Bottomonia Suppression in Heavy-Ion Collisions from AdS/CFT

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Modeling the Quark Gluon Plasma

- QCD phase diagram not well understood
- QGP formed by LHC successfully described by divergent frameworks


- Weak coupling
- LATTICE QCD

- Weak & strong coupling
- AdS/CFT

- Strong coupling
Matsui & Satz proposed that in QGP quarkonia (heavy $q\bar{q}$) can exist above $T_c$.

As $T$ increases → Debye screening length drops below size of quarkonia → $q\bar{q}$ dissociates.

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Sequential melting of quarkonia in QGP


Experimental Results for $\Upsilon(2S) & \Upsilon(3S)$ Melting

How do we produce theoretical predictions to quantify this data?

Quarkonia Suppression: $R_{AA}$

$$R_{AA} \{x\} = \frac{\text{# observed in } A + A \text{ at } \{x\}}{\left(\text{# observed in } p + p \text{ at } \{x\}\right)\left(\text{# of } p + p\text{-like collisions at } \{x\}\right)}$$

$R_{AA} < 1$ indicates suppression of quarkonia
Quarkonia Suppression: $R_{AA}$

$$R_{AA}(\{x\}) = \frac{\# \text{ observed in } A + A \text{ at } \{x\}}{(\# \text{ observed in } p + p \text{ at } \{x\})(\# \text{ of } p + p\text{-like collisions at } \{x\})}$$

We may predict $R_{AA}$ from a suppression model

$$R_{AA}(p_T, b) = \frac{\int d^2 x_\perp d\phi \, T_{AA}(x_\perp, b) \, R_{AA}(p_T, y, x_\perp, b)}{2\pi N_{\text{coll}}$$

BACKGROUND

DISSOCIATION MODEL
Suppression Model

We may predict $R_{AA}$ from a suppression model

$$R_{AA}(p_T, b) = \int d^2x_\perp d\phi \frac{T_{AA}(x_\perp, b) R_{AA}(p_T, y, x_\perp, b)}{2\pi N_{\text{coll}} d(T_{AA})}$$

Background

- Optical limit of the Glauber model
- Gives qualities of medium through which quarkonia propagates
  ($T_{AA}, N_{\text{coll}},$ temperature profile of QGP)

Dissociation model
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**BACKGROUND**

Optical limit of the Glauber model

Gives qualities of medium through which quarkonia propagates ($T_{AA}, N_{\text{coll}},$ temperature profile of QGP)

**DISSOCIATION MODEL**

Survival of quarkonia based on imaginary part of binding energy
Dissociation Model

Schematically:

\[ R_{AA} \sim e^{-L_S[E_{bind}]} \]


We find \( E_{bind} \) from complex potential \( V \)

Theoretical Frameworks \( \rightarrow \) Heavy Quark Potential \( V \) \( \rightarrow \) Binding Energy \( E_{bind} \) \( \rightarrow \) \( R_{AA} \) for Comparison
Calculating Quarkonia Binding Energies

**NRTDSWE**

\[ i\partial_t \Psi(r, t) = H \Psi(r, t) = \left[ -\frac{1}{2m} \nabla^2 + V(r) \right] \Psi(r, t) \]

\[ \Psi(r, t) = \sum_{n=0}^{\infty} c_n \psi_n(r) e^{-iE_n t} \]

**Wick rotation**

\[ \tau \equiv it \]

\[ \lim_{\tau \to \infty} \Psi(r, t) \to c_0 \psi_0(r) e^{-E_0 \tau} \]

Large \( m_Q \) and small relative \( \nu \) of heavy quarks → non-relativistic quantum mechanics
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Real time evolution operator of wave function \( \sim e^{-iEt} \)
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**Large** \( m_Q \) and **small relative** \( v \) of heavy quarks \( \rightarrow \) **non-relativistic quantum mechanics**

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**Psi**

\[ \Psi(r, t) = \sum_{n=0}^{\infty} c_n \psi_n(r) e^{-iE_n t} \]

**Wick rotation**

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**Limit**

\[ \lim_{\tau \to \infty} \Psi(r, t) \to c_0 \psi_0(r) e^{-E_0 \tau} \]

- Large \( m_Q \) and small relative \( v \) of heavy quarks \( \rightarrow \) **non-relativistic quantum mechanics**
- Real time evolution operator of wave function \( \sim e^{-iEt} \)
- Imaginary time evolution operator \( \sim e^{-E\tau} \)
- At large tau, only ground state wave function survives
Calculating Quarkonia Binding Energies

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At large \( \tau \), only ground state wave function survives.
Having found the ground state wave function, we can calculate $E_0$ using

$$E_0 = \frac{\int r^2 \, dr \, \psi_0(r)^* \, H \, \psi_0(r)}{\int r^2 \, dr \, |\psi_0|^2}$$

The binding energy $E_{\text{bind}}$ follows from

$$E_{\text{bind}} \equiv E_0 - \Re[V(|r| \to \infty)]$$
Potential Models

**pQCD**


**AdS/CFT**

1.0 1.5 2.0 2.5 3.0
T / T_c

-\mathcal{E}_{\text{bind}} (\text{GeV})

\begin{align*}
\text{Re}[-\mathcal{E}_{\text{bind}}] & \quad \alpha_s = 0.27 \\
\text{Re}[-\mathcal{E}_{\text{bind}}] & \quad \lambda = 5.5 \\
\text{Im}[-\mathcal{E}_{\text{bind}}] & \quad \alpha_s = 0.27 \\
\text{Im}[-\mathcal{E}_{\text{bind}}] & \quad \lambda = 5.5 \\
\end{align*}


No single obvious map between parameters in QCD and \mathcal{N} = 4 SYM
Y(1S) Suppression

\[ \alpha_s = 0.27 \]

\[ \lambda = 5.5 \]

\[ 0 \leq p_T \leq 40 \]
The plot shows the $Y(1S)$ suppression as a function of $N_{\text{part}}$, the number of participants, and $p_T$, the transverse momentum. The data from CMS is represented by the orange circles. The blue line represents the pQCD prediction with $\alpha_s = 0.27$, while the green line is the AdS/CFT prediction with $\lambda = 5.5$. The dashed red line represents the pQCD prediction with KRS $E_{\text{bind}}$, and the black dashed line is the AdS/CFT prediction with $\lambda = 5.5$.

- **Y(1S) suppression from pQCD consistent with data**
- **AdS/CFT overpredicts $Y(1S)$ suppression compared to data**
- Large sensitivity to background - $R_{AA}$ for KRS $E_{\text{bind}}$ using Glauber smaller than that found using aHydro
- If our pQCD $E_{\text{bind}}$ more correct, better background may $\rightarrow R_{AA}$ inconsistent with data
- AdS/CFT may be consistent with more sophisticated suppression model & $p_T$-dependent potential

Conclusions & Outlook

Quantifying dissociation of quarkonia in a medium useful probe of QGP properties

Suppression ($R_{AA}$) calculated from binding energies of quarkonia by solving NRTDSWE in imaginary time

First results for suppression of strongly coupled $\Upsilon(1S)$ in isotropic QGP presented

AdS/CFT significantly overpredicts $\Upsilon(1S)$ suppression compared to data

Future work:

- Comparison of binding energies to those from independent semi-classical string theory calculations
- Use of velocity dependent potentials
- More sophisticated modeling of medium background & dissociation model
- Thorough investigation of systematic theoretical uncertainties in quarkonia $R_{AA}$
BACK-UP SLIDES
Complex Ritz Variational Method

c-product: \( (\psi|\phi) = \int_{\mathbb{R}^n} \psi(\bar{x})\phi(\bar{x}) \, d^n x \)

Given eigenvalue problem \( H\psi(\bar{x}) = E\psi(\bar{x}) \) where \( (\psi|\psi) \neq 0 \)

Rayleigh quotient: \( R(\bar{\alpha}) \equiv \frac{(\psi|\hat{H}|\psi)}{(\psi|\psi)} \)

where \( \psi(\bar{x}; \bar{\alpha}) \) parameter-dependent trial wave function \( \bar{\alpha} \in \mathbb{C}^m \)

If \( H\psi(\bar{x}; \bar{\alpha}_0) = E\psi(\bar{x}; \bar{\alpha}_0) \) for \( \psi(\bar{x}; \bar{\alpha}_0) \)

given \( (\psi(\bar{\alpha}_0)|\psi(\bar{\alpha}_0)) \neq 0 \), then \( \frac{\partial R(\bar{\alpha}_0)}{\partial \alpha_i^0} = 0 \)
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stationary state wave function of eig problem!
Y(1S) Suppression vs $p_T$

Cent. 0–100%

- **pQCD**
- **pQCD (KRS $E_{\text{bind}}$)**
- **pQCD (KRS Full)**
- **AdS/CFT $\alpha_s = 0.27$**
- **AdS/CFT $\lambda = 5.5$**
- **Y(1S) (CMS)**
\[ \Re[V(r)] = -\frac{\alpha}{r}(1 + \mu r)e^{-\mu r} + \frac{2\sigma}{\mu}(1 - e^{-\mu r}) - \sigma r e^{-\mu r} - \frac{0.8\sigma}{m_Q^2 r} \]

\[ \Im[V(r)] = \alpha T \left\{ \phi(\hat{r}) - \xi [\psi_1(\hat{r}, \theta) + \psi_2(\hat{r}, \theta)] \right\} \]

where \[ \phi(\hat{r}) \equiv 2 \int_0^\infty dz \frac{z}{(z^2 + 1)^2} \left[ 1 - \frac{\sin (z\hat{r})}{z\hat{r}} \right] \]


\[ \alpha = 0.385 \]
\[ \sigma = 0.223 \text{ GeV}^2 \]
\[ m_Q = 1.3 \text{ GeV} \]
\[ \mu = m_D \simeq 3p_{\text{hard}} \]
\[ \hat{r} = m_D r \]
\[ T = p_{\text{hard}} \]
Strongly Coupled Potential from AdS/CFT

\[ V_s(r) = \frac{\sqrt{\lambda}}{2c_0\pi} \left[ -\frac{1}{z_{\text{max}}} \left( 1 - \frac{z_{\text{max}}^4}{z_h^4} \right) F\left( \frac{1}{2}, \frac{3}{4}; \frac{1}{4}; \frac{z_{\text{max}}^4}{z_h^4} \right) + \frac{1}{z_h} \right] \]

where \( c_0 = \Gamma^2 \left( \frac{1}{4} \right) / (2\pi)^{3/2} \), \( z_h = 1/\pi T \),

and \( z_{\text{max}} \) is found using

\[ rc_0 = \frac{z_{\text{max}}}{z_h^2} \sqrt{z_h^4 - z_{\text{max}}^4} \left( \frac{1}{2}, \frac{3}{4}; \frac{1}{4}; \frac{z_{\text{max}}^4}{z_h^4} \right) \]


Solid root chosen over complex conjugate to ensure \( \text{Im}[V_s] < 0 \rightarrow \text{probability of state} < 1 \)
Strongly Coupled Potential from AdS/CFT

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and \[ z_{\text{max}} \] is found using

\[ r c_0 = \frac{z_{\text{max}}^2}{z_h^2} \sqrt{z_h^4 - z_{\text{max}}^4} F \left( \frac{1}{2}, \frac{3}{4}, \frac{5}{4}, \frac{z_{\text{max}}^4}{z_h^4} \right) \]