Current Status of Smooth Quantum Gravity

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smooth quantum gravity - motivations

In dim. 4 there are (typically) continuum infinite many different smoothness structures on open 4-manifolds like $\mathbb{R}^4$ or $S^3 \times \mathbb{R}$. These standard manifolds are extensively used in physics. Do their exotic smoothness is physically (especially QG) valid)?

Or: Given exotic R^4 it is Riemannian smooth 4-manifold homeomorhic to $\mathbb{R}^4$. Its Riemannian curvature tensor can not vanish! So exotic R^4 has to have non-zero curvature and density of gravitational energy is non-zero as well. Is this curvature physical?

i) YES, the Riemannian curvature of exotic R^4's leads directly to QG.

ii) Embeddings of exotic R^4's determine topological 3-D invariants with cosmological meaning.

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SQG IS A THEORETICAL ATTEMPT TO UNDERSTAND AND DERIVE SOME FREE PARAMETERS IN PHYSICS BASED ON TOPOLOGY UNDERLYING SMOOTH EXOTIC $\mathbb{R}^4$

smooth quantum gravity - some results

a. Large exotic R^4's are gravitational instantons. Any theory of QG has to deal with them [G. Etesi, 2019].

b. The path integral of GR is dominated by contributions from exotic R^4's [TAM, 2016].

c. The curvature of R^4 embnedded in $K3\#\overline{CP2}$ determines the realistic (small) value of the Cosm. Const. [TAM, JK, 2018]

d. This CC is a topological invariant [TAM, JK, 2018].

e. The cosmological FRW model on exotic $S^3 \times \mathbb{R}$ (from $R^4 \hookrightarrow K3\#\overline{CP2}$) predicts and explains the realistic values of inflation parameters [TAM, JK, 2019].

f. The electroweak and GUT's scales and the bound on neutrino masses are also predicted [TAM, JK, 2019].

g. Exotic R^4's determine von Neuman algebras containing factor III_1 [G. Etesi, 2018, TAM 2016].

The realistic (small) value of the Cosm. Constant

- THE FORMULA FOR CC (curvature of exotic R^4 embedded in $K3\#\overline{CP2}$)

$$\Omega_\Lambda = \frac{c^5}{24hGH_0^2} \cdot \exp\left(- \frac{3}{CS(\Sigma(2,5,7))} - \frac{3}{CS(P\#P)} - \frac{\chi(A)}{4}\right)$$

here $\chi(A) = 1$ is the Euler characteristic of the Akbulut cork A, $\Sigma(2,5,7)$ is the Brieskorn homology 3-sphere, $CS(\Sigma(2,5,7))$ - the Chern-Simons invariant of Σ, $P\#P$ - the connected sum of two copies of the Poincaré 3-spheres. The CC value follows

$$\Omega_\Lambda \approx 0,7029$$

which agrees with PLANCK

THIS CC IS A TOPOLOGICAL INVARIANT!

- Why it is so? The derivation follows from hyperbolic geometry of 3 and 4-manifolds (cobordisms) in the embedding $R^4 \hookrightarrow K3\#\overline{CP2}$ [T.Asselmeyer-Maluga,JK,Phys.Dark.Univ.2018].

Two topology changes

Exotic R^4's are topologically trivial, where does the nontrivial topology come from?

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The embedding $R^4 \hookrightarrow K3\#\overline{CP2}$ determines two topology changes (3-dimensional):

$$S^3 \xrightarrow{1} \Sigma(2,5,7) \xrightarrow{2} P\#P$$

- The evolution in $K3$ must be smooth through both topology changes, so we need to glue in 3 Casson handles at the second step and one in the first.

Based on this we can determine energy scales of both transitions...

1^{st} TOPOLOGY CHANGE

Inside K3 there is a cork (Akbulut) - a compact, contractible smooth submanifold $A \subset K3$.

- THE BOUNDARY ∂A IS A HOMOLOGY 3-SPHERE $\Sigma(2,5,7)$ (HYPERBOLIC).
- $\Sigma(2,5,7)$ can not be replaced by S^3: inside A there is smoothly embedded S^3 but there is no smooth $S^3 \subset \mathrm{K3}$ such that $A \subset S^3$.

But

There exists smooth 4-cobordism in K3 between $S^3 \subset A$ and $\Sigma(2,5,7)$

Smooth cob $S^3 \to \Sigma(2,5,7)$

To go smoothly from S^3 to $\Sigma(2,5,7)$ we need to glue in flexible handles – Casson handles (CH). Then we get smooth 4-cobordism $W(S^3, \Sigma) \subset A \subset \mathrm{K3}$.

- $\Sigma(2,5,7)$ is hyperbolic and it is rigid – one can not scale it and its volume V is an invariant.
- thus Σ determines characteristic length $\sqrt[3]{V} = L$.
- Expressing via CS invariant and taking the radius of 3-sphere as r_{S^3} we obtain the rescaling

$$a = r_{S^3} \cdot \exp\left(\frac{3}{2 \cdot CS(\Sigma(2,5,7))}\right).$$

ENERGY AND TIME SCALES OF 1

- FREEDMAN: EVERY CH IS EMBEDABLE IN ITS 1ST 3-STAGES.

Then (for $\theta = \frac{3}{2 \cdot CS(\Sigma(2,5,7))}$)

$$E_1 = \frac{E_P}{1 + \theta + \frac{\theta^2}{2} + \frac{\theta^3}{6}} \simeq 10^{15} GeV \quad t_1 = t_P\left(1 + \theta + \frac{\theta^2}{2} + \frac{\theta^3}{6}\right) \simeq 10^{-39}$$

E_1 ($\sim$ GUT energy) – topologically supported

ENERGY AND TIME SCALES OF 2

- K3 is decomposed as

$$\mathrm{K3} = |\mathrm{E}_8 \oplus \mathrm{E}_8| \# (\mathrm{S}^2 \times \mathrm{S}^2) \# (\mathrm{S}^2 \times \mathrm{S}^2) \# (\mathrm{S}^2 \times \mathrm{S}^2) \, .$$

- $|E_8|$ has boundary P – the Poincaré sphere. $|E_8 \oplus E_8|$ can not be realized as smooth closed 4-manifold (Donaldson). It has boundary $P \# P$ (3-submanifold of K3).
- $\Sigma(2,5,7) \xrightarrow{2} P \# P$ here we need full infinite 3CH's since $|E_8 \oplus E_8|$ is not smooth (Donaldson). After gluing CH's we have smooth evolution 2 within $\mathrm{K3} \# \overline{\mathrm{CP}}(2)$.

ENERGY AND TIME SCALES OF 2

Again $\theta = \frac{3}{2\cdot CS(\Sigma(2,5,7))} = \frac{140}{3}$ and taking $\Delta_0 t$ as the time for 1-level CH we have for the total time and energy after 2^{nd} change $(CS(P\#P) = \frac{1}{60})$:

$$\Delta t = \frac{t_{Pl}\cdot \exp\left(\frac{-1}{2CS(P\#P)}\right)}{1+\theta_2+\frac{{\theta_2}^2}{2}+\frac{{\theta_2}^3}{6}}, \quad \Delta E = \frac{E_{Pl}\cdot \exp\left(\frac{-1}{2CS(P\#P)}\right)}{1+\theta_2+\frac{{\theta_2}^2}{2}+\frac{{\theta_2}^3}{6}} \simeq 63GeV\,.$$

2^{nd} energy scale is close to the electroweak scale and it is topologically supported

MASSIVE NEUTRINOS

Seesaw mechanism for generating the masses of neutrinos:

$$\begin{pmatrix} 0 & M \\ M & B \end{pmatrix}, \; M \ll B, \; \lambda_1 = B, \; \lambda_2 = -\frac{M^2}{B} \, .$$

Let us fix the energy scales $E_1 = 0,67 \cdot 10^{15} \, GeV$ as B and $E_2 = 63 \, Gev$ as M. Then $m_n = \frac{M^2}{B} \simeq 0,006 \, eV$ – agrees with current limitation for the sum of 3 neutrino masses from PLANCK $\sim 0,12 \, eV$.

Topology determines the realistic neutrino masses.

LEFT-RIGHT NEUTRINOS

- 1^{st} topology change: $S^3 \to \Sigma(2,5,7)$ with the GUT scale gives rise to 2 Dirac operators on Σ, since the mapping class group of Σ is nontrivial
$$\pi_0(\mathrm{Diff}(\Sigma(2,5,7))) = \mathbb{Z}_2 \,.$$
hence left and right-handed neutrinos are there.
- The 2^{nd} topology change $\Sigma(2,5,7) \to P\#P$ gives rise only to left-handed neutrinos (EW energy scale) since
$$\pi_0(\mathrm{Diff}(\mathrm{P})) = 1 \,.$$

We found 'first principles' allowing for derivation of certain free parameters in Cosmology and Particle Physics. This is $R^4 \hookrightarrow K3\#\overline{CP2}$, R^4 exotic.

AGAIN 1^{st} TOPOLOGY CHANGE

Let $r_{S^3} \sim P_L$ to be of Planck length thus (with $CS(\Sigma) = 9/280$)

$$10^{-34}[m] \to 10^{-15}[m], \quad N = \frac{3}{2 \cdot CS(\Sigma(2,5,7))} + \ln 8\pi^2 \simeq 51\,.$$

- The topology of the Akbulut cork for exotic K3 codes the inflationary expansion of the Universe.

Starobinsky model for inflation

$$S = \int_{M^4} d^4x\sqrt{-g}(R + \alpha \cdot R^2),\ \alpha \text{ free param.}$$

$\alpha \cdot M_{Pl}^{-2} = \frac{\Delta E_{\text{infl}}}{E_{Pl}}$ so we get from 1^{st} topology change:

$$\alpha \sim 10^{-5};\ \left(\alpha \cdot M_{Pl}^{-2} = \frac{1}{1+\theta+\frac{\theta^2}{2}+\frac{\theta^3}{6}},\ \ \theta = \frac{3}{2 \cdot CS(\Sigma(2,5,7))}\right)$$

the spectral tilt n_s and the tensor-scalar ratio r follow

$$n_s = 1 - \frac{2}{\theta + \ln(8\pi^2)} \approx 0,961,\ \ r = \frac{12}{(\theta + \ln(8\pi^2))^2} \approx 0,0046\,.$$

α, n_s, r are topologically supported due to the K3 smoothness structure

Smooth quantum gravity

Exclusively in dimension 4:

- NONSTANDARD SMOOTHNESS $\Rightarrow$ NONZERO CURVATURE OF R^4
- NONSTANDARD SMOOTHNESS $\Rightarrow$ QUANTIZATION IN SPACETIME

Large exotic R^4 embedded in $K3\#\overline{CP2}$ is Ricci flat, hyperkähler, self-dual, hence gravitational instanton [Etesi, 2019]. Any QG theory has to deal with them.

Smooth quantum gravity

- the 3-sphere $S^3 \hookrightarrow A \hookrightarrow$ K3 is widely embedded – represents quantum state.
- Connes: wild S^3 generates the factor III_1 von Neuman algebra and the Fock space of certain QFT.
- When smoothness of K3 and R^4 are standard the sphere S^3 is tame and no quantum algebras result.

SQG explores the overlapping contact space of QM and GR via exotic differentiable structure on K3 and R^4.

Thank You

for Your Attention !!

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