PAUL SCHERRER INSTITUT

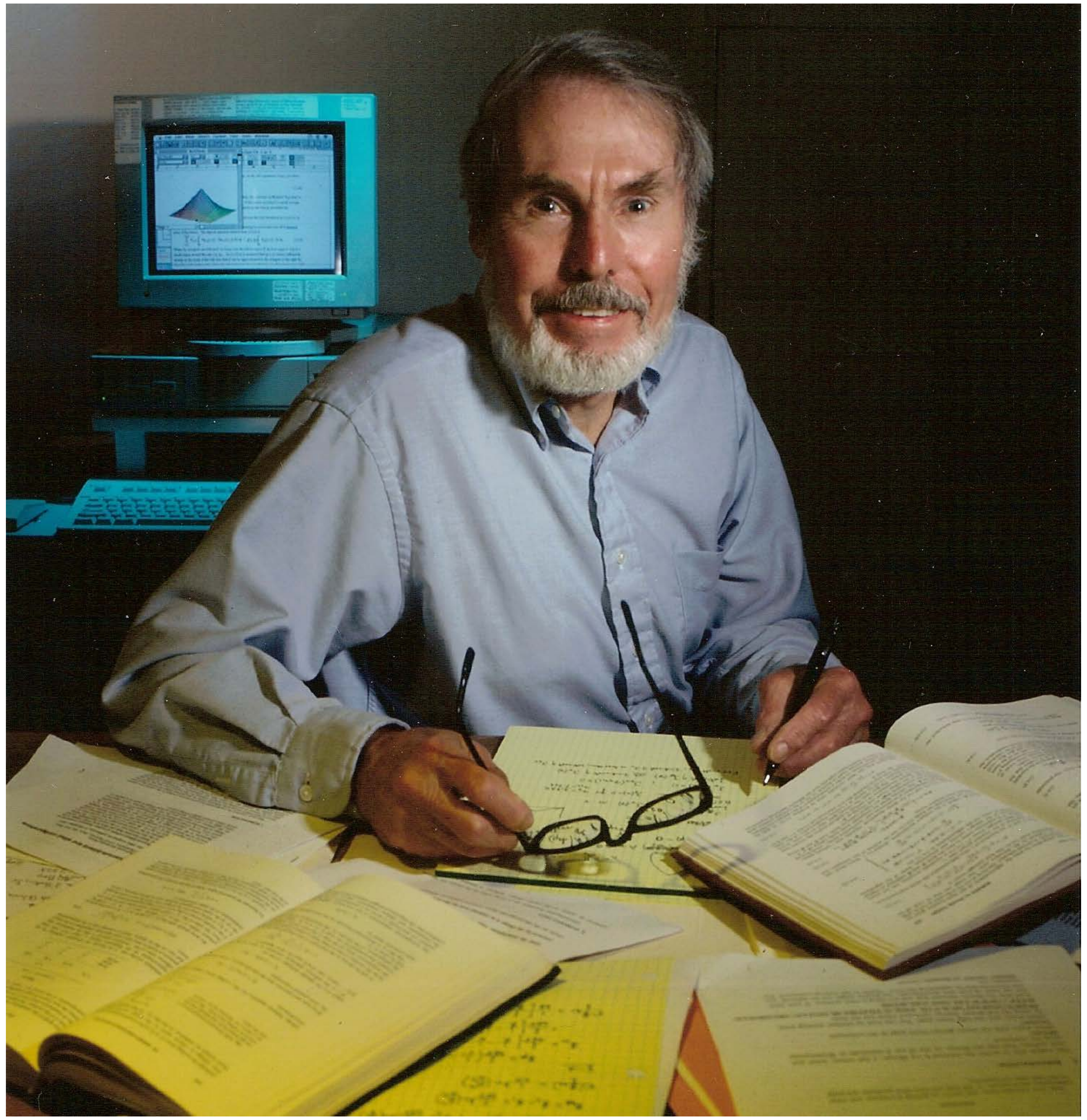
Rasmus Ischebeck

Machine Physics 1

Joint Universities Accelerator School

Radiation Emitted by Electrons in a Synchrotron

- Radiation emitted by electrons in a magnetic fields can be calculated from Maxwell's equations



Lienard-Wiechert potentials (I)

We want to compute the em field generated by a charged particle in motion on a given trajectory $\bar{x} = \bar{r}(t)$

The charge density and current distribution of a single particle read

$$\rho(\bar{x},t) = q\delta^{(3)}(\bar{x} - \bar{r}(t)) \qquad \bar{J}(\bar{x},t) = q\bar{v}(t)\delta^{(3)}(\bar{x} - \bar{r}(t))$$

We have to solve Maxwell equations driven by such time varying charge density and current distribution.

The general expression for the wave equation for the em potentials (in the Lorentz gauge) reads

$$\bar{\nabla}^2\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\frac{\rho}{\varepsilon_0} \qquad \bar{\nabla}^2\bar{A} - \frac{1}{c^2}\frac{\partial^2\bar{A}}{\partial t^2} = -\mu_0\bar{J}$$

Lienard-Wiechert potentials (II)

The general solutions for the wave equation driven by a time varying charge and current density read (in the Lorentz gauge) [Jackson Chap. 6]

$$\Phi(\bar{x},t) = \frac{1}{4\pi\varepsilon_0}\int d^3\bar{x}'\int dt' \frac{\rho(\bar{x}',t')}{|\bar{x}-\bar{x}'|}\delta\left(t' + \frac{|\bar{x}-\bar{x}'|}{c} - t\right) \qquad A(\bar{x},t) = \frac{1}{4\pi\varepsilon_0 c^2}\int d^3\bar{x}'\int dt' \frac{\bar{J}(\bar{x}',t')}{|\bar{x}-\bar{x}'|}\delta\left(t' + \frac{|\bar{x}-\bar{x}'|}{c} - t\right)$$

Integrating the Dirac delta in time we are left with

$$\Phi(\bar{x},t) = \frac{1}{4\pi\varepsilon_0}\iiint_V \frac{\rho(\bar{x}',t_{ret})}{|\bar{x}-\bar{x}'|} d^3\bar{x}' \qquad \bar{A}(\bar{x},t) = \frac{\mu_0}{4\pi}\iiint_V \frac{\bar{J}(\bar{x}',t_{ret})}{|\bar{x}-\bar{x}'|} d^3\bar{x}'$$

where ret means retarted $t_{ret} = t - \frac{|\bar{x}(t) - \bar{x}(t_{ret})|}{c}$ **(see next slide)**

Now we use the charge density and current distribution of a single particle

$$\rho(\bar{x},t) = q\delta^{(3)}(\bar{x}-\bar{r}(t)) \qquad \bar{J}(\bar{x},t) = q\bar{v}(t)\delta^{(3)}(\bar{x}-\bar{r}(t))$$

Lienard-Wiechert potentials (III)

Substituting we get

$$\Phi(\bar{x},t)=\frac{q}{4\pi\varepsilon_0}\iiint_V \frac{\delta^{(3)}[\bar{x}'-\bar{r}(t_{ret})]}{|\bar{x}-\bar{x}'|}d^3\bar{x}' \qquad \bar{A}(\bar{x},t)=\frac{q\mu_0}{4\pi}\iiint_V \frac{\bar{v}(t_{ret})[\bar{x}'-\bar{r}(t_{ret})]}{|\bar{x}-\bar{x}'|}d^3\bar{x}'$$

Using again the properties of the Dirac deltas we can integrate and obtain the Lienard-Wiechert potentials

$$\Phi(\bar{x},t)=\frac{1}{4\pi\varepsilon_0}\left[\frac{e}{(1-\bar{\beta}\cdot\bar{n})R}\right]_{ret} \qquad \bar{A}(\bar{x},t)=\frac{1}{4\pi\varepsilon_0 c}\left[\frac{e\bar{\beta}}{(1-\bar{\beta}\cdot\bar{n})R}\right]_{ret}$$

These are the potentials of the em fields generated by the charged particle in motion.

The trajectory itself is determined by external electric and magnetic fields

Which Simplification Have I Tacitly Made?
Where Does This Simplification Fail?

Nick Veasey

Critical Frequency and Critical Angle

$$\frac{d^3I}{d\Omega\,d\omega} = \frac{e^2}{16\pi^3\varepsilon_0 c}\left(\frac{2\omega\rho}{3c\gamma^2}\right)^2\left(1+\gamma^2\theta^2\right)^2\left[K_{2/3}^2(\xi)+\frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}K_{1/3}^2(\xi)\right]$$

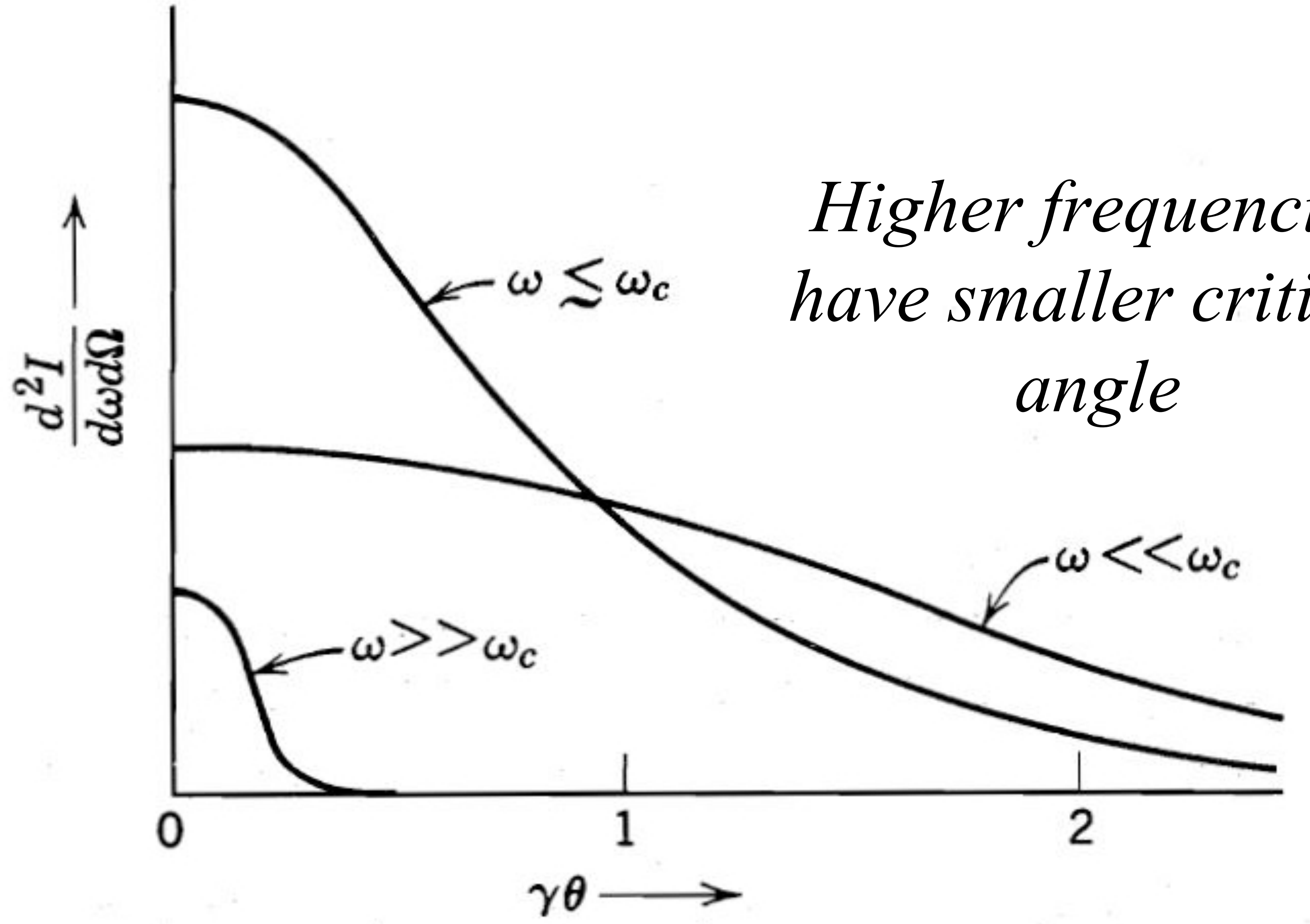
Properties of the modified Bessel function ==> radiation intensity is negligible for x >> 1

$$\xi = \frac{\omega\rho}{3c\gamma^3}\left(1+\gamma^2\theta^2\right)^{3/2} >> 1$$

Critical frequency $\omega_c = \frac{3}{2}\frac{c}{\rho}\gamma^3$

$\approx \omega_{rev}\gamma^3$

Critical angle $\theta_c = \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3}$



Higher frequencies have smaller critical angle

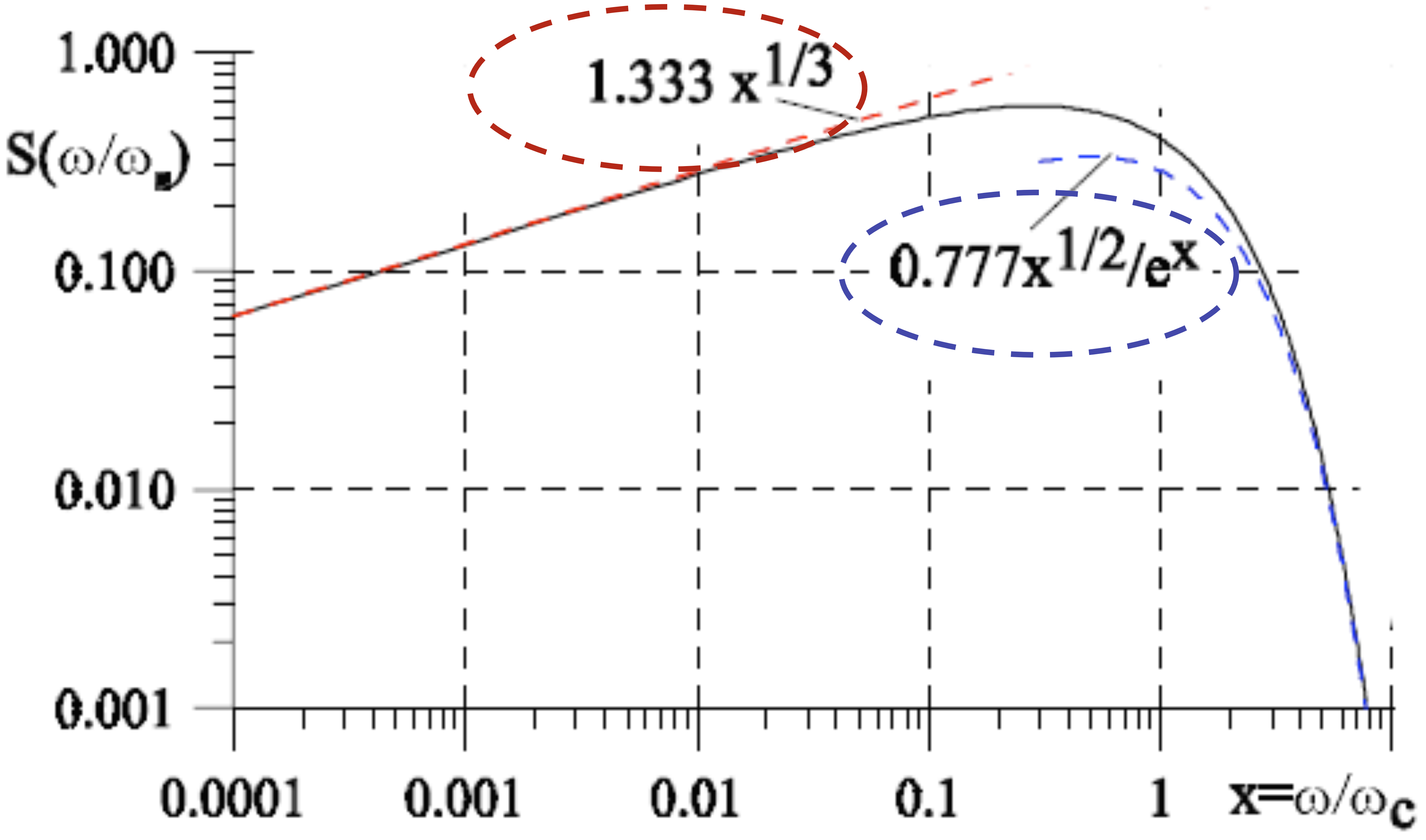
For frequencies much larger than the critical frequency and angles much larger than the critical angle the synchrotron radiation emission is negligible

Spectrum of Synchrotron Radiation

$$\frac{dI}{d\omega} = \iint_{4\pi} \frac{d^3 I}{d\omega\, d\Omega} d\Omega = \frac{\sqrt{3} e^2}{4\pi\varepsilon_0 c} \gamma \frac{\omega}{\omega_C} \int_{\omega/\omega_C}^{\infty} K_{5/3}(x) dx$$

$$\frac{dI}{d\omega} \approx \frac{e^2}{4\pi\varepsilon_0 c} \left(\frac{\omega\rho}{c} \right)^{1/3} \quad \omega << \omega_c$$

$$\frac{dI}{d\omega} \approx \sqrt{\frac{3\pi}{2}} \frac{e^2}{4\pi\varepsilon_0 c} \gamma \left(\frac{\omega}{\omega_c} \right)^{1/2} e^{-\omega/\omega_c} \quad \omega >> \omega_c$$



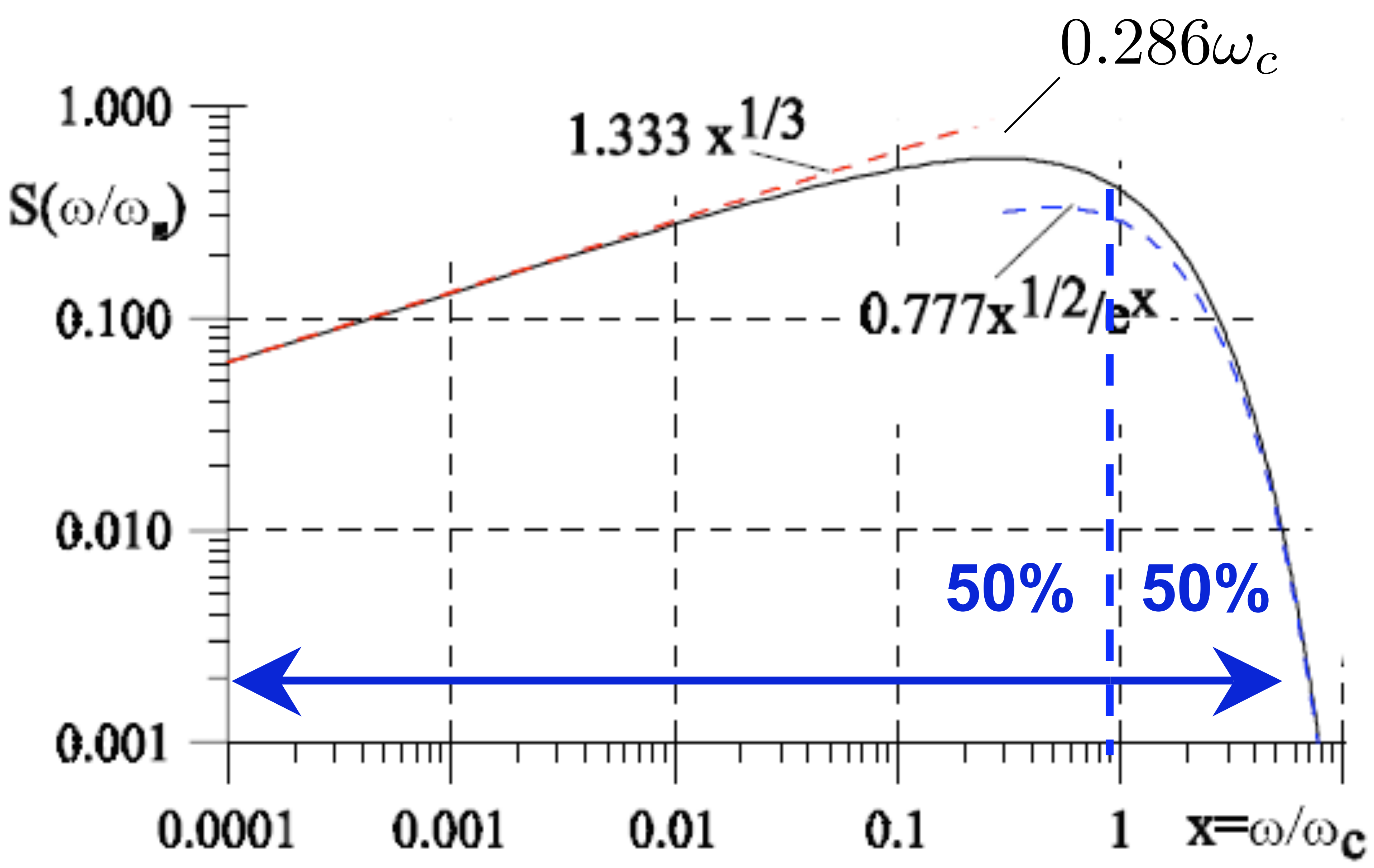
Bill Barletta

Frequency Distribution of Radiation

The integrated spectral density up to the critical frequency contains half of the total energy radiated, the peak occurs approximately at $0.286\omega_c$

where the critical photon energy is

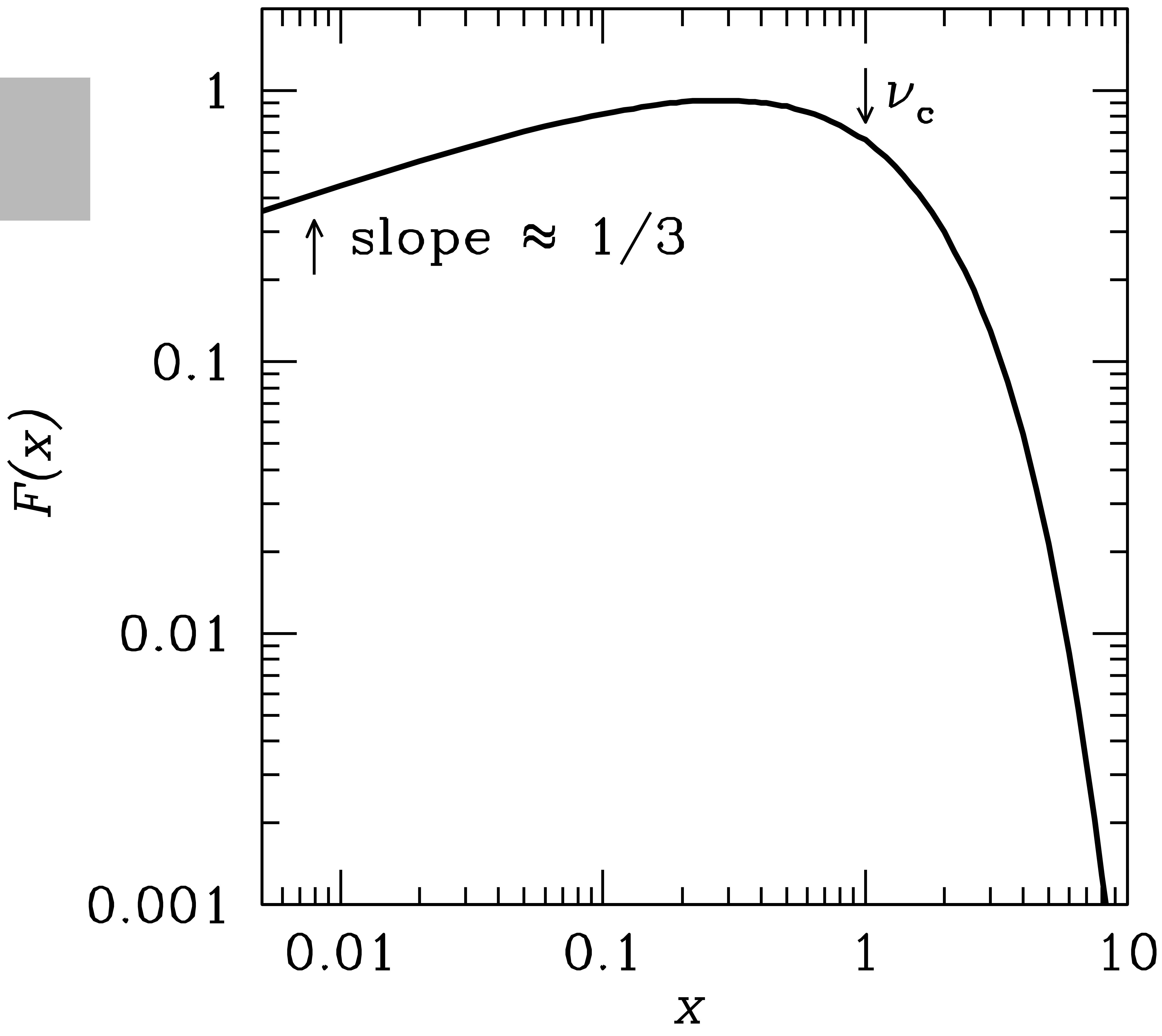
$$\varepsilon_c = \hbar\omega_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3$$

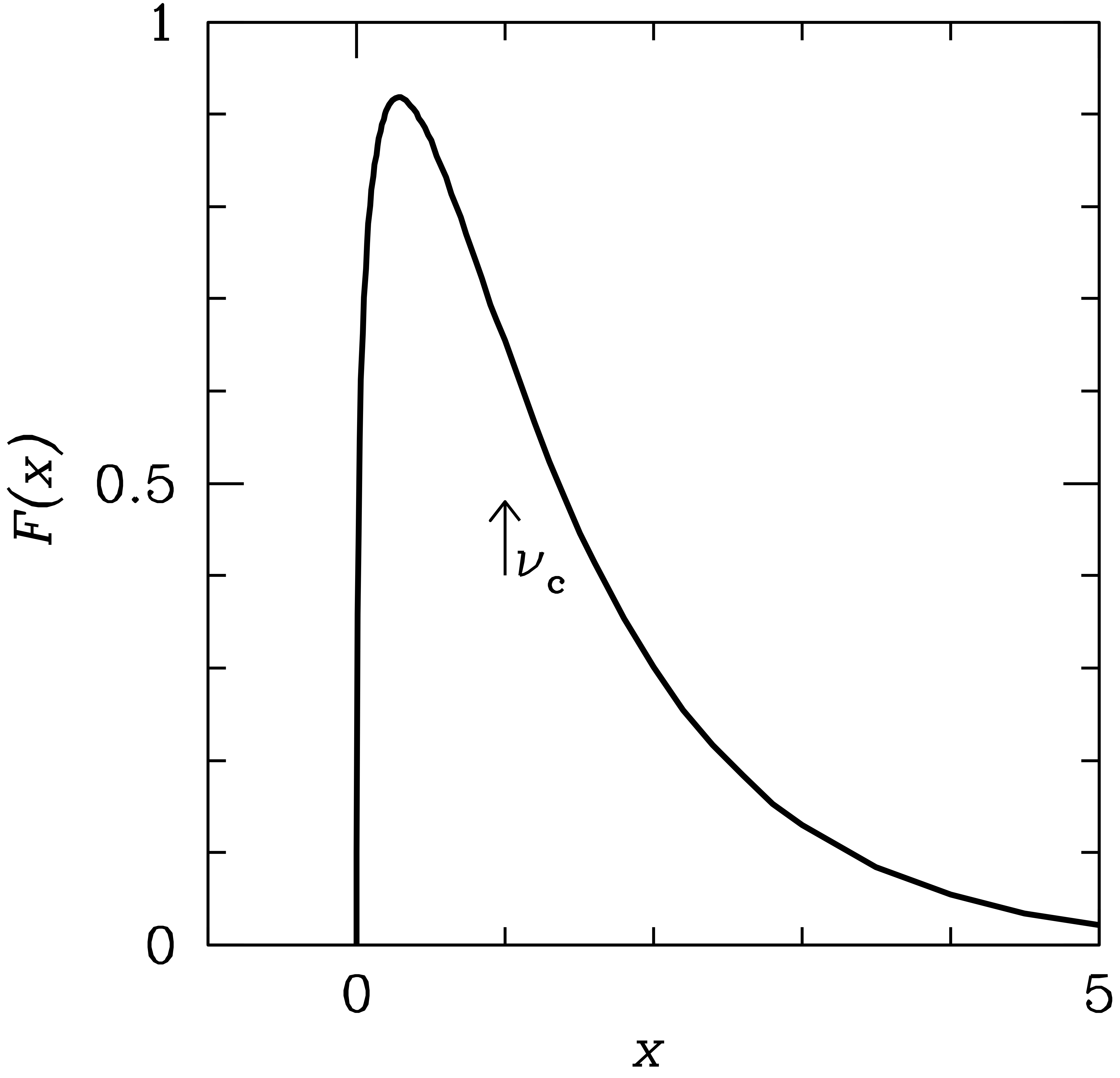


For *electrons*, the critical energy in practical units is

$$\varepsilon_c[keV] = 2.218\frac{E[GeV]^3}{\rho[m]} = 0.665 \cdot E[GeV]^2 \cdot B[T]$$

Applying a Linear Scale





https://www.cv.nrao.edu/course/astr534/SynchrotronSpectrum.html

Number of Photons Emitted

- Since the energy lost per turn is

$$U_0 = \frac{e^2\gamma^4}{3\varepsilon_0\rho}$$

- And average energy per photon is the

$$\langle \varepsilon_\gamma \rangle \approx \frac{1}{3}\varepsilon_c = \frac{\hbar\omega_c}{3} = \frac{1}{2}\frac{\hbar c}{\rho}\gamma^3$$

- The average number of photons emitted per revolution is

$$\langle n_\gamma \rangle \approx 2\pi\alpha_{fine}\gamma$$

Counting Electrons...

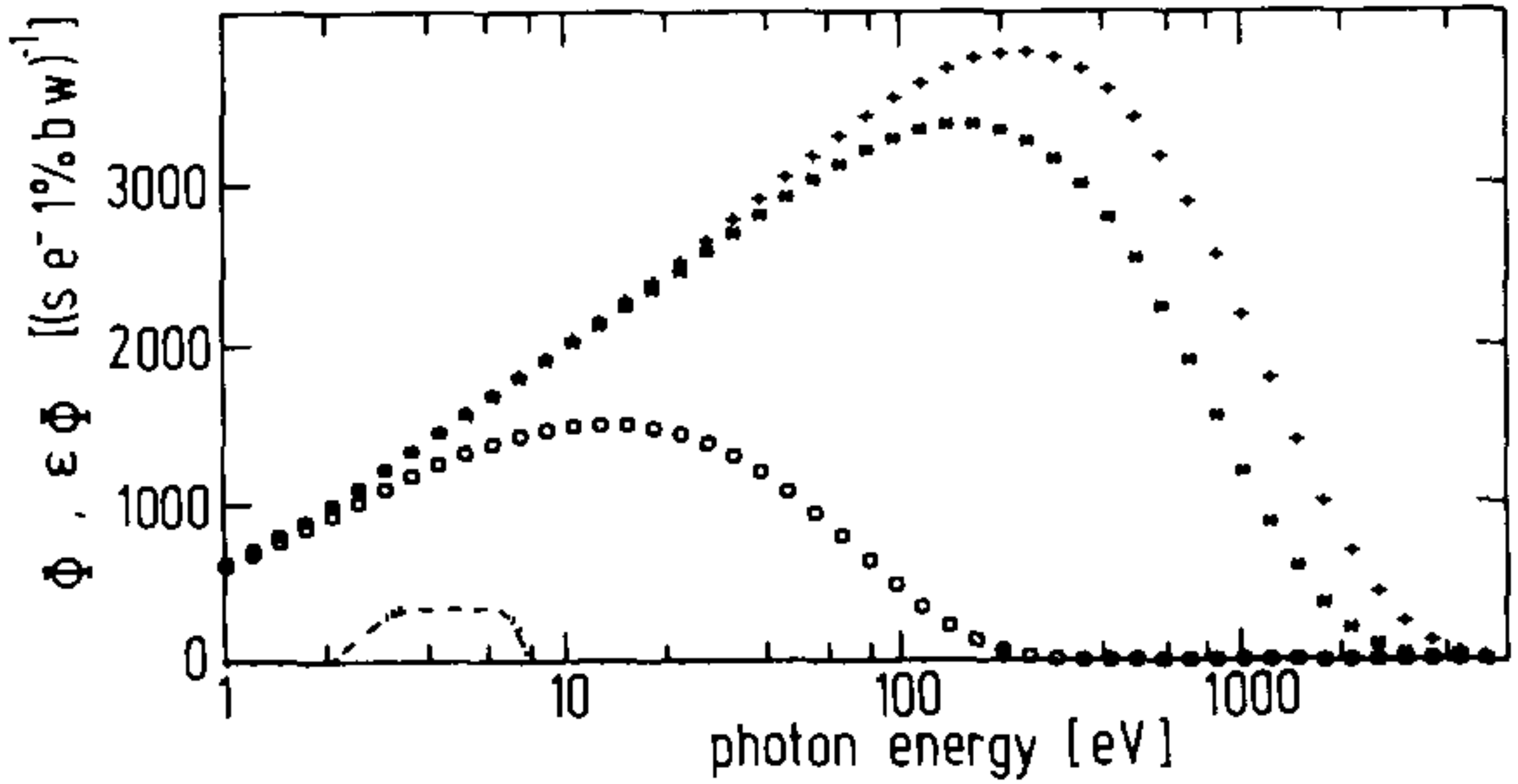


Fig. 2. Calculated spectral photon flux Φ (photons per second and electron and 1% bandwith) of BESSY at electron energies $E = 336$ MeV (□), $E = 755$ MeV (∗) and $E = 856$ MeV (+) as accepted by the multiplier system. The corresponding multiplier count rates can be evaluated by integrating the product function $\Phi \cdot \epsilon$ ($E = 336$ MeV (·–·–·) and $E = 856$ MeV (······) respectively).

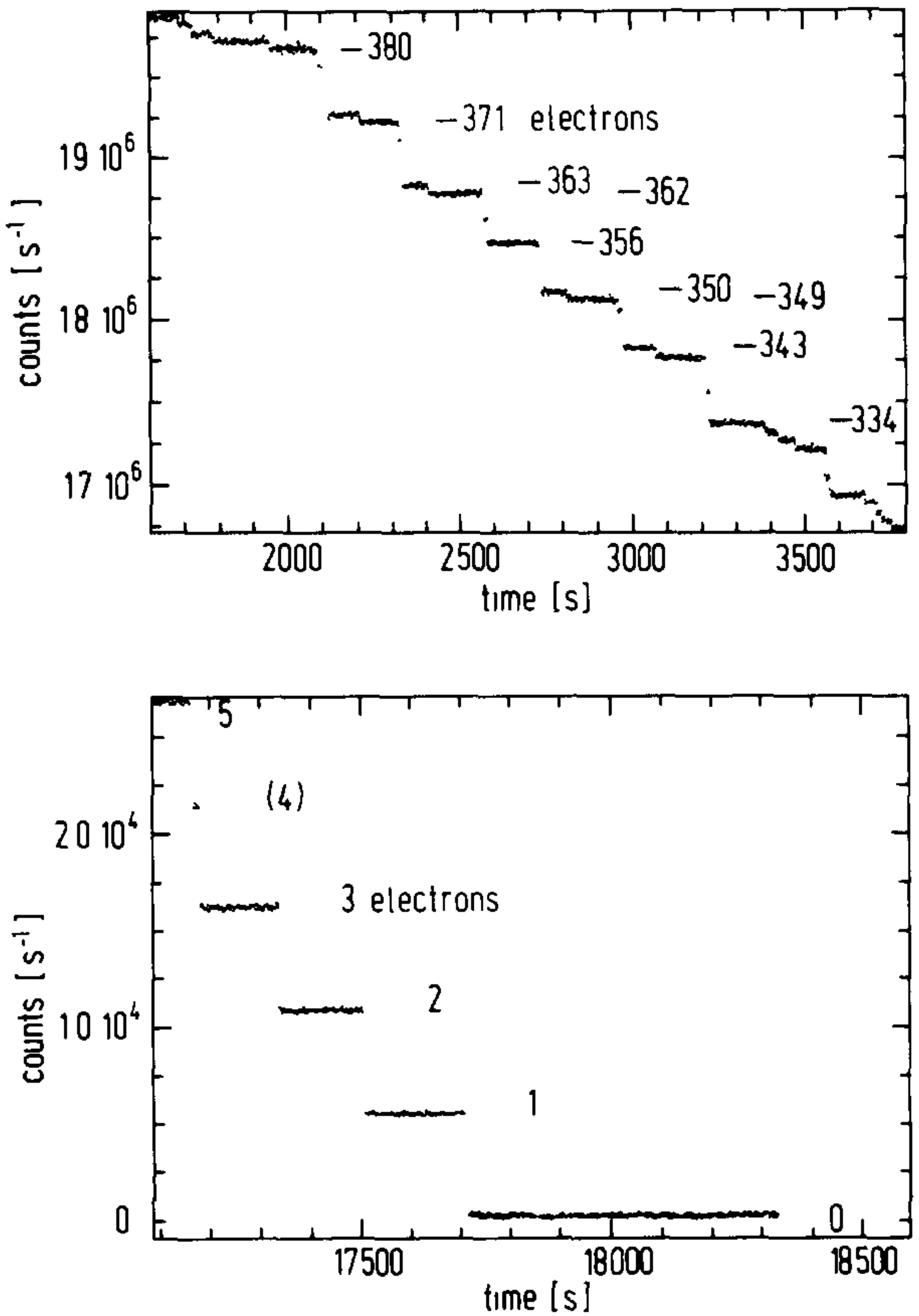


Fig. 3. Count rate of the multiplier system irradiated by the SR of 384 → 321 electrons (above) and the last 5 electrons (below).

F. Riehle et al., NIM–A 268, 262–269 (1988)

Undulators

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