The EW schemes for precision physics at the Z resonance

E. Richter-Was

- EW schemes: from LEP to LHC
- Comparison of predictions (different EW schemes) for $\sin^2\theta_W$ measurement
- Comment on EW schemes for multi-boson production at LHC

Material partly in collaboration with
F. Piccinini (Powheg_ew), S. Bondarenko &L. Kalinovskaya (MCSANC), A. Armbruster (DYTURBO)

EW schemes

- **LEP legacy: ($\alpha(0)$, G_μ, M_Z)**
  - Inputs are very precisely measured physics quantities
  - M_Z, M_W are on-shell masses
  - Genuine EW and lineshape corrections in form of (multiplicative) form-factors to LO couplings

D. Bardin et al.
arXiv:9908433

- **LHC paradigm: (G_μ, M_Z, M_W).**
  - M_Z, M_W are pole-masses or complex masses.
  - Absorbs most of universal corrections into lowest-order couplings
  - Higher-order corrections redefine couplings in non-multiplicative manner

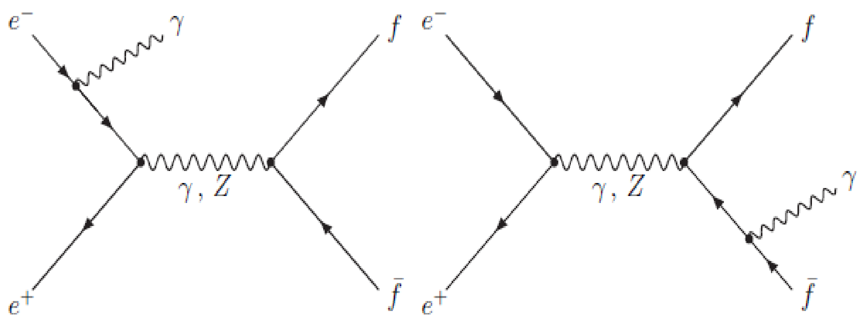
S. Dittmaier, M. Huber
arXiv:0911.2329

LEP legacy: QED (radiative) corrections

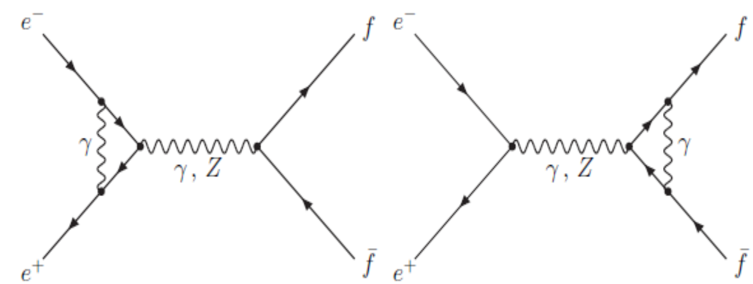
NOT discussed here.

QED FSR can be simulated by PHOTOS (exponentiated multi-photon emission) implemented as after-burner step on already generated event.

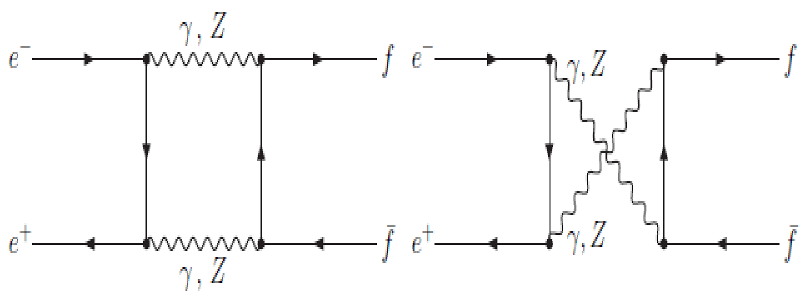
Real emission + pairs creation



Vertex corrections



$\gamma\gamma$ and γZ box diagrams



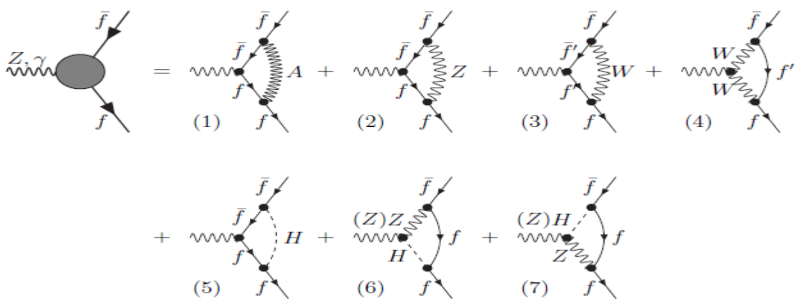
It is QED gauge-invariant set of diagrams (D. Bardin, hep-ph/9908433) which can be factorised out and/or convoluted with QCD corrections.

Calculated with fixed value of α_{QED}
α_{QED} = 1./137.0359895

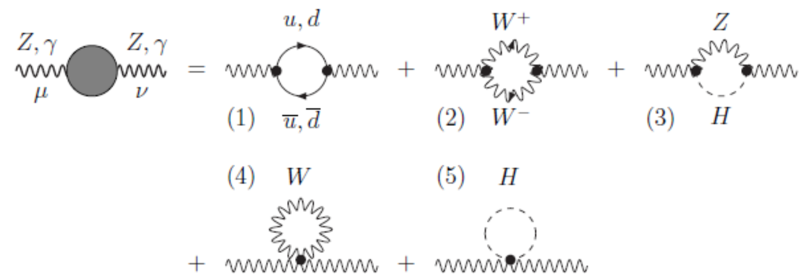
LEP legacy: Genuine EW and lineshape corrections

Also gauge-invariant set of diagrams. Calculated as form-factor corrections to couplings, propagators and masses. Eg. running $\alpha_{QED}(s)$, $\alpha_{QED}(M_Z)$ = 1./128.86674175

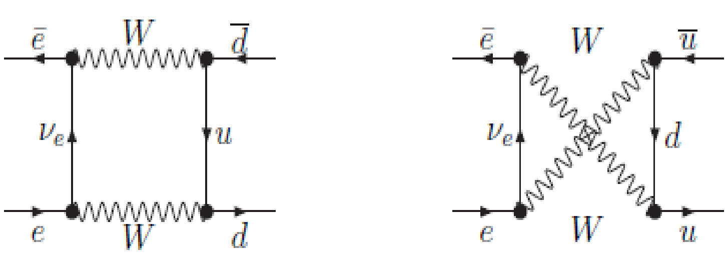
Zff and γ ff vertices



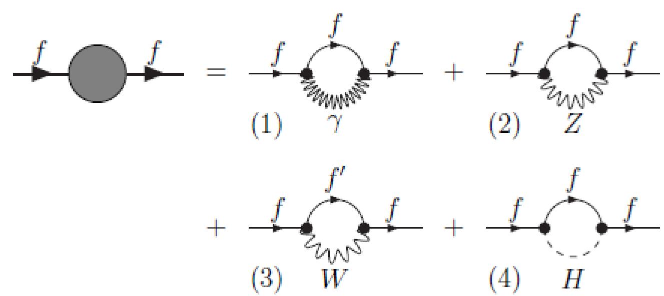
Bosonic self-energies



WW, ZZ boxes (shown only WW diagrams)



Fermionic self-energies



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From Zfitter/Dizet documentation

D. Bardin et al.
arXiv:9908433

Zfitter is a **semi-analytical program** for calculating total cross-sections and pseudo-observables (eg. A_{fb}, $\sin^2\theta_W^{eff}$), used by LEP1, and to a lesser degree by LEP2.

DIZET is a library for calculating form-factors and some other corrections. Provides complete EW $O(\alpha)$ weak-loop corrections supplemented with selected higher order terms (eg. vacum polarisation, $\alpha_{QED}(Q^2)$).

For analyses at LEP1, LEP2 used aways in parallel with **MC generators (KoralZ, KoralW)** eg. to evaluate systematics of simplified cuts used in analysis integration.

one loop amplitude

$$\mathcal{A}_Z^{OLA}(s,t) = i\sqrt{2}G_\mu I_e^{(3)} I_f^{(3)} M_Z^2 \chi_Z(s) \boxed{\rho_{ef}(s,t)} \Big\{ \gamma_\mu(1+\gamma_5)\otimes\gamma_\mu(1+\gamma_5) - 4|Q_e|s_W^2 \boxed{\kappa_e(s,t)} \gamma_\mu \otimes \gamma_\mu(1+\gamma_5) - 4|Q_f|s_W^2 \boxed{\kappa_f(s,t)} \gamma_\mu(1+\gamma_5)\otimes\gamma_\mu + 16|Q_e Q_f| s_W^4 \boxed{\kappa_{e,f}(s,t)} \gamma_\mu\otimes\gamma_\mu \Big\}. \quad \text{(A.4.75)}$$

$$\mathcal{A}_\gamma^{OLA} = i\chi_\gamma(s) \boxed{\alpha(s)} \gamma_\mu \otimes \gamma_\mu \,. \quad \text{(2.2.36)}$$

Dyson summation leads to the change of α into $\alpha(s)$:

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha^{\rm fer}(s)} = \frac{\alpha(0)}{1-\boxed{\Delta\alpha^{(5)}(s) - \Delta\alpha^{t}(s) - \Delta\alpha^{\alpha\alpha_s}(s)}}, \quad \text{(2.2.37)}$$

Vacuum polarisation corrections

LEP legacy: from Zfitter/Dizet documentation

After some trivial algebra one derives the final expressions:

$$\boxed{\rho_{ef}} = 1 + \frac{g^2}{16\pi^2}\Big\{ -\Delta\rho_Z^F + \mathcal{D}_Z^F(s) + \frac{5}{3}B_0^F(-s; M_W, M_W) - \frac{9}{4}\frac{c_W^2}{s_W^2}\ln c_W^2 - 6$$
$$+\frac{5}{8}c_W^2\left(1+c_W^2\right) + \frac{1}{4c_W^2}\left(3v_e^2 + a_e^2 + 3v_f^2 + a_f^2\right)\mathcal{F}_Z(s) + \hat{\mathcal{F}}_W^0(s) + \hat{\mathcal{F}}_W(s)$$
$$-\frac{r_t}{4}\left[B_0^F(-s; M_W, M_W) + 1\right] - c_W^2\left(R_Z - 1\right)s\mathcal{B}^d_{WW}(s,t)\Big\}, \qquad (A.4.80)$$

$$\boxed{\kappa_e} = 1 + \frac{g^2}{16\pi^2}\Big\{ -\frac{c_W^2}{s_W^2}\Delta\rho^F - \Pi^F_{Z\gamma}(s) - \frac{1}{6}B_0^F(-s; M_W, M_W) - \frac{1}{9} - \frac{v_e\sigma_e}{2c_W^2}\mathcal{F}_Z(s)$$
$$-\hat{\mathcal{F}}_W^0(s) + \left(R_Z - 1\right)\Big[\frac{|Q_f|}{2}\left(1 - 4|Q_f|s_W^2\right)\mathcal{F}_Z(s) + c_W^2\Big[\hat{\mathcal{F}}_{W_n}(s)$$
$$-|Q_{f'}|\mathcal{F}_{W_a}(s) + s\hat{\mathcal{B}}^d_{WW}(s,t)\Big]\Big]\Big\}, \qquad (A.4.81)$$

$$\boxed{\kappa_f} = 1 + \frac{g^2}{16\pi^2}\Big\{ -\frac{c_W^2}{s_W^2}\Delta\rho^F - \Pi^F_{Z\gamma}(s) - \frac{1}{6}B_0^F(-s; M_W, M_W) - \frac{1}{9} - \frac{v_f\sigma_f}{2c_W^2}\mathcal{F}_Z(s)$$
$$-\hat{\mathcal{F}}_W(s) + \left(R_Z - 1\right)\Big[\frac{|Q_e|}{2}\left(1 - 4|Q_e|s_W^2\right)\mathcal{F}_Z(s) + c_W^2\Big[\hat{\mathcal{F}}^0_{W_n}(s)$$
$$-|Q_{e'}|\mathcal{F}_{W_a}(s) + s\hat{\mathcal{B}}^d_{WW}(s,t)\Big]\Big] - \frac{r_t}{4}\left[B_0^F(-s; M_W, M_W) + 1\right]\Big\}, \qquad (A.4.82)$$

interference

$$\boxed{\kappa_{ef}} = 1 + \frac{g^2}{16\pi^2}\Big\{ -2\frac{c_W^2}{s_W^2}\Delta\rho^F - 2\Pi^F_{Z\gamma}(s) - \frac{1}{3}B_0^F(-s; M_W, M_W) - \frac{2}{9}$$
$$-\frac{1}{4c_W^2}\left[\frac{\delta_e^2 + \delta_f^2}{s_W^2}\left(R_W - 1\right) + 3v_e^2 + a_e^2 + 3v_f^2 + a_f^2\right]\mathcal{F}_Z(s)$$
$$-\hat{\mathcal{F}}_W^0(s) - \hat{\mathcal{F}}_W(s) - \frac{r_t}{4}\left[B_0^F(-s; M_W, M_W) + 1\right]$$
$$+c_W^2\left(R_Z - 1\right)\left[\frac{2}{3} - \hat{\Pi}^{\mathrm{bos},F}_{\gamma\gamma}(s) + s\hat{\mathcal{B}}^d_{WW}(s,t)\right]\Big\}. \qquad (A.4.83)$$

BOX

Fermionic loops in γ propagator

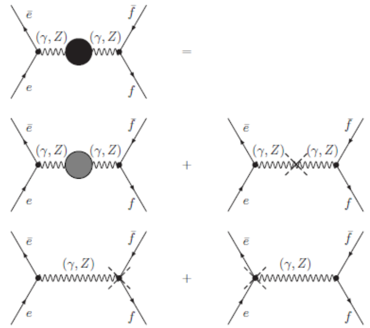


Figure A.11. *Bosonic self-energies and bosonic counter-terms for* $e\bar{e} \to (Z,\gamma) \to f\bar{f}$

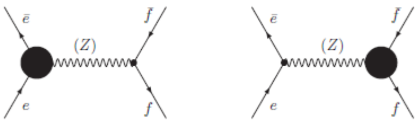


Figure A.10. *Electron (a) and final fermion (b) vertices in* $e\bar{e} \to (Z) \to f\bar{f}$

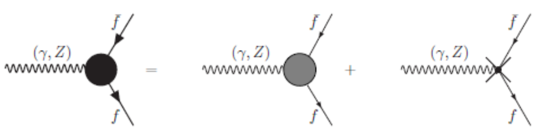


Figure A.6. *Off-shell* $Zf\bar{f}$ *and* $\gamma f\bar{f}$ *vertices*

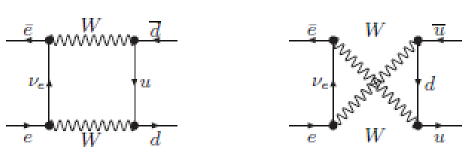


Figure A.7. *The WW boxes*

etc. etc.

LEP legacy: effective weak mixing angle

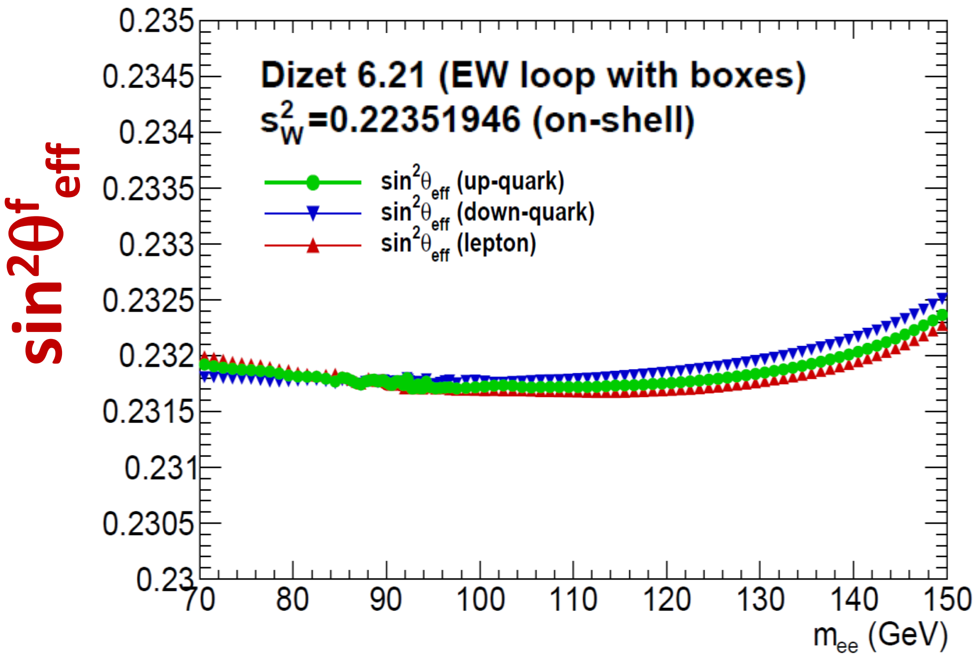
Here convoluted with line-shape and cosθ* distribution of MC events.

$$\sin^2\theta^f_{eff} = Re(K^f_Z)s^2_W + I^2_f$$

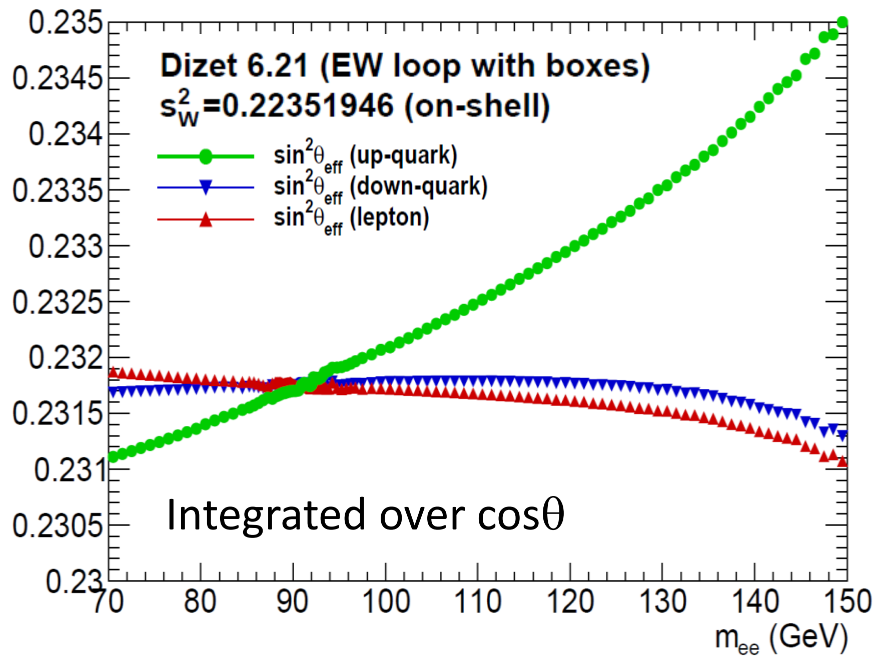
K(s,t)

$$I^2_f = \alpha^2(s)\frac{35}{18}[1 - \frac{8}{3}Re(K^f_Z)s^2_W]$$ = ~ 10 -4

Without box corrections



With box corrections



LHC paradigm

S. Dittmaier, M. Huber
arXiv:0911.2329

- **New schemes for input parameters:**
  ($\alpha(0)$, M_Z, M_W); ($\alpha(M_Z)$, M_Z, M_W); (G_μ, M_Z, M_W)
- **New treatment of Z-boson prop.**
  „complex mass scheme (CMS)",
  „pole mass scheme (PS)",
  „factorisation scheme (FS)"
- **Two scales for α_{QED}: $\alpha_{G\mu}$, $\alpha(0)$**
- **More emphasis on split into:**
  - **NLO corrections**
  - **Universal two-loop contributions**
- **EW correction terms organised differently, eg. $\sin^2\theta_{eff}$ not anymore transparent in the calculations**

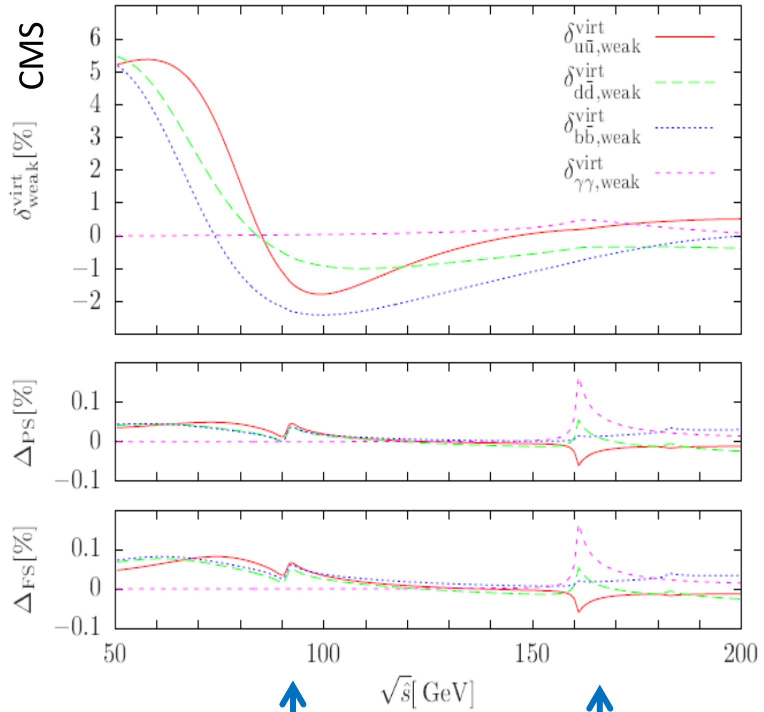


Figure 6: Weak corrections $\delta^{\rm virt}_{q\bar{q},\rm weak}$ and $\delta^{\rm virt}_{\gamma\gamma,\rm weak}$ to the total partonic cross sections for the different initial states and the differences Δ_X between scheme X and the CMS for treating the Z resonance.

Z threshold **WW threshold**

$$s_W^2 \to \bar{s}_W^2 \equiv s_W^2 + \Delta\rho\, c_W^2\,, \qquad c_W^2 \to \bar{c}_W^2 \equiv 1 - \bar{s}_W^2 = (1-\Delta\rho)\, c_W^2\,.$$

New paradigm for EW corrections, cont.

S. Dittmaier, M. Huber
arXiv:0911.2329

$\alpha(0)$

$\alpha(M_Z)$

G_μ

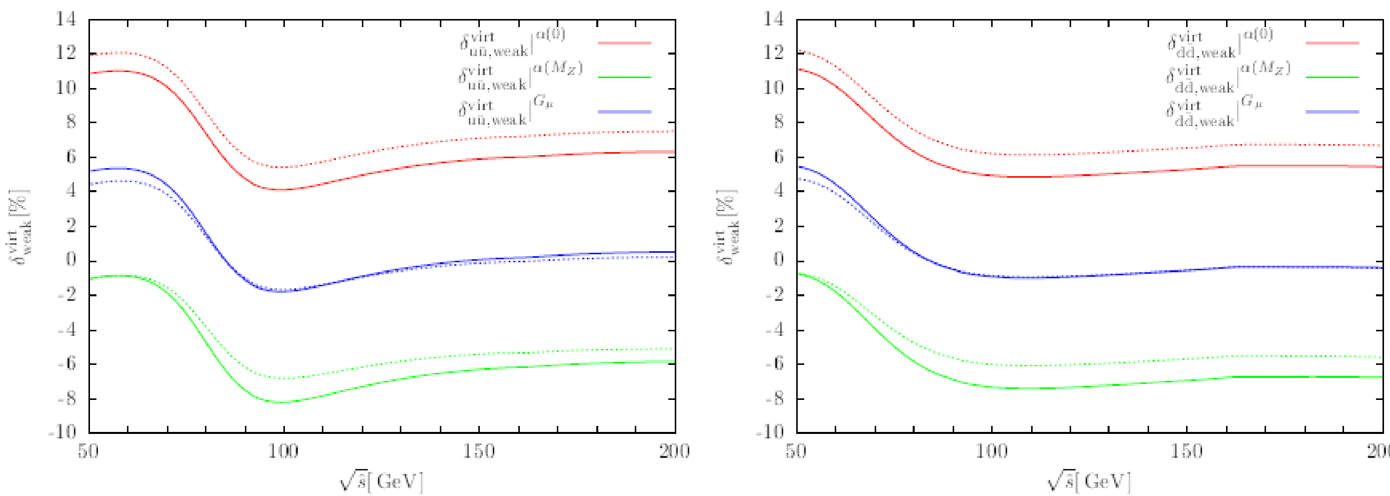


Figure 7: Weak corrections $\delta^{\rm virt}_{u\bar{u},{\rm weak}}$ and $\delta^{\rm virt}_{d\bar{d},{\rm weak}}$ to the partonic cross sections for the different input-parameter schemes, with (dashed lines) and without leading higher-order corrections due to $\Delta\alpha$ and $\Delta\rho$.

- **G_μ scheme the most stable w.r.t. higher-order electroweak effects among discussed input-parameter schemes.**
- **Desire to absorb the effects of HO corrections into the LO predictions.**
- **Two scales for $\alpha_{\rm QED}$**

EW schemes: pros and cons

- **EW scheme $\alpha(0)$ v0: input $\alpha(0)$, M_Z, $G\mu$**
  - **Pros:**
    - Precisely measured physics input, LEP legacy EW scheme
  - **Cons:**
    - Moderate NLO and HO corrections (few %) calculated theoretically or taken from low-energy measurements ($\alpha_{had}^{(5)}$)

- **EW scheme $\alpha(0)$ v1: input $\alpha(0)$, M_Z, M_W**
  - **Pros:**
    - Moderate NLO corrections (few %), small HO corrections (<1%)
  - **Cons:**
    - Input M_W with ±15 MeV uncertainties (=> 20-30 10-5 on s2w)
    - requires shifting G_μ far from its measured value.

- **EW scheme G_μ: input G_μ, M_Z, M_W**
  - **Pros:**
    - Small NLO (1%) and very small HO (0.2%) corrections
  - **Cons:**
    - Input M_W with ±15 MeV uncertainties (=> 20- 30 10-5 on s2w)
    - Requires two definitions for em coupling: $\alpha(0)$ for ISR/FSR/IFI and $\alpha_{G\mu}$ for matrix elements.

We are now establishing level of agreement between predictions calculated in three EW schemes, after including EW NLO+HO corrections.

EW schemes: input parameters

SM fundamental relation used to calculate EW parameters at LO in different EW schemes, on-mass-shell definition.

EW scheme:	G_μ, α, M_Z	α, M_W, M_Z	G_μ, M_W, M_Z
	LEP legacy		LHC standard
	$\alpha(0)$ v0	$\alpha(0)$ v1	$G\mu$
M_Z	91.1876 GeV	91.1876 GeV	91.1876 GeV
Γ_Z	2.4952 GeV	2.4952 GeV	2.4952 GeV
Γ_W	2.085 GeV	2.085 GeV	2.085 GeV
α	1/137.03599	1/137.03599	1/132.23323
G_μ	$1.1663787 \cdot 10^{-5}$ GeV^{-2}	$1.1254734 \cdot 10^{-5}$ GeV^{-2}	$1.1663787 \cdot 10^{-5}$ GeV^{-2}
M_W	80.93886 GeV	80.385 GeV	80.385 GeV
s_W^2	0.2121517	0.2228972	0.2228972
$\frac{G_\mu \cdot M_Z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha}$	1.0	1.0	1.0

$$G_\mu = \frac{\pi\alpha}{\sqrt{2} M_W^2 s_W^2}$$

$$s_W^2 = 1 - m_W^2 / m_Z^2$$

Be aware: $\alpha(0)$ v1 comes with unphysical value of $G\mu$

DIZET library
exact $O(\alpha)$+ higher order terms

Powheg_ew, MCSANC
EW NLO, NLO+HO

EW schemes: details

EW schemes: come with „on-shell” or „pole” definitions!

Table 44: The EW parameters used at tree-level EW, with on-mass-shell definition (LEP convention).

Parameter	$\alpha(0)$ v0	$\alpha(0)$ v1	G_μ
M_Z	91.1876 GeV	91.1876 GeV	91.1876 GeV
Γ_Z	2.4952 GeV	2.4952 GeV	2.4952 GeV
Γ_W	2.085 GeV	2.085 GeV	2.085 GeV
α	1/137.03599	1/137.03599	1/132.23323
G_μ	$1.1663787 \cdot 10^{-5}$ GeV^{-2}	$1.1254734 \cdot 10^{-5}$ GeV^{-2}	$1.1663787 \cdot 10^{-5}$ GeV^{-2}
M_W	80.93886 GeV	80.385 GeV	80.385 GeV
s_W^2	0.2121517	0.2228972	0.2228972
$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha}$	1.0	1.0	1.0

Runing Γ_Z in Z-propagator

Shift:

- **-30 MeV for M_Z**
- **change on Γ_Z**
- **-0.00006 for s^2w**

Scaling

- **0.99906 for α**

Table 45: The EW parameters used at tree-level EW, with pole definition of the Z, W masses.

Parameter	$\alpha(0)$ v0	$\alpha(0)$ v1	G_μ
M_Z	91.15348 GeV	91.15348 GeV	91.15348 GeV
Γ_Z	2.494266 GeV	2.494266	2.494266 GeV
Γ_W	2.085 GeV	2.085 GeV	2.085 GeV
α	1/137.03599	1/137.03599	1/132.3572336357709
G_μ	$1.1663787 \cdot 10^{-5}$ GeV^{-2}	$1.126555497 \cdot 10^{-5}$ GeV^{-2}	$1.1663787 \cdot 10^{-5}$ GeV^{-2}
M_W	80.91191 GeV	80.35797 GeV	80.35797 GeV
s_W^2	0.21208680	0.22283820939	0.22283820939
$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha}$	1.0	1.0	1.0

Fixed Γ_Z in Z-propagator

Strategy for comparison

- **Scope:**
  - Genuine EW and lineshape corrections to Drell-Yan production at NLO QCD.
  - Three EW LO schemes chosen to allow for straightforward interpretation of results. We tuned EW LO parameters, otherwise out-of-the-box.
  - The highest available corrections in a given approach used.
  - QED FRS/ISR not included here.
- **Observables:**
  - Lineshape (cross-section) and forward-backward asymmetry A_{FB} in the full phase-space.
  - Compared ratios or absolute differences between different EW LO schemes and/or between NLO, NLO+HO predictions within each EW scheme and same MC generator. Allows to minimize sensitivity to QCD details.
- **Goals:**
  - Check if reweighting with wt^{EW} (TauSpinner) works for NLO QCD MC's. Compared distributions at EW LO (DYTURBO, Powheg_ew).
  - Establish how consistent are predictions between different EW schemes with EW NLO corrections (Powheg_ew, MCSANC).
  - Establish how consistent are EW NLO+HO corrections of Dizet 6.21 form-factors implemented in wt^{EW} and those of Powheg_ew.

What we have so far .....

PowhegZj: QCD NLO, Z+j
wt^{EW} : TauSpinner + Dizet 6.21

wt^{EW}

EW LO → $\alpha(0)$ v0: LO, NLO+HO
EW LO → $\alpha(0)$ v1: LO
EW LO → G_μ: LO

Arbitrary
EW setup

Powheg_ew: QCD LO, Z

- $\alpha(0)$ v0: LO
- $\alpha(0)$ v1: LO, NLO, NLO+HO
- G_μ: LO, NLO, NLO+HO

DYTURBO: QCD LO, NLO, Z

- $\alpha(0)$ 0: LO
- $\alpha(0)$ v1: LO
- G_μ : LO

MCSANC: QCD LO, Z

- $\alpha(0)$ v1: LO, NLO, HO
- G_μ: LO, NLO, HO

Constructing wt^{EW}: EW Improved Born (IBA)

ERW and Z.Was, arXiv: 1808.08616

$$\mathscr{A}^{Born+EW} = \frac{\alpha}{s}\{ \quad [\bar{u}\gamma^{\mu}v g_{\mu\nu}\bar{v}\gamma^{\nu}u]\cdot(q_{\ell}\cdot q_f)\cdot \boxed{\Gamma_{V_{\Pi}}}\cdot\chi_{\gamma}(s)$$
$$+[\bar{u}\gamma^{\mu}v g_{\mu\nu}\bar{v}\gamma^{\nu}u\cdot(v_{\ell}\cdot v_f\cdot vv_{\ell f}) + \bar{u}\gamma^{\mu}v g_{\mu\nu}\bar{v}\gamma^{\nu}\gamma^{5}u\cdot(v_{\ell}\cdot a_f)$$
$$+\bar{u}\gamma^{\mu}\gamma^{5}v g_{\mu\nu}\bar{v}\gamma^{\nu}u\cdot(a_{\ell}\cdot v_f) + \bar{u}\gamma^{\mu}\gamma^{5}v g_{\mu\nu}\bar{v}\gamma^{\nu}\gamma^{5}u\cdot(a_{\ell}\cdot a_f)]\cdot \boxed{Z_{V_{\Pi}}}\cdot\chi_Z(s) \quad \}$$

$$\chi_{\gamma}(s) = 1$$

$$\chi_Z(s) = \frac{G_{\mu}\cdot M_z^2\cdot\Delta^2}{\sqrt{2}\cdot 8\pi\cdot\alpha}\cdot\frac{s}{s-M_Z^2+i\cdot\Gamma_Z\cdot M_Z}$$

$$v_{\ell} = (2\cdot T_3^{\ell} - 4\cdot q_{\ell}\cdot s_W^2\cdot \boxed{K_{\ell}(s,t)})/\Delta$$
$$v_f = (2\cdot T_3^{f} - 4\cdot q_f\cdot s_W^2\cdot \boxed{K_f(s,t)})/\Delta$$
$$a_{\ell} = (2\cdot T_3^{\ell})/\Delta$$
$$a_f = (2\cdot T_3^{f})/\Delta$$

$$\Delta = \sqrt{16\cdot s_W^2\cdot(1-s_W^2)}$$

$$\boxed{Z_{V_{\Pi}}} = \rho_{e,f}(s,t)$$

EW form-factors , functions of (s,t)=(m_{ll}, cosθ)
Calculated with Dizet 6.21 library.

$$\boxed{\Gamma_{V_{\Pi}}} = \frac{1}{2-(1+\boxed{\Pi_{\gamma\gamma}(s)})}$$

$$vv_{\ell f} = \frac{1}{v_{\ell}\cdot v_f}[(2\cdot T_3^{\ell})(2\cdot T_3^{f}) - 4\cdot q_{\ell}\cdot s_W^2\cdot \boxed{K_f(s,t)}(2\cdot T_3^{\ell}) - 4\cdot q_f\cdot s_W^2\cdot \boxed{K_{\ell}(s,t)}(2\cdot T_3^{f})$$
$$+(4\cdot q_{\ell}\cdot s_W^2)(4\cdot q_f\cdot s_W^2)\boxed{K_{\ell f}(s,t)}]\frac{1}{\Delta^2}$$

Vacuum polarisation corrections, used low-energy experiment input.
Warning: problem for analytic continuation.

Constructing wt^{EW}: per-event weight

ERW and Z.Was, arXiv: 1808.08616

Define per event electroweak weight

$wt^{EW} = \sigma_{Born}^{new} / \sigma_{Born}^{old}$

Approach developed in TauSpinner, arXiv:1802.05459

$$wt^{EW} = \frac{d\sigma_{Born+EW}(x_1, x_2, \hat{s}, \cos\theta\ , s_W^2)}{d\sigma_{Born}(x_1, x_2, \hat{s}, \cos\theta\ , s_W^2)}$$

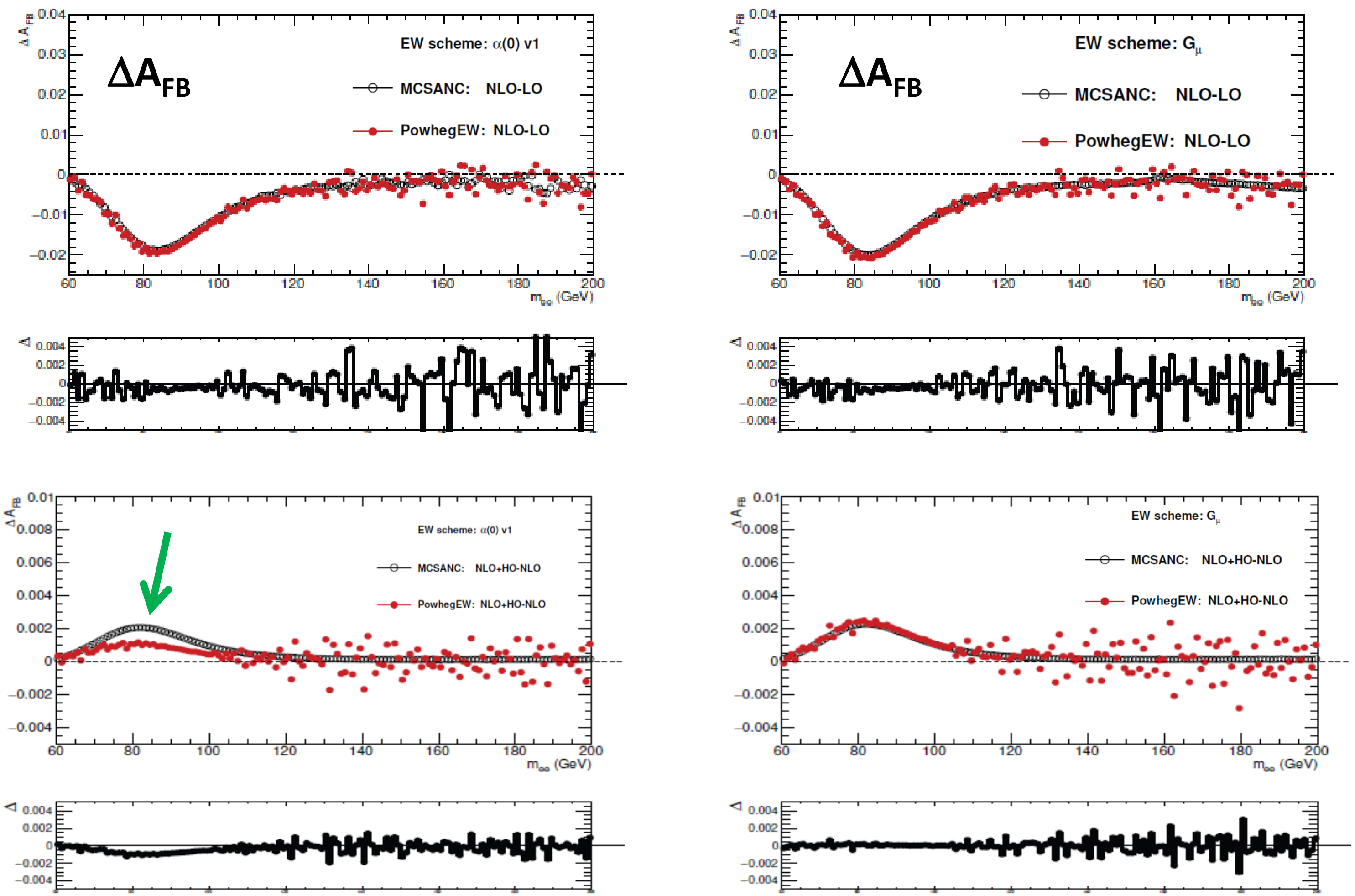
$$d\sigma_{Born}(x_1, x_2, \hat{s}, \cos\theta^*, s_W^2) = \sum_{q_f, \bar{q}_f} [f^{q_f}(x_1, ...) f^{\bar{q}_f}(x_2, ...) d\sigma_{Born}^{q_f \bar{q}_f}(\hat{s}, \cos\theta\ , s_W^2) + f^{q_f}(x_2, ...) f^{\bar{q}_f}(x_1, ...) d\sigma_{Born}^{\bar{q}_f q_f}(\hat{s}, -\cos\theta\ , s_W^2)]$$

x_1, x_2, $\cos\theta$ (symmetrised) calculated using 4-momenta of outgoing leptons; asymmetry in sign of $\cos\theta$ from weighted average over PDFs

Allows to reweight MC event generated between different EW LO scheme and to Improved Born Approximation in EW scheme used for form-factors calculation.

Theory predictions: EW LO, NLO, NLO+HO

NEW!!



m_{ll} (GeV)

F. Piccinini et al.

Powheg_ew

S. Bondarenko,

L. Kalinovskaya

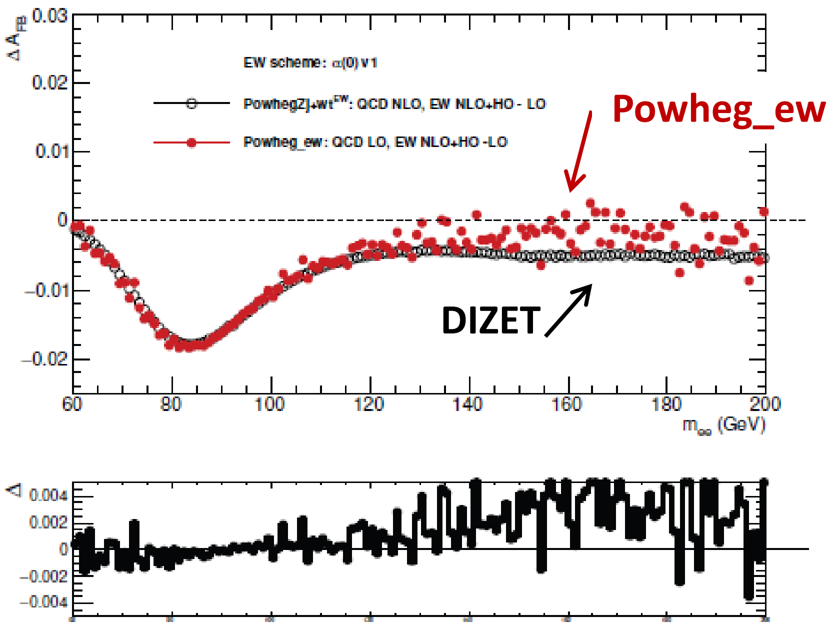
MCSANC

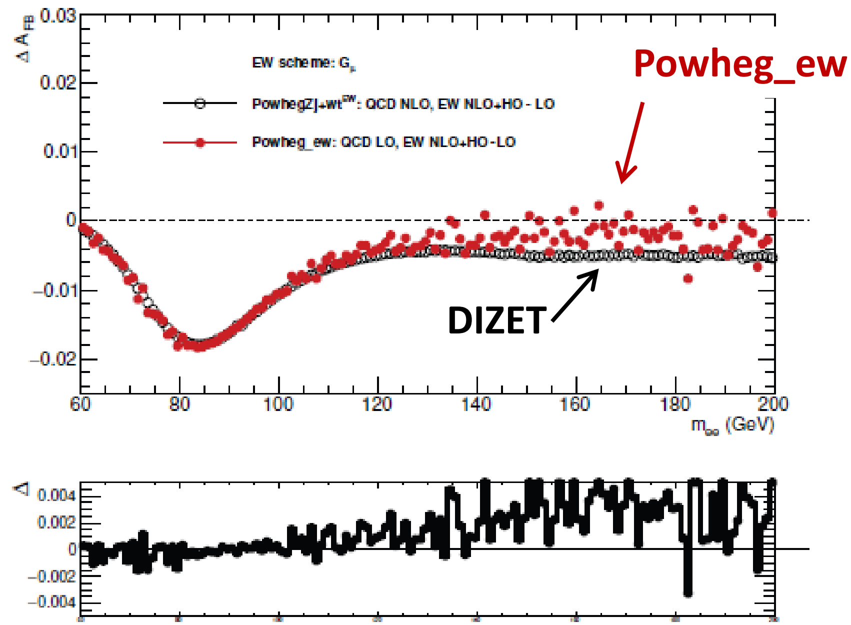
Investigating now this discrepancy:

0.001 shift on ΔA_{FB} at Z-pole corresponds to shift ~ 30 10-5 on $\sin^2\theta_{eff}$

Theory predictions: EW LO, NLO+HO

NEW!!





Good agreement between Powheg_ew and DIZET around Z-pole

At higher masses, DIZET predicts stable shift of 0.005 while both PowhegEW and MCSANC predicts (NLO+HO – LO) being close to zero.

Powheg_ew: EW LO, NLO, NLO+HO

	EW order	m_{ee} = 89 - 93 GeV	m_{ee} = 80 - 100 GeV	m_{ee} = 70 - 120 GeV
$A_{FB}\ \alpha(0)$ v0	LO	0.06691361	0.06392369	0.06253754
$A_{FB}\ \alpha(0)$ v1	LO	0.04653886	0.04343789	0.04212883
$A_{FB}\ G_\mu$	LO	0.04653886	0.04343789	0.04212883
$A_{FB}\ \alpha(0)$ v1	NLO	0.03004289	0.02690785	0.02569858
$A_{FB}\ G_\mu$	NLO	0.02905841	0.02592168	0.02471918
$A_{FB}\ \alpha(0)$ v1	NLO+HO	0.03083234	0.02770533	0.02649700
$A_{FB}\ G_\mu$	NLO+HO	0.03090286	0.02777783	0.02656851
$\Delta A_{FB}\ \alpha(0)$ v1	NLO-LO	-0.0164959	-0.0165300	-0.0164302
$\Delta A_{FB}\ G_\mu$	NLO-LO	-0.0174805	-0.0175162	-0.0174096
$\Delta A_{FB}\ \alpha(0)$ v1	NLO+HO-LO	-0.0157065	-0.0157326	-0.0156318
$\Delta A_{FB}\ G_\mu$	NLO+HO-LO	-0.0156360	-0.0156596	-0.0155603

A_{FB}

ΔA_{FB} (NLO – LO)

ΔA_{FB} (NLO+HO – LO)

ΔA_{FB}	EW order	m_{ee} = 89 - 93 GeV	m_{ee} = 80 - 100 GeV	m_{ee} = 70 - 120 GeV
$\alpha(0)$ v1 - $\alpha(0)$ v0	LO	-0.020375	-0.020486	-0.020487
G_μ - $\alpha(0)$ v0	LO	-0.020375	-0.020486	-0.0204871
G_μ - $\alpha(0)$ v1	LO	0.0	0.0	0.0
G_μ - $\alpha(0)$ v1	NLO	-0.00098	-0.00098	-0.00098
G_μ - $\alpha(0)$ v1	NLO + HO	-0.00007	-0.00007	-0.00007

ΔA_{FB} between EW schemes at LO, NLO, NLO+HO

Better than 0.0001 agreement on A_{FB} at NLO+HO between two EW schemes !

Multi-boson precision physics at LHC

- The theoretical calculations for multi-boson processes at LHC use as default the G_μ scheme.
  - M_W, M_Z at the on-shell value
  - Z-couplings to fermions not at value measured at LEP, s2w = 0.22289 should be s2w= 0.23152
  - Two scales of α_{QED} :
    - $\alpha(0)$ = 1/137 , used for radiative corrections, Z γ hard processes
    - $\alpha_{G\mu}$ = 1/132 , used for γ couplings in matrix elements

    Requires attention to avoid breaking gauge-cancellations

- EW genuine and lineshape corrections often not available in MC's tools: eg. MATRIX. Requires using correcting scalings calculated with „effective" couplings instead eg. for predicting Z-polarisation in WZ events.

- More advanced technique, i.e. reweighting with wt^{EW} could provide pragmatic/operational solution.

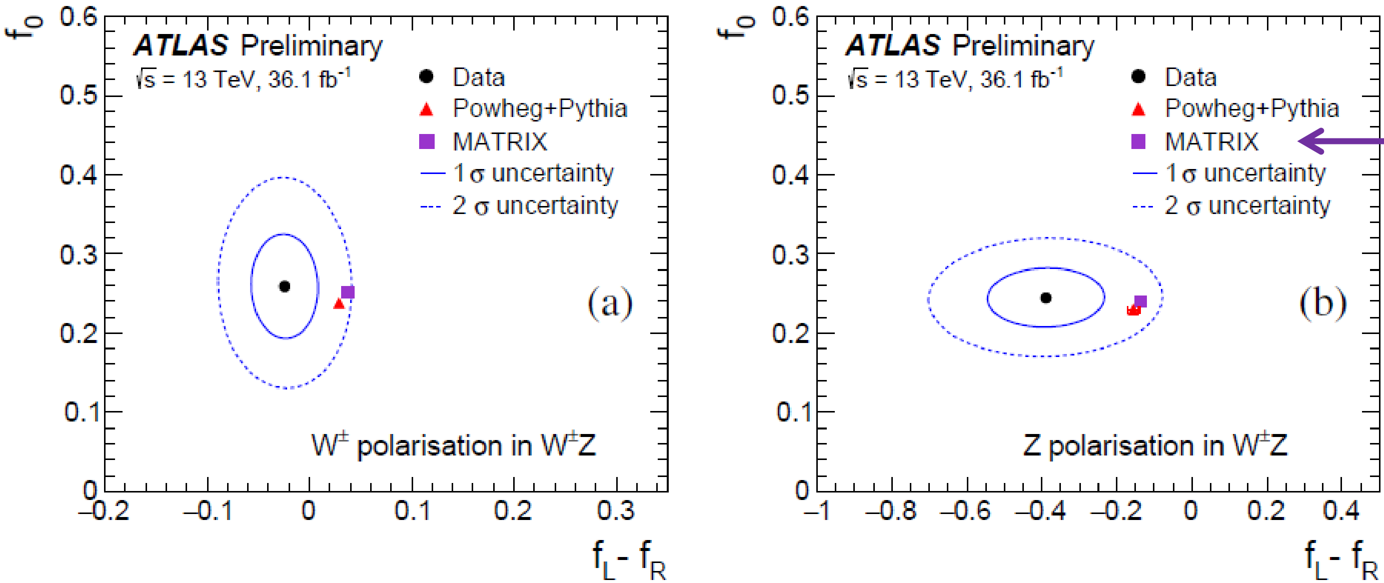
Gauge boson polarisation in WZ events

ATLAS-CONF-2018-034

$$\frac{1}{\sigma_{W^\pm Z}}\frac{d\sigma_{W^\pm Z}}{d\cos\theta_{\ell,W}} = \frac{3}{8}f_L(1\mp\cos\theta_{\ell,W})^2 + \frac{3}{8}f_R(1\pm\cos\theta_{\ell,W})^2 + \frac{3}{4}f_0\sin^2\theta_{\ell,W}\,,$$

$$\frac{1}{\sigma_{W^\pm Z}}\frac{d\sigma_{W^\pm Z}}{d\cos\theta_{\ell,Z}} = \frac{3}{8}f_L(1+2\alpha\cos\theta_{\ell,Z}+\cos^2\theta_{\ell,Z}) + \frac{3}{8}f_R(1+\cos^2\theta_{\ell,Z}-2\alpha\cos\theta_{\ell,Z}) + \frac{3}{4}f_0\sin^2\theta_{\ell,Z}$$

$$\alpha = \frac{2c_v c_a}{c_v^2+c_a^2} \qquad c_v = -\tfrac{1}{2} + 2\sin^2\theta_W^{\mathrm{eff}} \qquad c_a = -\tfrac{1}{2}$$



Predictions corrected to effective coupling

s2w = 0.22289

s2w = 0.23153

Predictions at EW LO, using effective $\sin^2\theta_W^{\mathrm{eff}} = 0.23152$

Summary

- **The LEP legacy EW scheme should be kept as a reference to allow for continuity with so far best measured SM parameters definitions.**
- **Keeping the standard of splitting genuine EW+lineshape corr. and FSR/ISR/IFI corrections is mandatory, because of experimental analyses complexity and required precision of theoretical predictions.**
- **Choice of the EW scheme: a trade-off between parametric uncertainty and correction size. Optimal choice depends on measurement and its accuracy.**
  - **Be aware that „G_μ scheme" for input parameters comes with large parametric uncertainty on M_W input parameter known to ± 15 MeV only (=> 20 - 30 10-5 on $\sin^2\theta_{eff}$)**
- **For multi-bosons: to get correct Z-polarisation mandatory to obtain $\sin^2\theta_{eff}$=0.23153 whichever input parameters one starts from.**

SPARES slides

Powheg_ew: EW LO, NLO, NLO+HO

Cross-section

	EW order	m_{ee} = 89 - 93 GeV	m_{ee} = 80 - 100 GeV	m_{ee} = 70 - 120 GeV
$\alpha(0)$ v0	LO	630.848722	906.156051	959.658977
$\alpha(0)$ v1	LO	571.411296	821.363274	870.729908
G_μ	LO	612.514433	880.446121	933.363827
$\alpha(0)$ v1	NLO	600.185042	863.142557	915.580114
G_μ	NLO	607.142292	873.173294	926.253246
$\alpha(0)$ v1	NLO+HO	607.551746	873.717147	926.761229
G_μ	NLO+HO	607.515354	873.655348	926.681425
$\alpha(0)$ v1	NLO/LO	1.050350	1.05087	1.05151
G_μ	NLO/LO	0.991230	0.99174	0.99238
$\alpha(0)$ v1	NLO+HO/LO	1.063247	1.063740	1.064349
G_μ	NLO+HO/LO	0.991038	0.992287	0.992840
$\alpha(0)$ v1 / $\alpha(0)$ v0	LO	0.90578	0.906426	0.90733
G_μ / $\alpha(0)$ v1	LO	1.07193	1.07193	1.07193
G_μ / $\alpha(0)$ v1	NLO	1.01159	1.01162	1.01166
G_μ / $\alpha(0)$ v1	NLO+HO	0.99994	0.99993	0.99991
G_μ / $\alpha(0)$ v0	LO	0.97094	0.97163	0.97260

σ (pb)

σ_{NLO}/σ_{LO}

$\sigma_{NLO+HO}/\sigma_{LO}$

Ratios between EW schemes LO, NLO, NLO+HO

Better than 0.01% agreement on σ between EW schemes at NLO+HO !

EW schemes: input parameters

SM fundamental relation used to calculate EW parameters at LO in different EW schemes, on-mass-shell definition.

EW scheme:	G_μ, α, M_Z	α, M_W, M_Z	G_μ, M_W, M_Z
Parameter	α(0) v0	α(0) v1	Gμ
M_Z	91.1876 GeV	91.1876 GeV	91.1876 GeV
Γ_Z	2.4952 GeV	2.4952 GeV	2.4952 GeV
Γ_W	2.085 GeV	2.085 GeV	2.085 GeV
α	1/137.03599	1/137.03599	1/132.23323
G_μ	$1.1663787 \cdot 10^{-5}$ GeV^{-2}	$1.1254734 \cdot 10^{-5}$ GeV^{-2}	$1.1663787 \cdot 10^{-5}$ GeV^{-2}
M_W	80.93886 GeV	80.385 GeV	80.385 GeV
s_W^2	0.2121517	0.2228972	0.2228972
$\frac{G_\mu \cdot M_Z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha}$	1.0	1.0	1.0

PowhegZj
91.1876 GeV
2.4952 GeV
2.085 GeV
1/128.88859
$1.16638 \cdot 10^{-5}$ GeV^{-2}
79.958 GeV
0.2311300
1.0

MC events used for reweighting

$$s_W^2 = 1 - m_W^2/m_Z^2$$

$$G_\mu = \frac{\pi\alpha}{\sqrt{2} M_W^2 s_W^2}$$

EW schemes: α(0) v0, α(0) v1 – same value of α
G_μ, α(0) v1 – same value of s^2w

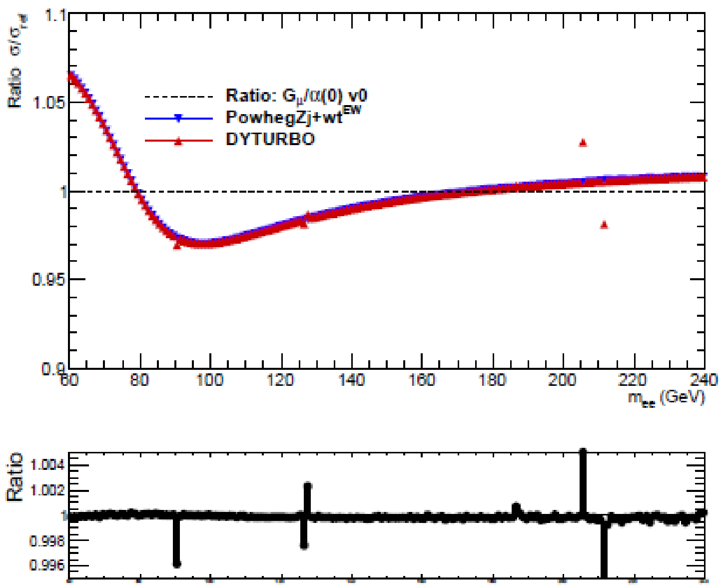
Validating reweighting with wt^{EW}: EW LO

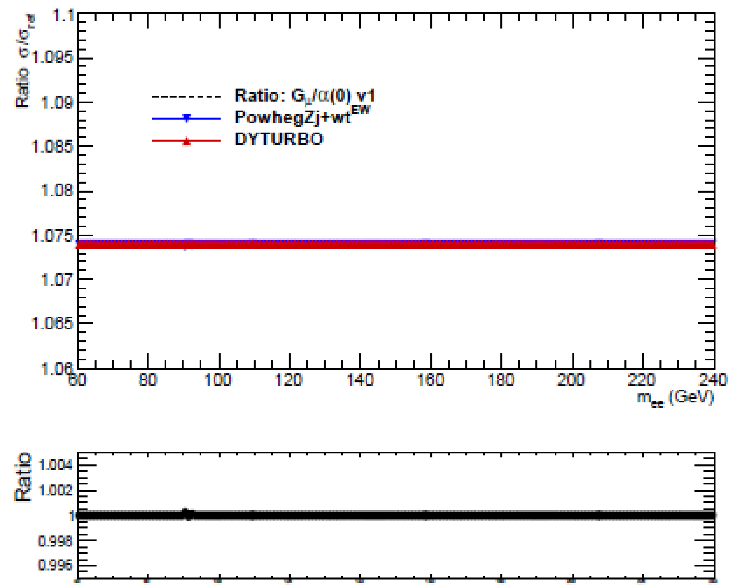
- Ratio of differential cross-sections (lineshapes) driven by relative balance between Z and γ contributions.
- EW $\alpha(0)$ v1 and G_μ schemes chosen as such that ratio of cross-sections is equal to ratio of QED couplings squared.

σ/σ_{ref}

PowhegZj+wt^{EW}
DYTURBO

Benchmark for wt^{EW} reweighting



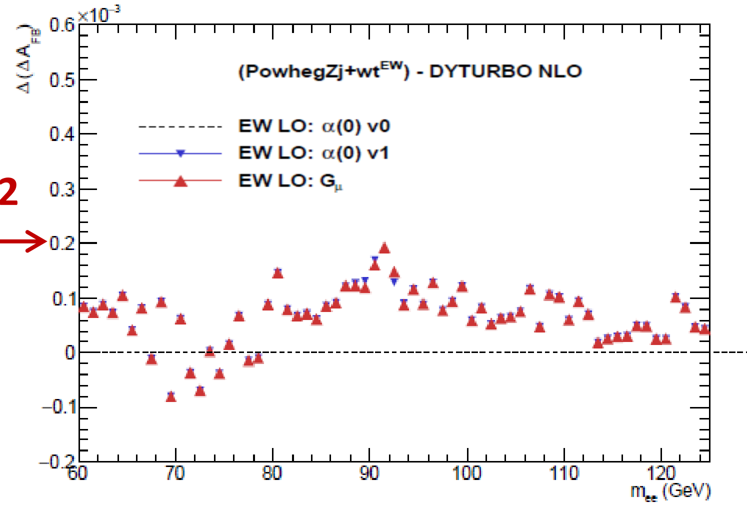


Validating reweighting with wt^{EW}: EW LO

$\Delta\, A_{FB}$: driven by s^2_W value (same for $\alpha(0)$ v1 and G_μ schemes)

Benchmark for wt^{EW} reweighting

0.0002



Double difference:

ΔA_{FB} (DYTURBO) - ΔA_{FB} (PowhegZj+wt^{EW})

$\alpha(0)$ v1 - $\alpha(0)$ v0

G_μ - $\alpha(0)$ v0

Agreement on $\Delta(\Delta\, A_{FB})$ within ± 0.0002

Should redo it with much finer binning around Z-pole to better estimate precision.

EW LO schemes in practice

- **SM fundamental relations used to calculate EW parameters in EW LO schemes**

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}M_W^2 s_W^2} \longrightarrow \frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2}\cdot 8\pi\cdot\alpha} = 1 \qquad \Delta^2 = 16\cdot s_W^2\cdot(1-s_W^2)$$

EW scheme: G_μ, α, M_Z **α(0) v0**	EW scheme: α, M_W, M_Z **α(0) v1**	EW scheme: G_μ, M_W, M_Z **G_μ**
$d2 = \frac{\sqrt{2}\cdot 8\pi\cdot\alpha}{G_\mu\cdot M_z^2}$ $\boxed{s_W^2} = (-1+\sqrt{1-d2/4})/2$	$\boxed{s_W^2} = 1 - m_W^2/m_Z^2$ $c_W^2 = m_W^2/m_Z^2$ $g2 = 4\cdot\pi\cdot\alpha/s_W^2$ $\boxed{G_\mu} = \sqrt{2}\cdot g2/8/m_W^2$	$\boxed{s_W^2} = 1 - m_W^2/m_Z^2$ $c_W^2 = m_W^2/m_Z^2$ $g2 = 8\cdot G_\mu\cdot m_W^2/\sqrt{2}$ $\boxed{\alpha} = g2\cdot s_W^2/4/\pi$

$\boxed{}$ **calculated**

EW LO schemes: details

Running and fixed Z-boson width in the propagator: taking into account photonic - loop corrections to Γ_Z

- **Fixed width** $$\chi_Z(s) = \frac{1}{s - M_Z^2 + i \cdot \Gamma_Z \cdot M_Z}.$$

- **Running width (LEP legacy)**

$$\chi'_Z(s) = \frac{1}{s - M_Z^2 + i \cdot \Gamma_Z \cdot s/M_Z}$$

$$\begin{aligned}
\chi'_Z(s) &= \frac{1}{s(1 + i \cdot \Gamma_Z/M_Z) - M_Z^2} \\
&= \frac{(1 - i \cdot \Gamma_Z/M_Z)}{s(1 + \Gamma_Z^2/M_Z^2) - M_Z^2(1 - i \cdot \Gamma_Z/M_Z)} \\
&= \frac{(1 - i \cdot \Gamma_Z/M_Z)}{(1 + \Gamma_Z^2/M_Z^2)} \frac{1}{s - \frac{M_Z^2}{1+\Gamma_Z^2/M_Z^2} + i \cdot \frac{\Gamma_Z M_Z}{1+\Gamma_Z^2/M_Z^2}} \\
&= N_Z \frac{1}{s - {M'_Z}^2 + i\Gamma'_Z M'_Z} \\
M'_Z &= \frac{M_Z}{\sqrt{1 + \Gamma_Z^2/M_Z^2}} \\
\Gamma'_Z &= \frac{\Gamma_Z}{\sqrt{1 + \Gamma_Z^2/M_Z^2}} \\
N_Z &= \frac{(1 - i \cdot \Gamma_Z/M_Z)}{(1 + \Gamma_Z^2/M_Z^2)} = \frac{(1 - i \cdot \Gamma'_Z/M'_Z)}{(1 + {\Gamma'_Z}^2/{M'_Z}^2)}
\end{aligned}$$

Both equivalent if redefined parameters m_Z, Γ_Z, N_Z (normalization). Change in the normalisation can (?) be absorbed into G_μ redefinition. In case of „pole" convention (last slide) it was absorbed into α.

Impact of $\Delta\alpha_{had}^{(5)}(M_Z^2)$

Predictions from Dizet 6.21 library

Parameter	$\Delta\alpha_h^{(5)}(M_Z^2) = 0.0280398$	$\Delta\alpha_h^{(5)}(M_Z^2) = 0.02753$	Ratio
$\alpha(M_Z^2)$	0.00775884	0.00775463	
$1/\alpha(M_Z^2)$	128.885224	128.95522	0.99932
s_W^2	0.22351946	0.22331458	1.00092
$sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23175990	0.23157062	1.00082
$sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23164930	0.23146414	1.00080
$sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23152214	0.23133715	1.00080
M_W	80.35281 GeV	80.36341 GeV	1.00013
Δr	0.03694272	0.03631342	1.01733
Δr_{rem}	0.01169749	0.01170244	0.99958
ρ_{eu}	1.005408	1.005426	0.99998
K_e	1.036649	1.036770	0.99988
K_u	1.036172	1.036293	0.99988
K_{eu}	1.074146	1.074397	0.99977
ρ_{ed}	1.005894	1.005906	0.99999
K_e	1.036649	1.036699	0.99995
K_d	1.035603	1.035719	0.99989
K_{ed}	1.073556	1.073859	0.99972

shift of about -0.00020 due to corrections to M_W

shift by +11 MeV

ATLAS measurement
M_W = 80370 ± 19 MeV

$$M_W = \frac{M_Z}{\sqrt{2}}\sqrt{1+\sqrt{1-\frac{4A_0^2}{M_Z^2(1-\Delta r)}}}$$

$$\Delta r = \Delta\alpha(M_Z^2) + \Delta r_{EW}$$

$$A_0 = \sqrt{\frac{\pi\alpha(0)}{\sqrt{2}G_\mu}}$$

Impact of m_t

Parameter	m_t = 171 GeV	m_t = 173 GeV	m_t = 175 GeV
$\alpha(M_Z^2)$	0.00775882	0.00775884	0.00775885
$1/\alpha(M_Z^2)$	128.888558	128.885224	128.885079
s_W^2	0.22375411	0.22351946	0.22328310
$sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23181756	0.23175990	0.23169368
$sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23171096	0.23164930	0.23169368
$sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23158377	0.23152214	0.23145996
Δr	0.03766186	0.03694272	0.03621664
Δr_{rem}	0.01165959	0.01169749	0.01173500
ρ_{eu}	1.005229	1.005408	1.005589
K_e	1.035837	1.036649	1.037467
K_u	1.035361	1.036172	1.036990
K_{eu}	1.072465	1.074146	1.075843
ρ_{ed}	1.005714	1.005894	1.006075
K_e	1.035837	1.036649	1.037467
K_d	1.034792	1.035603	1.036420
K_{ed}	1.071876	1.073556	1.075252

±2 GeV shift in m_t corresponds to ±0.00005 shift in $\sin^2_{eff}{}^{lep}$

Dizet 6.21 -> 6.42-> 6.44

AMT4 = 4 – available in Dizet 6.21

Pragmatic question: is it indeed more precise estimate to use AMT4=5 or AMT4=6?

Or better stay with well tested AMT4=4 ? What uncertaintity attribute to this correction?

arXiv:1302.1395v3

Table 1: *ZFITTER v.6.44beta, with the input values $\alpha_s = 0.1184$, $M_Z = 91.1876$ GeV, $M_H = 125$ GeV, $m_t = 173$ GeV. The dependence on electroweak NNLO corrections is studied for IMOMS=1 (input values are α_{em}, M_Z, G_μ). AMT4=4: with two-loop sub-leading corrections and re-summation recipe of [23-28] of [13]; AMT4=5: with fermionic two-loop corrections to M_W according to [29,30,32] of [13]; AMT4=6: with complete two-loop corrections to M_W [37] and fermionic two-loop corrections to $\sin^2\theta_W^{\mathrm{lept,eff}}$ [52] of [13]. IBAIKOV=0 (no α_s^4 QCD corrections) or IBAIKOV=2012 [190].*

AMT4	4	5	6	Diff.	Exp. Err.
IBAIKOV=0					
$\Gamma_Z(\mu^+\mu^-)$, MeV	83.9782	83.9748	83.9807	0.0059	0.086
Γ_Z, MeV	2494.7863	2494.6019	2494.8688	0.2669	2.3
$\Gamma_W(l\nu)$, MeV	226.3185	226.2877	226.2922	0.0308	1.9
Γ_W, MeV	2090.3308	2090.0465	2090.0882	0.2843	42
M_W, GeV	80.3578	80.3541	80.3546	0.0037	0.015
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$	0.231722	0.231791	0.231670	0.000121	0.00012
IBAIKOV=2012					
$\Gamma_Z(\mu^+\mu^-)$, MeV	83.9782	83.9748	83.9807	0.0059	0.086
Γ_Z, MeV	2494.5591	2494.3747	2494.6416	0.2669	2.3
$\Gamma_W(l\nu)$, MeV	226.3185	226.2877	226.2922	0.030	1.9
Γ_W, MeV	2090.1117	2089.8274	2089.8691	0.2843	42
M_W, GeV	80.3578	80.3541	80.3546	0.0037	0.015
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$	0.231722	0.231791	0.231670	0.000121	0.00012

± 0.00005 around nominal value of $\sin^2\theta_{\mathrm{eff}}$ with AMT4=4