GW sources for the ground-based detector network

Thomas Dent

Galician Gravitational Wave Week 2019

Lecture 1

Plan of lecture

- What could possibly produce detectable GW?
  - Frequency, compactness, order-of-magnitude estimate
  - Overview of potential GW sources
- Reminder of detector noise properties
- Detectability for different morphologies of GW signal
  - Burst
  - Continuous wave
  - Stochastic background
  - (Compact binary merger – next lecture!)

GW frequency : back of envelope

- Gravitationally bound system, total mass *M*, size *R*
  - Characteristic maximum *dynamical frequency*

$$R^2\omega_d^2 \sim \frac{GM}{R} \qquad \omega_d \sim \sqrt{\frac{GM}{R^3}} \sim (G\rho)^{1/2}$$

  - Sensitive frequency band of ground-based detectors
    10 Hz < $f_{GW} \sim \omega_d/\pi$ < few × 10^3 Hz

- Only *very dense* objects emit GW visible by LIGO
  - MS stars/planets : $\omega_d \sim 10^{-3} - 10^{-6}$ Hz
  - WD : 0.1 − 10 Hz
  - NS : 1000 − 2000 Hz
  - BH ... ?

Frequency of emission from BH

- Orbiting a Schwarzschild black hole **at** event horizon $R_S = 2\ GM/c^2$ not possible – closest stable orbit is $3\ R_S$
  - Source orbital frequency $$(\omega_s)_{ISCO} = \frac{1}{6\sqrt{6}}\frac{c^3}{GM}$$
  - GW emission frequency $$(f_{gw})_{ISCO} \simeq 4.4\mathrm{kHz}\left(\frac{M_\odot}{M}\right)$$
  - Beyond this point object quickly merges with BH (possibly at higher frequency)
  - Black hole QNM : still higher frequencies

GW amplitude : back of envelope

- 'Quadrupole formula' for strain at distance r from source

$$h(r) \sim \frac{1}{r}\frac{G}{c^4}\ddot{Q}$$

  - Q is quadrupole moment

$$Q \sim \int \mathrm{d}^3x\, x^2 \rho(x) \lesssim M R^2$$

  - (Maximum) rate of change described by dynamical frequency

$$\ddot{Q} \lesssim \omega_d^2 Q \sim \frac{GM^2}{R}$$

GW amplitude vs. compactness

- (Order-of-magnitude) bound on possible GW strain

$$h(r) \lesssim \frac{1}{r}\frac{G}{c^4}\frac{GM^2}{R} = \left(\frac{GM}{Rc^2}\right)\left(\frac{GM}{rc^2}\right)$$

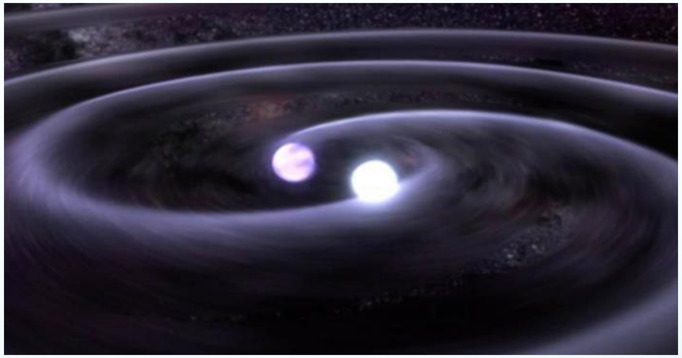
  - Scales as M/R (not as ρ)
- Recall $R_S = 2\,GM/c^2$:

$$h(r) \lesssim \left(\frac{R_S}{R}\right)\left(\frac{GM}{rc^2}\right)$$

  - Object cannot be smaller than its own Schwarzschild radius (to avoid collapse into BH!)
  - 'Compactness' R_S/R is strictly <1

GW are really small !

- Closest known NS are $10^2 - 10^3$ pc away (scale of Galaxy ~10^4 kpc)
- Most efficient GW emitters : *compact binaries* eg binary NS



$$h(r) \approx 10^{-22} \left(\frac{M}{2.8\, M_\odot}\right)^{5/3} \left(\frac{0.01\,\mathrm{s}}{P}\right)^{2/3} \left(\frac{100\ \mathrm{Mpc}}{r}\right)$$

GW source morphologies

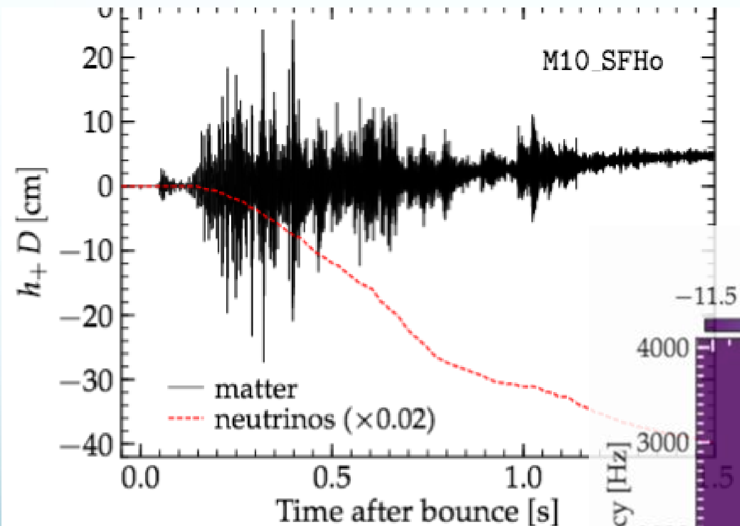
Transient vs. long duration GW

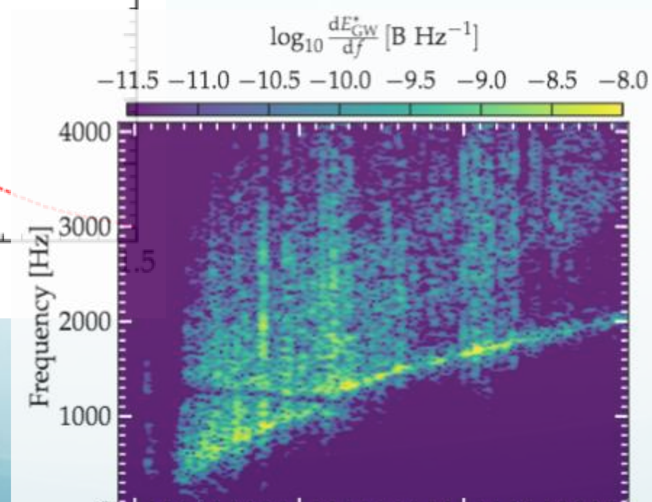
- Transient GW signals : cataclysmic events of compact astrophysical objects
  - – 'things that happen once'
  - birth or merger of neutron stars/black holes (associated with GammaRayBursts?)
  - transient (fast decaying) excitations of NS/BH
  - other exotic objects ?

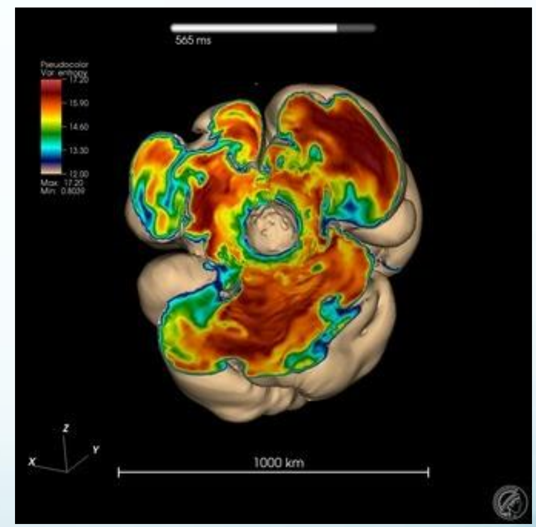
- Long duration / continuous signals : (nearly) stable GW emitting systems
  - isolated or long period binary neutron stars
  - 'stochastic' GW : weak, randomly overlapping signals from large number of sources at large distance

‘Burst’ sources

- GW emission with finite extent in time & frequency
- Form of *h*(*t*) not necessarily well known or modeled
  - e.g. core collapse supernova



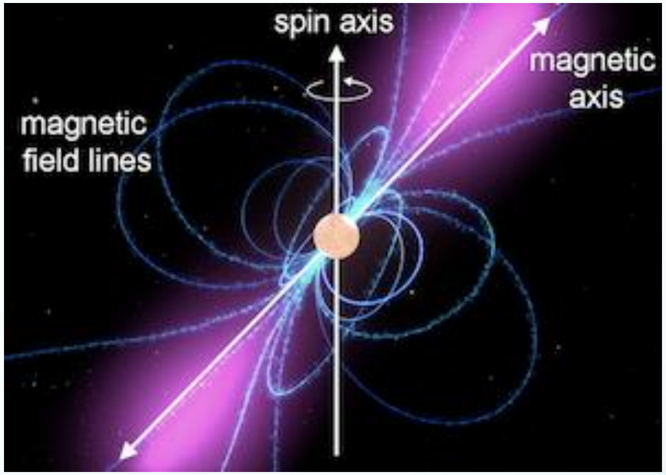




Simulation: F. Hanke et al. (MPIA Garching)

Continuous wave sources

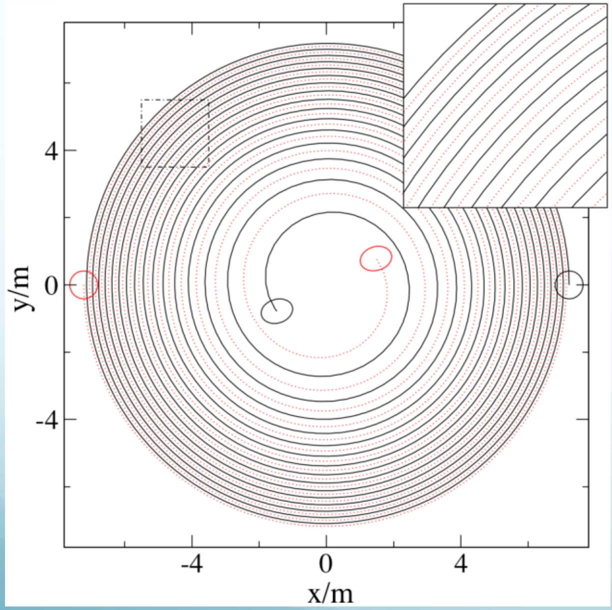
- Long-lived 'stable' sources : rotating neutron stars
- If NS not perfectly axisymmetric it emits GW at $f_{gw} = 2\, f_{rot}$
- Typical rotation periods ~0.1s to ~10^{-3} s (from known pulsars)

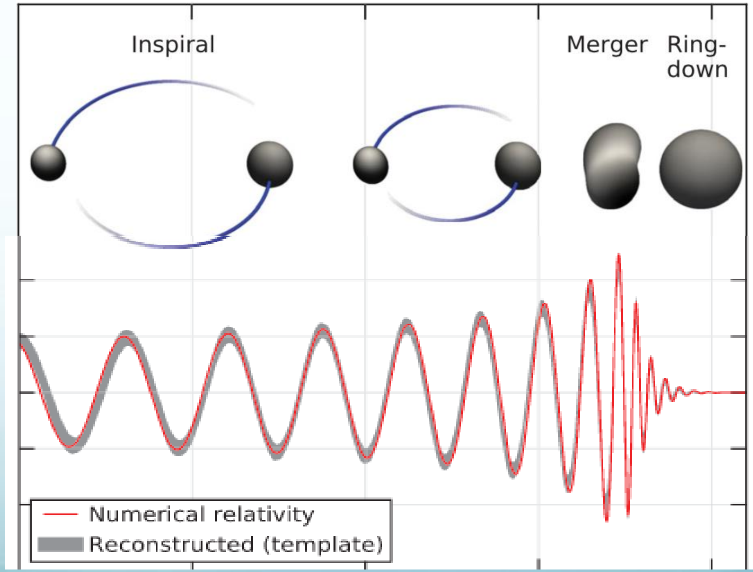


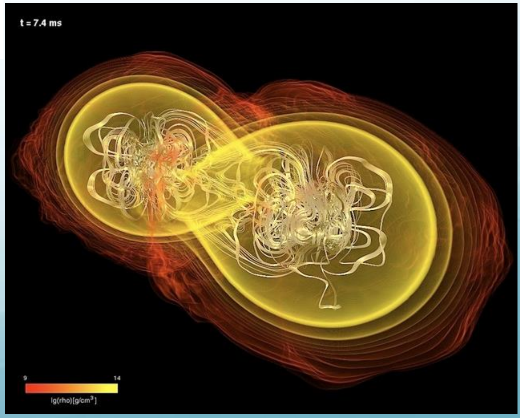
- GW strain at detector **not** a pure sinusoid
  - frequency modulation due to Doppler motions
  - orbital motion of NS (in binary system ..)
  - amplitude modulation due to Earth rotation

Compact binary mergers

- Binaries of NS / BH emit GW due to orbital motion
  - Orbit decays due to GW emission
  - Objects eventually collide / merge
  - Waveform predicted in GR given NS, BH masses/spins

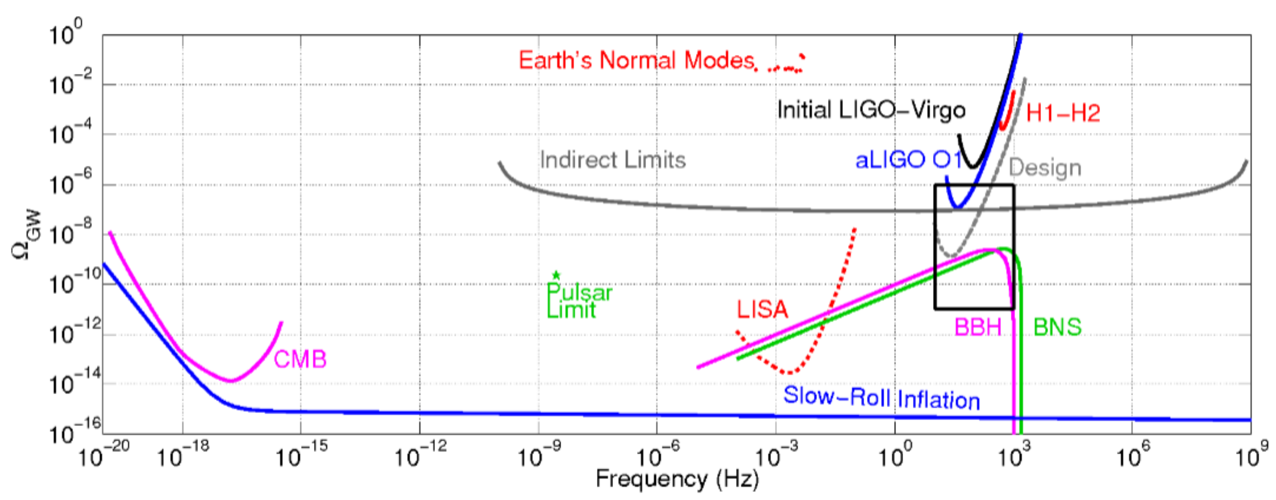






Stochastic background

- Continuous but *random* (unpredictable) gravitational wave field, expected to be isotropic
- Superposition of large number of distant (weak) sources *or* relic from inflation / hot early Universe
- Describe via *spectrum* of GW : energy density vs *f*



$$\Omega_{\mathrm{GW}}(f) = \frac{f}{\rho_c}\frac{d\rho_{\mathrm{GW}}}{df}$$

Describing instrumental noise : the power spectrum

Describing noise time series

- Measured strain ‘s(t)’ has contributions from many different processes – thermal, seismic, quantum fluctuations ...

$$s(t) = h(t) + n(t)$$

- Called ‘noise’ if they are
  a) unpredictable (random / stochastic / ..)
  b) not what you want to measure

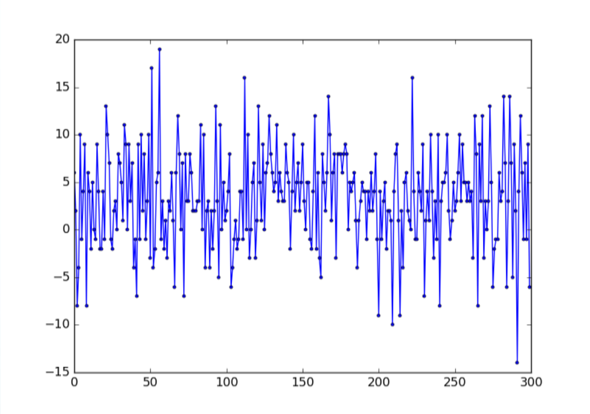
- “Stationary noise” : statistical properties do not change from one period of time to another

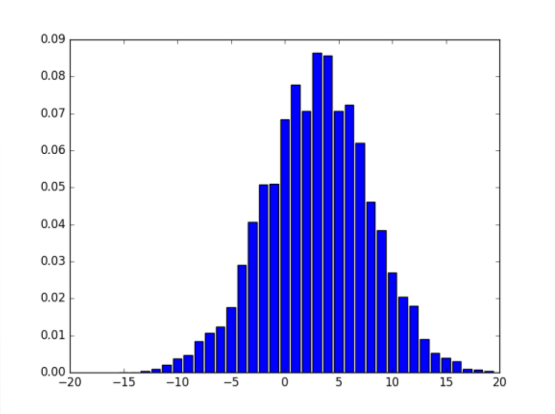
- Describe via statistics of single sample n[t_i] and correlations between different times n[t_i], n[t_j]

Noise distribution & autocorrelation

- Mean noise sample value $\langle n_j \rangle$ (can set = 0)
- Noise sample PDF e.g. Gaussian :

$$p(n_j) = (2\pi\sigma)^{-1/2} \exp(-n_j^2/2\sigma)$$





- Autocorrelation : $R(\tau) = \langle n(t+\tau)\, n(t) \rangle$
  - how noise at one time is related to other times

Noise fluctuation power

- How large are fluctuations about mean value ?

"Power" in given time interval T :

$$\int_{-T/2}^{T/2} \mathrm{d}t \, |n(t)|^2$$

Grows without limit as T → ∞

(nb *not* analogous to electrical/mechanical power!)

For stationary noise, use mean power per time

$$P_n \equiv \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \mathrm{d}t \, |n(t)|^2 \equiv \langle |n(t)|^2 \rangle$$

Go to freq. domain ..

$$P_n \equiv \lim_{T\to\infty} \frac{2}{T} \int_{0}^{\infty} \mathrm{d}f \, |\tilde{n}_T(f)|^2$$

Power spectral density

- Get total noise mean power by summing up frequency components

$$P_n = \int_0^\infty \mathrm{d}f \, S_n(f)$$

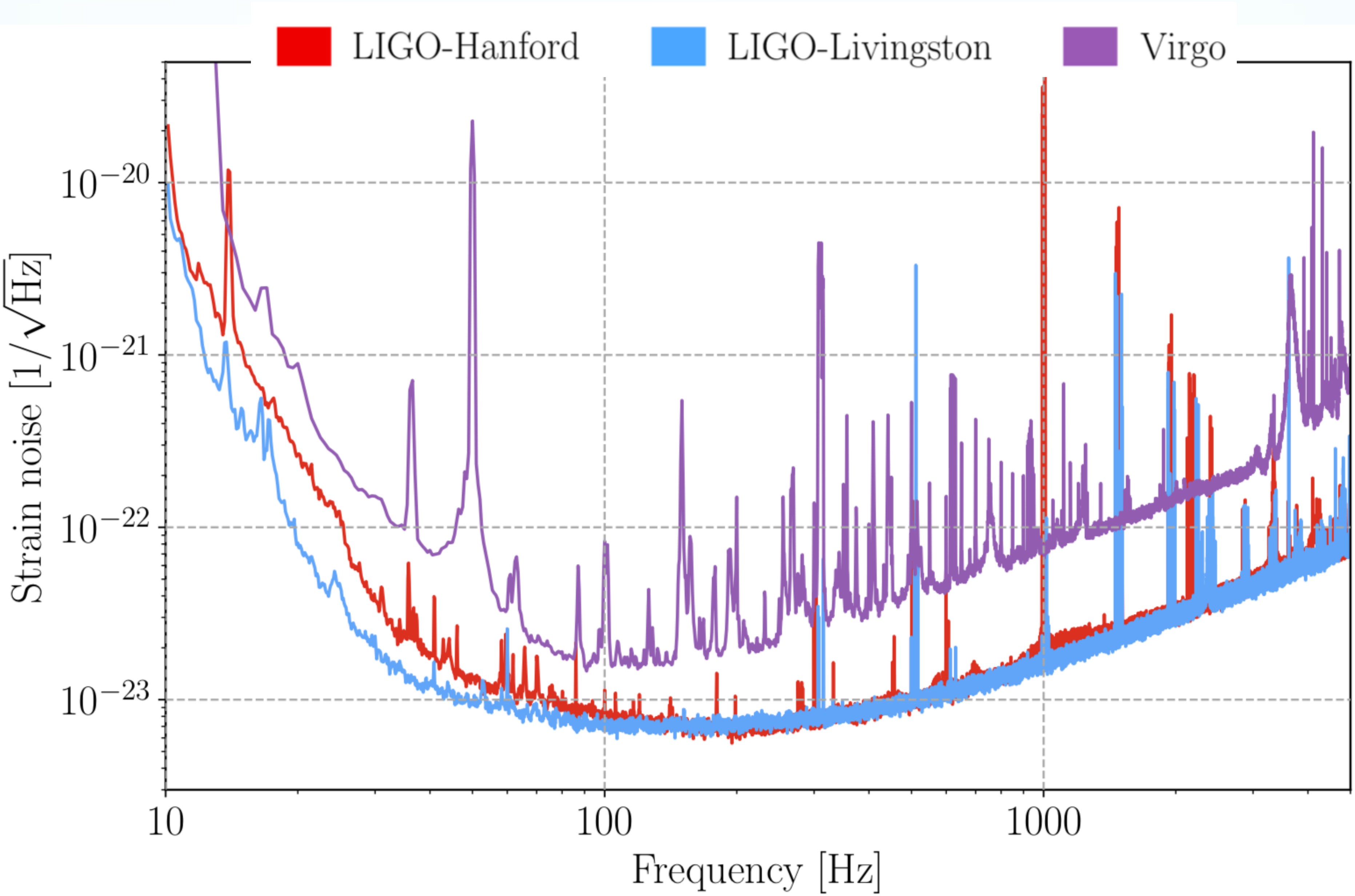
S_n is noise *power spectral density*

$$S_n(f) \equiv \lim_{T\to\infty} \frac{2}{T} \left| \int_{-T/2}^{T/2} \mathrm{d}t \, n(t) e^{2\pi i f t} \right|^2$$

Units : strain^2 / Hz

Quantity linear in GW strain :
amplitude spectral density ('ASD')

$$\sqrt{S_n(f)} \qquad \text{units}: \quad \texttt{strain}/\mathrm{Hz}^{1/2}$$



LIGO-Hanford
LIGO-Livingston
Virgo
10^{-20}
10^{-21}
10^{-22}
10^{-23}
Strain noise $[1/\sqrt{\text{Hz}}]$
10
100
1000
Frequency [Hz]

PSD and noise autocorrelation

- PSD is F.T. of the autocorrelation function R(τ)
- stationary noise ⇒ uncorrelated between different frequencies

$$\langle \tilde{n}^*(f)\tilde{n}(f')\rangle = \delta(f-f')\frac{1}{2}S_n(f)$$

- Alternative derivation of mean noise power

$$\langle |n(t)|^2\rangle = \langle |n(t=0)|^2\rangle = \iint \mathrm{d}f\,\mathrm{d}f'\,\langle \tilde{n}^*(f)\tilde{n}(f')\rangle$$

$$= \int_0^\infty \mathrm{d}f\, S_n(f)$$

Narrow band noise fluctuations

- Suppose we are interested in GW with frequency f
- isolate only the component of noise close to f
  - For data covering a time Δt, best possible frequency resolution is $\Delta f = 1/\Delta t$
- Calculate mean square fluctuation of remaining 'narrow band' noise :

$$[\Delta n(\Delta t, f)]^2 = \frac{S_n(f)}{\Delta t} = S_n(f)\Delta f$$

- RMS fluctuation

$$\Delta n(\Delta t, f)_{\mathrm{rms}} = \sqrt{S_n(f)\Delta f}$$

Detectability of bursts

- Generic parameters of GW burst
  - central frequency f
  - duration Δt
  - bandwidth Δf (may be > $1/\Delta t$)
- Characterize by mean square amplitude

$$\bar{P}_h = \frac{1}{\Delta t} \int_0^{\Delta t} \mathrm{d}t \, |h(t)|^2 \quad \equiv h_c^2$$

'characteristic strain'

- Compare with mean square noise fluctuation over same frequency range Δf

Burst SNR

- Ratio of signal to noise power

$$\left(\frac{S}{N}\right)^2 = \frac{\bar{P}_h}{\Delta f\, S_n(f)} = \frac{h_c^2}{\Delta f\, S_n(f)}$$

For 'broad band' burst $\Delta f \sim f$:

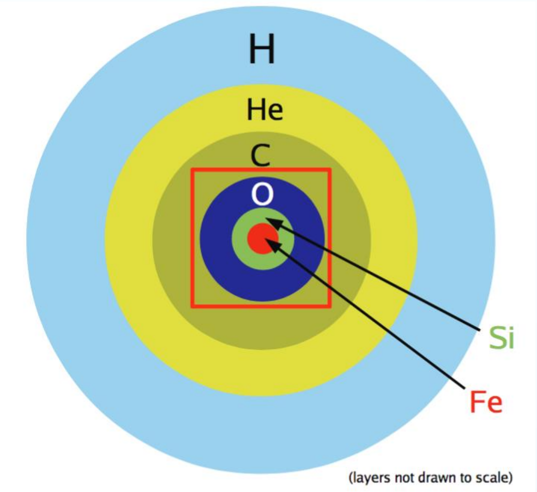
$$\left(\frac{S}{N}\right)^2 = \frac{h_c^2}{f\, S_n(f)}$$

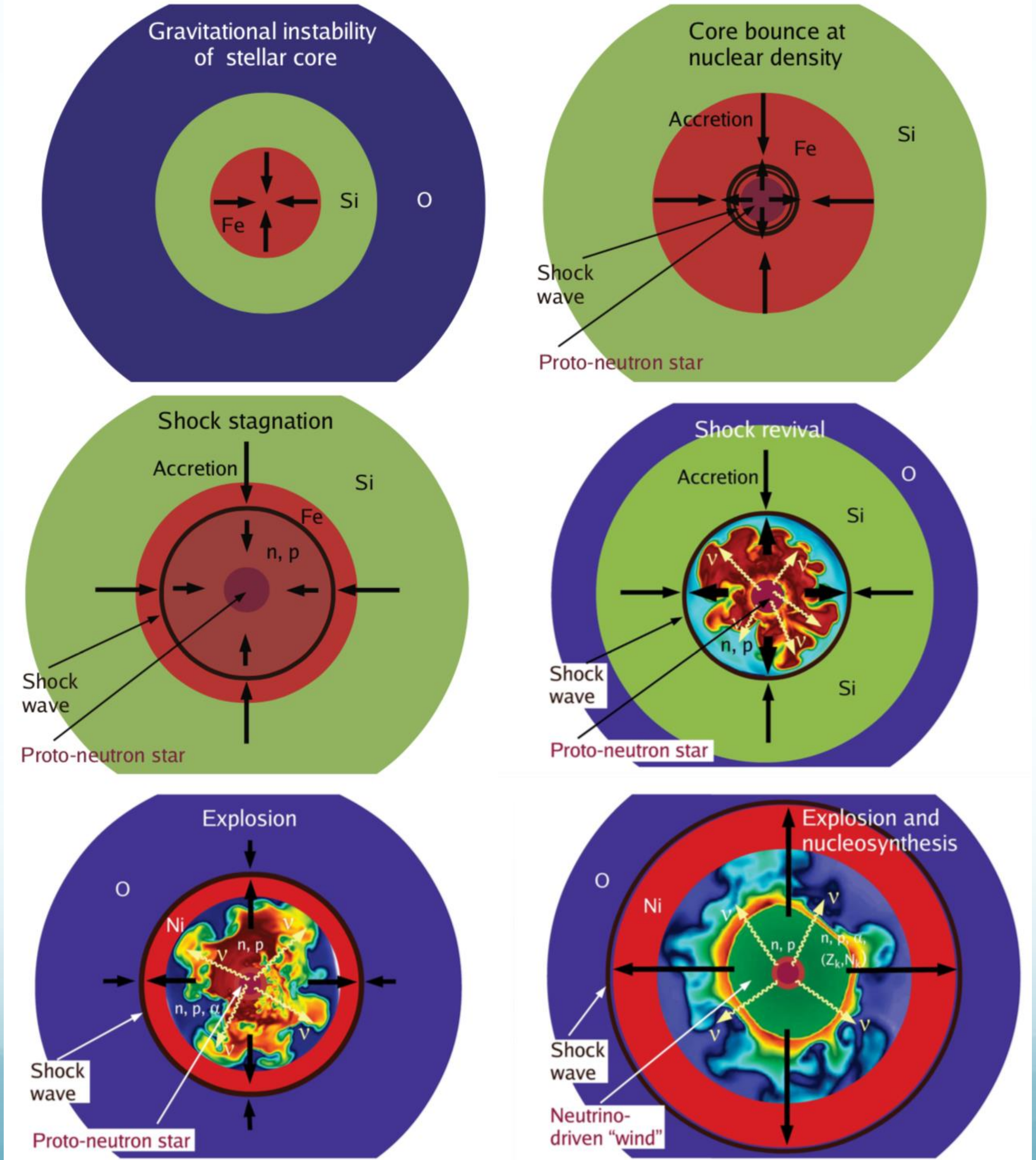
⇒ compare h_c with dimensionless quantity $\sqrt{f\, S_n(f)}$

'strain spectral density'

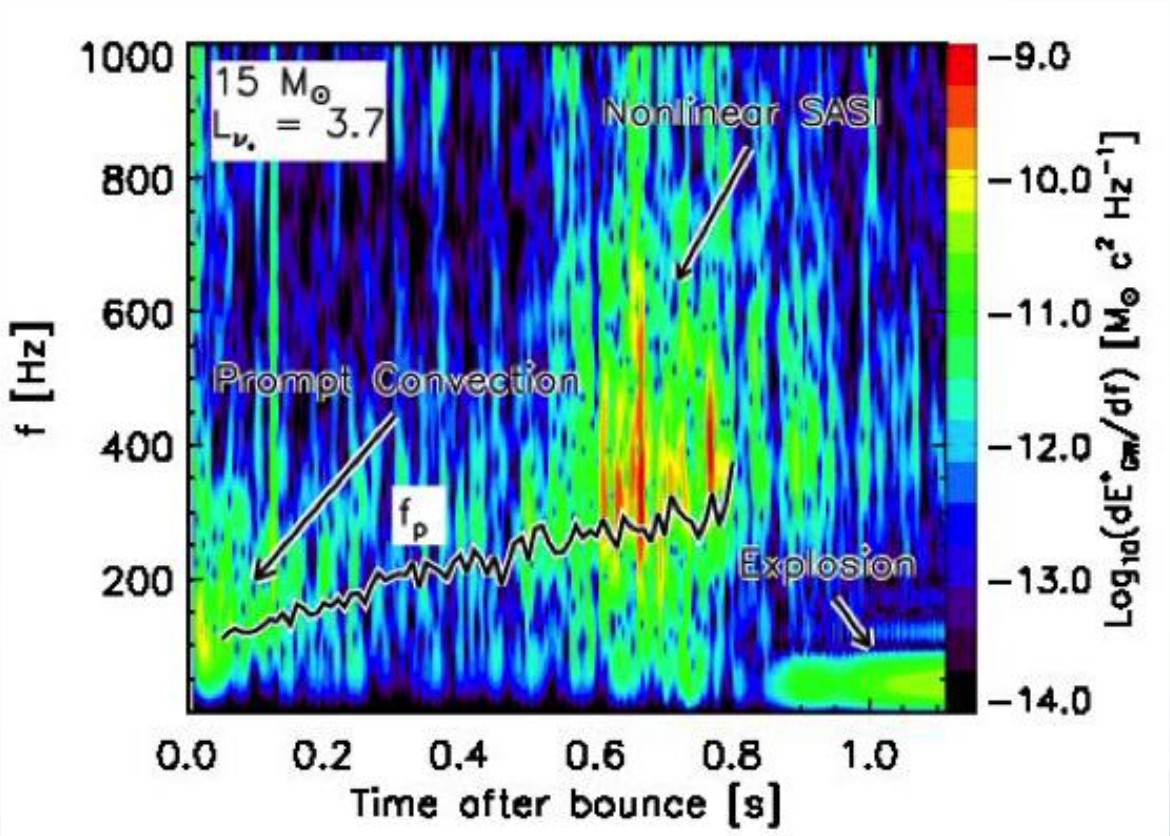
Bursts from core collapse supernova

- Massive stars (few × 10 $M_\odot$) quickly fuse all available light elements
- Sustain against gravitational collapse by continuing to 'burn' heavier elements
- Iron core cannot survive growing beyond Chandrasekhar limit ~1.4 $M_\odot$
- Core collapses to NS or BH, subsequent 'bounce' & shock wave blow off outer layers & lead to light emission




Gravitational instability of stellar core
Fe
Si
O
Core bounce at nuclear density
Accretion
Fe
Si
Shock wave
Proto-neutron star
Shock stagnation
Accretion
Si
Fe
n, p
Shock wave
Proto-neutron star
Shock revival
Accretion
O
Si
ν
n, p
Si
Shock wave
Proto-neutron star
Explosion
O
Ni
n, p
ν
n, p, α
Shock wave
Proto-neutron star
Explosion and nucleosynthesis
O
Ni
ν
n, p
n, p, α, (Z_k, N_k)
Shock wave
Neutrino-driven "wind"

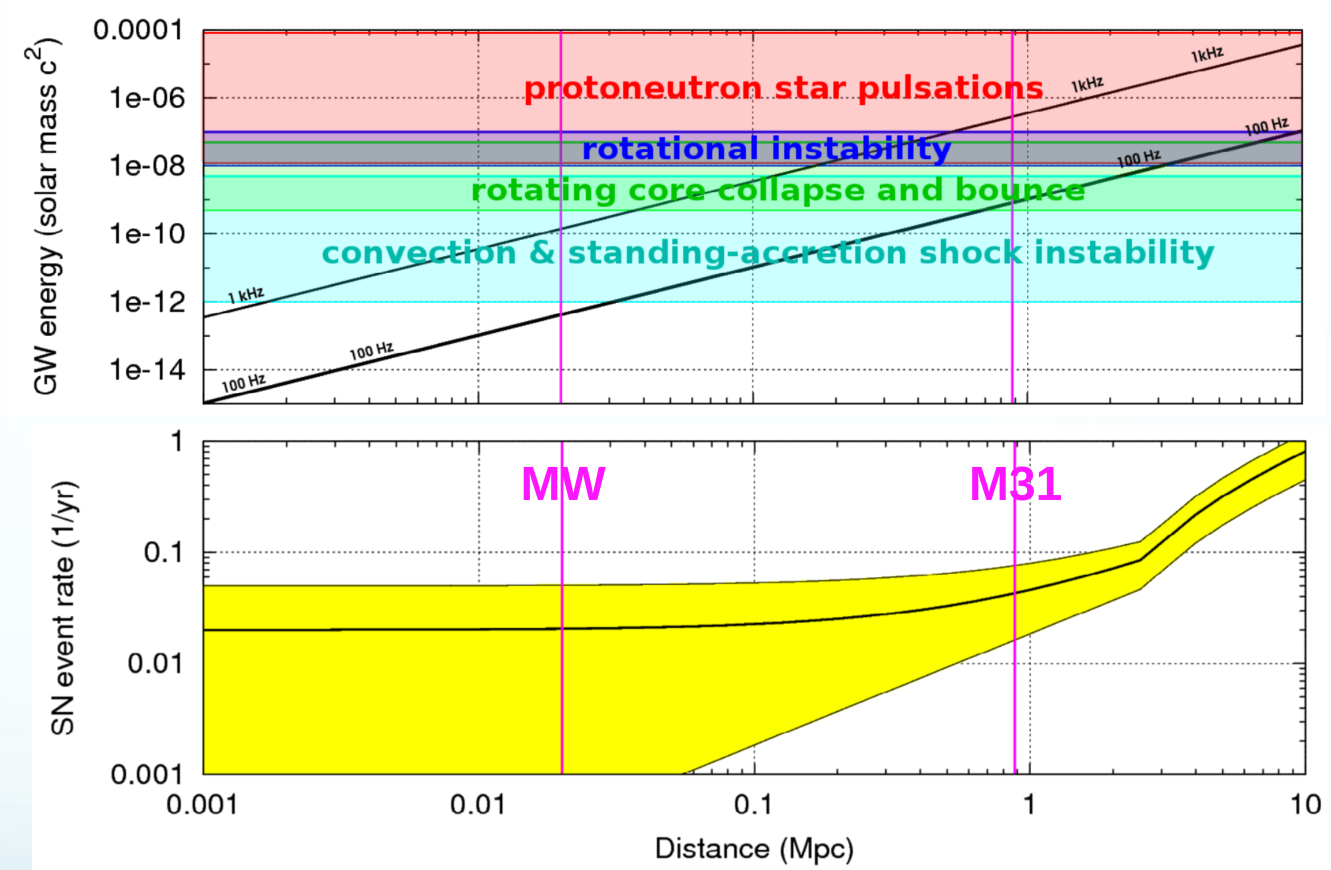
- Many different possible processes for GW emission



- Highly complex calculation : explosion mechanism still uncertain

https://www.youtube.com/watch?v=6r7YUj42SJ0
https://www.youtube.com/watch?v=yLub83WP3WA

Reach and rate of CCSN signals



Detectability in Einstein Telescope
(~10× lower noise than aLIGO)

Continuous wave GW sources

- Idealized picture : monochromatic source

$$h(t) = h_0 \exp(2\pi i\, f_0 t)$$

Constant signal power

$$\bar{P}_h = \lim_{T\to\infty} \int \mathrm{d}t\, |h(t)|^2 = \frac{1}{2} h_0^2$$

Power is concentrated at f_0

Finite length data : $\Delta f = T$ ⇒ Noise mean power ~ $\frac{S_n}{T}$

Thus S/N scales as $\sqrt{\frac{h_0 T}{S_n(f_0)}}$

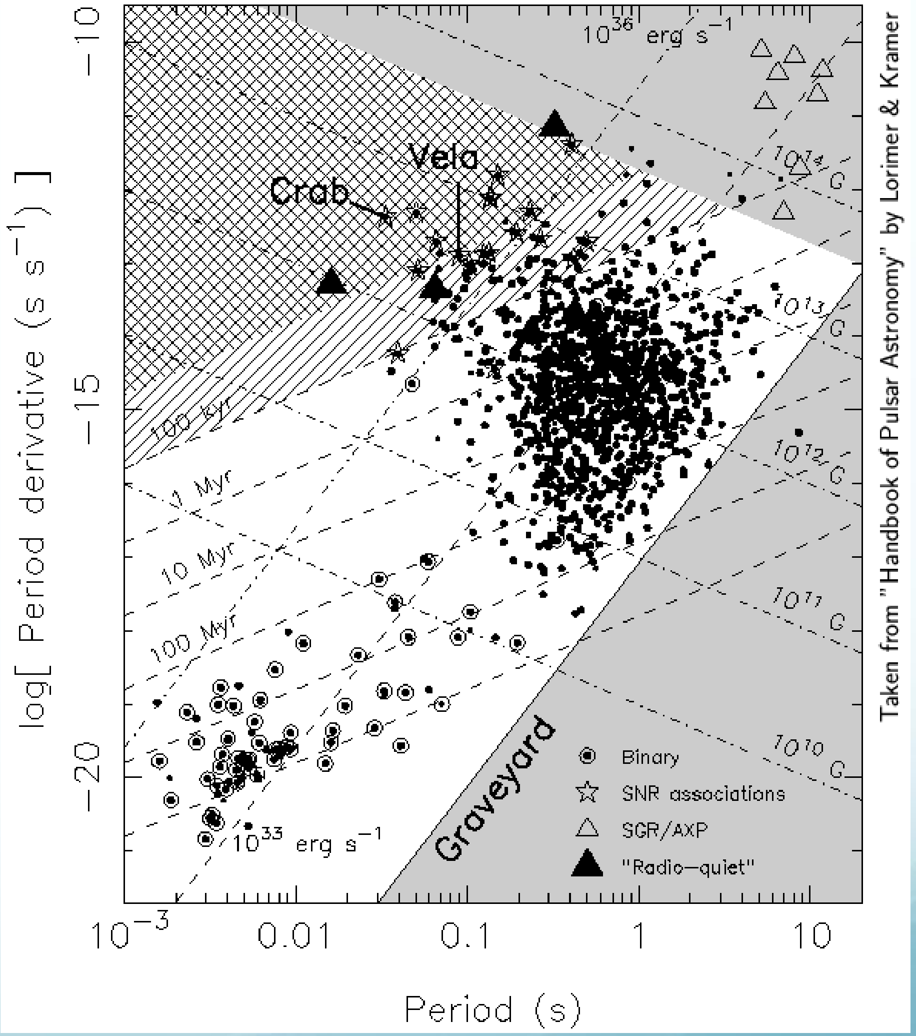
i.e. compare h_0 with effective noise level $\sqrt{S_n(f_0)/T}$

Neutron stars as CW sources

- Existence of NS known through pulsar emission
- Highly stable rotation, period ~1ms to ~10s
- If not perfectly axisymmetric, radiate GW at $f_{gw} = 2f_{rot}$
  - 'ellipticity'

$$\epsilon \equiv \frac{I_{xx} - I_{yy}}{I_{zz}}$$

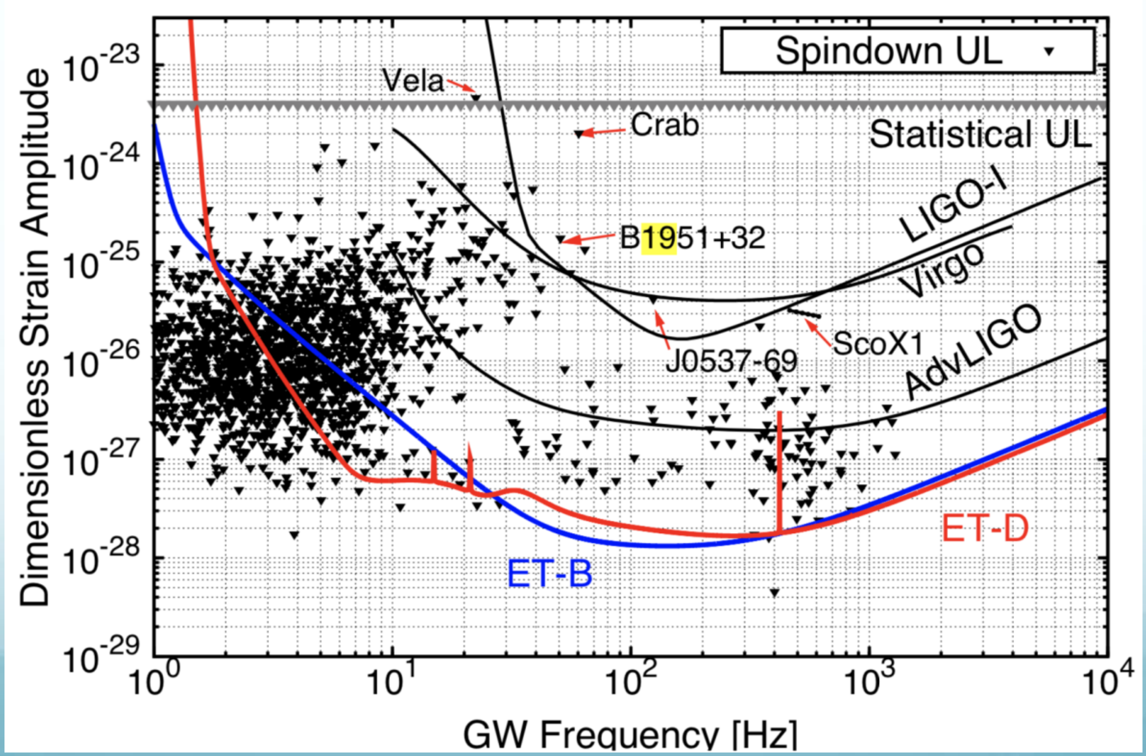
  - $I_{zz} \sim 10^{38}$ kg m^2



CW signal detectability

- Ellipticities are unknown, thought to be < 10^{-6}

$$h_0 = \frac{4\pi^2 G}{c^4}\frac{f_{gw}^2}{r} I_{zz}\epsilon \quad \simeq 10^{-24}\left(\frac{f_{gw}}{1\,\mathrm{kHz}}\right)^2 \frac{1\,\mathrm{kpc}}{r}\frac{I_{zz}}{I_0}\frac{\epsilon}{10^{-6}}$$



‘Spindown limit’ for pulsars

- Pulsars lose energy by many mechanisms – mainly EM emission (?)
- Rate of loss of rotational KE

$$\frac{dE_{\rm rot}}{dt} = I_{zz}\omega_{\rm rot}\dot{\omega}_{\rm rot}$$

Compare to energy lost via GW emission

$$\frac{dE}{dt}_{|\rm GW} = -\frac{32G}{5c^5} I_{zz}^2 \epsilon^2 \omega_{\rm rot}^6$$

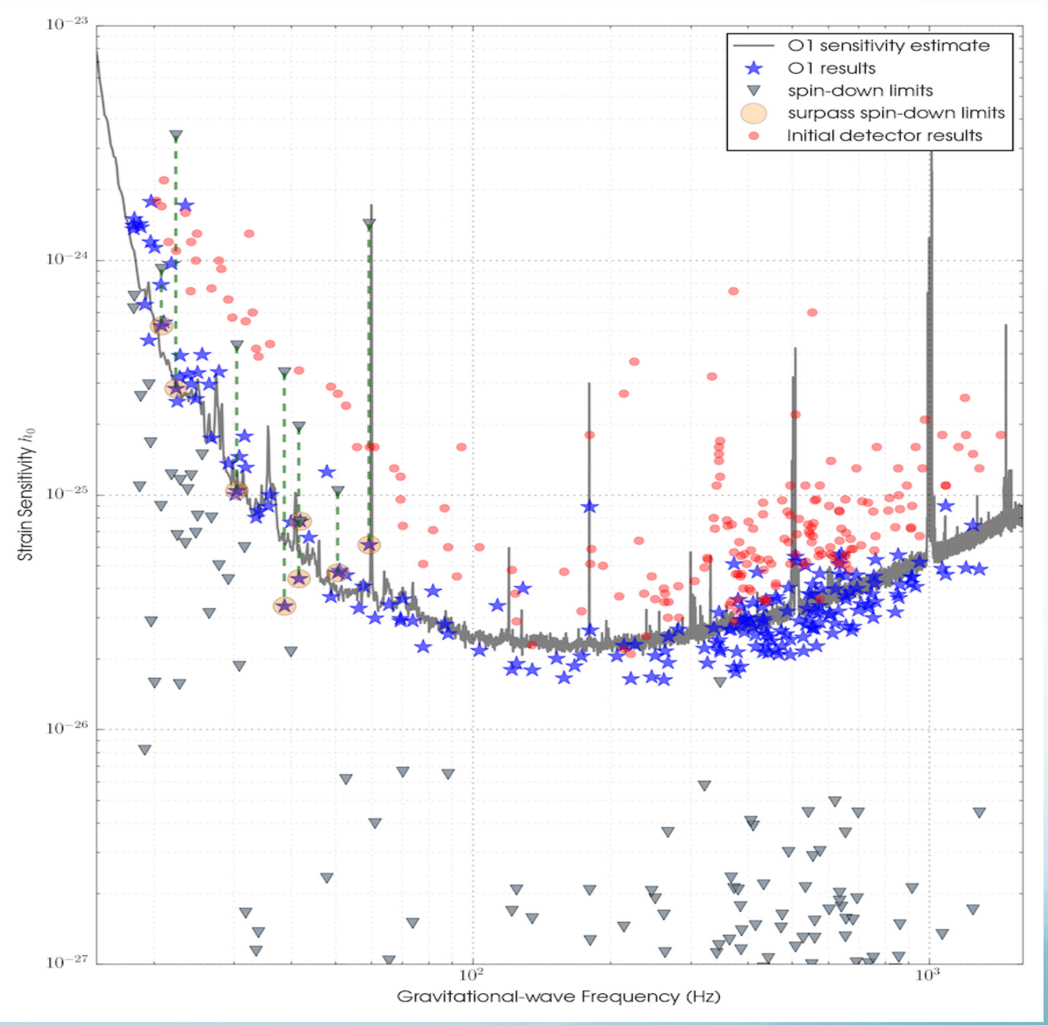
‘Spindown limit’ : set equal, find resulting ellipticity ϵ and GW amplitude

$$h_{\rm spin-down} = \frac{1}{r}\sqrt{-\frac{5}{2}\frac{G}{c^3} I_{zz} \frac{\dot{f}_{\rm GW}}{f_{\rm GW}}}$$

Recent results for known pulsars

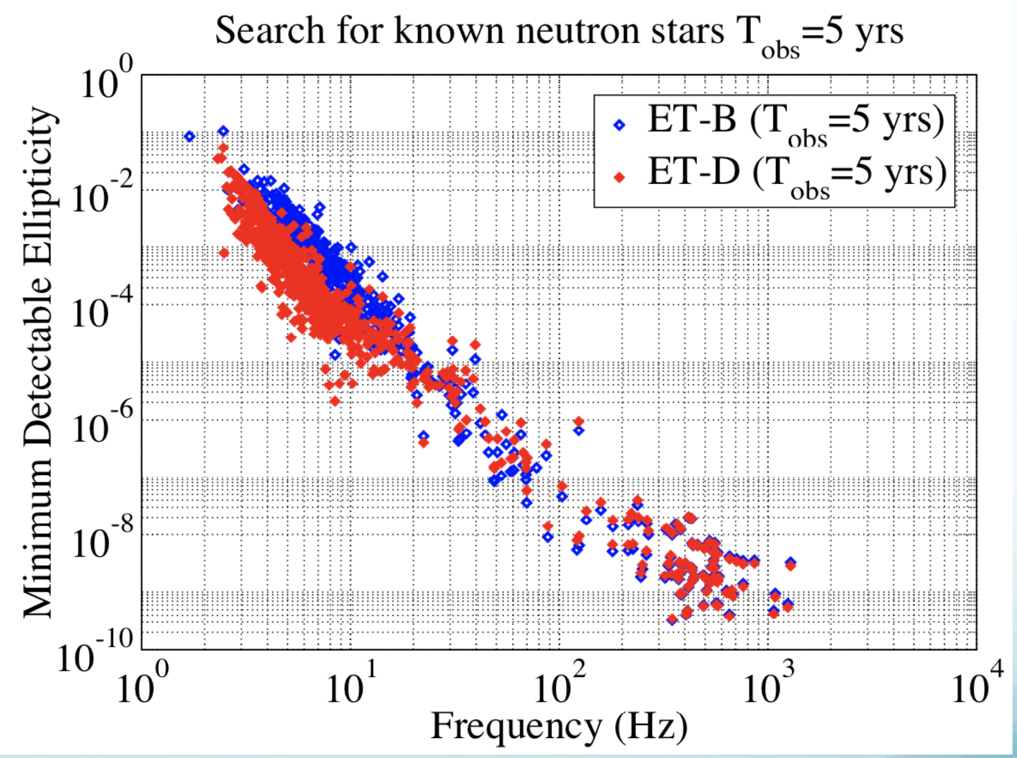
LIGO O1: beat spindown limit for some systems

⇒ *some* information about EM vs GW energy loss



How low can ϵ go?

- Empirical arguments for ellipticity to have lower limit ~10^{-9}
- Tough to detect .. even with Einstein Telescope !



Why CW search is difficult

GW from neutron stars are **not** monochromatic

- Spindown
- Doppler shift from Earth rotation $\Delta f/f \sim 10^{-6}$
- Doppler shift from Earth orbit $\Delta f/f \sim 10^{-4}$
- GR effects in Solar system – Sun potential well, Shapiro delays ...
- "Interesting" high–frequency pulsars are in binary systems (NS–NS or NS–WD)

To detect 'unknown' neutron stars need to get all of these factors right ...

Stochastic GW backgrounds

- 'Background' of GW existing through all of space
  – in all directions
  – with all polarizations
  – (in principle) at all frequencies
- Describe via amplitude coefficients $\hat{h}_A(f, \hat{\mathbf{n}})$
- Assume SB is stationary, Gaussian, isotropic, unpolarized ...

$$\langle \hat{h}_A(f, \hat{\mathbf{n}}) \hat{h}'_A(f', \hat{\mathbf{n}}') \rangle = \delta(f - f') \frac{\delta^2(\hat{\mathbf{n}}, \hat{\mathbf{n}}')}{4\pi} \delta_{AA'} \frac{1}{2} S_h(f)$$

Spectral density of S.B.

Energy density of SGWB

- Recall energy density of GW $\rho_{\rm gw} = \frac{c^2}{32\pi G}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$

  Express as fraction of 'critical energy density'

$$\Omega_{\rm gw} \equiv \frac{\rho_{\rm gw}}{\rho_c} \qquad \rho_c \equiv \frac{3c^2 H_0^2}{8\pi G}$$

- Distribution of $\rho_{\rm gw}$ over frequency is of interest

$$\rho_{\rm gw} \equiv \int_0^\infty \mathrm{d}(\log f)\,\frac{d\rho_{\rm gw}}{d(\log f)} \equiv \int_0^\infty \mathrm{d}(\log f)\,\Omega_{\rm gw}(f)$$

find : $\Omega_{\rm gw}(f) = \frac{4\pi^2}{3H_0^2} f^3 S_h(f)$

Detectable (?) SGWBs

- In single detector, SB looks like extra noise (maybe with different spectrum)
  ⇒ cannot detect ‘small’ GW backgrounds
- Need to cross-correlate 2 or more detector outputs
- SB dominated by astrophysical sources ?