Gamma-Ray Signals From Velocity-Dependent Dark Matter Annihilation

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collaborators

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- PRD **95**, 123008 (2017) [1702.00408]; KB, JK, LS, M-YW
- PRD **98**, 063012 (2017) [1805.08379]; KB, JK, LS
- 1905.03431; KB, JK, JR, LS

velocity-dependent DM annihilation

- the prompt photon flux from dark matter annihilation can be factorized into two pieces....
- ... a particle physics factor
  - depends on annihilation cross section, annihilation channel, particle mass
- and an astrophysics factor
  - depends on the dark matter density profile of the target
  - encoded in the J-factor
- but if dark matter annihilation is velocity-dependent
  - then velocity-distribution also come into play
- goal is to compute the effective J-factor (J_S) ...
- ... and see impact on gamma-ray searches for dark matter in subhalos
  - focusing on angular distribution

why subhalos?

- our previous work looked at relative flux from dSphs (1702.00408), and angular distribution from Galactic Center (1805.08379)
  - see also Robertson, Zentner (0902.0362); Ferrer, Hunter (1306.6586); Bergstrom, et al. (1712.03188); Petac, Ullio, Valli (1804.05052)
- focus now on angular distribution of emission from subhalos
- can constrain dark matter models with …
  - angular distribution from a single subhalo
  - anisotropies from a distribution of subhalos
- as more dSphs are found and new instruments make observations, measurements of angular distribution become more important
- there is an ongoing program to use grav. lensing surveys and numerical simulation to estimate the distribution of subhalos
- there could be new results ahead….

gameplan

- we will consider dark matter annihilation in subhalos
  - how does velocity-dependent annihilation affect the gamma ray angular distribution, and total flux?
  - effect on one subhalo, and on anisotropies from a distribution of subhalos

- we assume an spherically symmetric potential and isotropic velocity distribution with two parameters, ρ_s and r_s

- results are largely determined by dimensional analysis

- angular size of emission from each subhalo is rescaled by a universal factor, but flux from each subhalo is rescaled by a subhalo-dependent factor

what is the J-factor?

- the photon flux depends on ....
  - particle physics of the dark matter model
    - independent of target
  - astrophysics of the target
    - mostly independent of dark matter model
- J-factor is the astrophysics factor
  - larger J = larger flux, regardless of particles physics model
- but factorization based on an assumption
  - $\sigma_A v$ independent of v
- what happens for v-dependent annihilation? (assume power law)

$$\frac{d^2\Phi}{dE\,d\Omega} = \frac{1}{4\pi}\frac{dN}{dE}\int d\ell \int d^3v_1 \frac{f\left(\vec{r}\left(\ell,\Omega\right),\vec{v}_1\right)}{m_X}\int d^3v_2 \frac{f\left(\vec{r}\left(\ell,\Omega\right),\vec{v}_2\right)}{m_X} \times \frac{\sigma_A\left|\vec{v}_1-\vec{v}_2\right|}{2}$$

$$= \frac{\left(\sigma_A v\right)}{8\pi m_X^2}\frac{dN}{dE}\times J\left(\cos\theta\right)$$

$$J\left(\cos\theta\right) \equiv \int d\ell \left[\rho\left(\vec{r}\left(\ell,\Omega\right)\right)\right]^2$$

$$\rho\left(\vec{r}\right) = \int d^3v\, f\left(\vec{r},\vec{v}\right)$$

f = dark matter velocity distribution

defining J_S

- just need to absorb $S(v)$ into definition of astrophysical factor
- new factor, J_S, encodes astro. info needed to determine $d^2\Phi/dEd\Omega$ for velocity-dependent case
- assume $S(v) = (v/c)^n$
  - n=-1 (Sommerfeld (Coulomb))
  - n=0 (s-wave)
  - n=2 (p-wave)
  - n=4 (d-wave)
- need the DM velocity distribution
- assume potential is spherically symmetric and f is isotropic

$$\sigma_A v = (\sigma_A v)_0 \times S(v)$$

$$J_S(\cos\theta) \equiv \int d\ell \int d^3v_1 f(\vec{r}(\ell,\Omega),\vec{v}_1) \int d^3v_2 f(\vec{r}(\ell,\Omega),\vec{v}_2) \times S(|\vec{v}_1 - \vec{v}_2|)$$

$$\frac{d^2\Phi}{dEd\Omega} = \frac{(\sigma_A v)_0}{8\pi m_X^2}\frac{dN}{dE} \times J_S(\cos\theta)$$

dimensional analysis

- assume f depends on only three dimensionful quantities
  - scale density ρ_s
  - scale radius r_s
  - Newton's constant G_N
- only one quantity with units of velocity can be formed from these parameters… $(4\pi\, G_N\, \rho_s\, r_s^2)^{1/2}$
- so dependence on subhalo parameters determined by dimensional analysis
- can now express everything in terms of scale-free quantities

$$\tilde{r} \equiv r / r_s$$

$$\tilde{v} \equiv v / \left(4\pi G_N \rho_s r_s^2\right)^{1/2}$$

$$\rho(r) = \rho_s\ \tilde{\rho}(\tilde{r})$$

$$f(r,v) = \rho_s \left(4\pi G_N \rho_s r_s^2\right)^{-3/2} \tilde{f}(\tilde{r},\tilde{v})$$

$$D = \text{distance to target}$$

$$\theta_0 \equiv r_s / D = \text{naive angular size}$$

$$\tilde{\theta} \equiv \theta / \theta_0$$

scale-free distributions

- assume small angle approximation ($\theta_0 \ll 1$)
- can express effective J-factor in terms of scale-free quantities
- $\tilde{J}_{s(n)}(\tilde{\theta})/\tilde{J}_{s(n)}^{total}$ = scale-free angular distribution
  - depends on n and $\tilde{f}$
  - not on subhalo parameters
- normalization also scaled by $\tilde{J}_{s(n)}^{tot}\ (4\pi G_N \rho_s r_s^2/c^2)^{n/2}$
  - relative normalization depends on n and subhalo parameters, not on $\tilde{f}$

$$J_{S(n)}^{tot} = \frac{4\pi\rho_s^2 r_s^3}{D^2}\left(4\pi G_N \rho_s r_s^2 / c^2\right)^{n/2} \tilde{J}_{s(n)}^{tot}$$

$$\tilde{J}_{s(n)}^{tot} = \int_0 d\left(\tilde{\theta}^2/2\right)\ \tilde{J}_{s(n)}\left(\tilde{\theta}\right)$$

$$\tilde{J}_{s(n)}\left(\tilde{\theta}\right) = \int_{\tilde{\theta}}^{\infty} d\tilde{r}\left[1-\frac{\tilde{\theta}}{\tilde{r}}\right]^{-1/2} P_n^2\left(\tilde{r}\right)$$

$$P_n^2\left(\tilde{r}\right) = \int d^3\tilde{v}_1 \int d^3 v_2\ \tilde{f}\left(\tilde{r},\tilde{v}_1\right)\tilde{f}\left(\tilde{r},\tilde{v}_2\right)\ \left|\vec{\tilde{v}}_1 - \vec{\tilde{v}}_2\right|^n$$

$$\left\langle\theta\right\rangle = \theta_0 \frac{\int_0 d\left(\tilde{\theta}^2/2\right)\ \tilde{\theta}\ \tilde{J}_{s(n)}\left(\tilde{\theta}\right)}{\tilde{J}_{s(n)}^{tot}}$$

upshot

- for each halo, scale-free angular distribution ($\tilde{\theta}$) is the same
  - location and size come in when you rescale → $\theta = \tilde{\theta}\,(r_s/D)$
  - otherwise, angular distribution has no dependence on halo parameters
  - only depends on form of velocity-distribution and velocity-dependence
- but total J-factor (total flux) is rescaled by two factors
  - $\mathfrak{J}_{s(n)}^{tot}$ normalization factor is common to all halos, so it cannot be distinguished from a rescaling of the cross section
  - $(4\pi G_N \rho_s r_s^2)^{n/2}$ rescaling factor is determined by dimensional analysis only
    - doesn't matter what the velocity-distribution is, provided it depends only on the two parameters ρ_s and r_s, and is isotropic with spherically symmetric potential
- now, we will find the scale-free angular distribution for the NFW profile, after determining $f(r,v)$ using the Eddington formula

idea behind Eddington formalism

- velocity distribution f(**r**,**v**) is essentially the phase space density
- assume particles move only under a collective gravitational central potential (not two-body scattering)
- classical path depends only on integrals of motion, E and **L**
- Jean's theorem – phase space density depends only on integrals of motion → why?
  - if two phase space points have the same integrals of motion, any particles at one point will be (or once were) at the other
  - average phase space density is constant along path (Liouville's theorem)
  - so time-averaged phase space density depends only on the integrals of motion
- if f(**r**,**v**) depends on r, not **r** (spherically-symmetric potential) and depends on v, not **v** (isotropic), then f(r,v) depends only on E, not **L**
- so given ρ(r), can back out the v-dependence with Abel's integral equation

determining f(r,v)

- strategy
  - ansatz for DM density distrib.
    - NFW
  - fixes gravitational potential $\Phi(r)$
  - now Eddington inversion formula determines velocity distribution
- easy to write in scale-free form
- we'll use an NFW profile

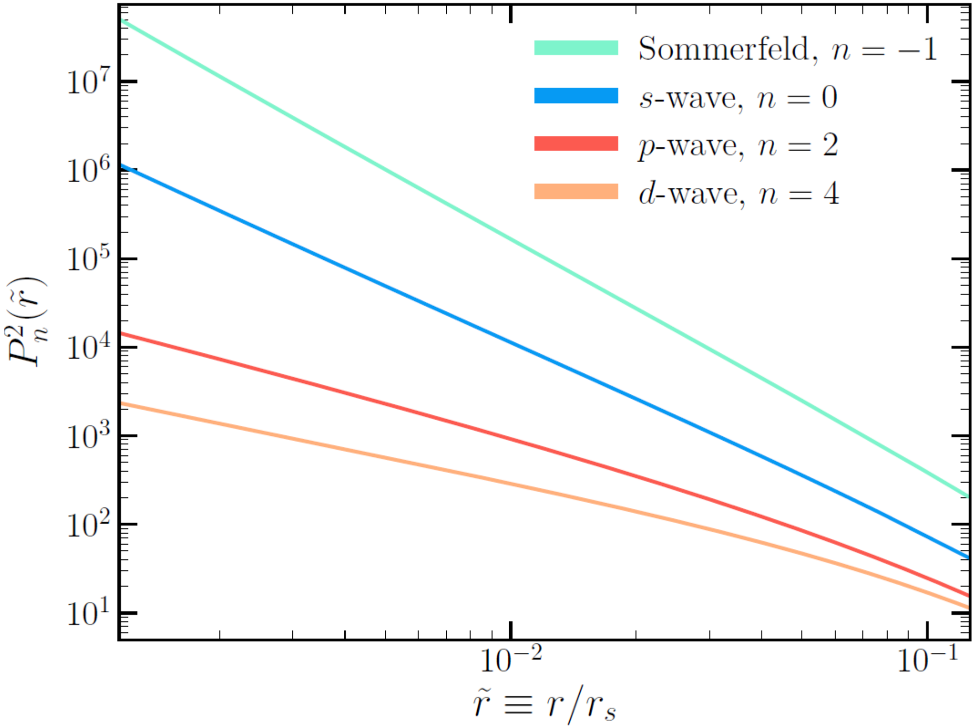
$$\tilde{\rho}(\tilde{r}) = \frac{1}{\tilde{r}^2 (1+\tilde{r})^2}$$

Eddington formula

$$f(\varepsilon) = \frac{1}{\sqrt{8}\pi^2} \int_{\varepsilon}^{0} \frac{d\Phi}{\sqrt{\Phi - \varepsilon}} \frac{d^2\rho}{d\Phi^2}$$

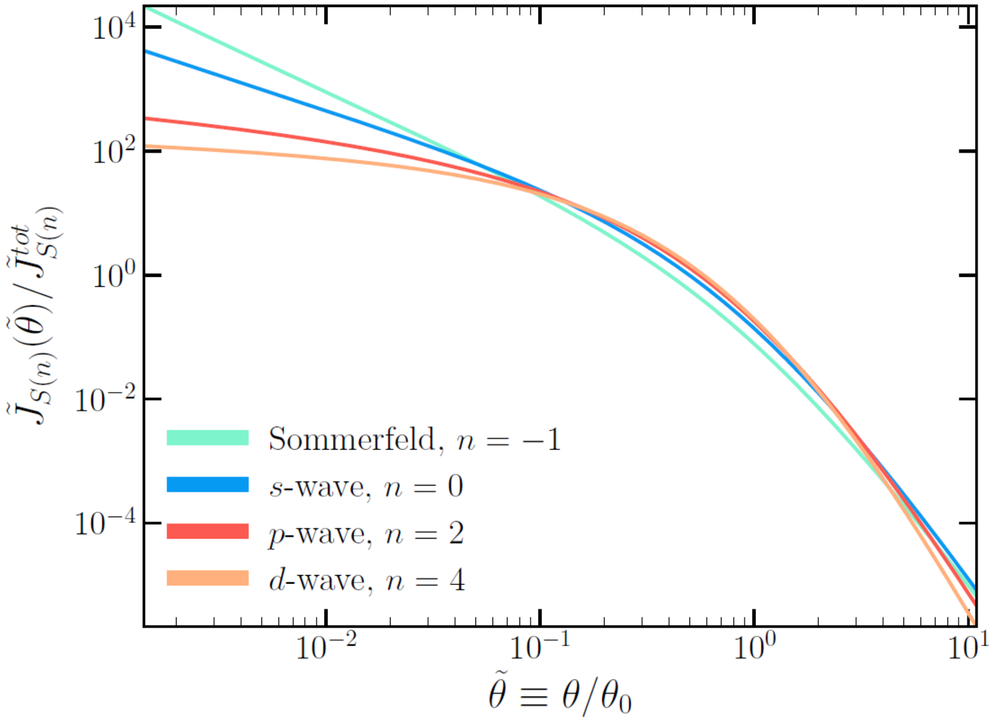
$$\varepsilon \equiv \frac{v^2}{2} + \Phi(r) < 0$$

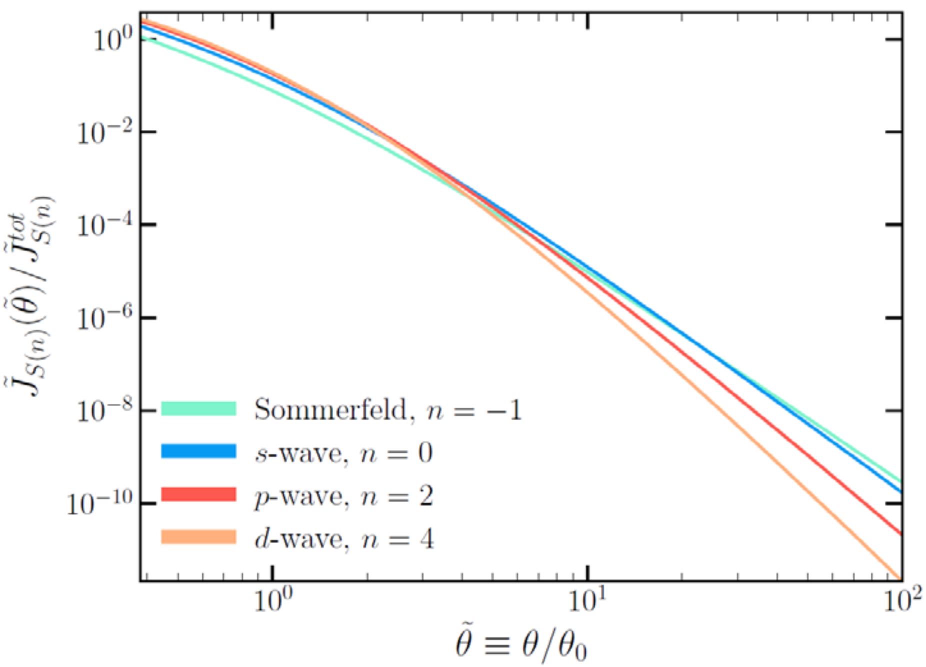
$$\rho(r) = 4\pi \int_0^{\sqrt{-2\Phi(r)}} dv\, v^2 f(r,v)$$



NFW

- velocity suppressed
  - far away – escape velocity smaller
  - near center – enclosed mass smaller and ang. mom. barrier
- biggest difference in angular distribution for Sommerfeld (~40%)

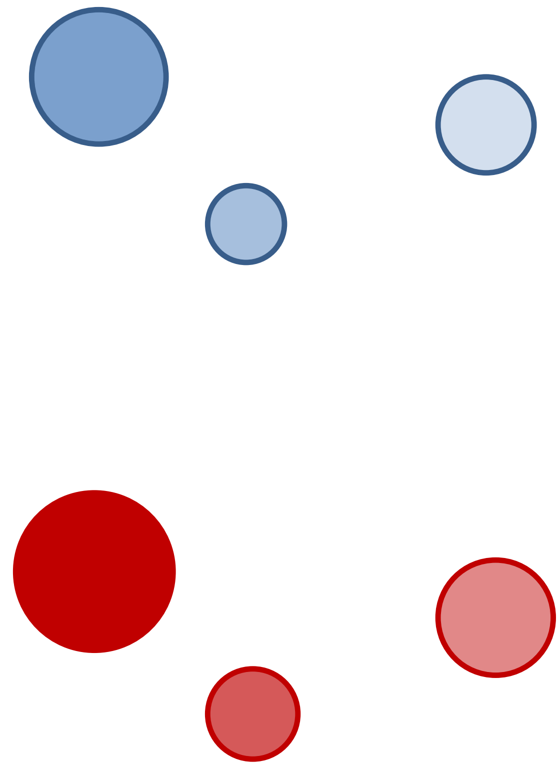




n	$\langle\theta\rangle/\theta_0$	$J_{s(n)}^{tot}$
-1	0.24	2.11
0	0.38	0.33
2	0.45	0.15
4	0.46	0.12

a distribution of halos

- now consider a distribution of many unresolved subhalos
- relative change in flux cannot be canceled by change in $\sigma_A v$
- more important than change in size of any particular subhalo….
- $M_s \equiv \rho_s \, r_s^3$
- flux scales as $(M_s/\theta_0 D)^{n/2}$
- p-, d-wave suppression favors massive subhalos which are nearer and smaller ang. size
- Sommerfeld favors opposite
- now need distribution



a single-parameter family

- halo has two parameters → ρ_s, r_s
- but can reduce them to one
  - simulations find tight correlation between subhalo mass and velocity dispersion → $M_s \propto \sigma^\alpha$
  - $\alpha \sim 3.3$
- amounts to $M_s \propto r_s^{\alpha/(\alpha-2)}$
- expected to hold for subhalos also
  - generally, they're tidally stripped
  - tidal radius much larger than r_s
  - annihilation rate small there, so relation is basically unchanged

- flux from halos of size θ_0 scales as $(\theta_0 D)^{n/(\alpha-2)}$
- probability distribution depends just on halo mass and distance
- in some examples, probability distribution factorizes
  - $\propto M^{-2} \times F(D)$
  - Han, Cole, Frenk, Jing MNRAS **457** 1208 (2016)
- then, flux from halos of size θ_0 scales as $(\theta_0)^{-3+(n+2)/(\alpha-2)}$
  - almost flat for p-wave, grows with θ_0 for d-wave, falls for s-wave or Sommerfeld

next steps

- we didn’t need to use NFW to get these general results
  - general formalism applies to any spherically symmetric potential and isotropic distribution with two dimensionful parameters
  - generalized NFW, Einasto, etc.
  - choice only affects the detailed form of the scale-free angular distribution
- didn’t need to use Eddington inversion formula to get velocity distribution
  - from simulation find $\rho/\sigma^3 \propto r^{-1.875}$ (Taylor, Navarro, ApJ **563**,483, 2001)
  - could use this for the p-wave case
  - but can’t use it for d-wave or Sommerfeld (only get $\sigma(\tilde{r})$, not $f(\tilde{r},\tilde{v})$)
- already get subhalo probability distributions from simulation
  - improvements possible
  - can use weak/strong lensing of other galaxies to get distributions
- can apply to gamma-ray data (single dSph, or to anisotropies)

conclusion

- J-factors of astrophysical objects change dramatically if dark matter annihilation is velocity-dependent
- changes size of gamma ray emission from individual subhalos, and relative flux from different halos
- set by dimensional analysis

- impact on data from gamma-ray observatories

Mahalo!

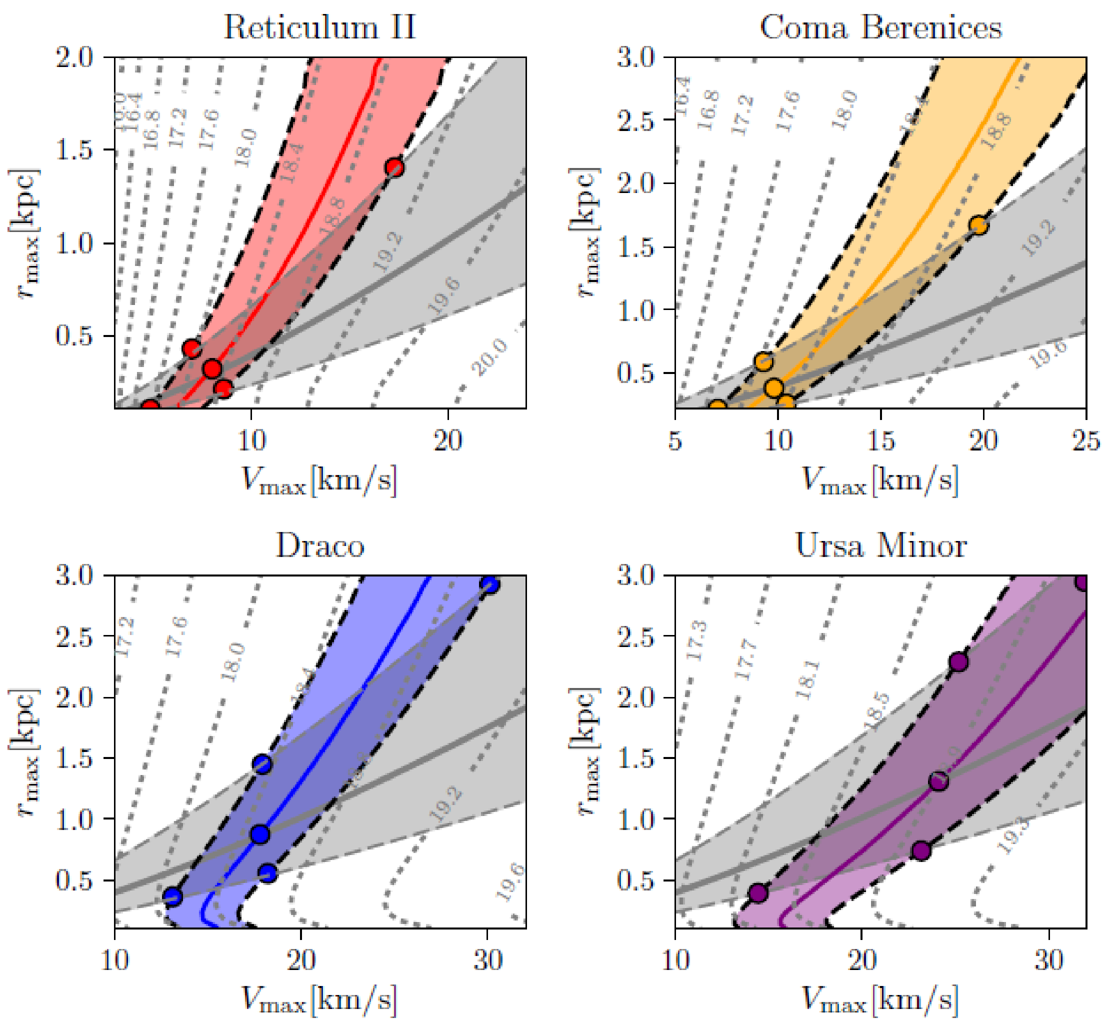
Back-up slides

upshot

- velocity-suppressed cross sections decrease annihilation rate at small distance and at large distance from GC
  - small distance – less enclosed mass
  - large distance – escape velocity is smaller
- for cuspy profiles, suppression at small distances dominates
  - angular distribution suppressed in inner 1°
- as inner slope becomes less steep, suppression at large distances becomes more important
  - suppresses angular distribution for > 50° degrees
- generally see increase at $\sim \mathcal{O}(10°)$ (~ 10-15% effect)
- not degenerate with changes to the inner slope
  - can fix small angle or large angle, but not both
- morphology can constrain velocity-dependence of signal from GC

dSph velocity profiles

V_{max} = max. circ velocity, at radius r_{max}



colored bands = fit from stellar velocity dispersion

$\sim V_{max} \propto r_{max}^{1/2}$

gray bands = fit to Aquarius

(Martinez, Bullock, Kaplinghat, Strigari, Trotta 0902.4715)

Sommerfeld-enhancement

- essential setup
  - dark matter annihilation is a contact interaction
  - but dark matter self-interacts through a long range force
    - mediator mass = m_ϕ
  - so have to rescale matrix element by wavefunction at the origin
- actual potential is Yukawa
  - can solve numerically
  - but can solve analytically if we approximate it with a Hulthén potential (within 10%)

- $\langle\sigma_A v\rangle \equiv \langle\sigma_A v\rangle_0 \times S(v)$
- $V(r) = -(\alpha_X / r)\exp(-m_\phi r)$
- four regimes for Hulthén
- $m_\Phi \gg \alpha_X m_X$: non-enhanced
  - $S = 1$
- $m_\Phi \ll v m_X \ll \alpha_X m_X$:Coulomb limit
  - $S(v) = 2\pi\alpha_X / v$
- $v m_X \ll m_\Phi \ll \alpha_X m_X$:saturation
  - $S(v) = 16\,\alpha_X\, m_X / m_\phi$
- $m_\Phi = 6\alpha_X m_X/(\pi^2 n^2) \ll \alpha_X m_X$: resonance
  - $S = 4\alpha_X^2 / v^2 n^2$ (cutoff at small v)
- focus: non-enhanced v. Coulomb