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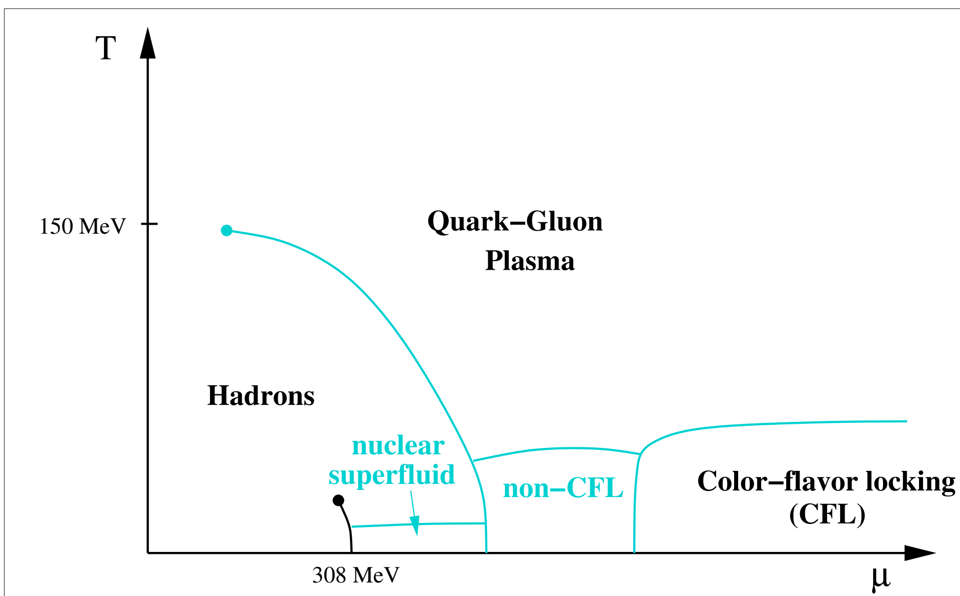
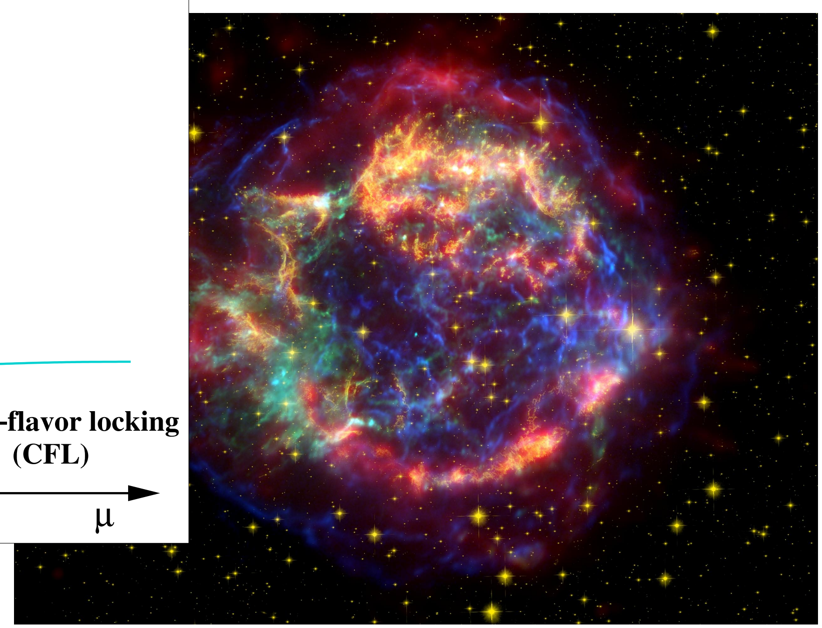
STAG RESEARCH CENTRE

Neutron stars as a laboratory for fundamental physics





Outline

- Introduction
  - Basic properties of neutron stars and QCD phase diagram
  - How to relate microscopic physics to astrophysical observables
- Dense quark matter
  - Non-interacting three-flavor quark matter
  - Brief view at interacting quark matter
- Dense nuclear matter
  - Non-interacting nuclear matter
  - Field-theoretical approach to interacting nuclear matter
- Connecting quark matter with nuclear matter
  - Nature and location of quark-hadron phase transition
  - Implications for mass/radius curve and neutron star mergers
- Transport in neutron stars (very briefly)

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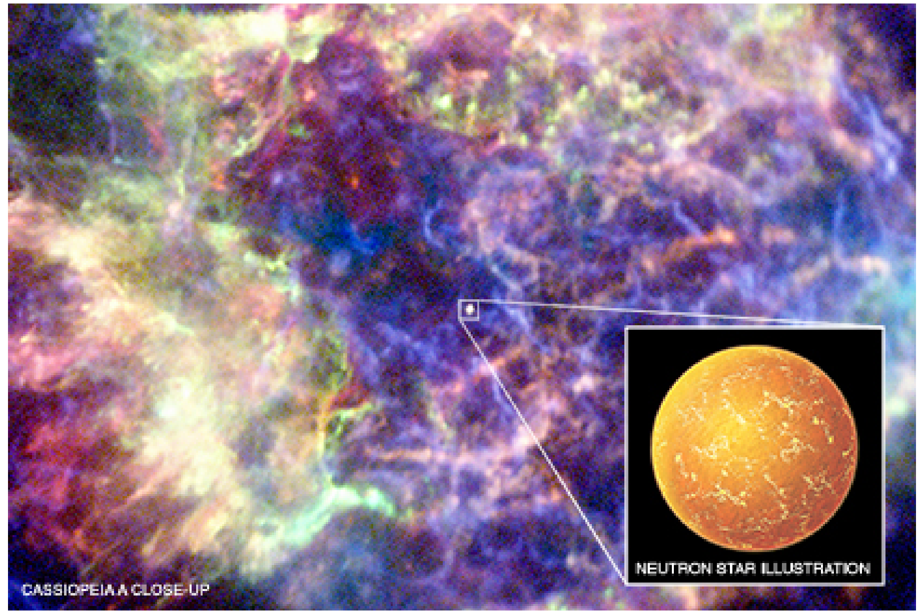
Neutron stars: densest matter in the universe

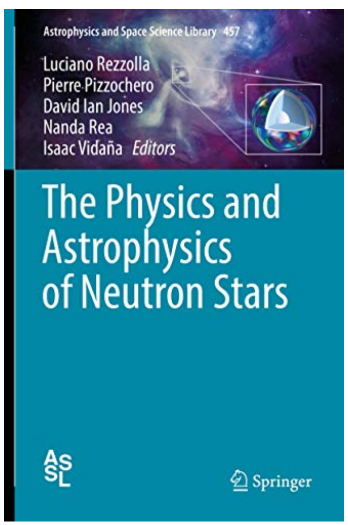
mass $\sim (1-2)M_\odot$

radius $\sim 10\,\mathrm{km}$

density $\lesssim 10\,n_0$

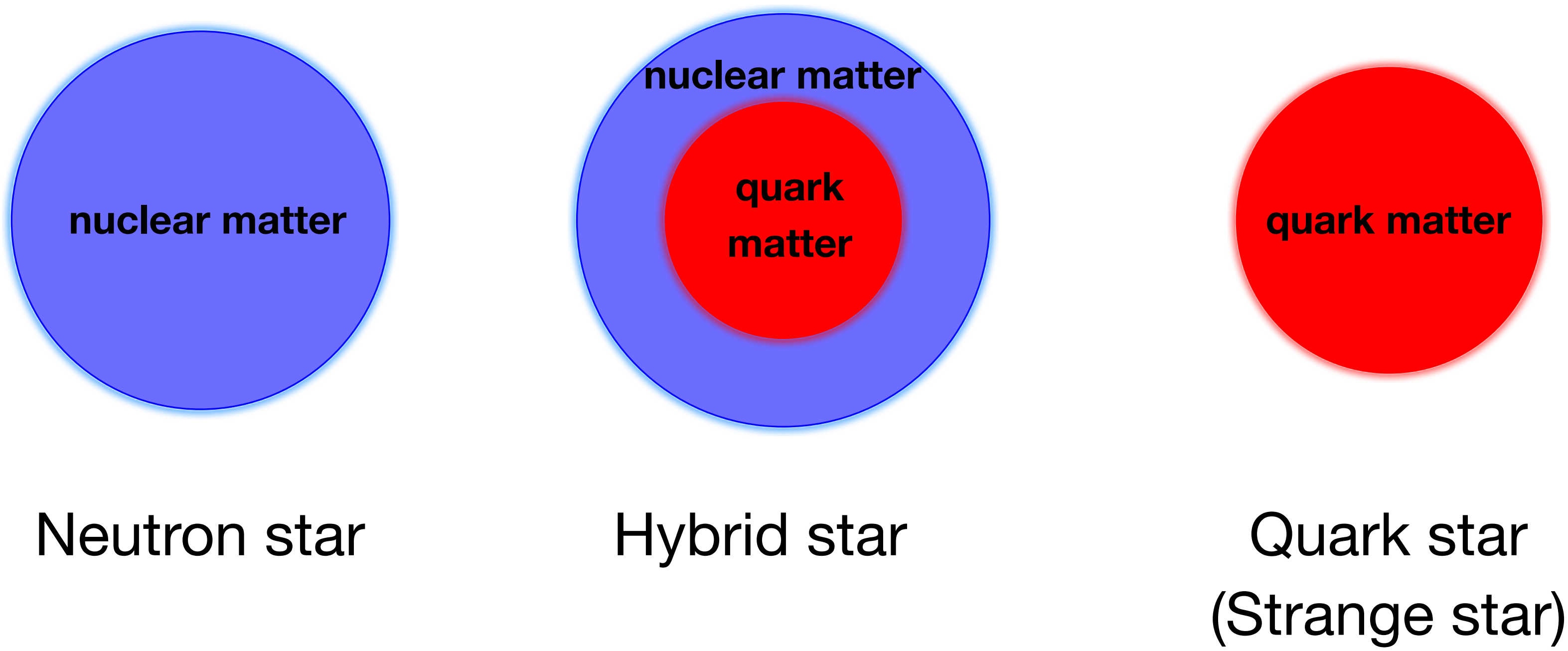


→ at these extreme densities, fundamental physics becomes relevant

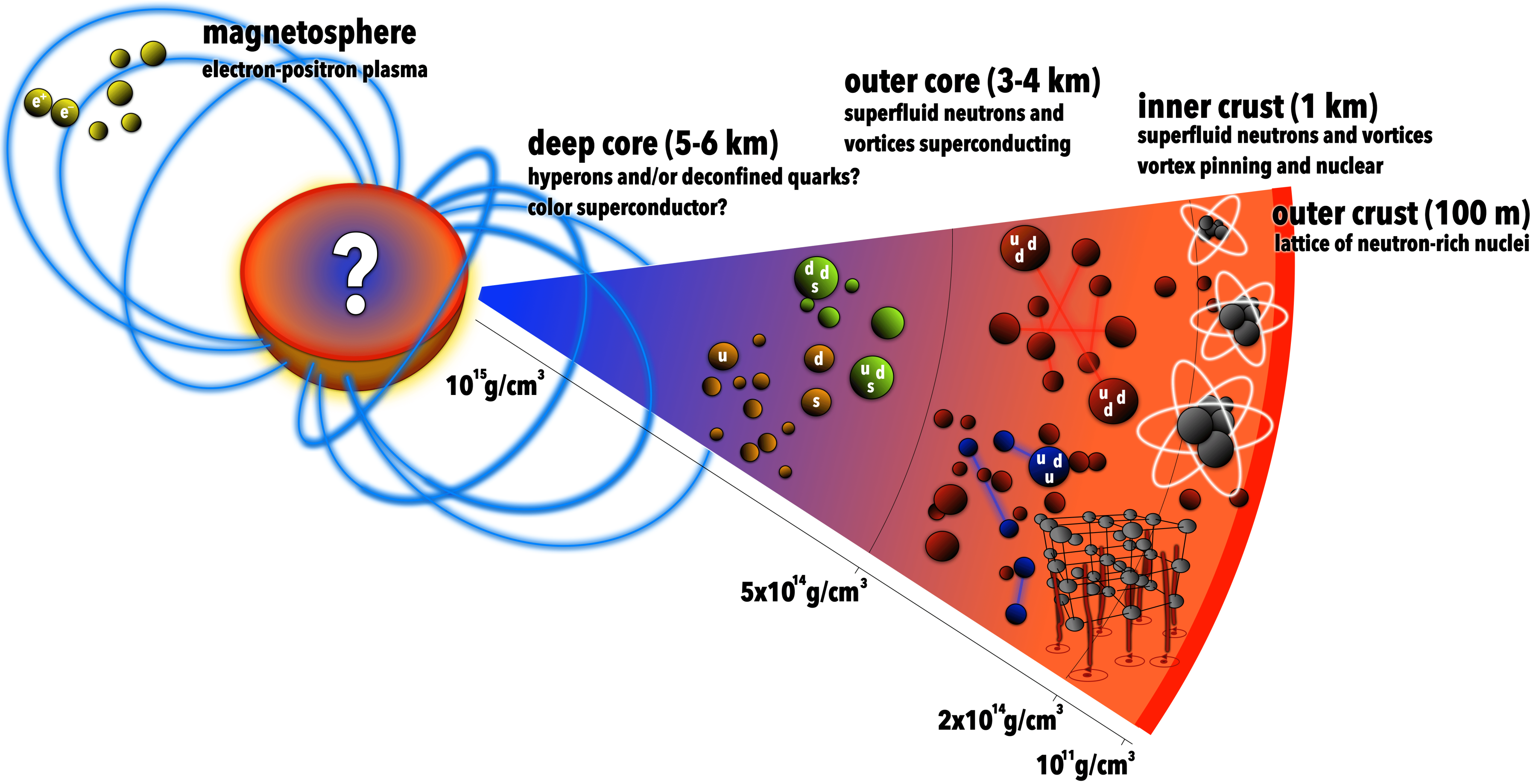


Recent collection of reviews: neutron star formation, gravitational waves (mergers and single neutron stars), equation of state, transport, ...

Compact star: simple view



Compact star: more detailed view

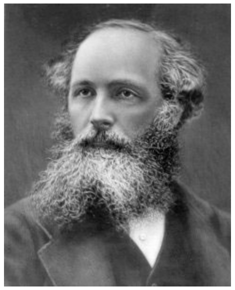


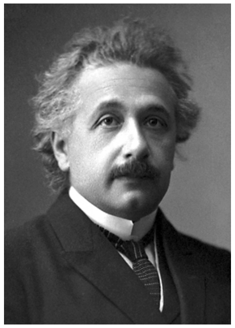
A. Watts *et al.*, PoS AASKA 14, 043 (2015)

Compact stars ...

... involve all fundamental forces

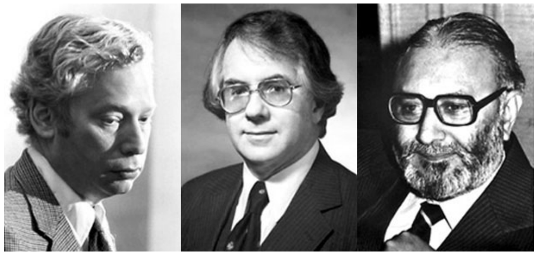
electromagnetism (magnetic field evolution, ...)





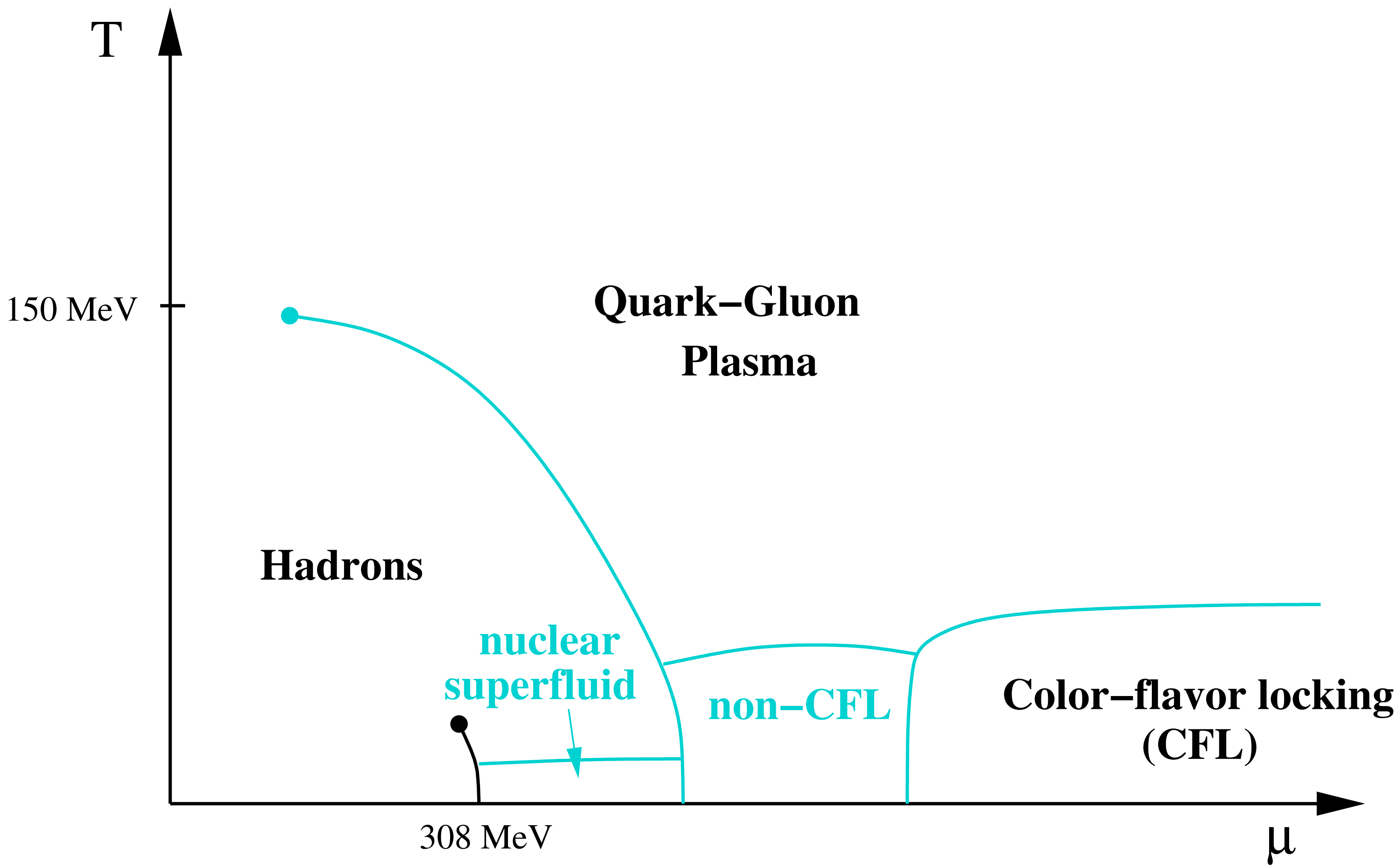
gravity (stability of the star, gravitational waves, ...)

weak interactions (neutrino emissivity, ...)

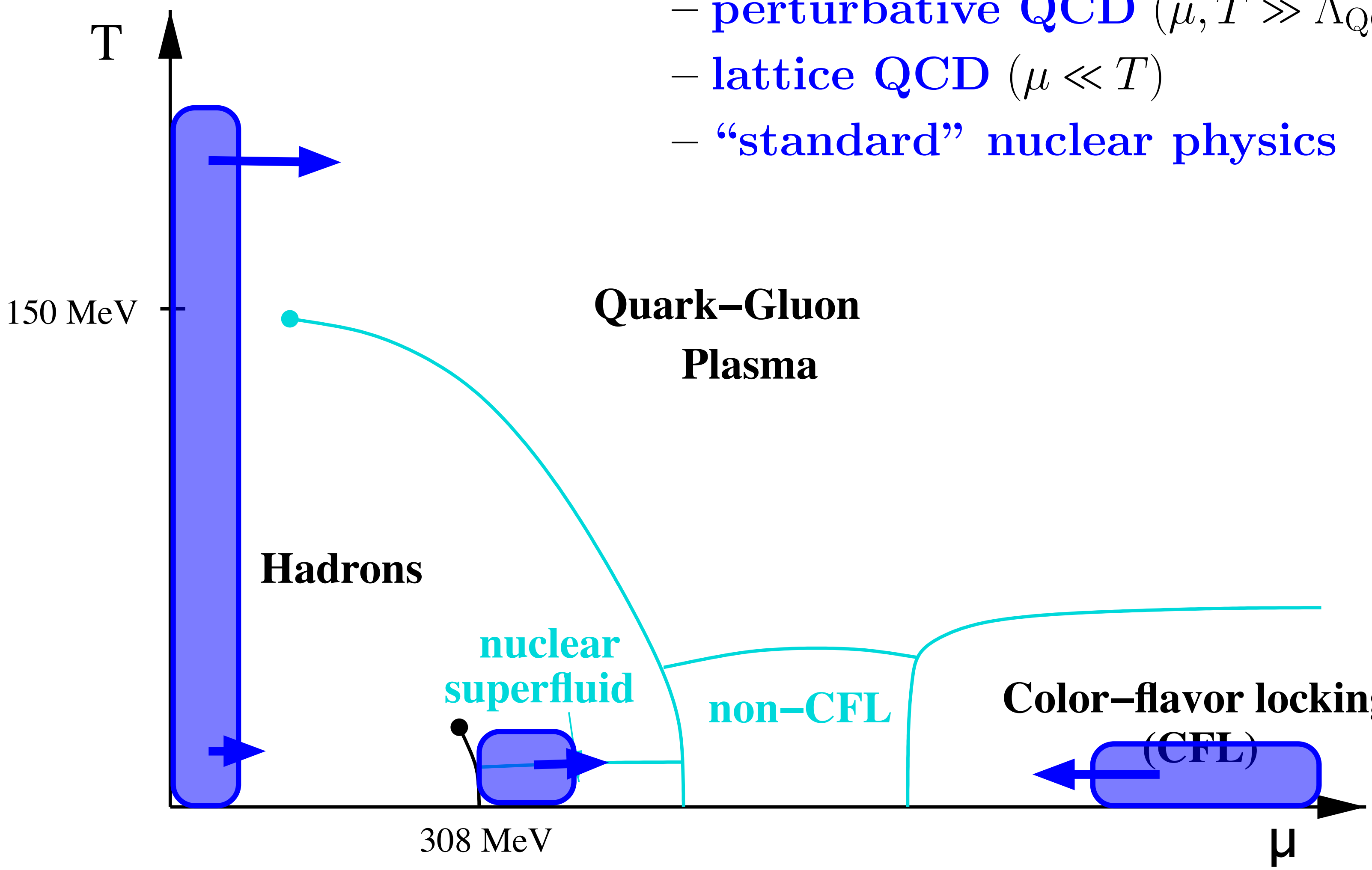


strong interactions (nuclear & quark matter, ...)

QCD at nonzero temperature and baryon density

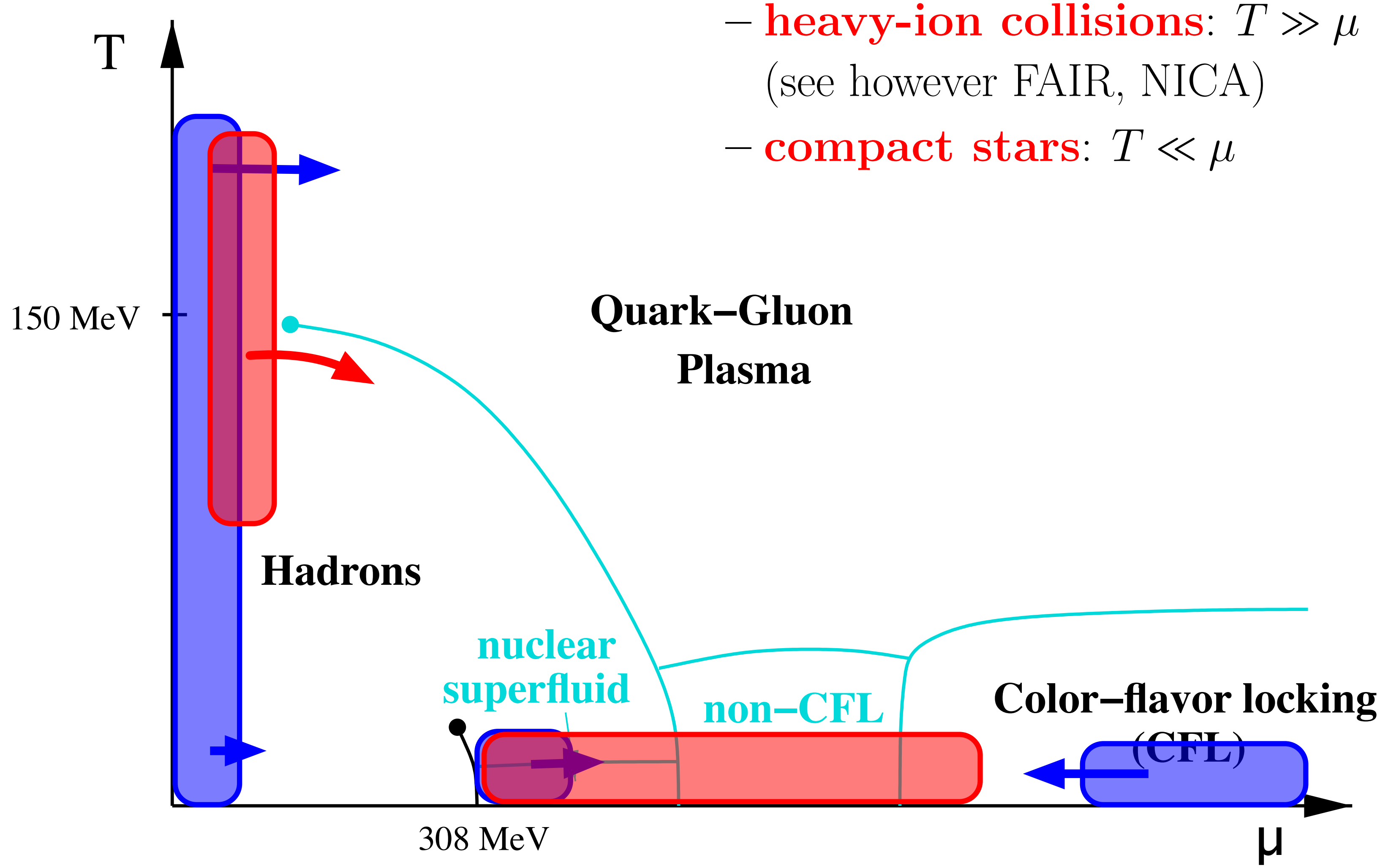


QCD at nonzero densities and temperatures



- **rigorous methods**
  - **perturbative QCD** $(\mu, T \gg \Lambda_{\mathrm{QCD}})$
  - **lattice QCD** $(\mu \ll T)$
  - **“standard” nuclear physics**

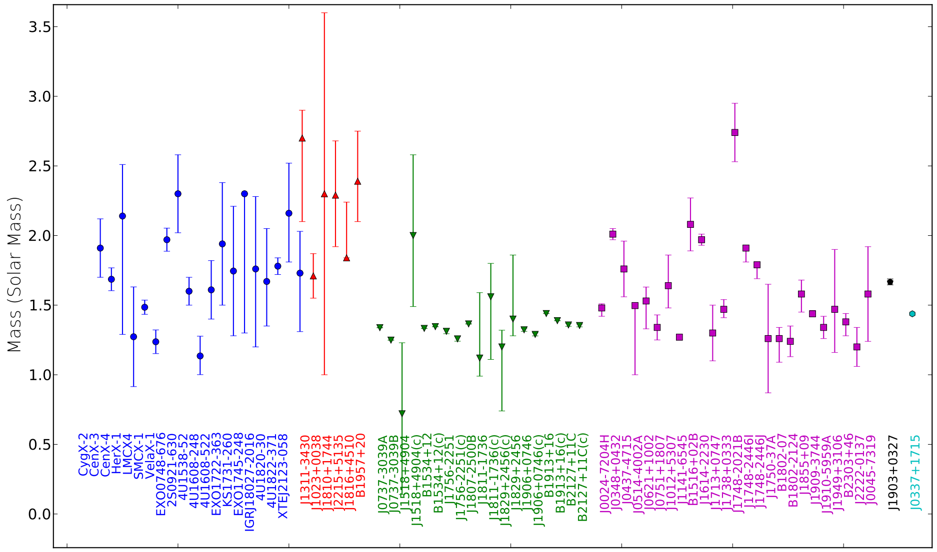
QCD at nonzero densities and temperatures



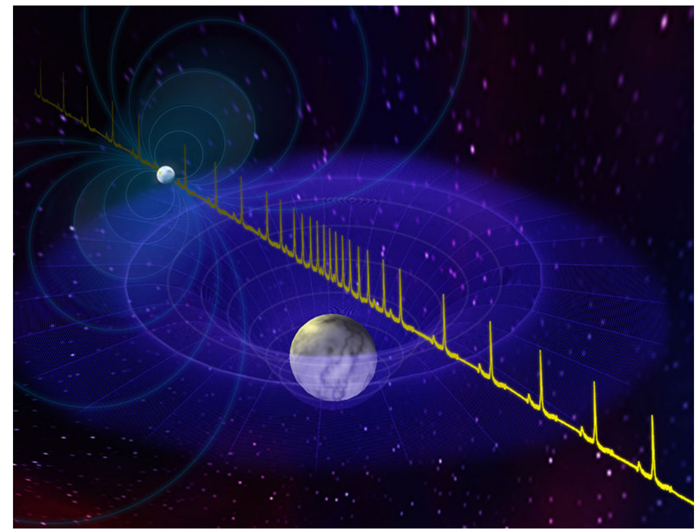
- data
  - heavy-ion collisions: $T \gg \mu$ (see however FAIR, NICA)
  - compact stars: $T \ll \mu$

Some astrophysical observations and their relation to fundamental physics

Neutron star masses (page 1/2): measurements



Neutron star masses
[A. Watts *et al.*, PoS AASKA 14, 043 (2015)]



Shapiro delay

- heaviest (accurately) known stars

$M = 1.97 \pm 0.04\, M_\odot$ P. Demorest et al., Nature 467, 1081 (2010)

$M = 2.01 \pm 0.04\, M_\odot$ J. Antoniadis et al. Science 340, 6131 (2013)

[see also $M = 2.27 \pm 0.15\, M_\odot$ M. Linares *et al.*, Astrophys. J. 859, 54 (2018)]

Neutron star masses (page 2/2): constraints on equation of state

equation of state $P(\epsilon)$ + TOV equation → $M(R)$ → maximal mass

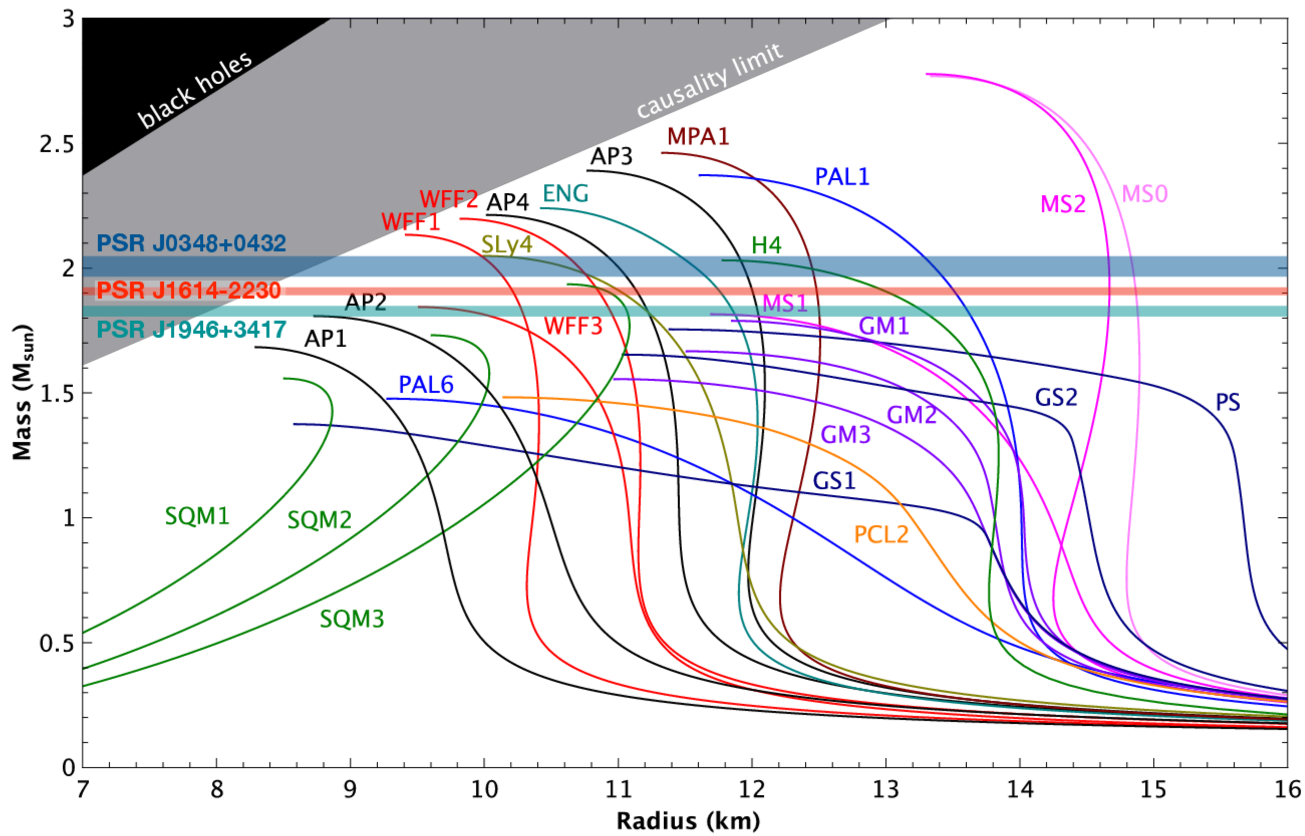
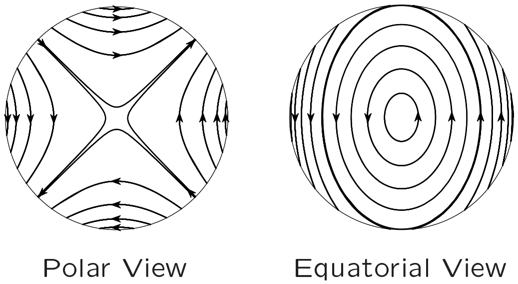


figure from http://www3.mpifr-bonn.mpg.de

r-mode instability (page 1/3): observational consequences

- r-modes: non-radial pulsation modes
  → unstable in a rotating star
  → star spins down by emitting gravitational waves
  N. Andersson, Astrophys. J. 502, 708-713 (1998)



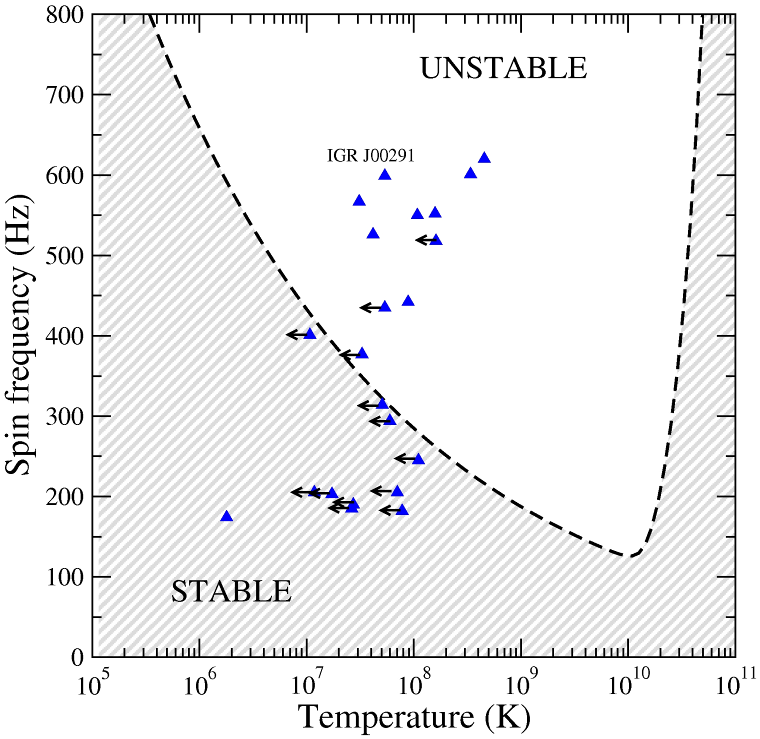
L. Lindblom, astro-ph/0101136

- observables: (i) continuous gravitational waves
  ($h \sim r$-mode saturation amplitude
  $f \sim \frac{4}{3} \times$rotation frequency of the star)

  (ii) stars should not be found in "instability window"

r-mode instability (page 2/3): puzzle

(ii) stars should not be found in instability window



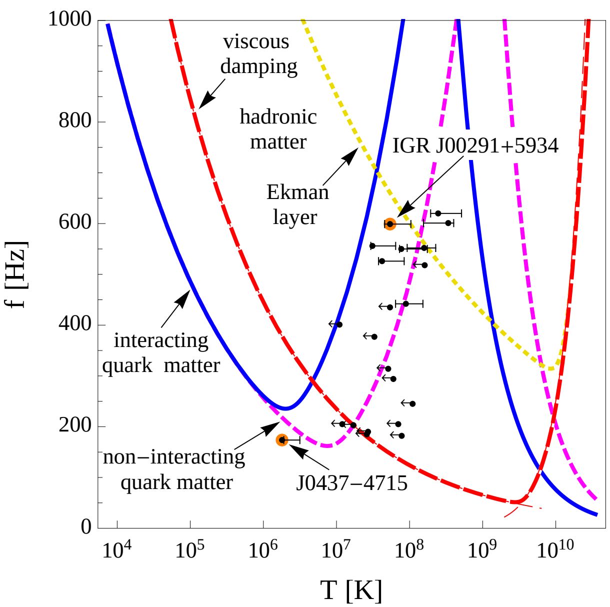
B. Haskell, et al., MNRAS 424, 93 (2012)

- instability curve from shear (low T) and bulk (high T) viscosity
- probes transport properties of nuclear or quark matter

r-mode instability (page 3/3): possible solutions

- small saturation amplitude due to cutting of superfluid vortices through superconducting flux tubes
  B. Haskell, K. Glampedakis and N. Andersson, MNRAS 441, 1662 (2014)

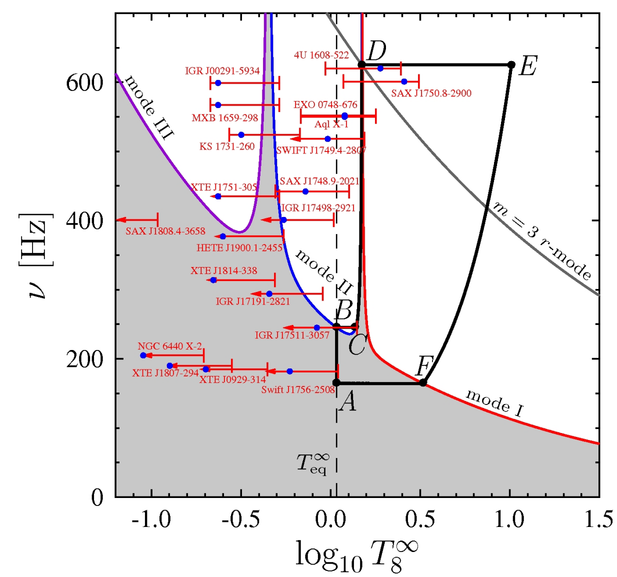
- quark matter (unpaired, non-Fermi liquid effects)



M. G. Alford, K. Schwenzer, PRL 113, 251102 (2014)

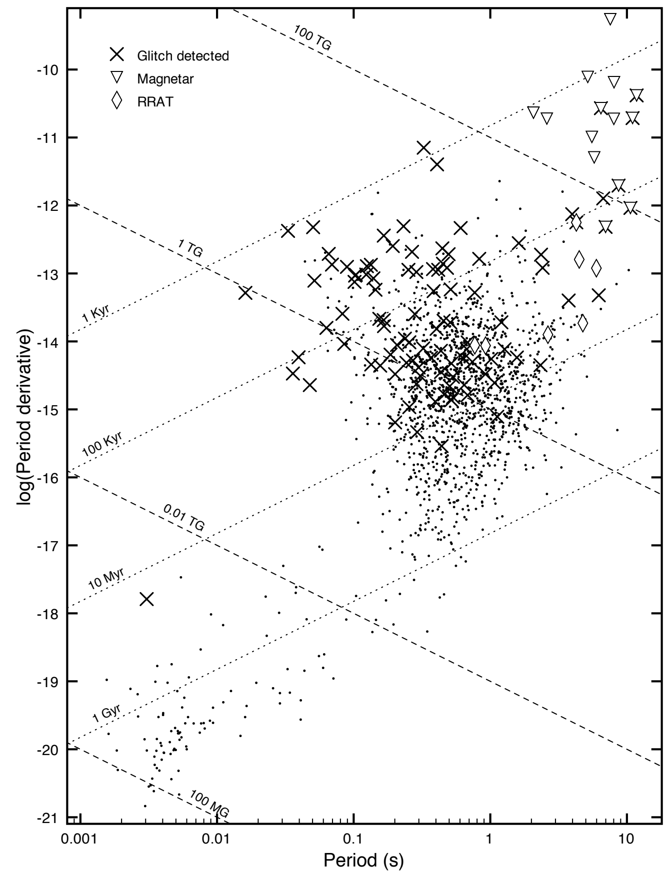
- coupling of "normal" r-mode to superfluid mode

M. E. Gusakov et al., PRL 112, 151101 (2014)



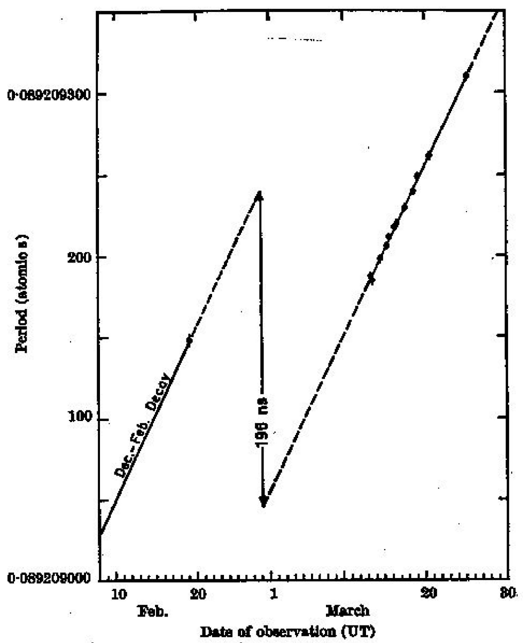
Pulsar glitches (page 1/3): observations

Espinoza *et al.*, MNRAS 414, 1679 (2011)



- pulsars usually spin-down steadily
- pulsar glitch = sudden spin-up
- first observed in Vela pulsar

  V. Radhakrishnan, R.N. Manchester, Nature 222, 228 (1969)

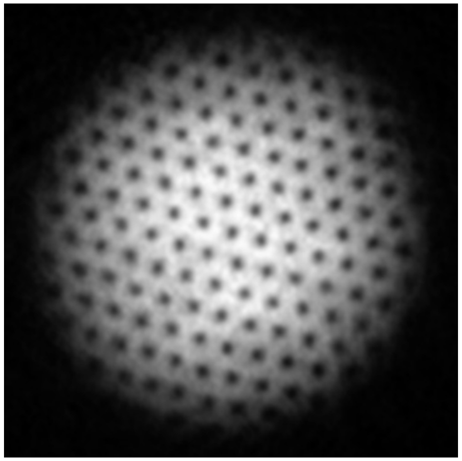


534 glitches observed in 188 pulsars (Jan 2019)

glitch table http://www.jb.man.ac.uk/pulsar/glitches.html

Pulsar glitches (page 2/3): explanation

- rotating superfluid → vortex array



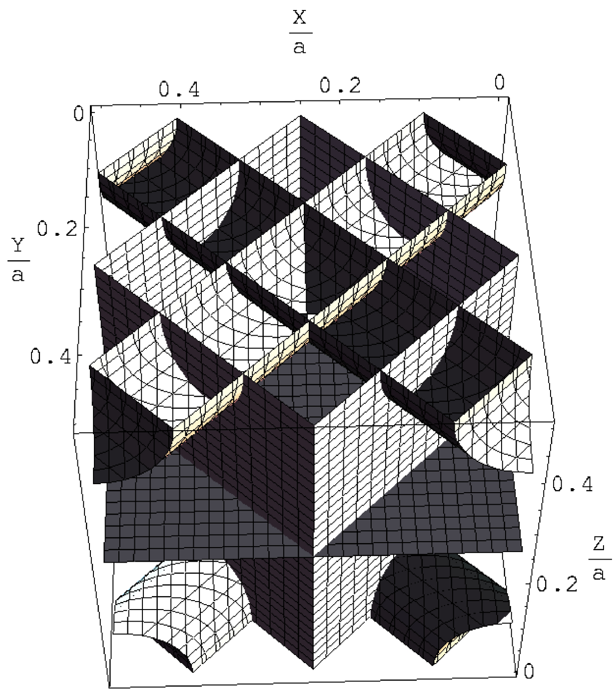
Vortices in rotating atomic superfluid
M. Zwierlein et al., Science 311, 492 (2006)

- crust: superfluid neutrons + ion lattice
- glitch mechanism:
  vortex pinning and sudden (collective) unpinning
  → sudden transfer of angular momentum
  from superfluid to rest of star
  P. W. Anderson, N. Itoh, Nature 256, 25 (1975)

Pulsar glitches (page 3/3): problems and alternatives

- huge glitches observed, $\Delta\Omega/\Omega \simeq 3 \times 10^{-5}$
  R.N. Manchester, G. Hobbs, Astrophys.J. 736, L31 (2011)

- uncompatible with superfluid entrainment in the crust?
  "The crust is not enough" N. Andersson, et al., PRL 109, 241103 (2012)
  "The crust may be enough" J. Piekarewicz, et al., PRC 90, 015803 (2014)



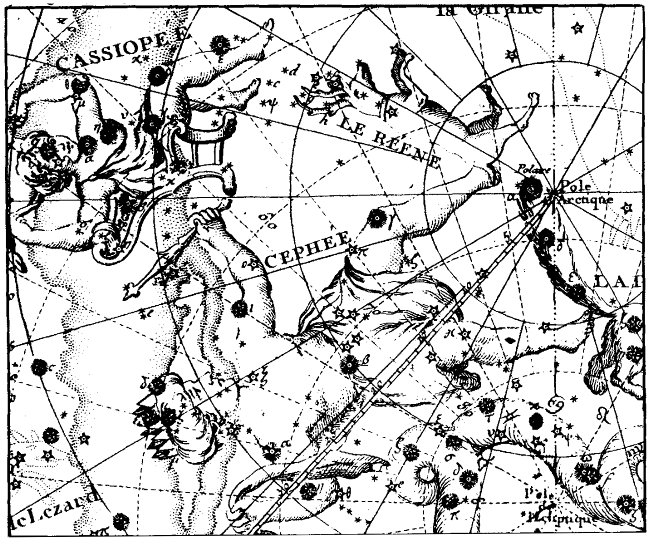
Crystalline CFL

- what triggers the collective unpinning? superfluid two-stream instability?
  N. Andersson, G.L. Comer, R. Prix, PRL 90, 091101 (2003)
  A. Schmitt, PRD 89, 065024 (2014)
  A. Haber, A. Schmitt, S. Stetina, PRD 93, 025011 (2016)

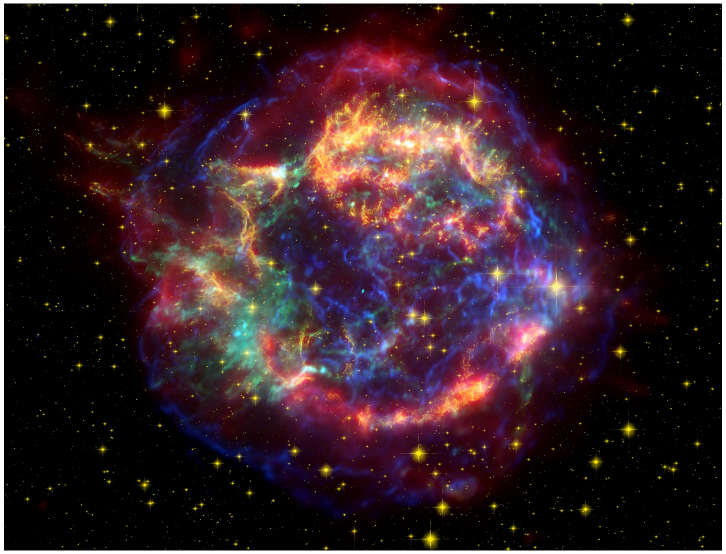
- alternative mechanism: crystalline CFL quark matter in the core?
  K. Rajagopal and R. Sharma, PRD 74, 094019 (2006)
  M. Mannarelli *et al.*, PRD 76, 074026 (2007)

Rapid cooling in Cas A (page 1/2)

- young compact star (~ 340 yr) at center of supernova remnant Cassiopeia A (Cas A)
  [supernova possibly observed historically D.W. Hughes, Nature 285, 132 (1980)]
  [compact star observed in 1999 H. Tananbaum, IAUC 7246, 1 (1999)]



From Atlas Céleste de Flamsteed, l'Académie Royale de Science, Paris, 1776



Cas A, combined image from Spitzer and Hubble Telescopes and Chandra X-ray

- rapid cooling observed: temperature decrease of 1% - 3% over 10 yr C. O. Heinke and W. C. G. Ho, Astrophys. J. 719, L167 (2010); K.G. Elshamouty, *et al.*, Astrophys. J. 777, 22 (2013)

Rapid cooling in Cas A (page 2/2)

- superfluidity: neutrino emission suppressed at low T
- Cooper pair breaking and formation → enhancement possible just below T_c
- rapid cooling due to transition to neutron superfluidity (in the presence of proton superc.)

  D. Page, *et al.* PRL 106, 081101 (2011)

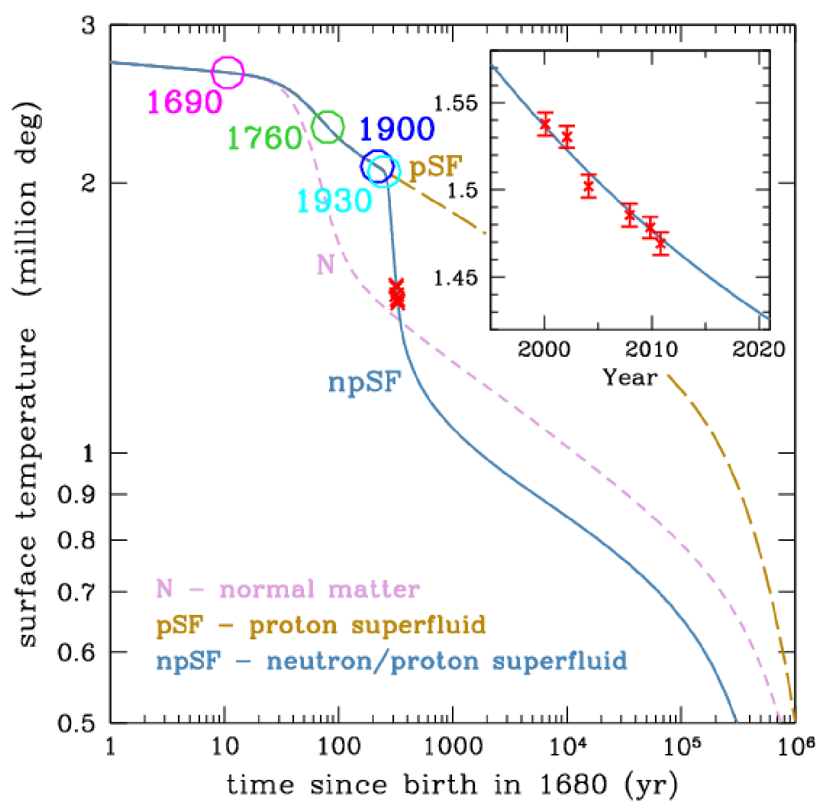
  P. S. Shternin, *et al.* MNRAS 412, L108 (2011)

  → "measurement" of

$$T_c \simeq (5-8) \times 10^8\,\mathrm{K}$$

- alternative explanation: 2SC → LOFF transition in quark matter

  A. Sedrakian, A&A 555, L10 (2013)



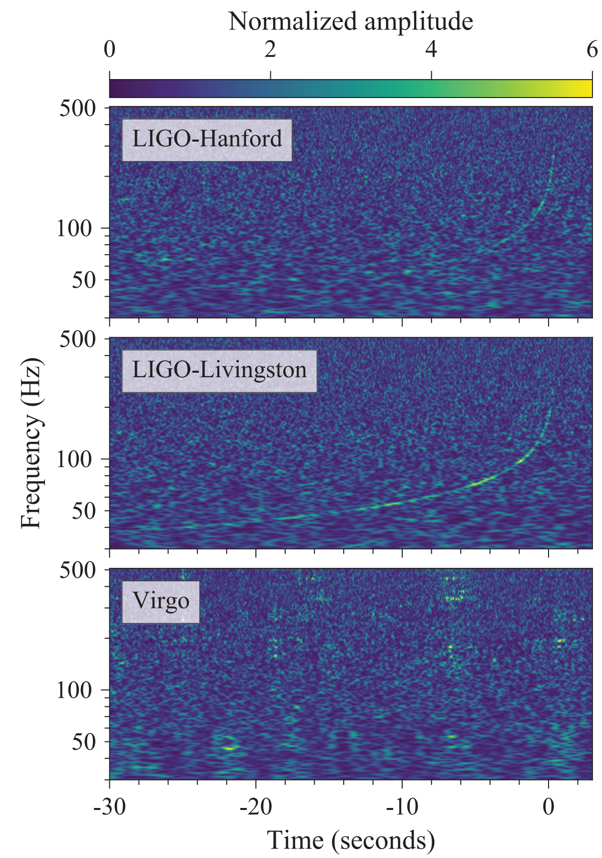
W.C.G. Ho, *et al.*, PoS ConfinementX, 260 (2012)

Gravitational waves (page 1/3: detection)

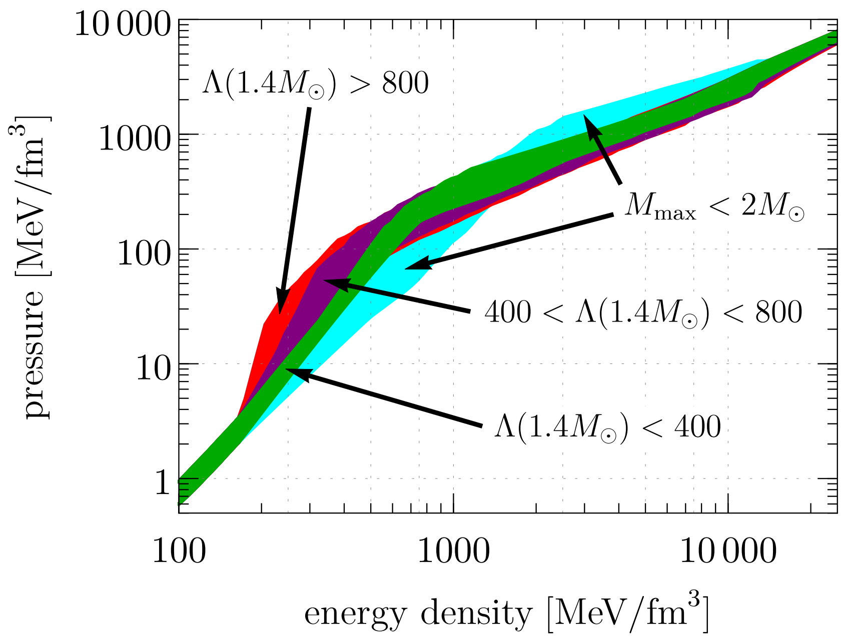
- gravitational waves: first detected by LIGO from black hole merger 2015 (Nobel Prize 2017)

- neutron stars as potential sources for gravitational waves:
  P. Lasky, Publ. Astr. Soc. of Australia, 32, E034 (2015)
  K. Glampedakis, L. Gualtieri, arXiv:1709.07049 [astro-ph.HE]
  - neutron star mergers
  - "mountains" (ellipticity + rotation)
  - oscillations (r-mode)

Gravitational waves (page 2/3: neutron star merger)



- gravitational waves detected from neutron star merger
  LIGO and Virgo, PRL 119, 161101 (2017)
  → upper limit for tidal deformability Λ

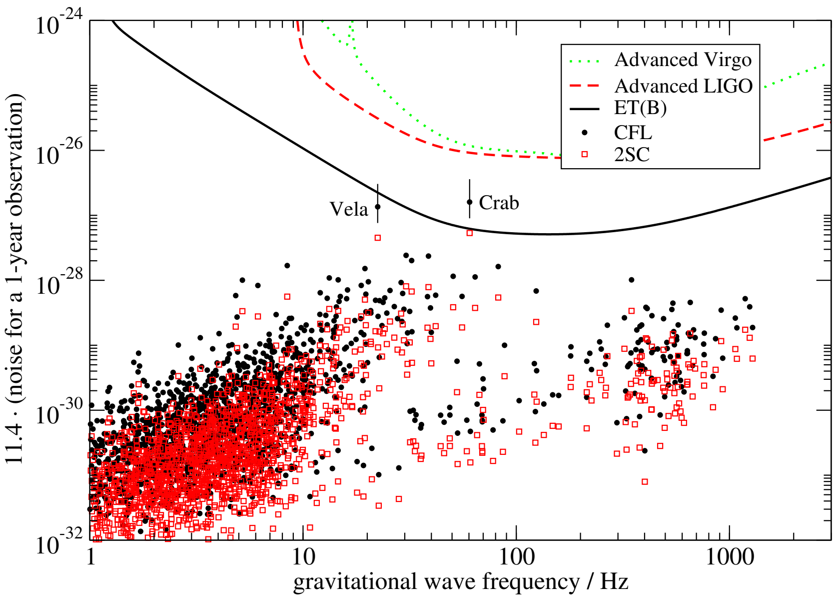


- 2-solar-mass stars:
  EoS must be sufficiently stiff
- upper limit for Λ:
  EoS must not be too stiff
  (stiff EoS → large stars → large Λ)
- constrain family of EoSs
  E. Annala *et al.*, PRL 120, 172703 (2018)

Gravitational waves (page 3/3: mountains)

- ellipticity of star (”mountains”):
  sustained by crystalline structures (e.g., crust of the star, mixed phases, LOFF phase, array of magnetic flux tubes, ...)
- misalignment of magnetic and rotational axis → gravitational waves

- for instance enhanced ellipticity of compact stars with flux tubes in quark matter core
  K. Glampedakis, D. I. Jones and L. Samuelsson, PRL 109, 081103 (2012)
  A. Haber and A. Schmitt, J. Phys. G 45, 065001 (2018)



Summary: compact stars are laboratories for fundamental physics

- matter inside compact stars is cold and dense ($\mu \gg T$) and very challenging to describe theoretically

- observations can be related to microscopic physics

mass/radius	$\leftrightarrow$	equation of state
r-mode instability	$\leftrightarrow$	shear/bulk viscosity
pulsar glitches	$\leftrightarrow$	superfluidity
cooling	$\leftrightarrow$	neutrino emissivity
grav. waves (mergers)	$\leftrightarrow$	tidal deformability (viscosity?)
grav. waves (r-mode instab.)	$\leftrightarrow$	shear/bulk viscosity
grav. waves (mountains)	$\leftrightarrow$	crystalline structures

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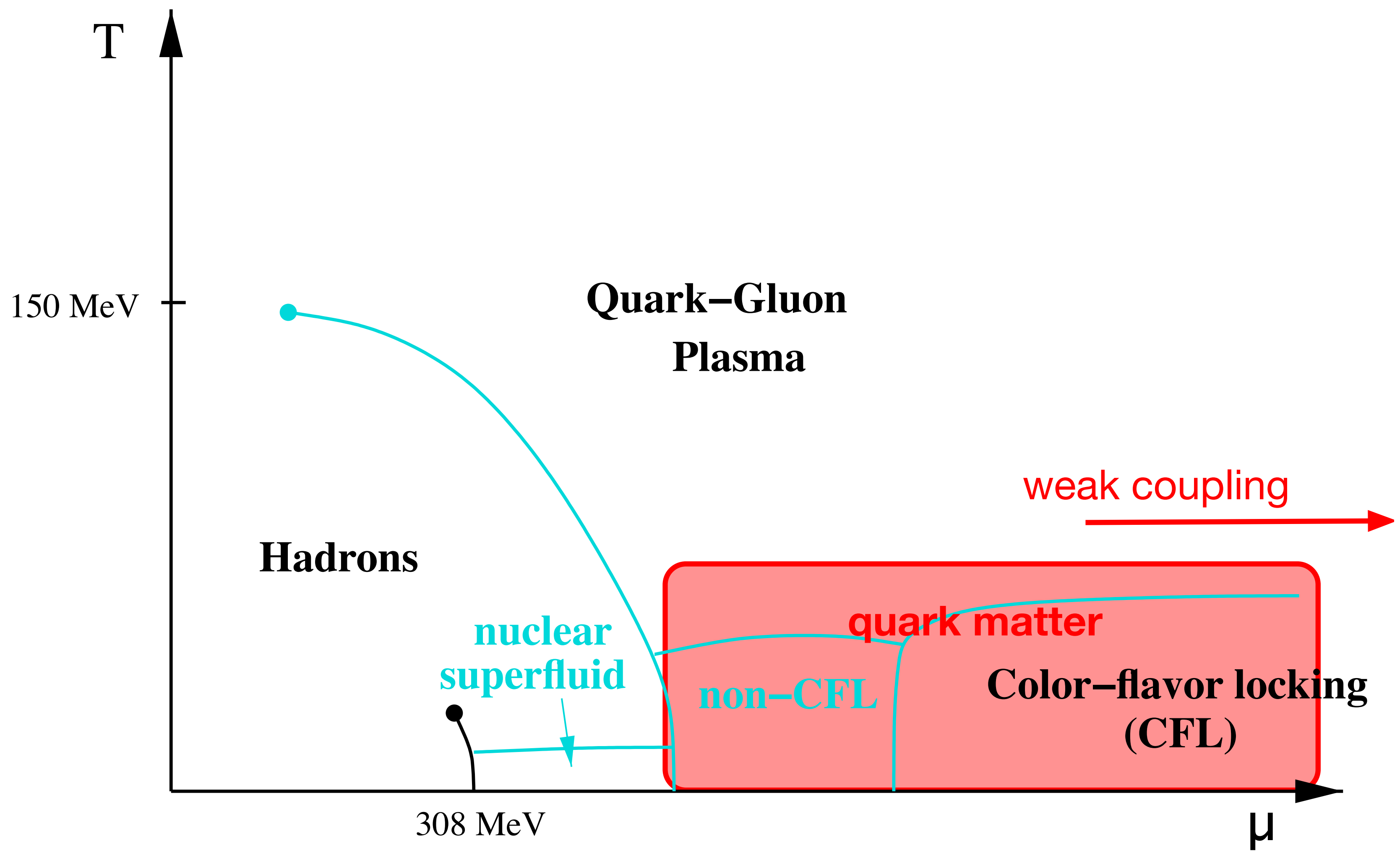
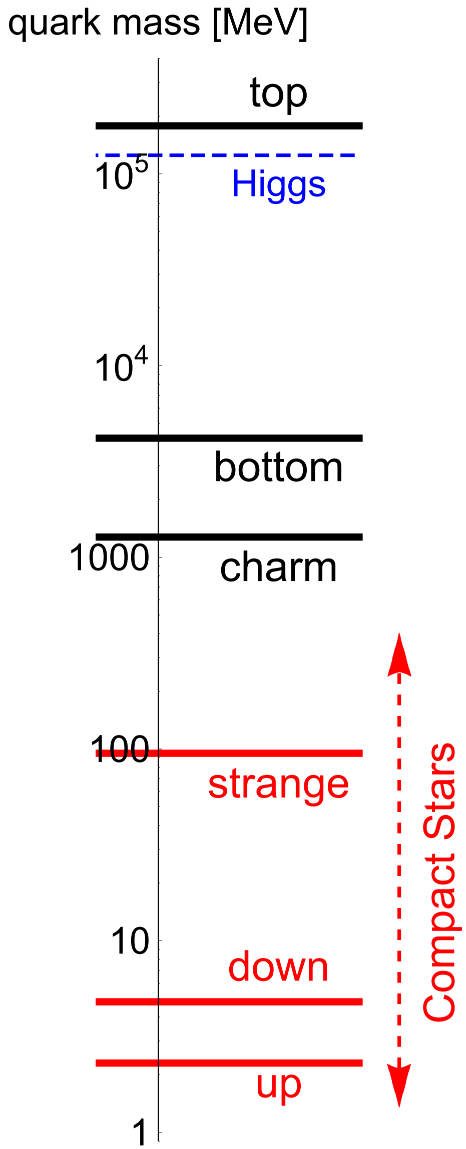
Noninteracting quark matter

see Sec. 2.2 in A. Schmitt, Lect. Notes Phys. 811, 1 (2010)



Three-flavor quark matter



- quark chemical potential in compact stars $300\,\text{MeV} \lesssim \mu \lesssim 500\,\text{MeV}$

> $\Rightarrow$ three-flavor quark matter
> (ignore c,b,t)

- $0 \simeq m_u \simeq m_d \ll \mu$, but m_s not negligible

- remember electric charges:

$$q_u = \frac{2}{3}e\,, \qquad q_d = q_s = -\frac{1}{3}e$$

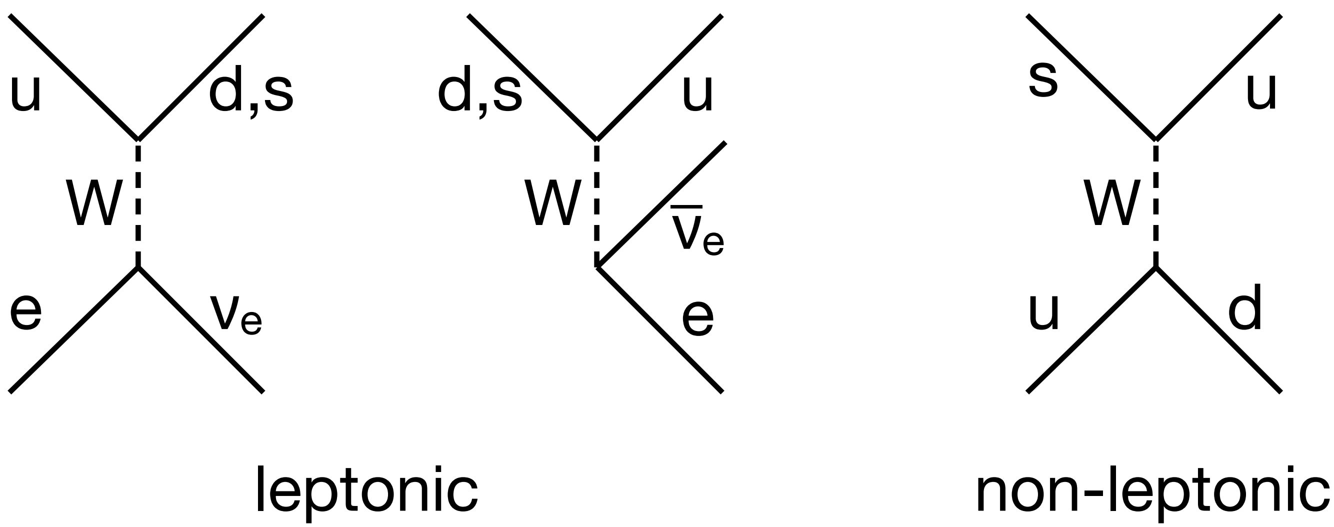
β-equilibrium and electric charge neutrality (page 1/2)

- pure QCD: quark chemical potentials μ_u, μ_d, μ_s independent
- include weak interactions: μ_u, μ_d, μ_s related through β-equilibrium

$$u + e \to d + \nu_e \qquad d \to u + e + \bar{\nu}_e$$

$$u + e \to s + \nu_e \qquad s \to u + e + \bar{\nu}_e$$

$$s + u \leftrightarrow d + u$$



β-equilibrium and electric charge neutrality (page 2/2)

- β-equilibrium

$$\mu_d = \mu_e + \mu_u\,, \qquad \mu_s = \mu_e + \mu_u$$

(this automatically implies $\mu_d = \mu_s$)

- electric charge neutrality

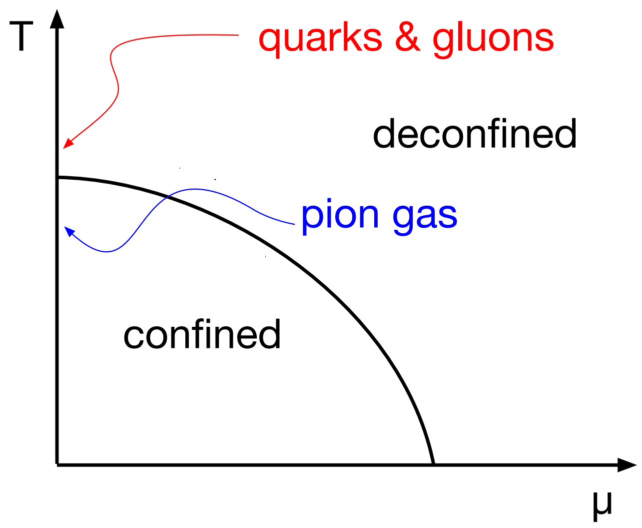
$$\sum_{f=u,d,s} q_f n_f - n_e = 0$$

(n_e electron density, q_f quark charges)

Bag model (page 1/2)

A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, PRD 9, 3471 (1974)

- for now consider $\mu = 0$ and nonzero T



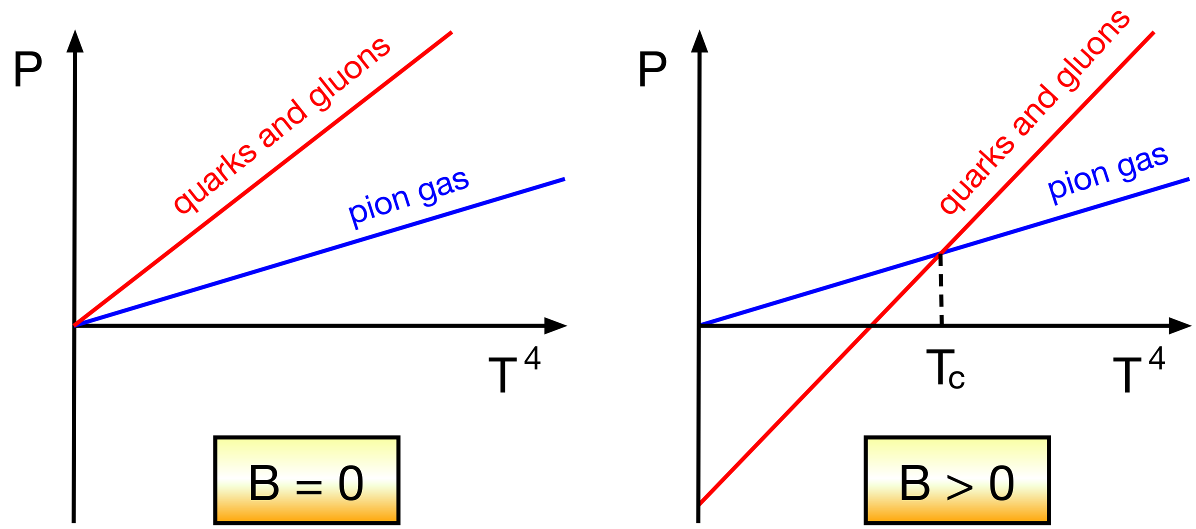
$$P_\pi = 3\,\frac{\pi^2 T^4}{90}$$

$$P_{q,g} = 37\,\frac{\pi^2 T^4}{90} - B$$

$$P_{\text{boson}} \simeq -T \int \frac{d^3\mathbf{k}}{(2\pi)^3} \ln\left(1 - e^{-k/T}\right) = \frac{\pi^2 T^4}{90}$$

$$P_{\text{fermion}} \simeq T \int \frac{d^3\mathbf{k}}{(2\pi)^3} \ln\left(1 + e^{-k/T}\right) = \frac{7}{8}\frac{\pi^2 T^4}{90}$$

Bag model (page 2/2)



- without bag constant B: quarks and gluons "too favored"
- bag constant B is a (very crude!) model for confinement: pressure of the "bag" counterbalances microscopic pressure of quarks

$$P + B = \sum_f P_f \,, \qquad \epsilon = \sum_f \epsilon_f + B$$

Equation of state (page 1/2)

- pressure

$$\sum_{i=u,d,s,e} P_i = \frac{\mu_u^4}{4\pi^2} + \frac{\mu_d^4}{4\pi^2} + \frac{3}{\pi^2}\int_0^{k_{F,s}} dk\, k^2 \left(\mu_s - \sqrt{k^2 + m_s^2}\right) + \frac{\mu_e^4}{12\pi^2}$$

with quark Fermi momenta $k_{F,u} \simeq \mu_u$, $k_{F,d} \simeq \mu_d$, $k_{F,s} = \sqrt{\mu_s^2 - m_s^2}$

and electron contribution $k_{F,e} \simeq \mu_e$

- write chemical potentials in terms of average quark chemical potential μ and μ_e (β-equilibrium)

$$\mu_u = \mu - \frac{2}{3}\mu_e\,, \qquad \mu_d = \mu + \frac{1}{3}\mu_e\,, \qquad \mu_s = \mu + \frac{1}{3}\mu_e$$

- solve charge neutrality

$$0 = \frac{\partial}{\partial \mu_e}\sum_{i=u,d,s,e} P_i = -\frac{2}{3}n_u + \frac{1}{3}n_d + \frac{1}{3}n_s + n_e$$

to lowest order in the strange quark mass

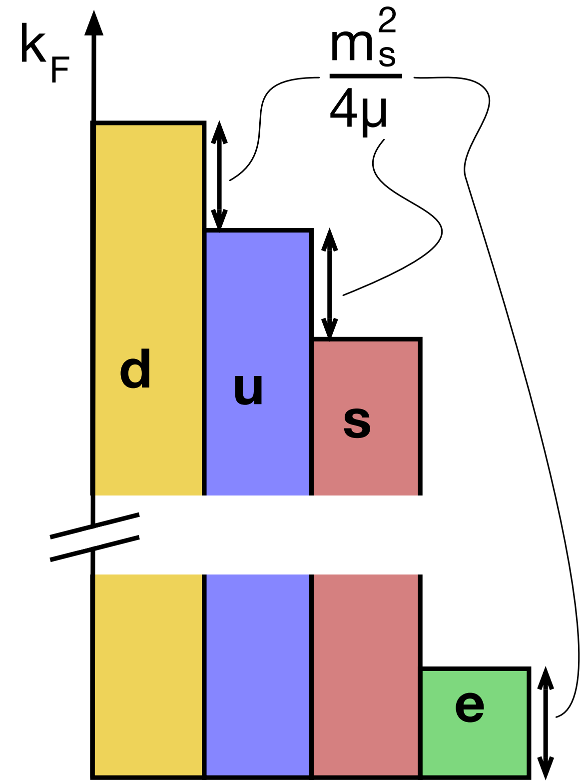
Equation of state (page 2/2)

$$\Rightarrow \qquad \mu_e \simeq \frac{m_s^2}{4\mu}$$

equation of state
(recall $P = -\epsilon + \mu n + sT$):

$$P(\epsilon) \simeq \frac{\epsilon - 4B}{3} - \frac{m_s^2\sqrt{\epsilon - B}}{3\pi}$$

sound speed $\quad c_s^2 = \frac{\partial P}{\partial \epsilon} \simeq \frac{1}{3}\left(1 - \frac{m_s^2}{3\mu^2}\right)$



- asymptotically large densities ($\mu \gg m_s$):
  equal Fermi surfaces, quark matter "automatically" neutral
- realistic densities: splitting of Fermi surfaces
  → "stressed" Cooper pairing

Including interactions and Cooper pairing

- including interactions between (unpaired) quarks perturbatively $\rightarrow$ corrections in powers of α_s

  G. Baym and S. A. Chin, PLB 62, 241 (1976)

  B. A. Freedman and L. D. McLerran, PRD 16, 1169 (1977)

$$k_F = \mu\left(1 - \frac{2\alpha_s}{3\pi}\right)$$

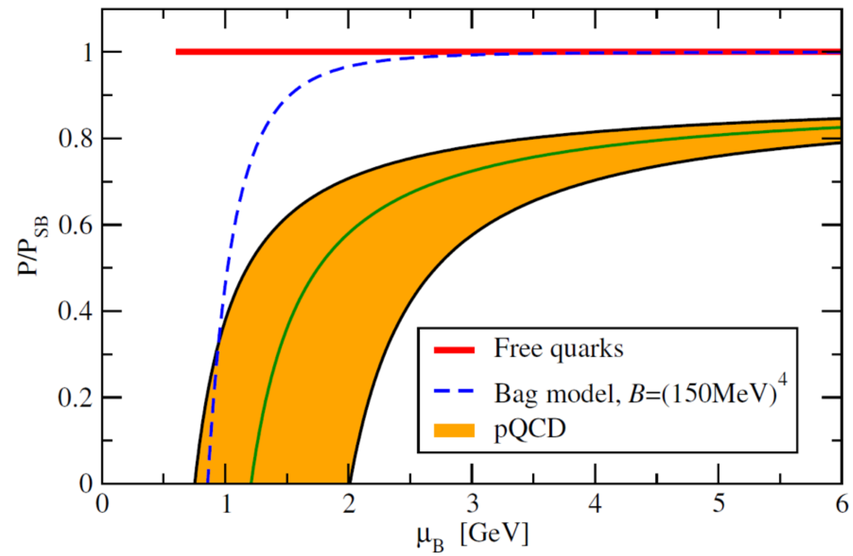
- include energy gap Δ from Cooper pairing

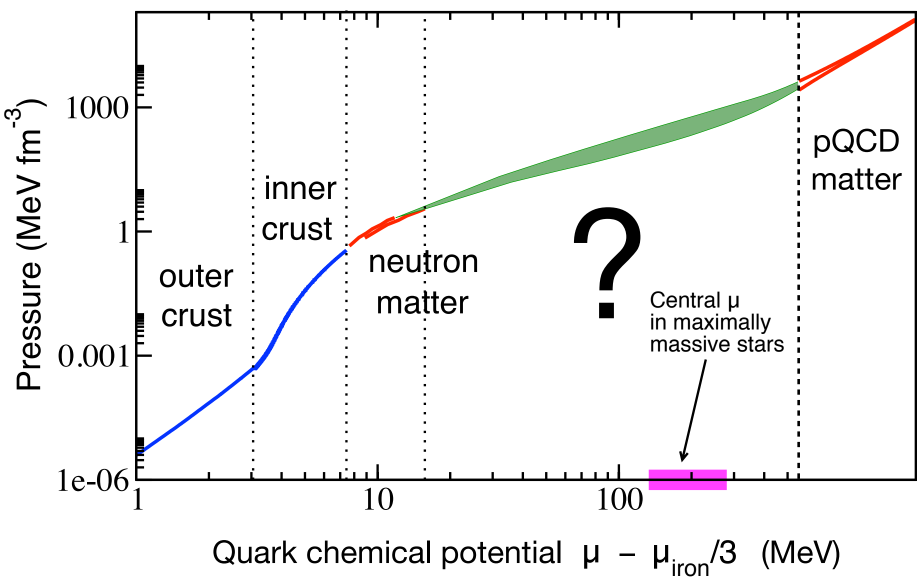
$$P \simeq \frac{3\mu^4}{4\pi^2}\left(1 - \frac{2\alpha_s}{\pi}\right) - \frac{3\mu^2}{4\pi^2}(m_s^2 - 4\Delta^2) - B$$

Recent studies of perturbative quark matter

- second-order corrections in α_s
  A. Kurkela, P. Romatschke, A. Vuorinen
  PRD 81, 105021 (2010)

- large corrections to bag model at all relevant densities!





- connect nuclear matter (low density) to perturbative QCD (high density)
  A. Kurkela, E. S. Fraga,
  J. Schaffner-Bielich, A. Vuorinen,
  Astrophys. J. 789, 127 (2014)

Summary: unpaired quark matter

- zero quark masses:

  quark matter is particularly symmetric:
  $n_u = n_d = n_s$ (and no electrons)

- nonzero strange quark mass:

  β-equilibrated, electrically neutral quark matter has $n_d > n_u > n_s$ (and nonzero n_e)

- perturbative results can be used to constrain equation of state at moderate densities

- strange quark matter hypothesis (not discussed here):

  A. R. Bodmer, PRD 4, 1601 (1971); E. Witten, PRD 30, 272 (1984)

  strange quark matter is the true ground state at zero pressure

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  - Implications for mass/radius curve and neutron star mergers
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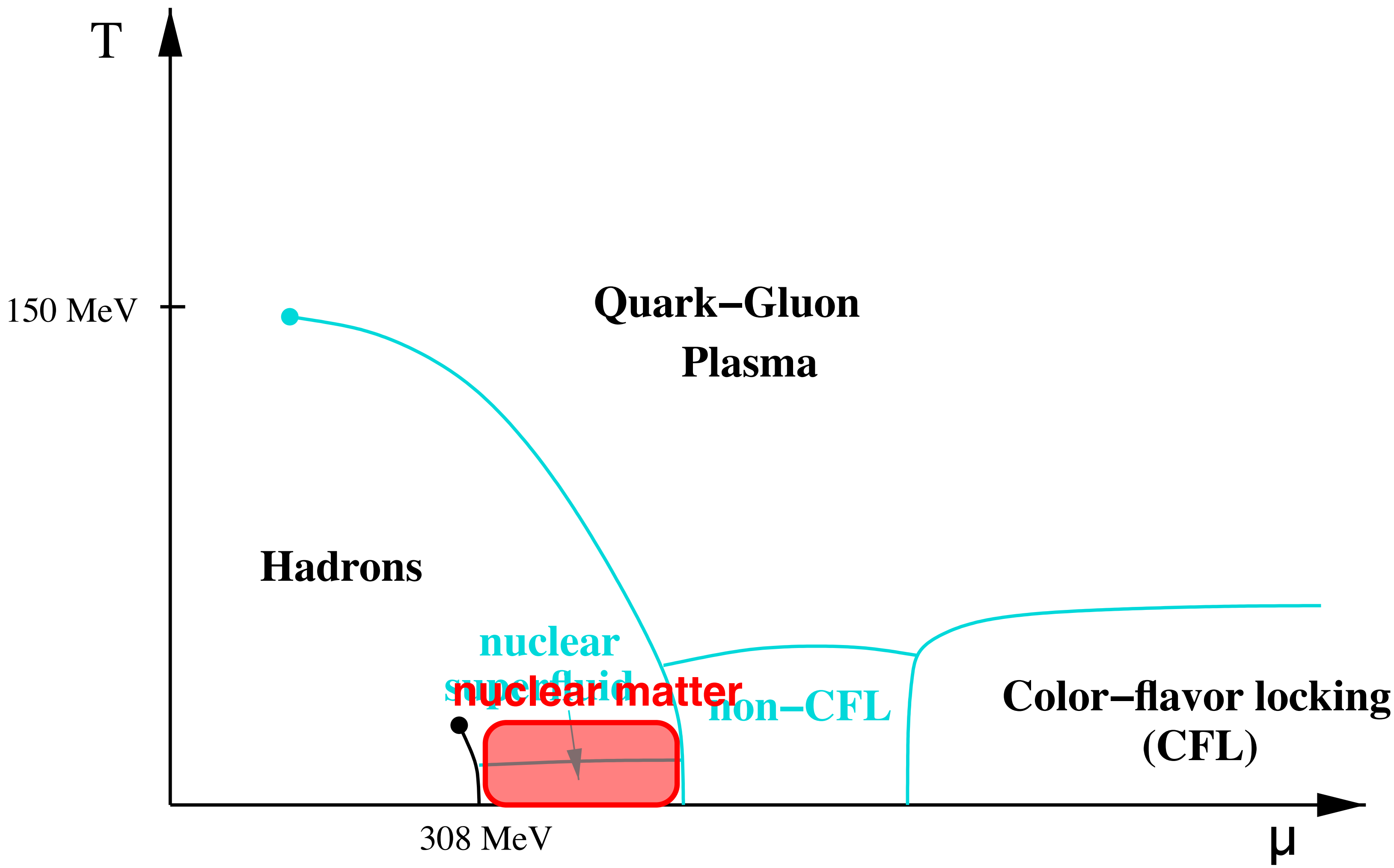
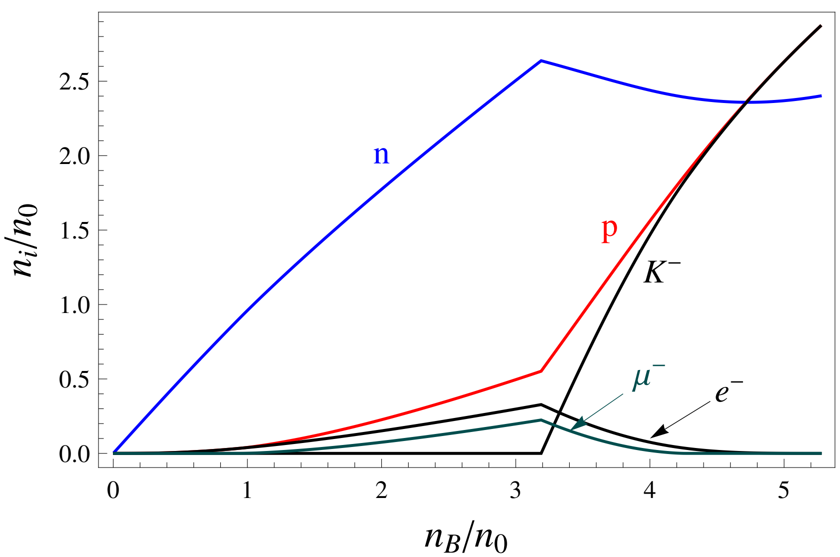
Nuclear matter



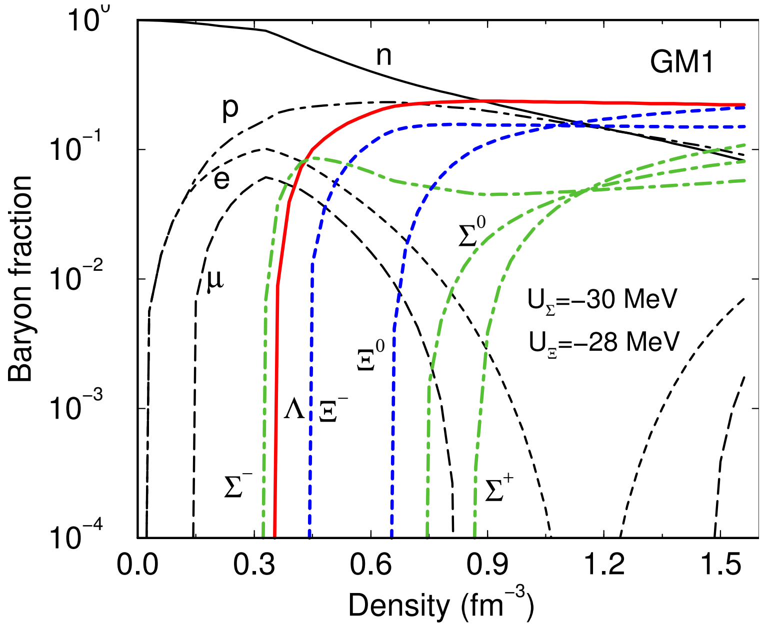
Nuclear matter

- "ordinary" nuclear matter: neutrons (n), protons (p), electrons (e)

- more exotic phases possible at high density:
  kaon condensation, hyperons, ...



A. Schmitt, Lect. Notes Phys. 811, 1 (2010)



J. Schaffner-Bielich, NPA 835, 279 (2010)

Non-interacting nuclear matter (page 1/3)

Consider npe matter at zero temperature

- neutrality: $n_e = n_p \quad \Rightarrow \quad k_{F,e} = k_{F,p}$ (since $n \propto k_F^3$)
- β-equilibrium: $\mu_e + \mu_p = \mu_n$ (assuming $\mu_\nu \simeq 0$)
- together:

$$\sqrt{k_{F,p}^2 + m_e^2} + \sqrt{k_{F,p}^2 + m_p^2} = \sqrt{k_{F,n}^2 + m_n^2} \qquad (*)$$

(i) npe matter must contain protons:

Suppose $k_{F,p} = 0$. Then, $(*)$ becomes

$$k_{F,n}^2 = (m_e + m_p)^2 - m_n^2$$

rhs is negative (that's why a neutron in vacuum decays)

$\Rightarrow$ no solution $\Rightarrow$ $k_{F,p} = 0$ and thus $n_p = 0$ can't be true

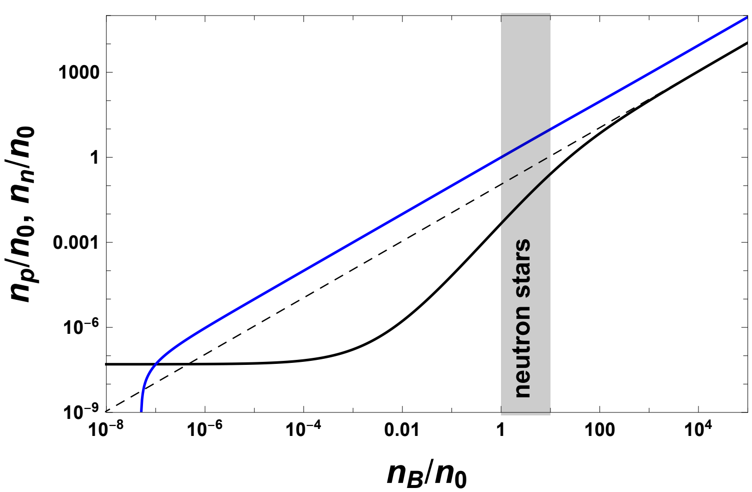
Non-interacting nuclear matter (page 2/3)

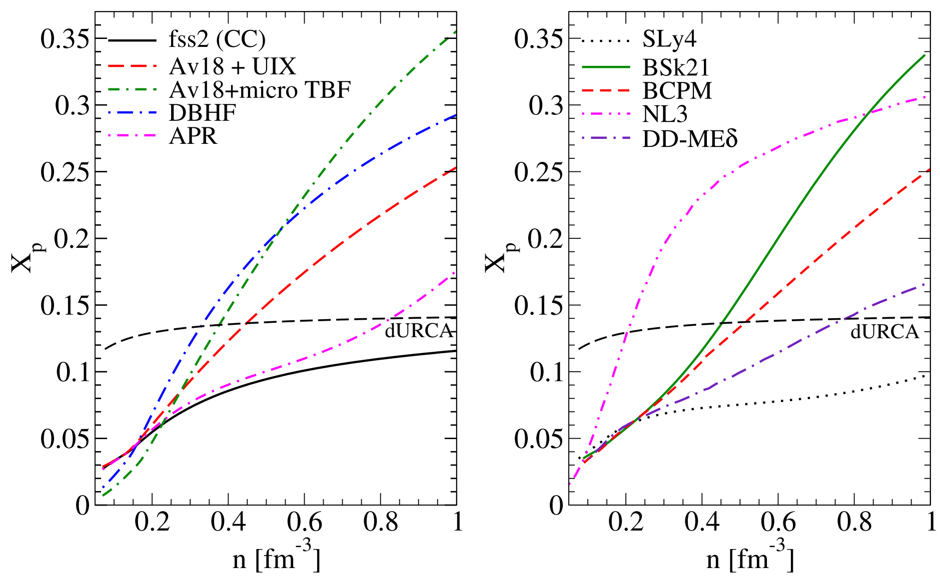
(ii) npe matter has proton fraction $\frac{n_p}{n_B} = \frac{1}{9}$ in the ultra-relativistic limit:

Assume $m_e \simeq m_n \simeq m_p \simeq 0$. Then $(\ast)$ becomes $2k_{F,p} = k_{F,n}$ and thus

$$8n_p = n_n \quad \Rightarrow \quad \frac{n_p}{n_B} = \frac{1}{9} \quad \text{with} \quad n_B = n_n + n_p$$

(iii) npe matter obeys $\frac{n_p}{n_B} < \frac{1}{9}$ except for very small n_B:





G. F. Burgio, A. F. Fantina, arXiv:1804.03020

Non-interacting nuclear matter (page 3/3)

(iv) non-relativistic, non-interacting, pure neutron matter has "polytropic" equation of state $P(\epsilon) = K\epsilon^p$:

Non-relativistic limit: $m \gg k_F$. Hence

$$\epsilon = \frac{1}{\pi^2}\int_0^{k_F} dk\, k^2\sqrt{k^2+m^2} \simeq \frac{m}{\pi^2}\int_0^{k_F} dk\, k^2\left(1+\frac{k^2}{2m}\right) = \frac{mk_F^3}{3\pi^2} + \mathcal{O}(k_F^5)$$

and

$$P = \frac{1}{\pi^2}\int_0^{k_F} dk\, k^2(\mu - \sqrt{k^2+m^2}) \simeq \frac{1}{\pi^2}\int_0^{k_F} dk\, k^2\left[m\left(1+\frac{k_F^2}{2m}\right) - m\left(1+\frac{k^2}{2m}\right)\right]$$

$$= \frac{1}{2m\pi^2}\int_0^{k_F} dk\, k^2(k_F^2 - k^2) = \frac{1}{2m\pi^2}\left(\frac{k_F^5}{3} - \frac{k_F^5}{5}\right) = \frac{k_F^5}{15m\pi^2}$$

Putting this together gives $P(\epsilon) = K\epsilon^p$ with

$$p = \frac{5}{3}, \qquad K = \left(\frac{3\pi^2}{m}\right)^{5/3}\frac{1}{15m\pi^2}$$

Basic properties of (interacting) nuclear matter

see Sec. 3.1 in A. Schmitt, Lect. Notes Phys. 811, 1 (2010)

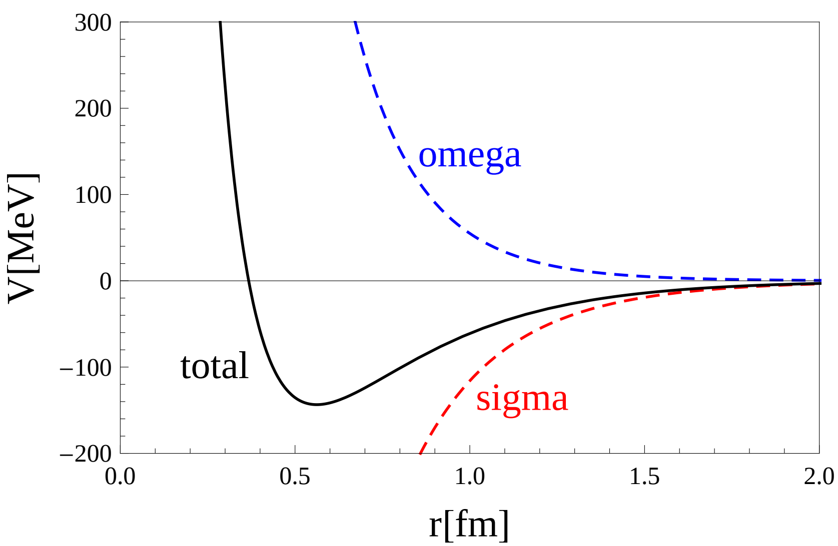
- relativistic, symmetric nuclear matter ("Walecka model")

$$\mathcal{L} = \bar\psi(i\gamma^\mu\partial_\mu - m_N + \mu\gamma^0)\psi + g_\sigma\bar\psi\sigma\psi - g_\omega\bar\psi\gamma^\mu\omega_\mu\psi$$

$$+\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$

(with μ introduced through $\mathcal{H} - \mu\mathcal{N}$)

- two parameters (to be fitted later): g_σ, g_ω
- attractive and repulsive interaction through sigma and omega exchange



Mean-field approximation

- replace meson fields by their vevs (space-time independent)

$$\sigma \to \langle\sigma\rangle\,, \qquad \omega_\mu \to \langle\omega_0\rangle\delta_{0\mu}$$

- mean-field Lagrangian

$$\mathcal{L}_{\text{mean-field}} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m_N^* + \mu^*\gamma_0\right)\psi - \frac{1}{2}m_\sigma^2\langle\sigma\rangle^2 + \frac{1}{2}m_\omega^2\langle\omega_0\rangle^2$$

with

$$m_N^* \equiv m_N - g_\sigma\langle\sigma\rangle\,, \qquad \mu^* \equiv \mu - g_\omega\langle\omega_0\rangle$$

→ looks like non-interacting Lagrangian: interaction absorbed in effective mass m_N^* and effective chemical potential μ^*

Pressure from partition function (page 1/2)

- partition function

$$Z = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\sigma\mathcal{D}\omega \, \exp \int_X \mathcal{L}$$

$$= e^{\frac{V}{T}\left(-\frac{1}{2}m_\sigma^2\langle\sigma\rangle^2+\frac{1}{2}m_\omega^2\langle\omega_0\rangle^2\right)} \underbrace{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp \int_X \bar{\psi}\left(i\gamma^\mu\partial_\mu - m_N^* + \mu^*\gamma_0\right)\psi}_{\det_{\mathrm{Dirac},K} \frac{-\gamma^\mu K_\mu - \gamma_0\mu^* + m_N^*}{T}}$$

with

$$\int_X \equiv \int_0^\beta d\tau \int d^3x \, , \qquad X^\mu = (-i\tau, \mathbf{x}) \, , \qquad K^\mu = (-i\omega_n, \mathbf{k})$$

Thermal field theory: $Z = \mathrm{Tr} e^{-\beta\hat{H}} = \int d\phi \langle\phi|e^{-\beta\hat{H}}|\phi\rangle \leftrightarrow \int d\phi\langle\phi|e^{-it_f\hat{H}}|\phi\rangle$
→ "imaginary time" τ and periodic boundary conditions for ϕ
(anti-periodic for fermions)
→ discrete energies → Matsubara frequencies $\omega_n = (2n+1)\pi T$ (fermionic)

Pressure from partition function (page 2/2)

- pressure

$$P = \frac{T}{V} \ln Z$$

- 4-momentum sum = sum over Matsubara frequencies & 3-momentum integral

$$\frac{T}{V} \ln \det_K \to \frac{T}{V} \sum_K \ln \to T \sum_n \int \frac{d^3\mathbf{k}}{(2\pi)^3} \ln$$

- determinant over Dirac space & summation over Matsubara sum & ignore "vacuum contribution" & neglect anti-baryons

$$P = -\frac{1}{2} m_\sigma^2 \langle \sigma \rangle^2 + \frac{1}{2} m_\omega^2 \langle \omega_0 \rangle^2 + \underbrace{4T \int \frac{d^3\mathbf{k}}{(2\pi)^3} \ln\left(1 + e^{-(E_k - \mu^*)/T}\right)}_{P_N}$$

with $E_k = \sqrt{k^2 + (m_N^*)^2}$

Stationarity equations

- compute meson vevs from

$$0 = \frac{\partial P}{\partial \langle \sigma \rangle} = -m_\sigma^2 \langle \sigma \rangle - g_\sigma \frac{\partial P_N}{\partial m_N^*} \equiv -m_\sigma^2 \langle \sigma \rangle + g_\sigma n_s$$

$$0 = \frac{\partial P}{\partial \langle \omega_0 \rangle} = m_\omega^2 \langle \omega_0 \rangle - g_\omega \frac{\partial P_N}{\partial \mu^*} \equiv m_\omega^2 \langle \omega_0 \rangle - g_\omega n_B$$

- for given n_B the equations decouple and we need to solve

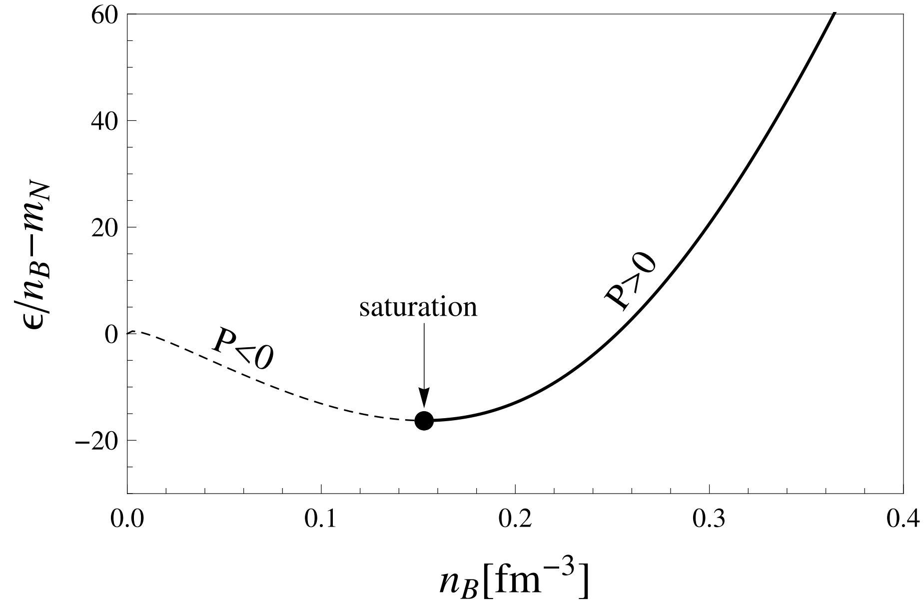
$$m_N^* = m_N - \frac{g_\sigma^2}{m_\sigma^2} n_s$$

for m_N^*

Saturation density and binding energy

- $\exists$ minimum of $\epsilon/n_B = E/A$ at "saturation density"

$$n_0 \simeq 0.15\,\mathrm{fm}^{-3}$$

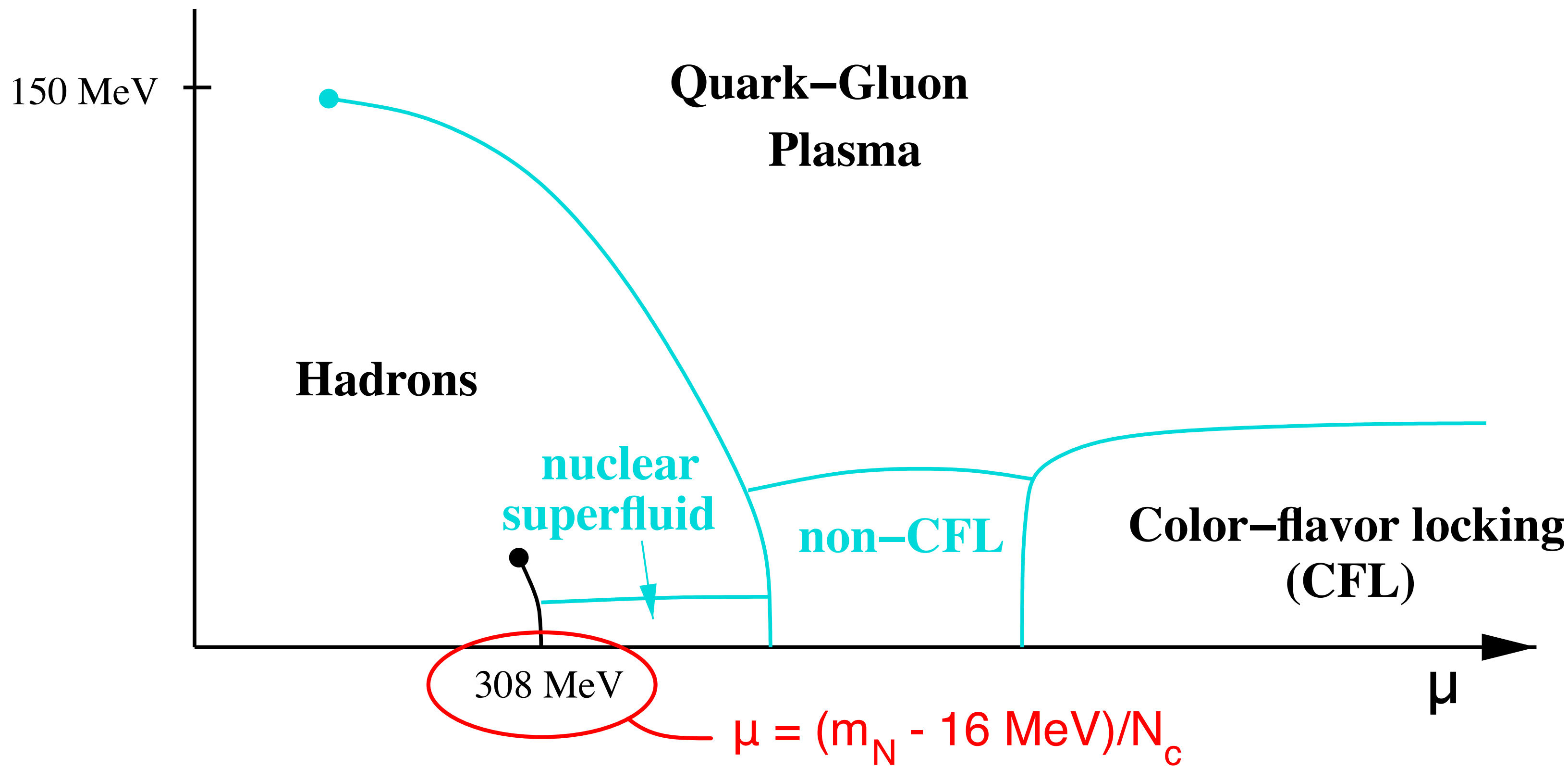


- semi-empirical energy

$$E = -a_1 A + \underbrace{a_2 A^{3/2}}_{\text{surface}} + \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\text{Coulomb}} + \underbrace{a_4 \frac{(A-2Z)^2}{A}}_{\text{(a)symmetry}}$$

- symmetric, infinite nuclear matter without EM has binding energy $E_0 \equiv E/A = -a_1 = -16\,\mathrm{MeV}$
- g_σ and g_ω fitted to reproduce n_0 and E_0

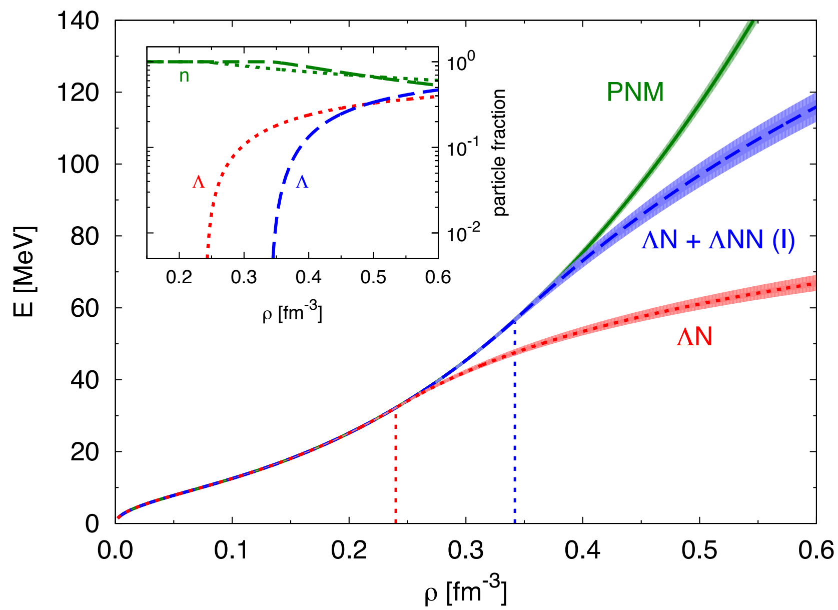
Saturation density in the QCD phase diagram



- $\mu_B < m_N - E_0$: vacuum with $P = 0$ and $n_B = 0$
- $\mu_B = m_N - E_0$: first-order phase transition to nuclear matter with $P = 0$ and $n_B = n_0$
- $\mu_B > m_N - E_0$: nuclear matter with $P > 0$ and $n_B > n_0$

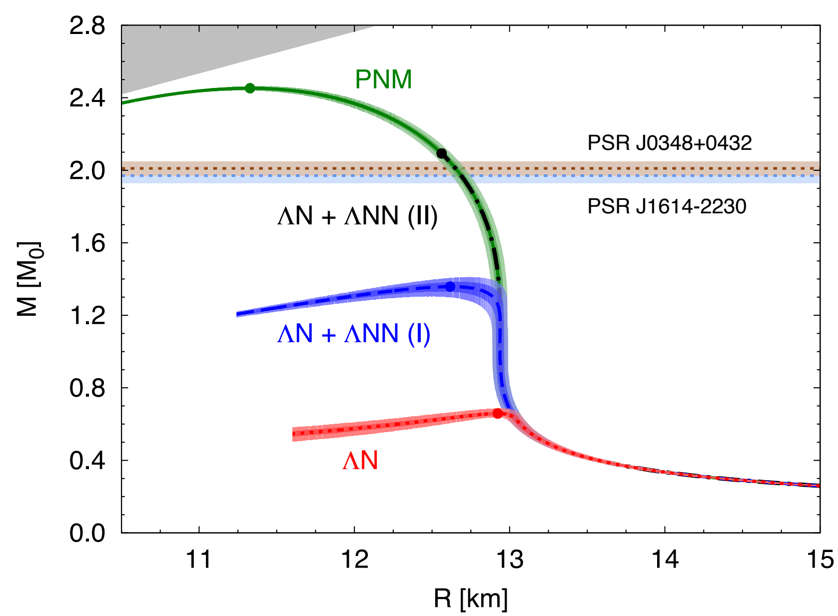
Current research (example 1/2): nuclear/hyperonic matter

D. Lonardoni, A. Lovato, S. Gandolfi and F. Pederiva, PRL 114, 092301 (2015)



- PNM = pure neutron matter
- ΛN = two-body Λ-nucleon int.
- ΛNN = three-body Λ-nucleon int.
- different interactions lead to (very) different high-density EoSs

- mass/radius relations
- no hyperons for "ΛN + ΛNN (II)"

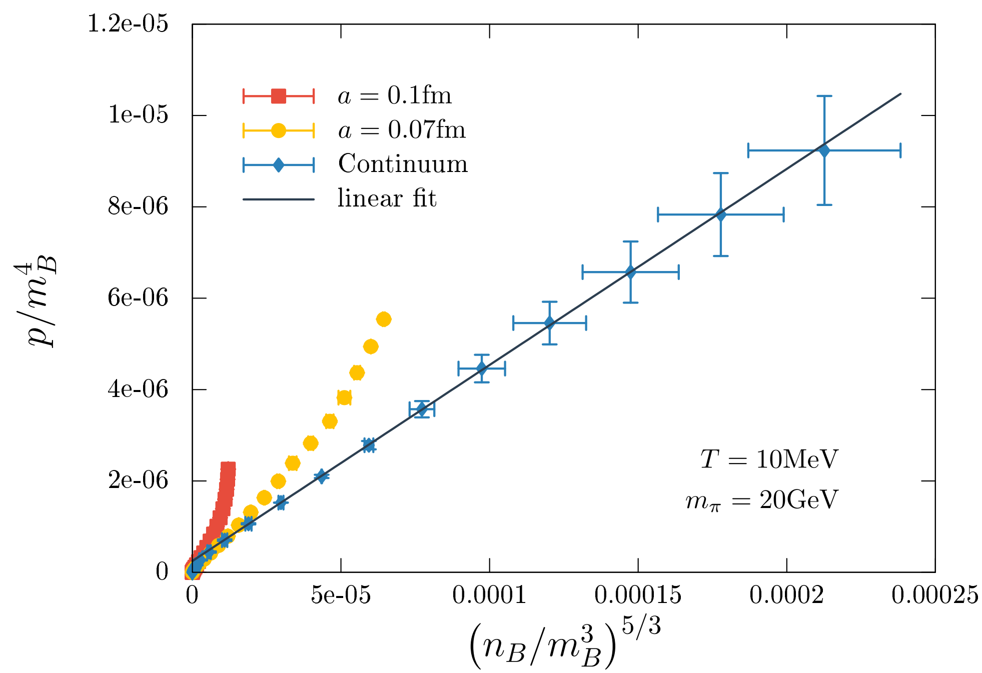


Current research (example 2/2): nuclear matter on the lattice

J. Glesaaen, M. Neuman and O. Philipsen, JHEP 1603, 100 (2016)

- lattice QCD: plagued by the "sign problem" at nonzero μ
- circumvent problem by strong-coupling expansion with (very!) heavy quarks

- baryon onset is seen
- equation of state can be extracted (polytropic?)



Summary: nuclear matter

- neutral nuclear matter in β-equilibrium is neutron-rich
  $\rightarrow$ "neutron star"

- symmetric nuclear matter has a "saturation density" n_0
  and a "binding energy" E_0

- as a consequence, there is a first-order baryon onset
  (liquid-gas transition) in the QCD phase diagram

- neutron star densities allow for "exotic" matter such as hyperons

- nuclear interactions at high densities are poorly constrained by experiments (hyperon-nucleon interaction even more so)
  (and currently they cannot be computed from first principles)

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Transition from nuclear to quark matter

At what μ does the transition from nuclear to quark matter occur?
What kind of transition is it: first order, crossover?

order parameter	Polyakov loop (confinement)	chiral condensate
spontaneously breaks	$\mathbb{Z}_{N_c}$	$SU(N_f) \times SU(N_f)$
symmetry exact for	pure Yang-Mills ($m_q = \infty$)	chiral limit ($m_q = 0$)

→ in real-world QCD no exact symmetry is spontaneously broken (ignoring Cooper pairing)

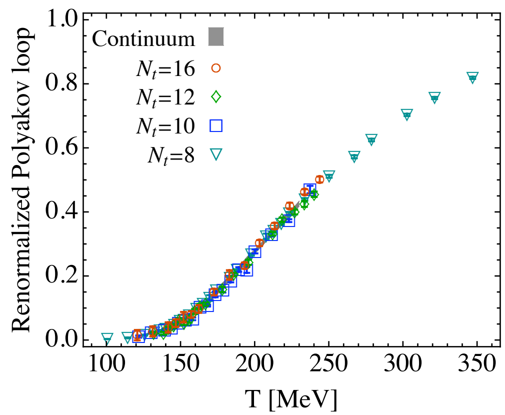
→ transition is allowed to be smooth (can still be first order)

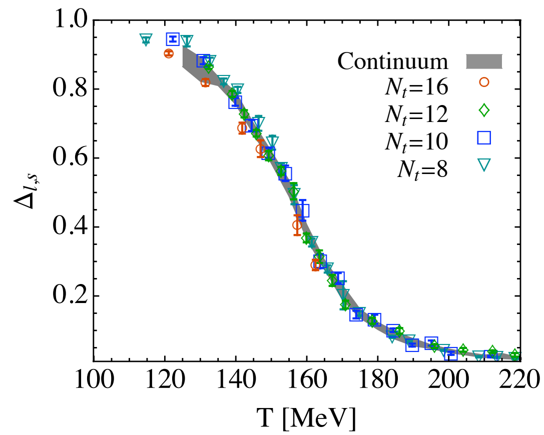
quark-hadron crossover at large densities: T. Hatsuda *et al.*, PRL 97, 122001 (2006)

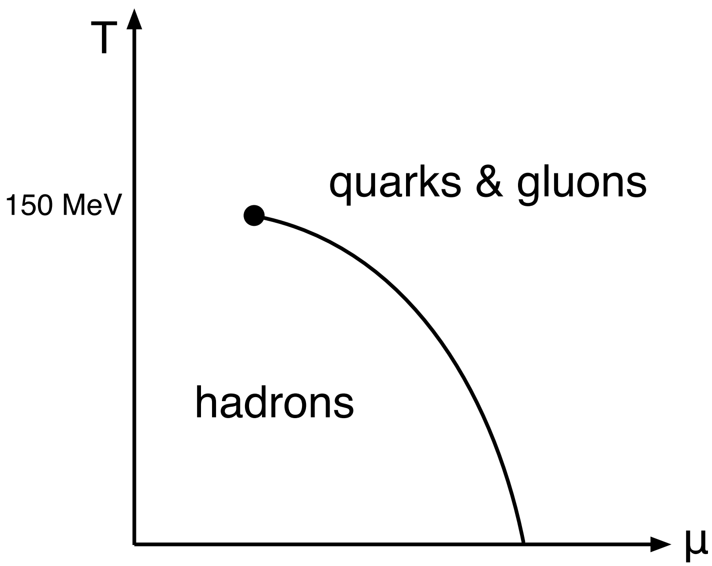
review: G. Baym *et al.*, Rept. Prog. Phys. 81, 056902 (2018)

Crossover at $\mu = 0$

Lattice QCD: smooth
"order parameters" at $\mu = 0$
S. Borsanyi *et al.* JHEP 1009, 073 (2010)







- nonzero μ: lattice methods don't work ("sign problem")

  recent progress (reviews):

  G. Aarts, J.Phys.Conf.Ser. 706, 022004 (2016)

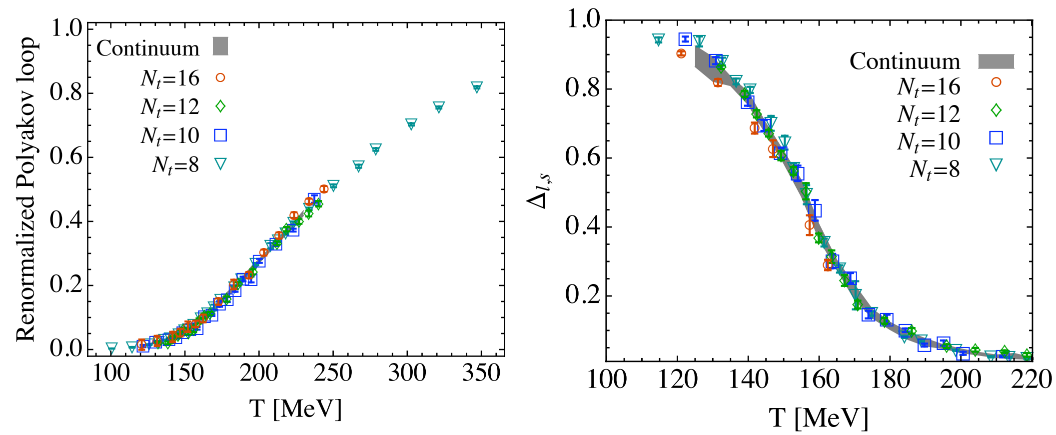
  O. Philipsen, EPJ Web Conf. 137, 03016 (2017)

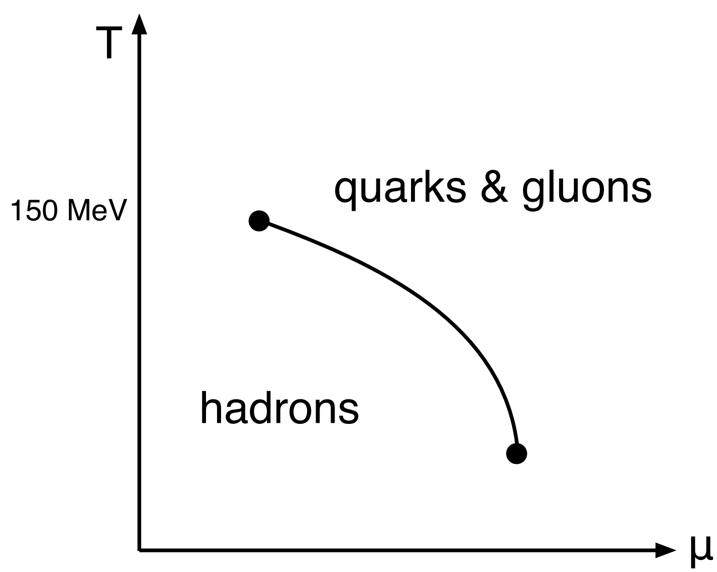
- quark-hadron continuity at $T = 0$?

Crossover at $\mu = 0$

Lattice QCD: smooth ”order parameters” at $\mu = 0$

S. Borsanyi *et al.* JHEP 1009, 073 (2010)





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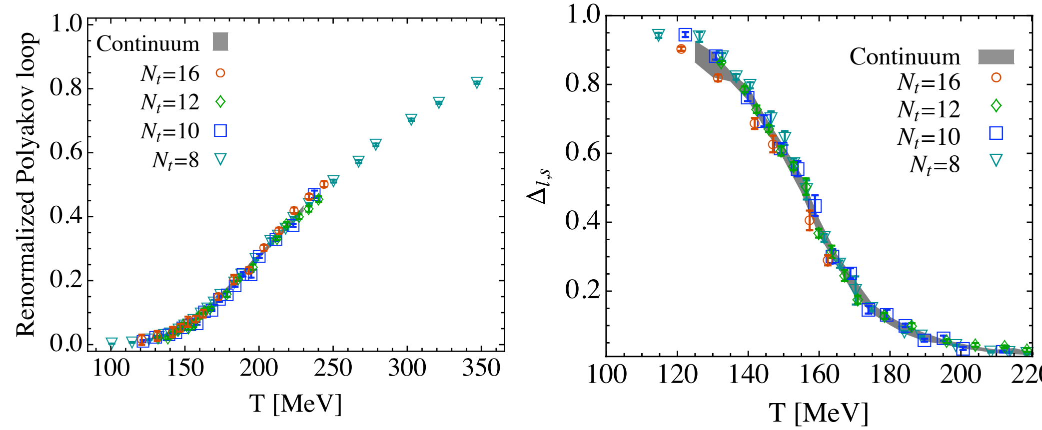

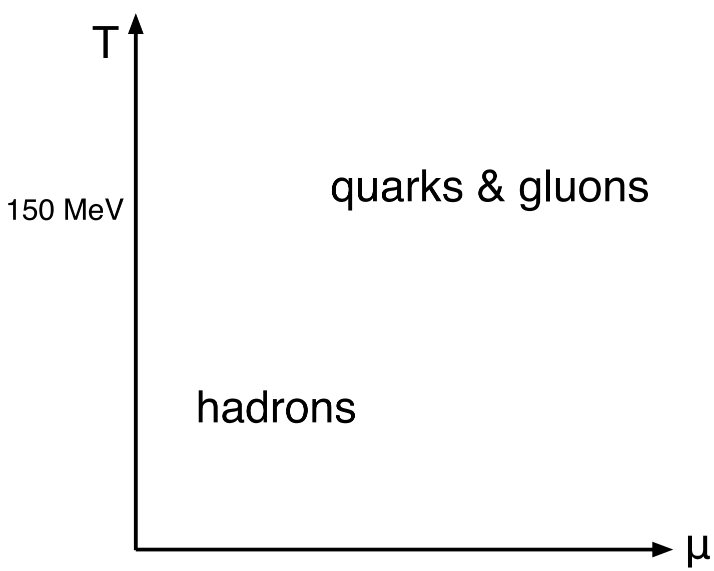

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  G. Aarts, J.Phys.Conf.Ser. 706, 022004 (2016)

  O. Philipsen, EPJ Web Conf. 137, 03016 (2017)

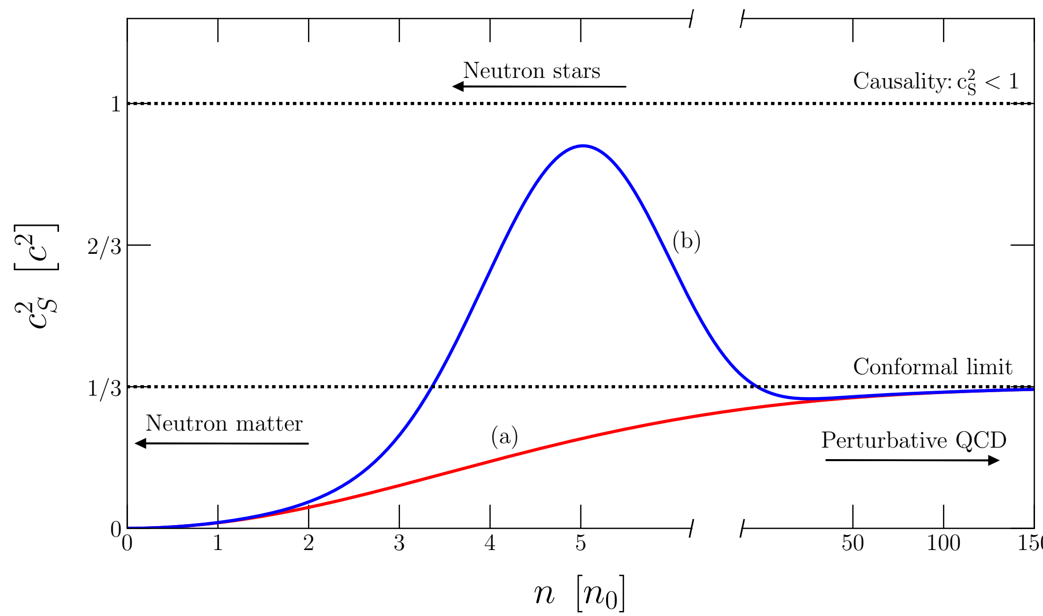
- quark-hadron continuity at $T = 0$?

Theoretical approaches

- first principles
  (currently too hard, sign problem)
- phenomenological models
  (usually either quark or nucleonic d.o.f., not both)
- patch together models
  (theoretically unsatisfying, many parameters)
- interpolate between low and ultra-high density
  (very general, no microscopic insight)
- gauge/gravity duality
  (distorted version of QCD at best, but consistent treatment of quark matter and nuclear matter possible, very few parameters)
  S. w. Li, A. Schmitt and Q. Wang, PRD 92, 026006 (2015)
  F. Preis and A. Schmitt, JHEP 1607, 001 (2016)
  K. Bitaghsir Fadafan, F. Kazemian, A. Schmitt, arXiv:1811.08698 [hep-ph]

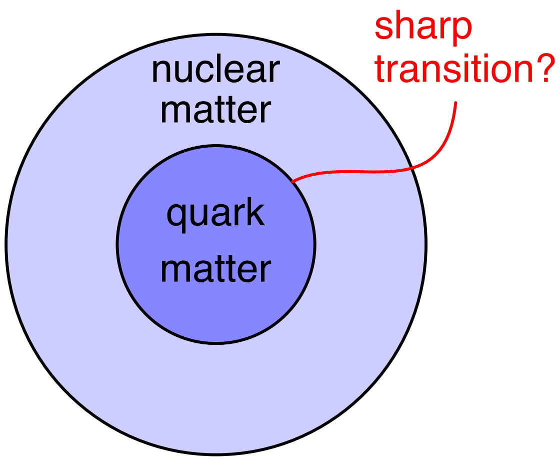
Speed of sound

I. Tews *et al.*,
Astrophys. J. 860, 149 (2018)



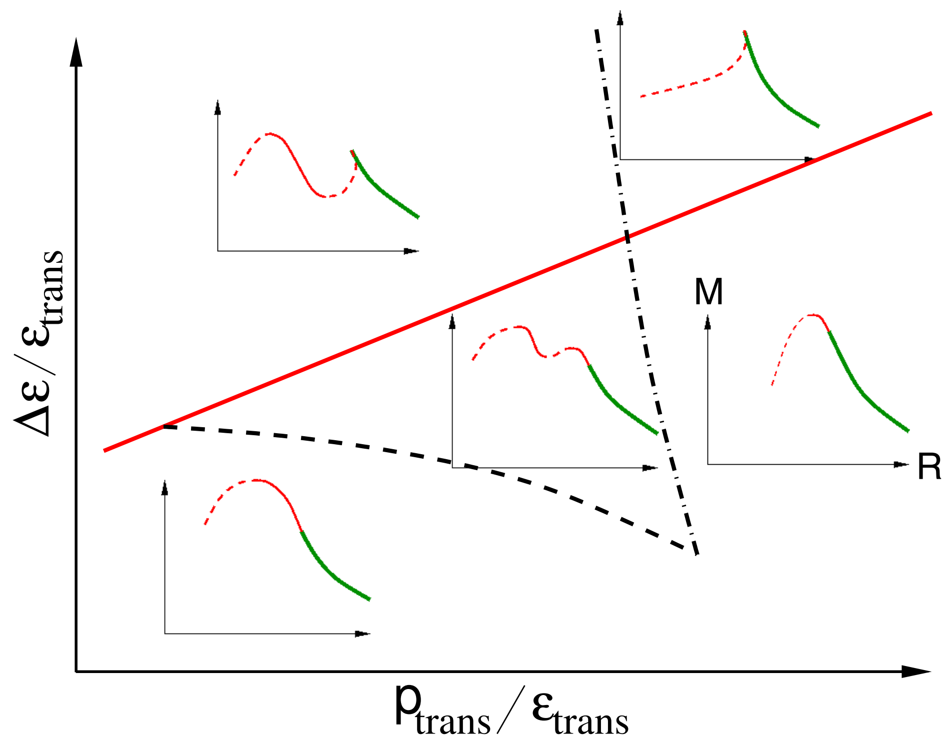
- asymptotic densities: $c_s^2 = \frac{\partial P}{\partial \epsilon} = \frac{1}{3}$ (conformal limit)
- perturbative corrections: $c_s^2 < \frac{1}{3}$
- low-density nuclear matter: non-relativistic $c_s^2 \ll 1$
- two-solar mass neutron star: need stiff equation of state → large speed of sound → non-monotonic behavior suggested (first-order phase transition: jump in c_s)
  P. Bedaque and A. W. Steiner, PRL 114, 031103 (2015)
- non-monotonic behavior from holography and quarkyonic model
  K. Bitaghsir Fadafan, F. Kazemian, A. Schmitt, arXiv:1811.08698 [hep-ph]
  L. McLerran and S. Reddy, arXiv:1811.12503 [nucl-th]

Implication for compact stars: mass/radius curve



smooth density profile? jump?
mixed phase (like "nuclear pasta")?
need surface tension
E. S. Fraga, M. Hippert and A. Schmitt
arXiv:1810.13226 [hep-ph]

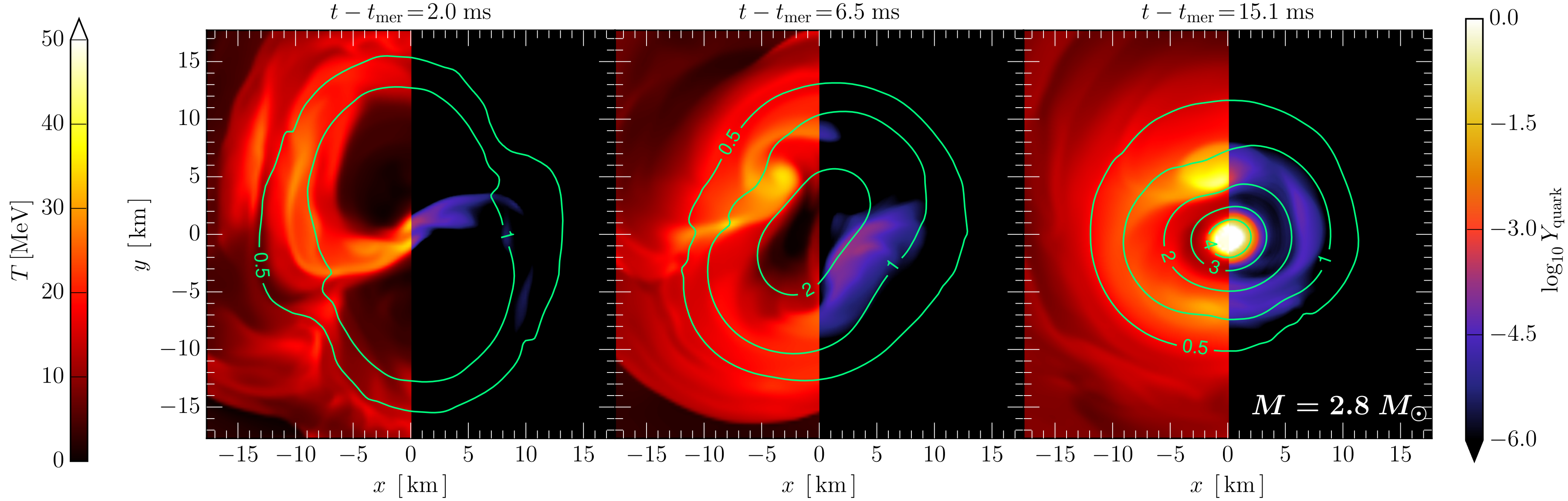
- qualitative difference in mass/radius curve
  M. G. Alford, S. Han and M. Prakash, PRD 88, 083013 (2013)
- sequential 1st-order transitions?
  M. G. Alford and A. Sedrakian, PRL 119, 161104 (2017)



Implication for compact stars: gravitational waves

Merger simulation with first-order phase transition to quark matter from phenomenological model

E. R. Most *et al.*, arXiv:1807.03684 [astro-ph.HE]



Gravitational waves from bubble nucleation during supernovae

G. Cao and S. Lin, arXiv:1810.00528 [nucl-th]

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Transport in neutron stars

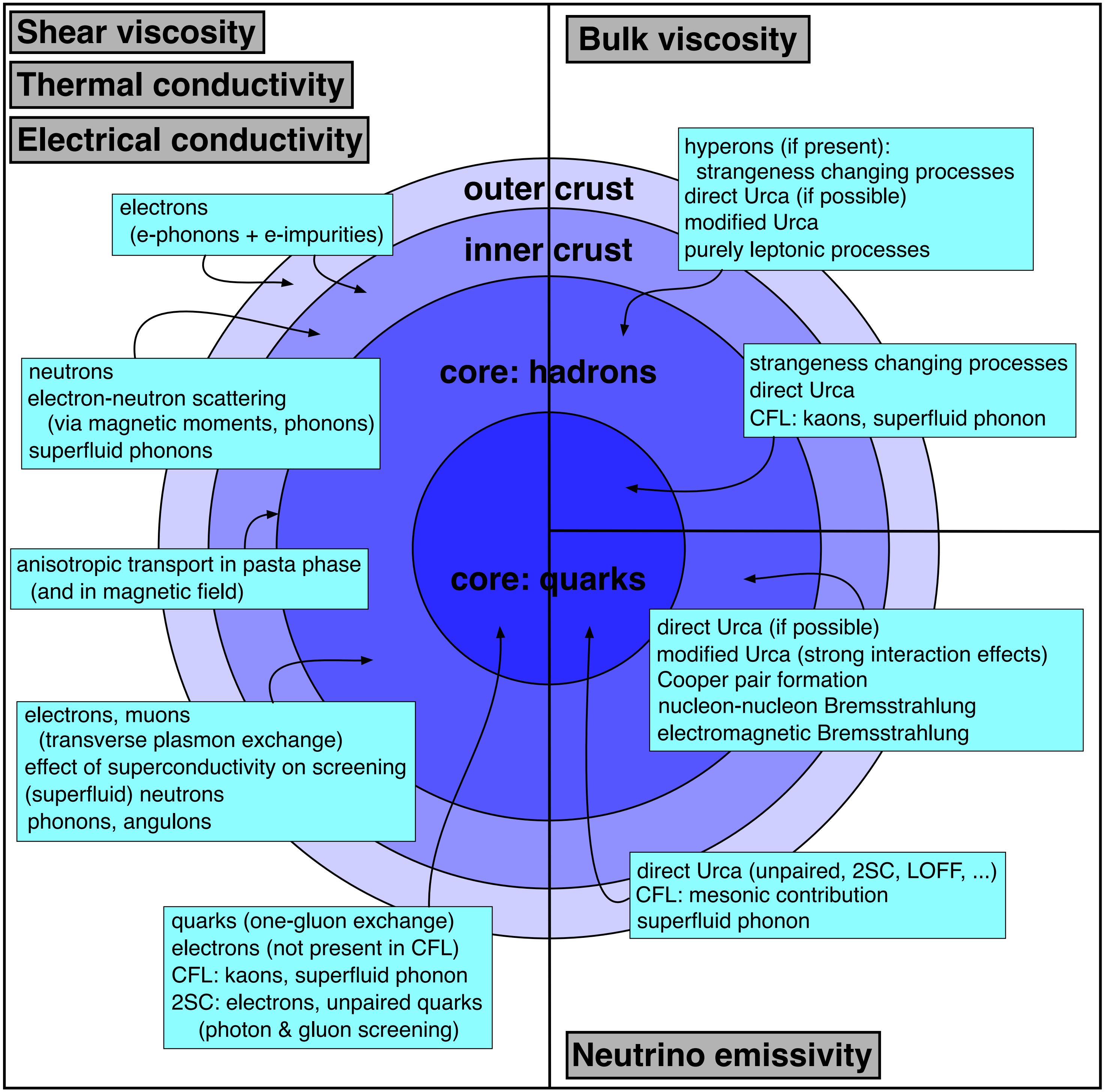
review: A. Schmitt and P. Shternin, arXiv:1711.06520 [astro-ph.HE]

"Transport": transfer of conserved quantities
(energy, momentum, particle number, electric charge, ...)
from one region to another due to non-equilibrium
(temperature gradient, non-uniform chemical composition, ...)

- general recipe: compute transport coefficients from some microscopic theory (e.g., Boltzmann eq) and insert into hydro eqs (if sufficiently close to equilibrium)
- complications in neutron star context:
  - (general) relativistic effects
  - magnetic field → magneto-hydrodynamics
  - two-fluid (multi-fluid) transport
    (electron-ion in the crust, *npe* matter in the core)
  - superfluid (two-fluid) transport
    → more transport coefficients, vortices, flux tubes ...

Transport and phenomenology

Phenomenon	**Transport properties**
oscillatory modes (r-modes)	shear & bulk viscosity
pulsar glitches	superfluid transport (vortex pinning)
thermal radiation	heat transport in outermost layers
cooling	neutrino emissivity, heat conductivity
magnetic field evolution	magnetohydrodynamics electrical & thermal conductivities
crust disruption (accretion, magnetar flares)	transport properties of the crust nuclear reactions ("deep crustal heating")
core-collapse supernovae	neutrino transport, neutrino-nucleus reactions
neutron star mergers	high-temperature transport (viscous) magnetohydrodynamics


Shear viscosity
Thermal conductivity
Electrical conductivity
Bulk viscosity
Neutrino emissivity
outer crust
inner crust
core: hadrons
core: quarks
electrons
(e-phonons + e-impurities)
neutrons
electron-neutron scattering
(via magnetic moments, phonons)
superfluid phonons
anisotropic transport in pasta phase
(and in magnetic field)
electrons, muons
(transverse plasmon exchange)
effect of superconductivity on screening
(superfluid) neutrons
phonons, angulons
quarks (one-gluon exchange)
electrons (not present in CFL)
CFL: kaons, superfluid phonon
2SC: electrons, unpaired quarks
(photon & gluon screening)
hyperons (if present):
strangeness changing processes
direct Urca (if possible)
modified Urca
purely leptonic processes
strangeness changing processes
direct Urca
CFL: kaons, superfluid phonon
direct Urca (if possible)
modified Urca (strong interaction effects)
Cooper pair formation
nucleon-nucleon Bremsstrahlung
electromagnetic Bremsstrahlung
direct Urca (unpaired, 2SC, LOFF, ...)
CFL: mesonic contribution
superfluid phonon

Conclusion

- compact stars are a laboratory for QCD, complementary to heavy-ion collisions ($\mu \gg T$ vs. $T \gg \mu$)
- gravitational waves provide new data from neutron star mergers ($\rightarrow$ equation of state, viscosity, heat conductivity, ...) and possibly continuous emission from isolated stars ($\rightarrow$ crystalline structures, flux tubes, shear and bulk viscosity, ...)