expanding bubble.
- $r < t$ describes the collapse.
- But expansion continues forever regardless of what is going on inside (information can't propagate from out of the $r < t$ region – peak of bubble wall moves at c)



De Sitter Bubbles

- So what happens to bubbles during inflation?
- For pure de Sitter tunnelling, get $a = \frac{1}{H}\sin H\chi$ for *Euclidean* (4-sphere) metric.
- $\Rightarrow f(\chi) = \tan\frac{H\chi}{2}$ and metric:
- $ds^2 = \frac{4}{H^2(1+\tilde{r}^2-\tilde{t}^2)^2}[-d\tilde{t}^2 + d\tilde{r}^2 + \tilde{r}^2\, d\Omega_2^2]$
- Via $\left.\begin{aligned} t &= \frac{1}{2H}\log\left|\frac{1-\tilde{r}^2+2\tilde{t}+\tilde{t}^2}{1-\tilde{r}^2-2\tilde{t}+\tilde{t}^2}\right| \\ r &= \frac{2\tilde{r}}{H(1-\tilde{r}^2-\tilde{t}^2)}\end{aligned}\right\} \Rightarrow$
- $ds^2 = -dt^2(1-H^2r^2) + dr^2(1-H^2r^2)^{-1} + r^2\, d\Omega_2^2$

De Sitter Bubbles

- We map the $(\tilde{r}, \tilde{t})$ parabolas onto (r, t) to see how a de Sitter bubble expands.
- Point with $\phi = \phi_0, r_0 = f\left(\phi_B^{-1}(\phi_0)\right)$ moves as:

$$r(t) = \frac{2}{H(1 + r_0^2)} \sqrt{r_0^2 + \frac{1}{4}(1 - r_0^2)^2 \tanh^2 Ht} \rightarrow \frac{1}{H} \text{ as } t \rightarrow \infty$$

- Main observation – bubble expands to *comoving* radius 1/H and no further. Fills a full Hubble Volume.

Conclusions

- Anti-de-Sitter region at bubble centre indeed collapses.
- But observers outside the bubble never see this. Bubble continues to expand at the speed of light.
- Fills an entire Hubble volume.
- Once filled, this volume expands at the same rate as the rest of the universe, rather than shrinking away.
- Hubble rates of order $10^8 - 10^{10}\mathrm{GeV}$ enough to be fatal. So new physics needed somewhere!