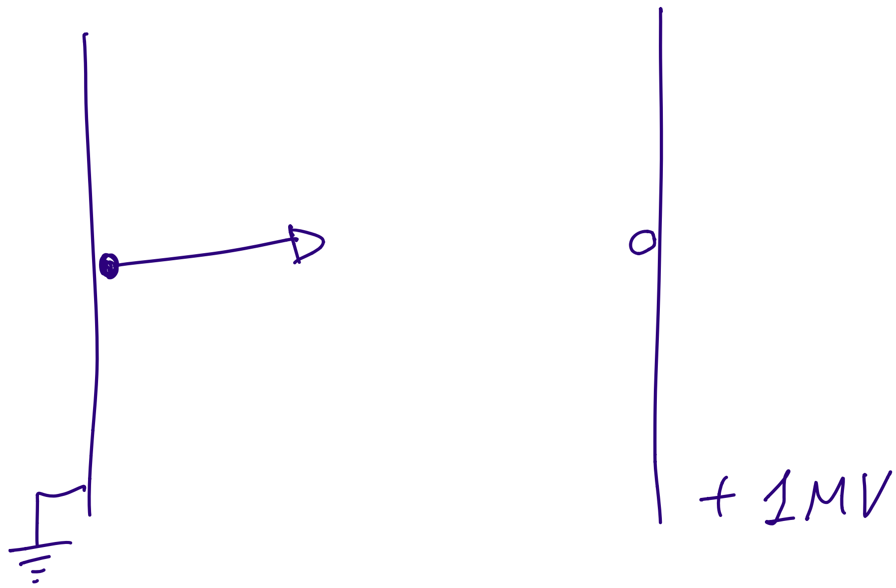
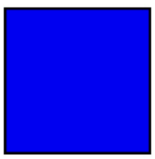
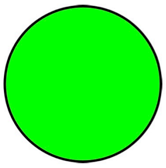
Acceleration by DC field

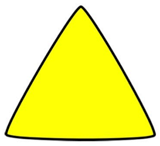




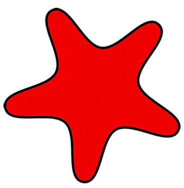
$E = 1\ \text{MeV}$



$E = 1\ \text{MeV} + 511\ \text{keV} = 1.511\ \text{MeV}$



$E = \sqrt{1^2 + 0.511^2}\ \text{MeV} = 1.123\ \text{MeV}$



This depends on the trajectory

Total energy $E = mc^2 + E_{kin}$

Momentum p: $E^2 = p^2c^2 + m^2c^4$

Only for $\gamma \gg 1$

$$E \approx E_{kin} \approx \frac{p}{c}$$

- Acceleration by AC field

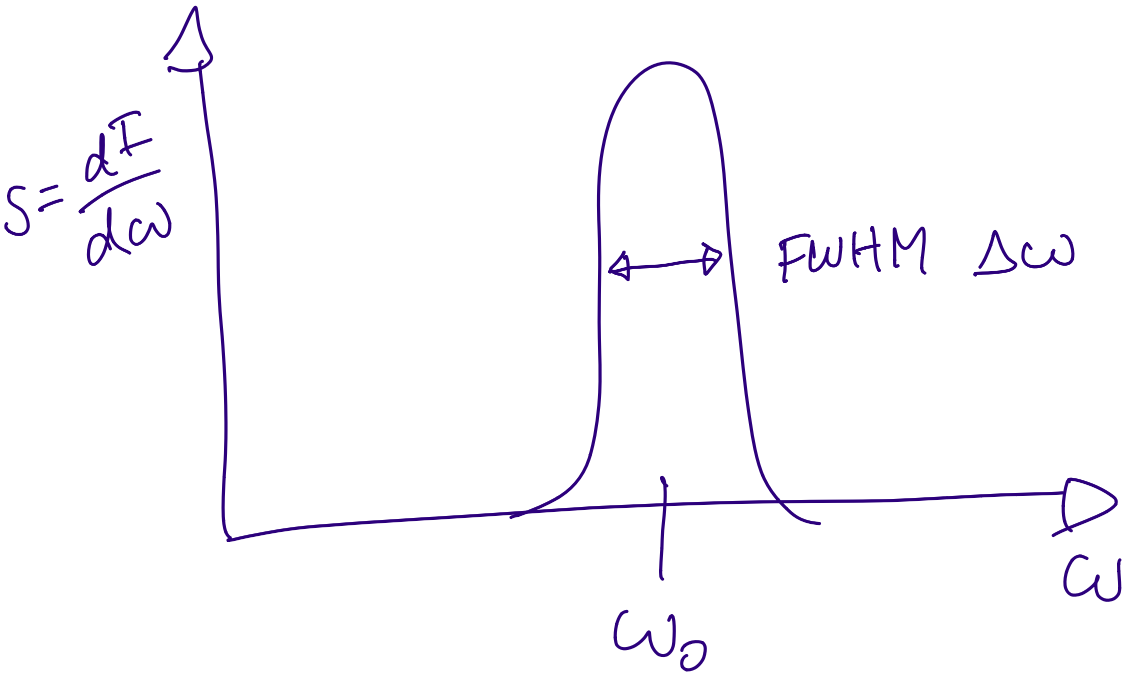
$$E_{kin} \oint_0^t -e\,\vec{E}(\vec{x}(t), t) \cdot \vec{n}\, dt$$

- Deflection in a dipole magnet

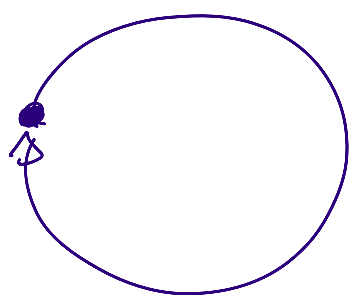
$$\rho = \frac{p}{eB}$$

Definition of Bandwidth

$$BW = \frac{\Delta\omega}{\omega_0}$$



non-relativistic particles



Radiation is emitted at the revolution frequency

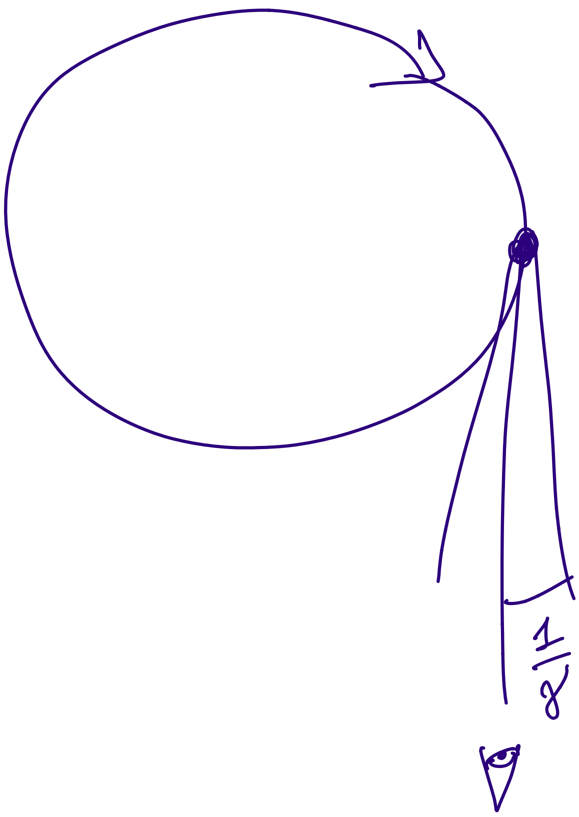
"cyclotron radiation"

$$P = \sigma_t \frac{B^2 v^2}{\mu_0 c}$$

where $\sigma_t = \frac{8\pi}{3} \left(\frac{q^2}{4\pi \varepsilon_0 m c^2} \right)^2$

$= 66.5 \, (fm)^2$

- Relativistic particles: "synchrotron radiation"



"critical" frequency

$$\omega_c = \frac{3}{2}\frac{c}{\rho}\gamma^3$$

corresponding photon energy

$$\varepsilon_c = \hbar\omega_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3$$

Total radiated power

$$P = \frac{e^2 c}{6\pi\varepsilon_0}\cdot\frac{\gamma^4}{\rho^2}$$

Energy lost per turn

$$U_0 = \frac{e^2 \gamma^4}{3 \varepsilon_0 \rho}$$

Undulators

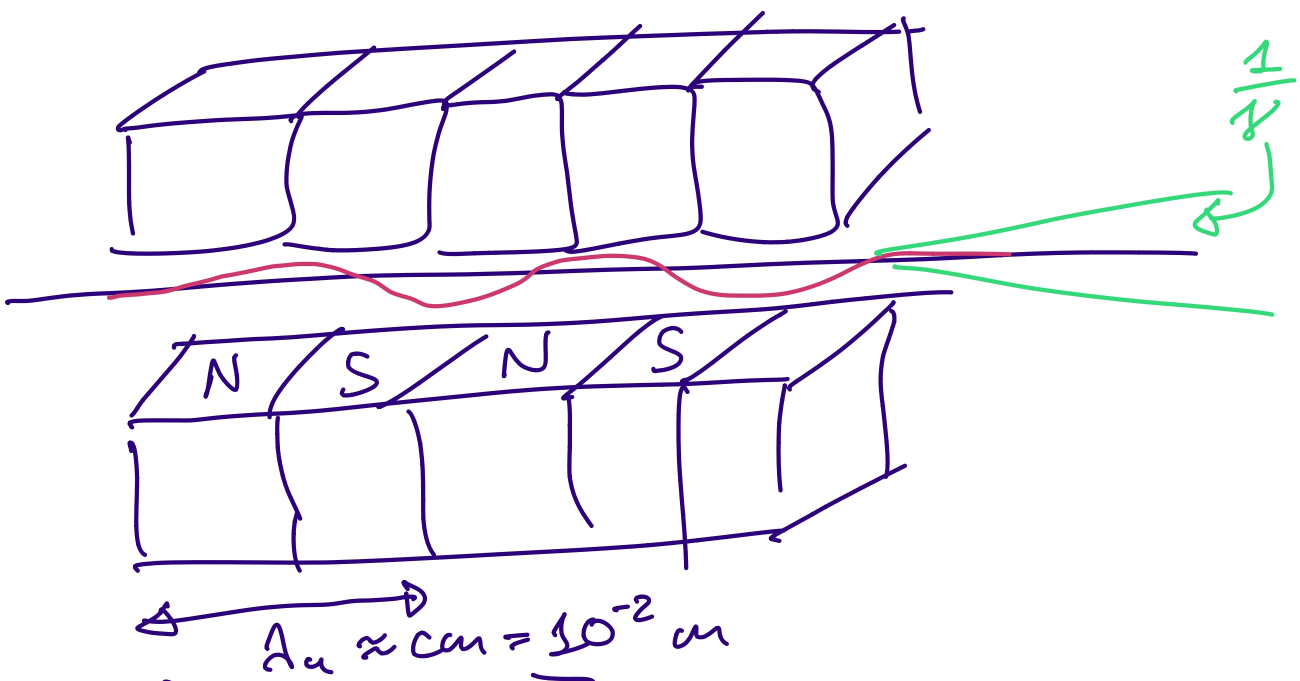
Simplifications:

- relativistic electrons: $\beta = \frac{v}{c} \approx 1$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \gg 1$$

- far-field radiation

Undulator



$$\vec{B} = \begin{bmatrix} 0 \\ B_0 \sin(k_u z) \\ 0 \end{bmatrix} \qquad k_u = \frac{2\pi}{\lambda_u}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = 0$$

1

$$\vec{B} = \begin{bmatrix} 0 \\ B_0 \cosh(k_u y)\sin(k_u z) \\ B_0 \sinh(k_u y)\cos(k_u z) \end{bmatrix}$$

small

0

Let's assume $y \approx 0$

$$m_e \gamma \frac{d\vec{v}}{dt} = \vec{F} = -e\vec{v}\times\vec{B}$$

$$m_e \gamma \frac{d\vec{v}}{dt} = e v_z B_y = e v_z B_0 \sin(k_u z)$$

$$\frac{dz}{dt} = v_z$$

$$\frac{dv_x}{dz} = \frac{e}{m_e \gamma} B_0 \sin(k_u z)$$

assume γ constant

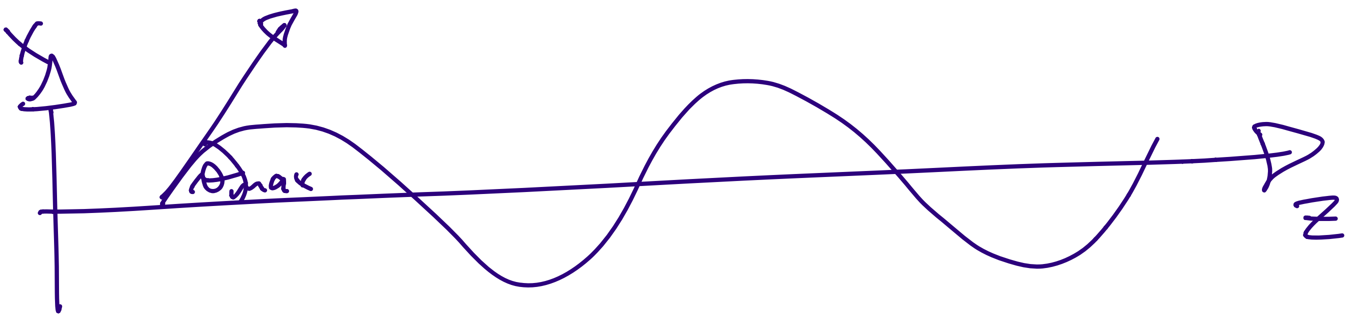
$$v_x(z) = -\frac{Kc}{\gamma}\cos(k_u z)$$

$$\boxed{K = \frac{eB_0}{m_e c k_u}}$$

"undulator K"
unitless

$$K = 0.934\, B_0[T] \cdot \lambda_u[cm]$$

$$\Rightarrow x(z) = -\frac{K}{k_u \gamma \beta_z}\sin(k_u z)$$



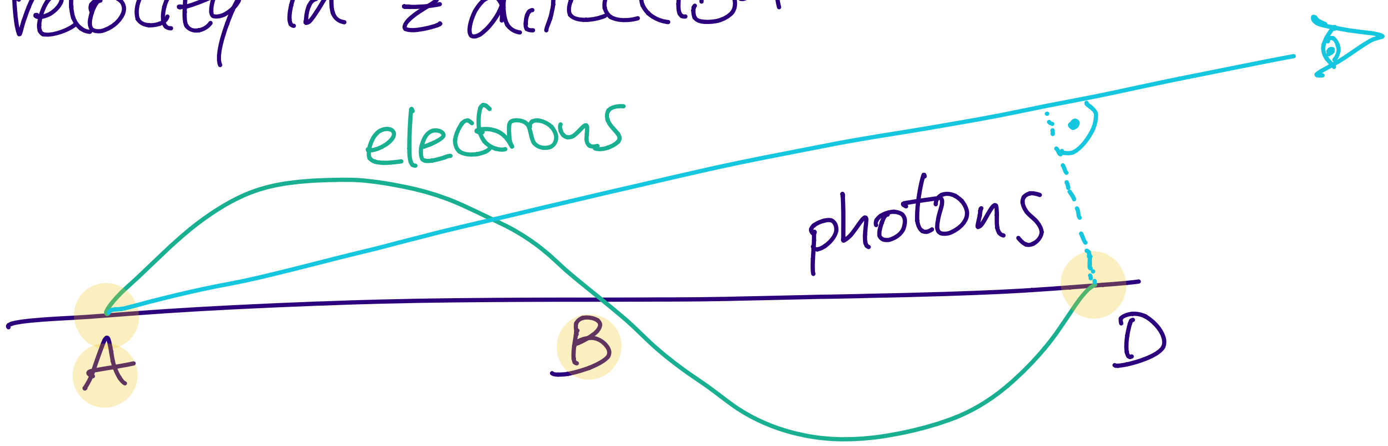
sinusoidal motion

- Radiation is emitted in a cone $\frac{1}{\gamma}$
- Maximum angular deviation is $\theta_{max} < \frac{1}{\gamma}$

- Definition of undulator: $\vartheta_{max} \lesssim \frac{1}{\gamma}$

  wiggler: $\vartheta_{max} \gtrsim \frac{1}{\gamma}$

- horizontal movement reduces average velocity in z direction



- Calculate the time difference between electrons and photons
  - ↳ slower: $\beta < 1$
  - ↳ longer path

- coherent emission occurs when:

$$\frac{\lambda}{c} = \frac{\widetilde{AD}}{\beta c} - \frac{\overline{AD}}{c}$$

$$\frac{\lambda}{2c} = \frac{\widetilde{AB}}{\beta c} - \frac{\overline{AB}}{c}$$

λ: photon wavelength

$\approx \mathring{A} = 10^{-10}\,m$

- Arc length

$$\widetilde{AB} = \int_0^{\lambda_u/2} \sqrt{1+\left(\frac{dx}{dz}\right)^2}\, dz$$

$$\ldots \quad = \frac{\lambda_u}{2}\left(1+\frac{K^2}{4\gamma^2}\right)$$

- Resonance condition for coherent emission:

$$\frac{\lambda}{2c} = \frac{\lambda_u}{2\beta c}\left(1+\frac{K^2}{4\gamma^2}\right) - \frac{\lambda_u}{2c}$$

$$\lambda = \frac{\lambda_u}{\beta}\left(1+\frac{K^2}{4\gamma^2}\right) - \lambda_u$$

$$\underbrace{\beta}_{\approx 1}\lambda = \underbrace{\lambda_u\left(1+\frac{K^2}{4\gamma^2}\right)}_{} - \underbrace{\beta}_{\approx 1-\frac{1}{2}\gamma^{-2}}\lambda_u$$

difference is small

That's why $\beta \not\approx 1$

on the right hand side of the eq.

$$\lambda = \lambda_u \left(1 - 1 + \frac{K^2}{4\gamma^2} + \frac{1}{2\gamma^2} \right)$$

$$= \lambda_u \left(\frac{K^2}{4\gamma^2} + \frac{2}{4\gamma^2} \right)$$

$$\boxed{\lambda = \frac{\lambda_u}{2\gamma^2} \left(\frac{K^2}{2} + 1 \right)}$$