

Joint Universities Accelerator School

JUAS 2019

Archamps, France, 18th – 20th February 2019

Normal-conducting accelerator magnets

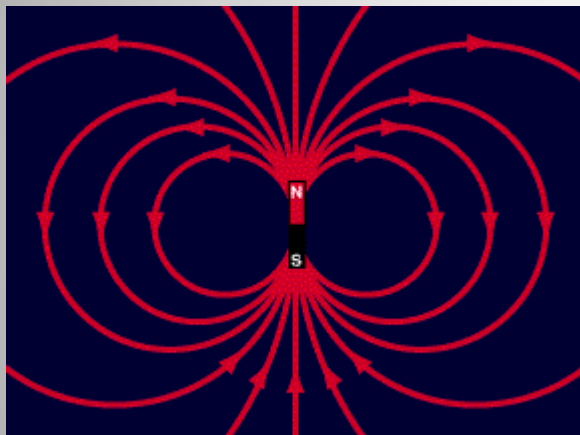
Lecture 4: Applied numerical design

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CERN



Lecture 4: Numerical design



Which code shall I use?
Introduction to 2D numerical design
How to evaluate the results
Field analysis
Typical application examples



Numerical design

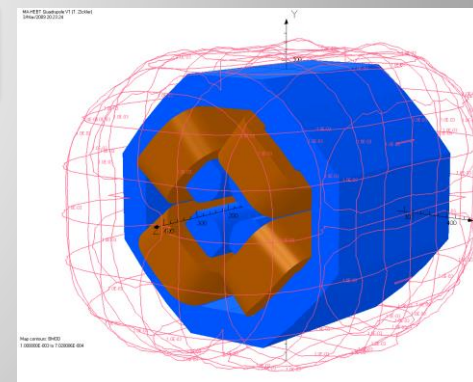
Common computer codes: Opera (2D) or Tosca (3D), Poisson, ANSYS, Roxie, Magnus, Magnet, Mermaid, Radia, **FEMM**, COMSOL, etc...

Technique is iterative

- calculate field generated by a defined geometry
- adjust geometry until desired distribution is achieved

Computing time increases for **high accuracy** solutions, **non-linear** problems and **time dependent** analysis → compromise between accuracy and computing time

2D	3D
<ul style="list-style-type: none">• 2D analysis is often sufficient• magnetic solvers allow currents only perpendicular to the plane• fast	<ul style="list-style-type: none">• produces large amount of elements• mesh generation and computation takes significantly longer• end effects included• powerful modeller



FEM codes are powerful tools, but be **cautious**:

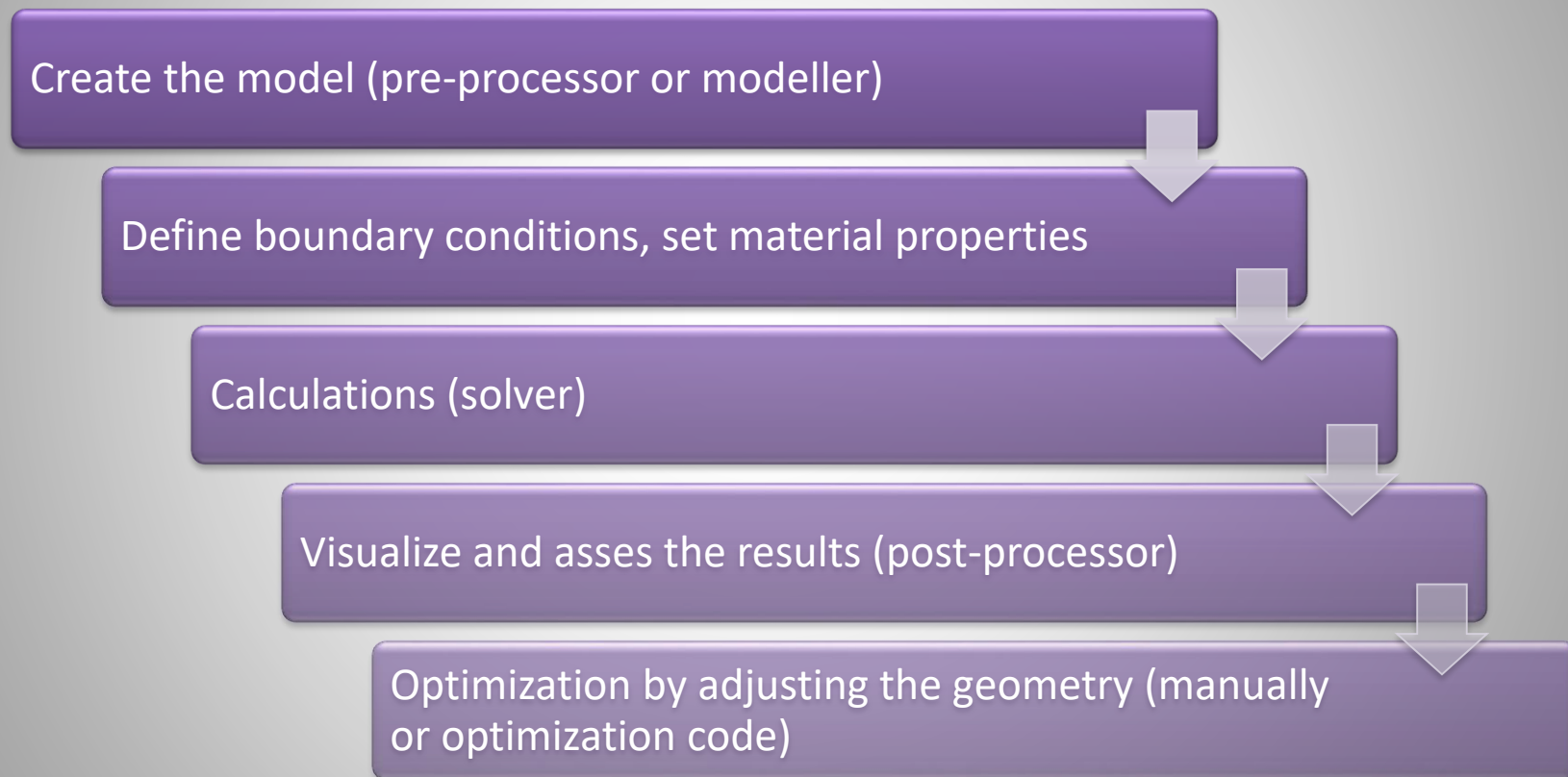
- Always check results if they are ‘**physical reasonable**’
- Use FEM for **quantifying**, not to qualify





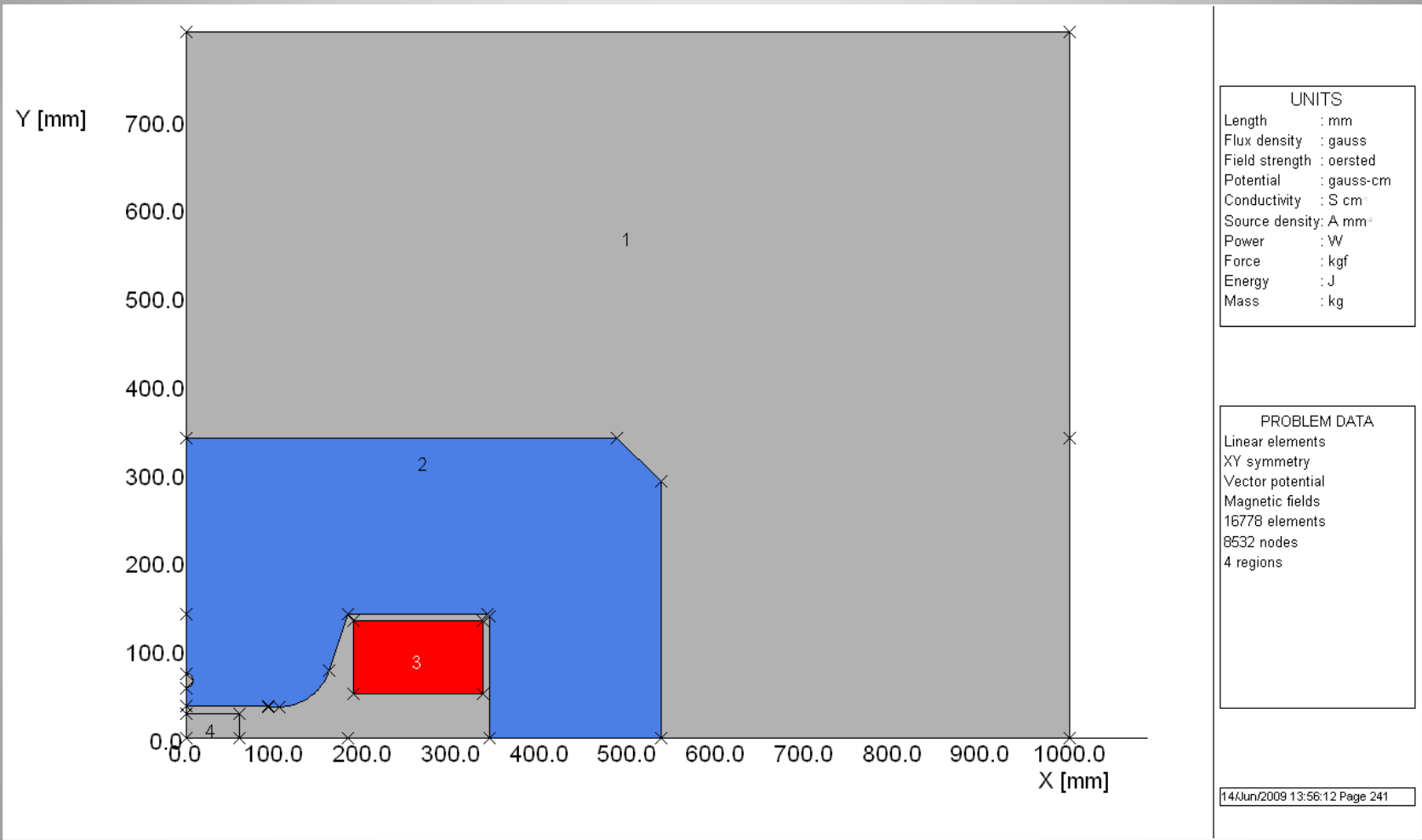
Numerical design process

Design process in 2D (similar in 3D):





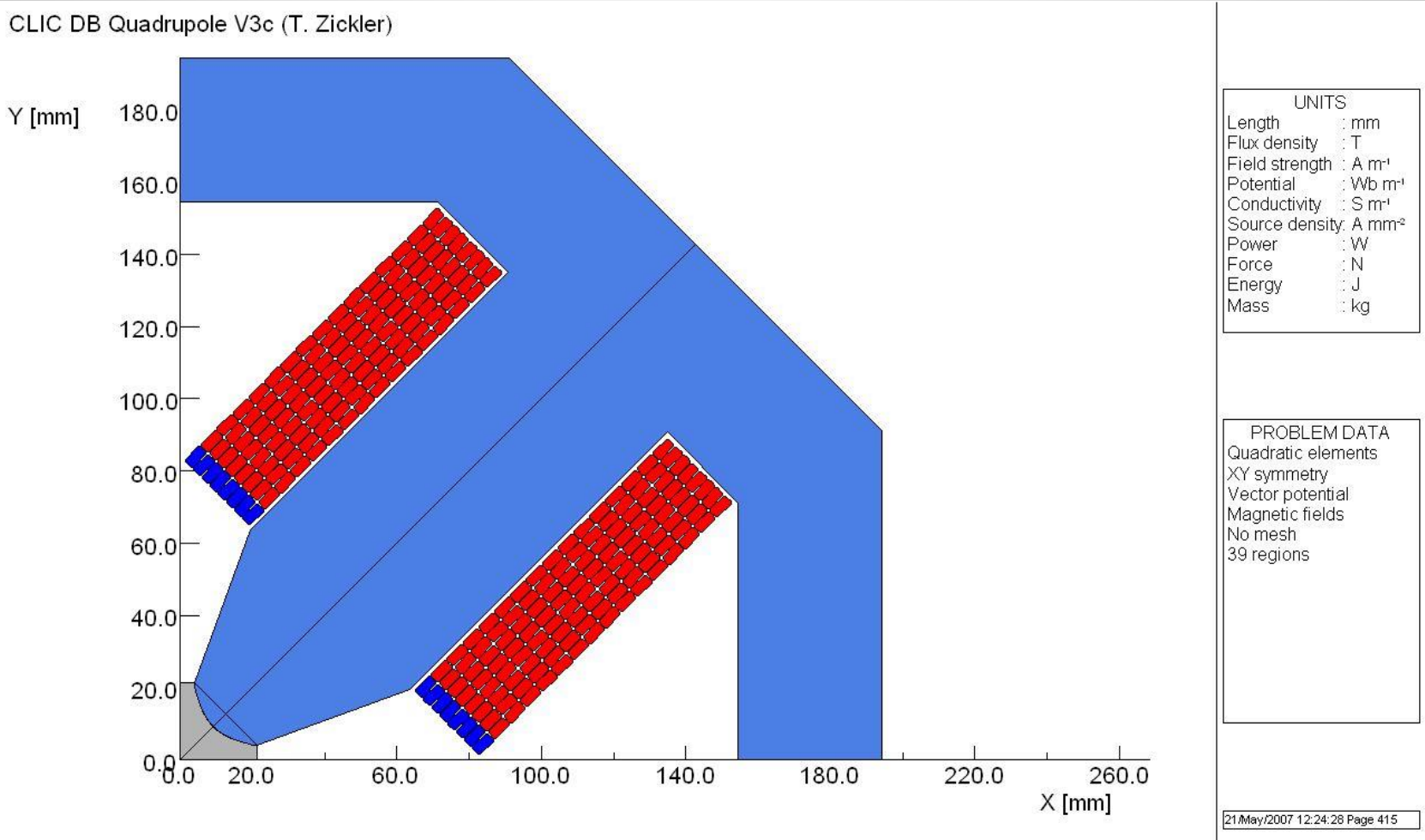
Creating the model



GUI vs. Script



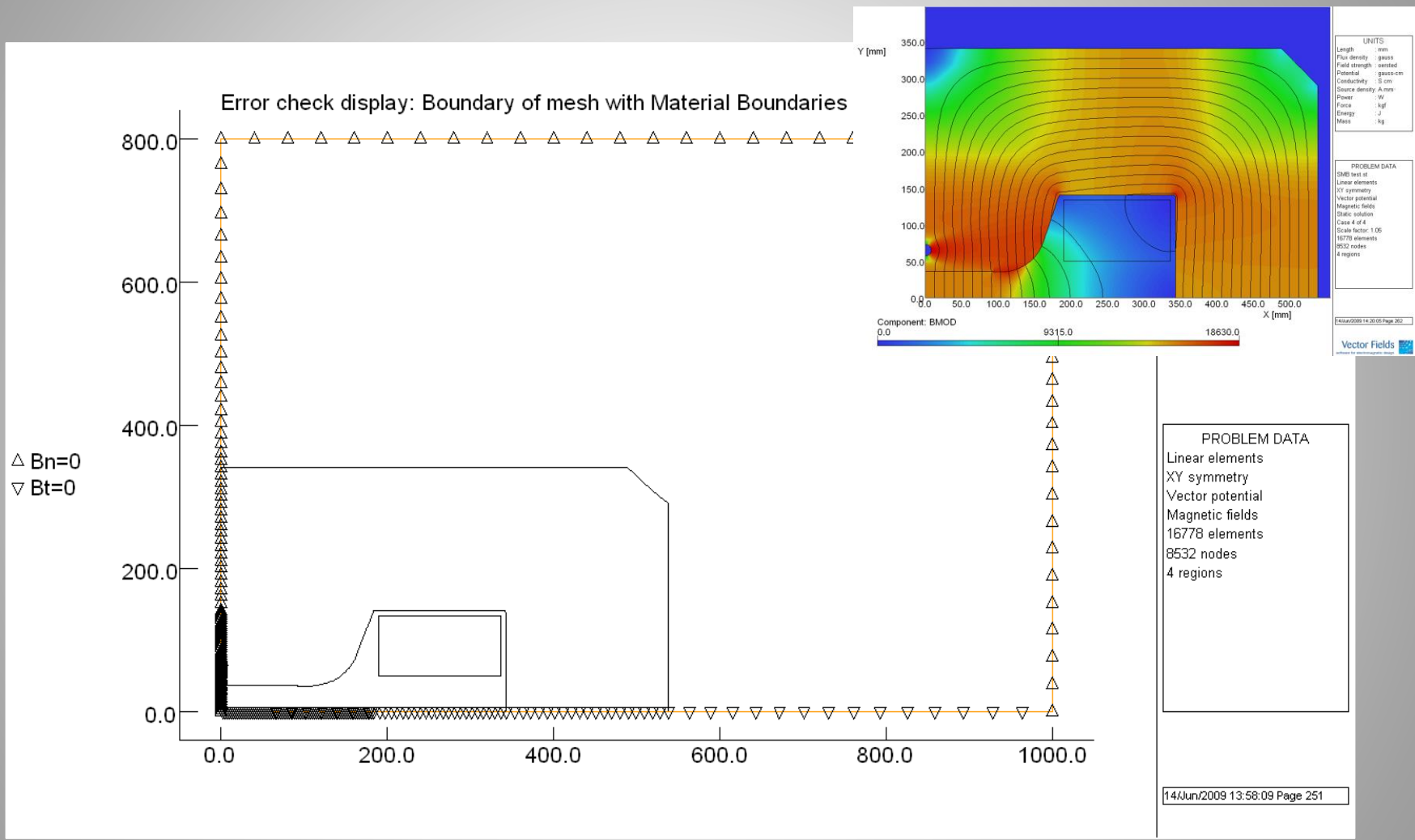
Model symmetries



Note: one eighth of quadrupole could be used with opposite symmetries defined on horizontal and $y = x$ axis

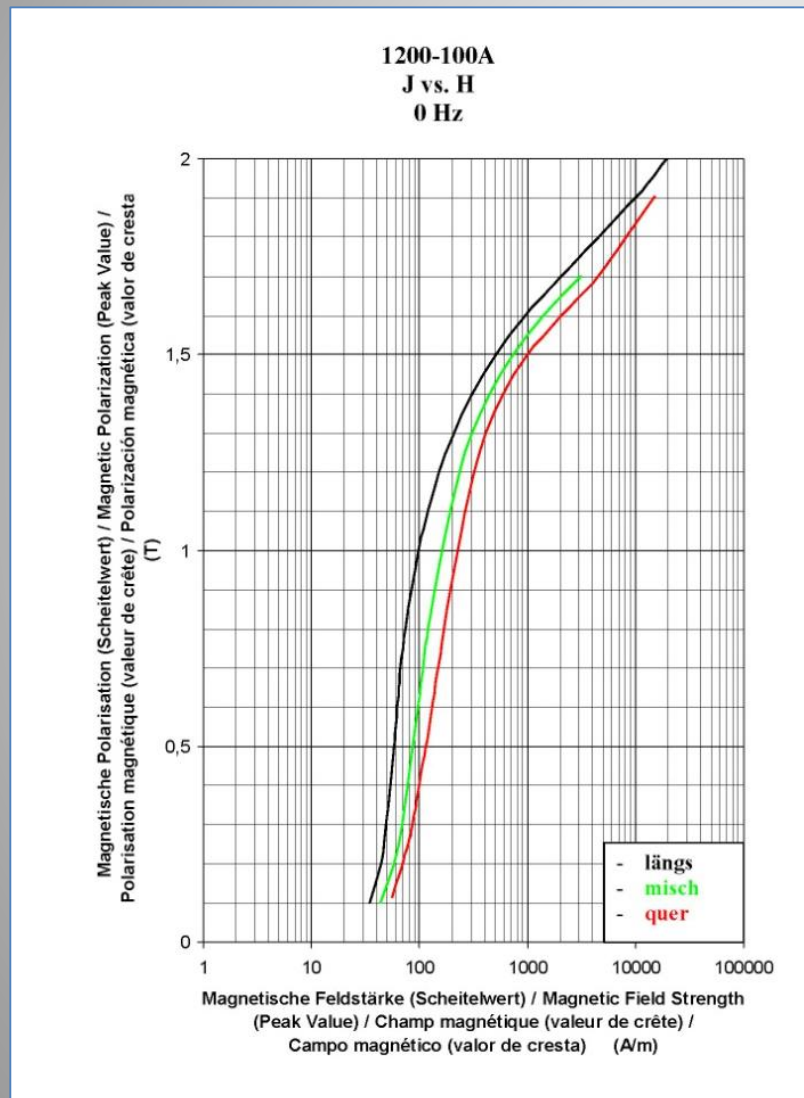


Boundary conditions





Material properties



Data source: Thyssen/Germany

Permeability:

- either fixed for linear solution
- or permeability curve for non-linear solution
- can be anisotropic
- apply correction for steel packing factor
- pre-defined curves available

Conductivity:

- for coil and yoke material
- required for transient eddy current calculations

Mechanical and thermal properties:

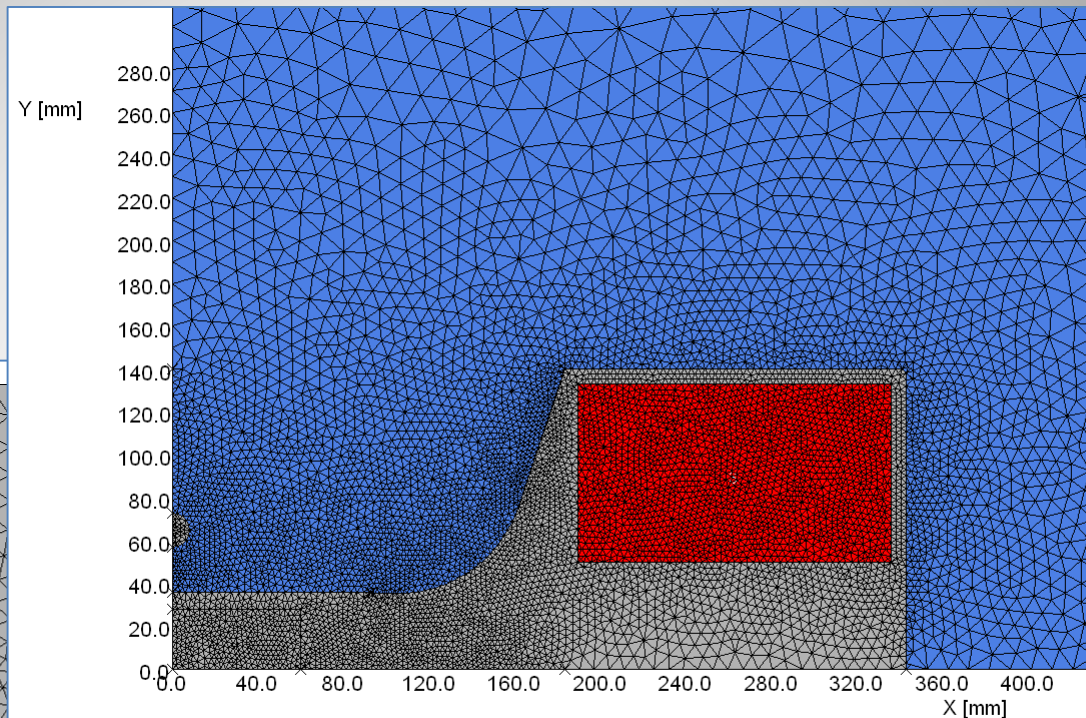
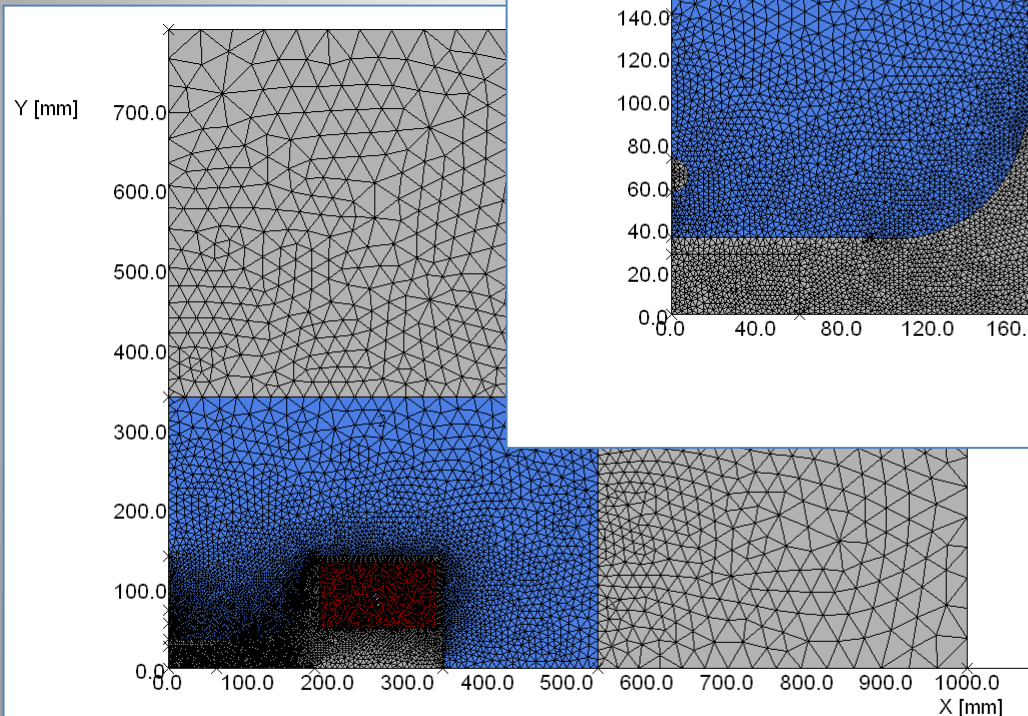
- in case of combined structural or thermal analysis

Current density in the coils



Mesh generation

- element shape
- element type
- element size



UNITS	
Length	: mm
Flux density	: gauss
Field strength	: oersted
Potential	: gauss-cm
Conductivity	: S cm
Source density	: A mm ²
Power	: W
Force	: kgf
Energy	: J
Mass	: kg

PROBLEM DATA	
Linear elements	
XY symmetry	
Vector potential	
Magnetic fields	
16778 elements	
8532 nodes	
4 regions	

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Magnetic fields
16778 elements
8532 nodes
4 regions

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Data processing

Solution

- linear: predefined constant permeability for a single calculation
- non-linear: permeability table for iterative calculations

Solver types

- static
- steady state (sine function)
- transient (ramp, step, arbitrary function, ...)

Solver settings

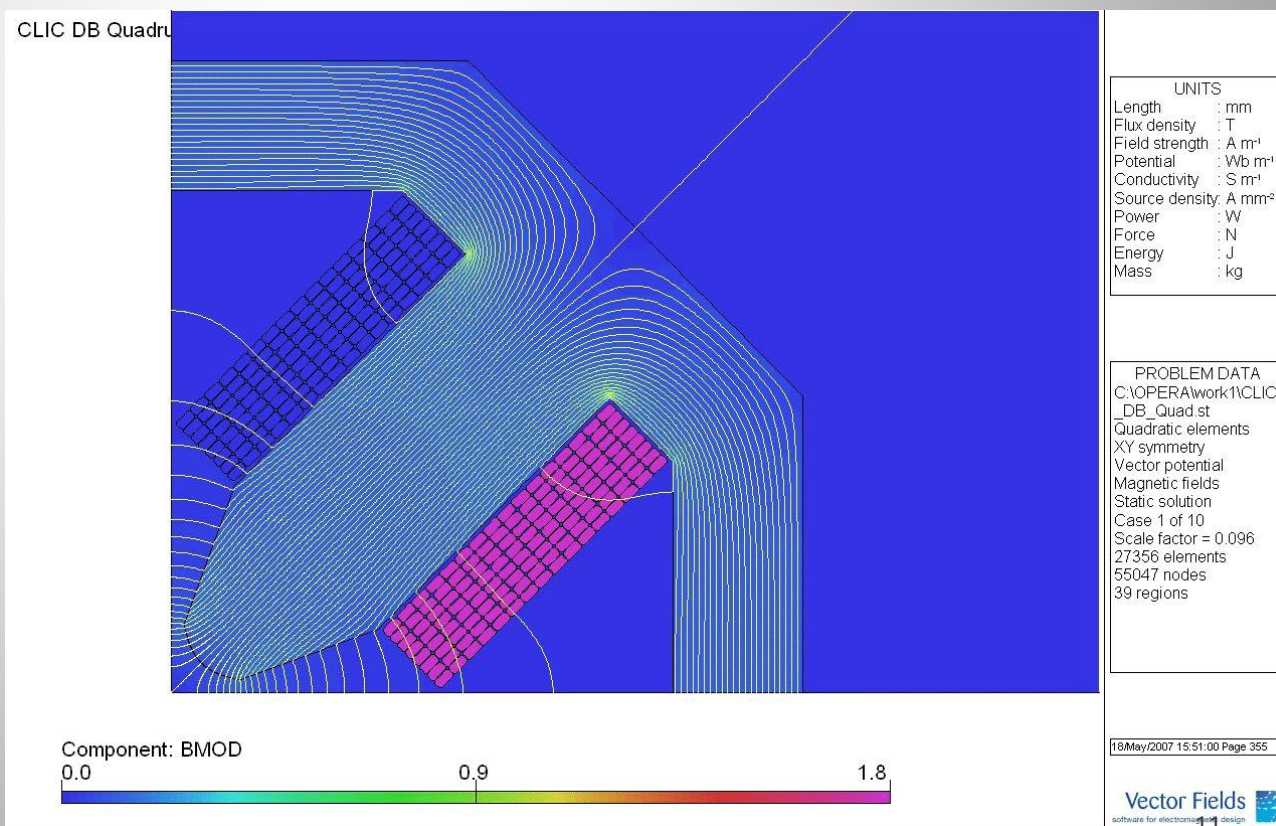
- number of iterations,
- convergence criteria
- precision to be achieved, etc...



Analyzing the results

With the help of the post-processor, field distribution and field quality and be visualized in various forms on the pre-processor model:

- Field lines and colour contours plots of flux, field, and current density
- Graphs showing absolute or relative field distribution
- Homogeneity plots

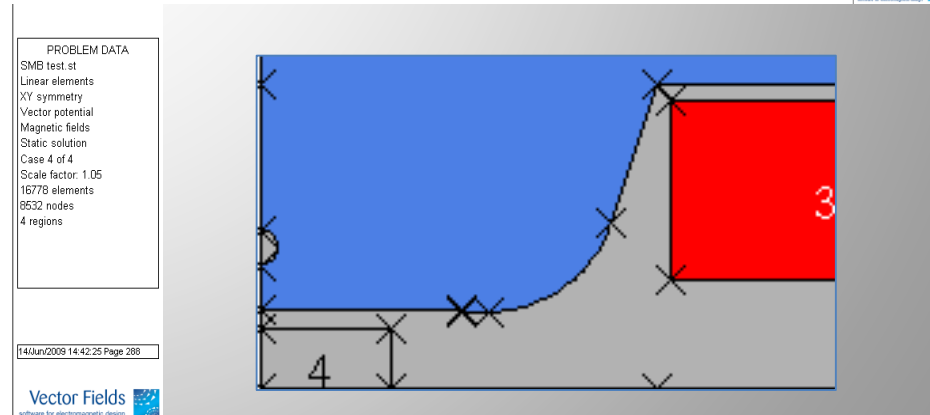
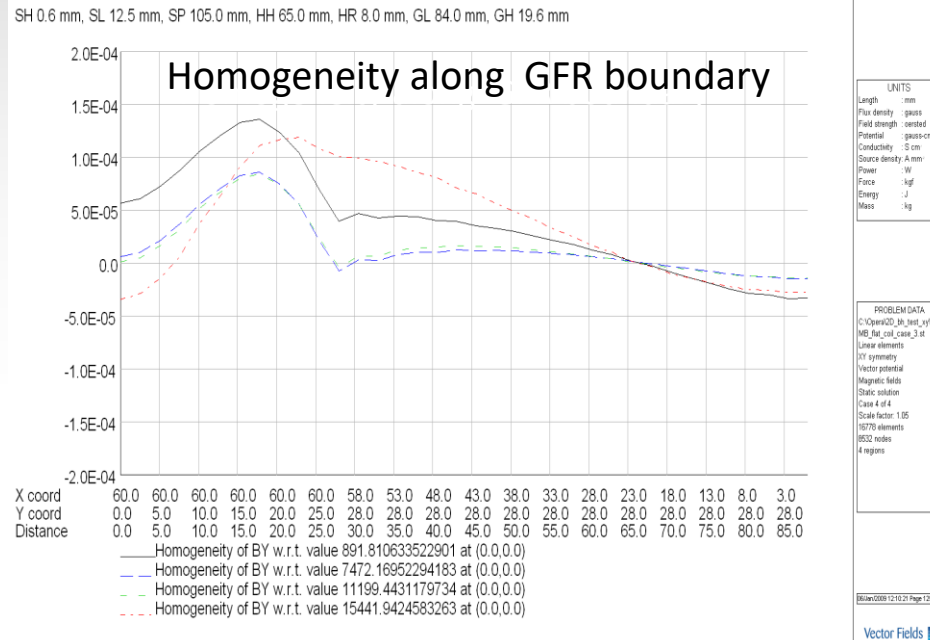
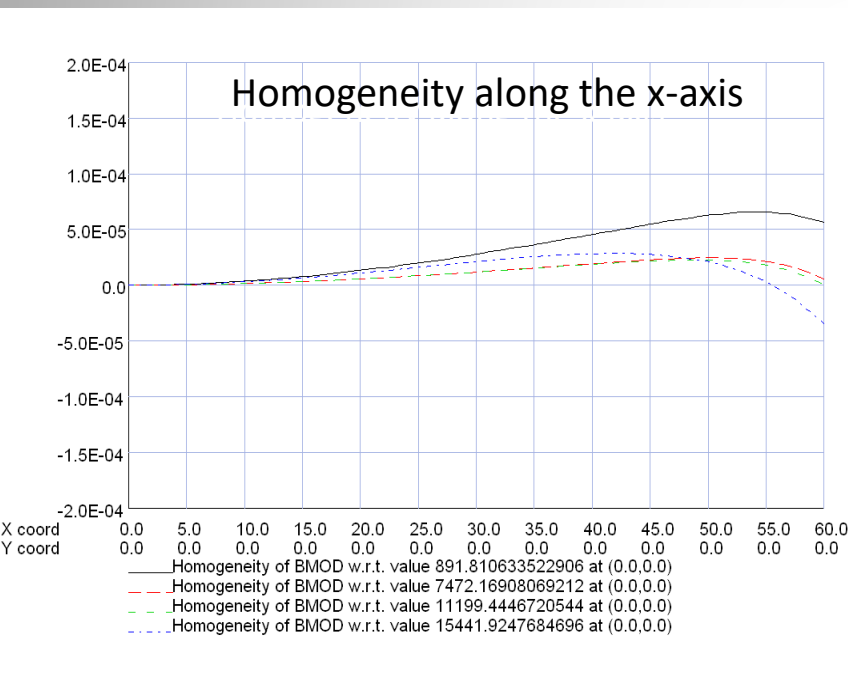




Field homogeneity in a dipole

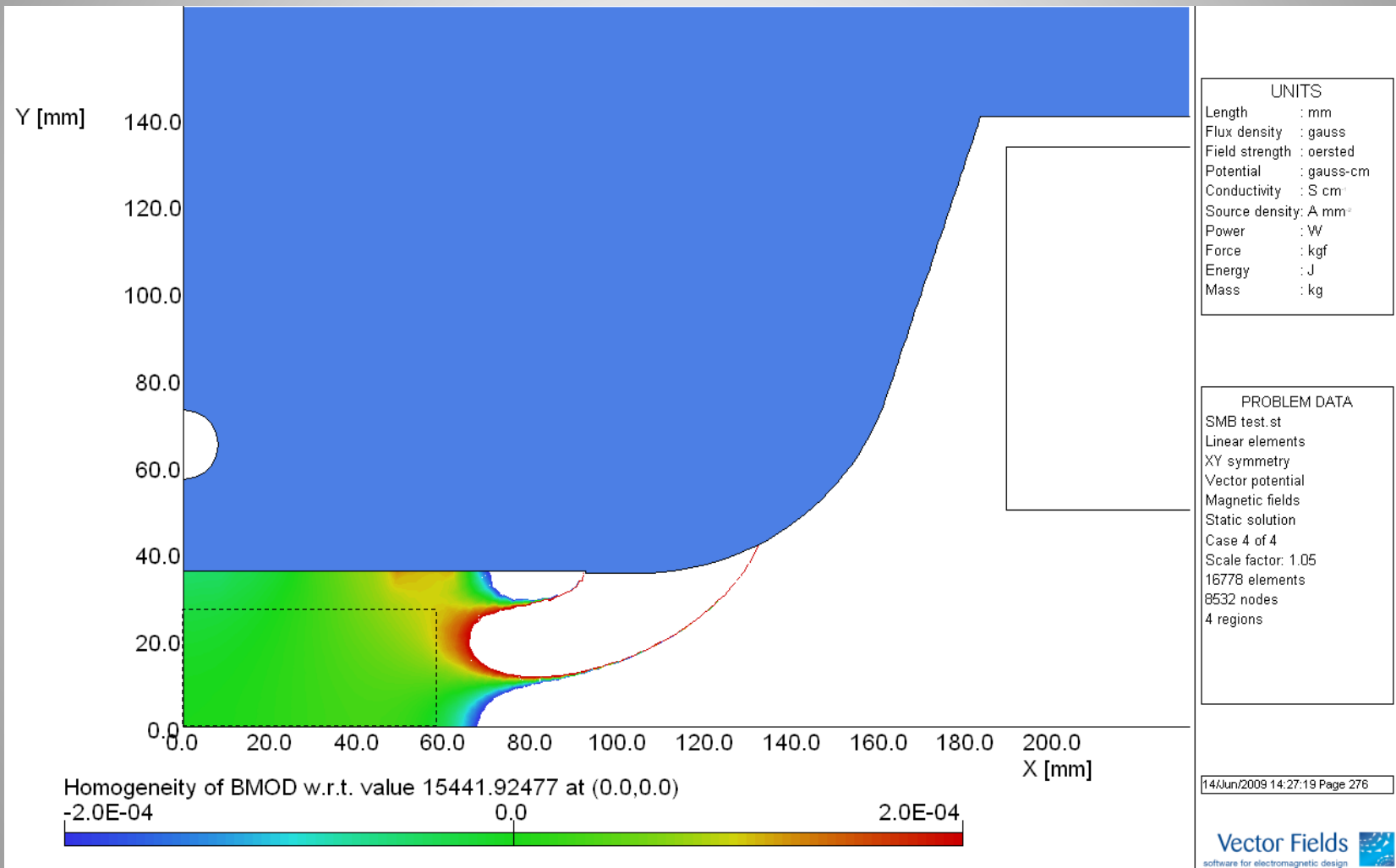
A simple judgment of the field quality can be done by plotting the field homogeneity

$$\frac{\Delta B}{B_0} = \frac{B_y(x, y)}{B_y(0,0)} - 1 \quad \frac{\Delta B}{B_0} \leq 0.01\%$$



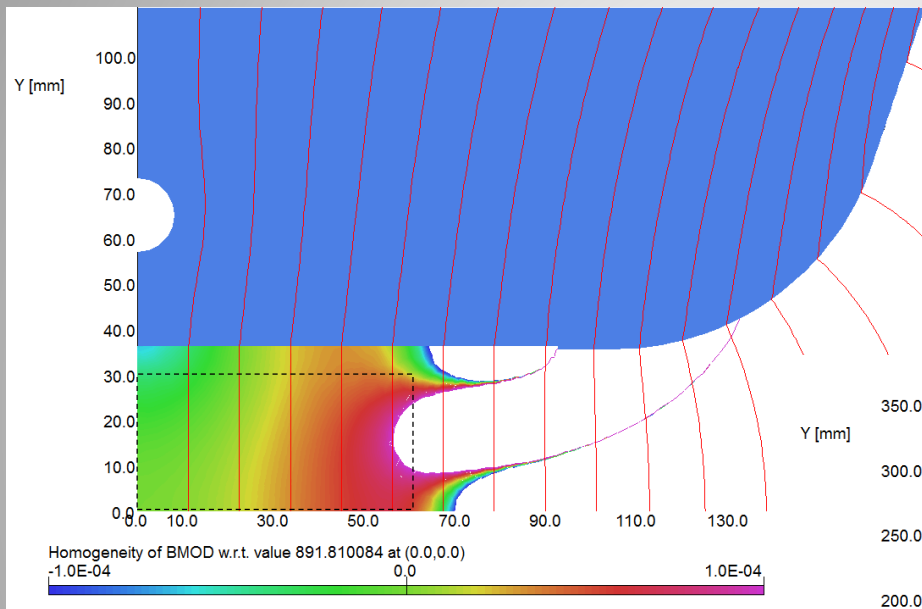


Field homogeneity in a dipole

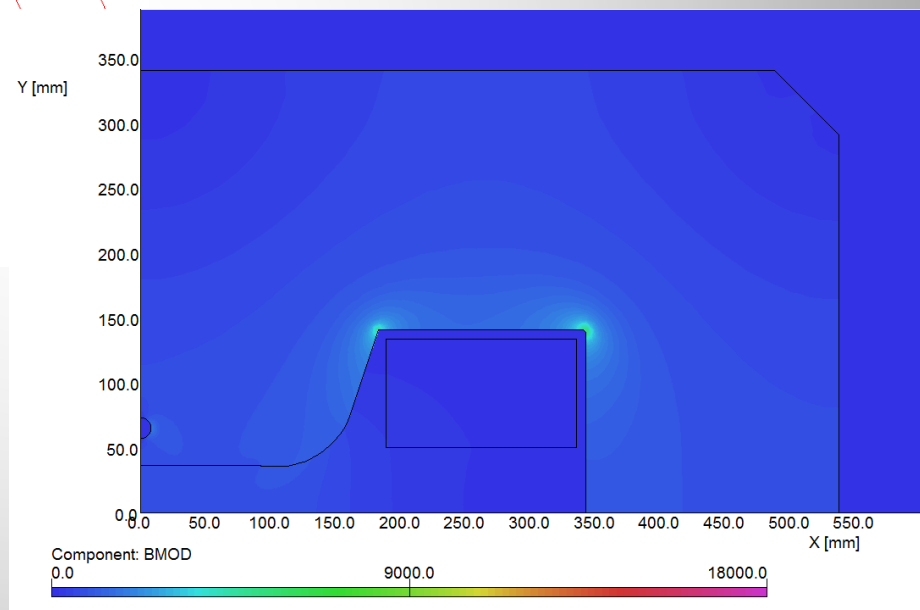




Saturation and field quality



Also very low fields can disturb the field quality significantly



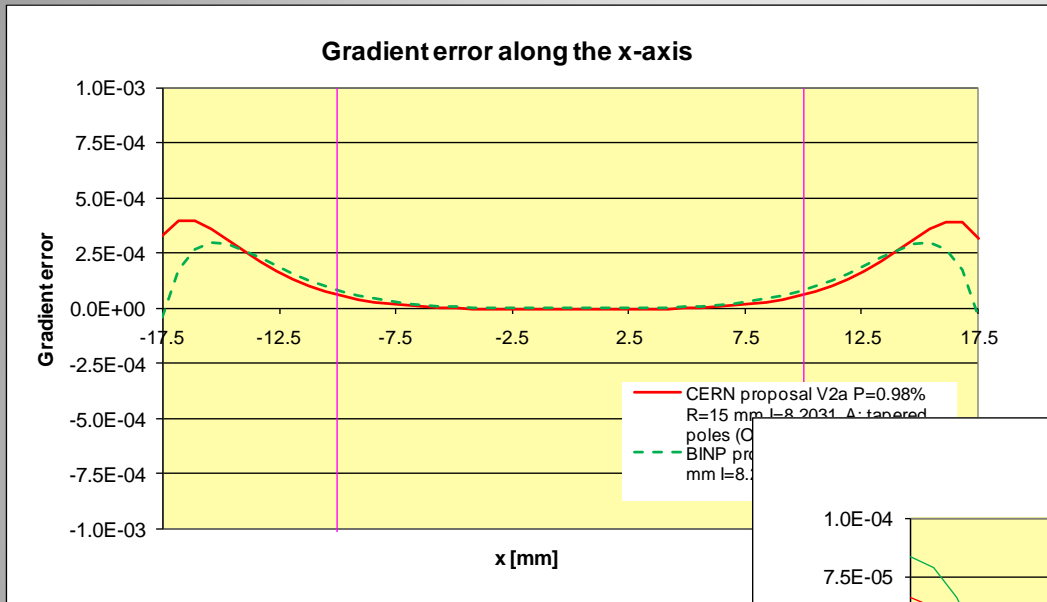
Field quality can vary with field strength due to saturation



Field quality in a quadrupole

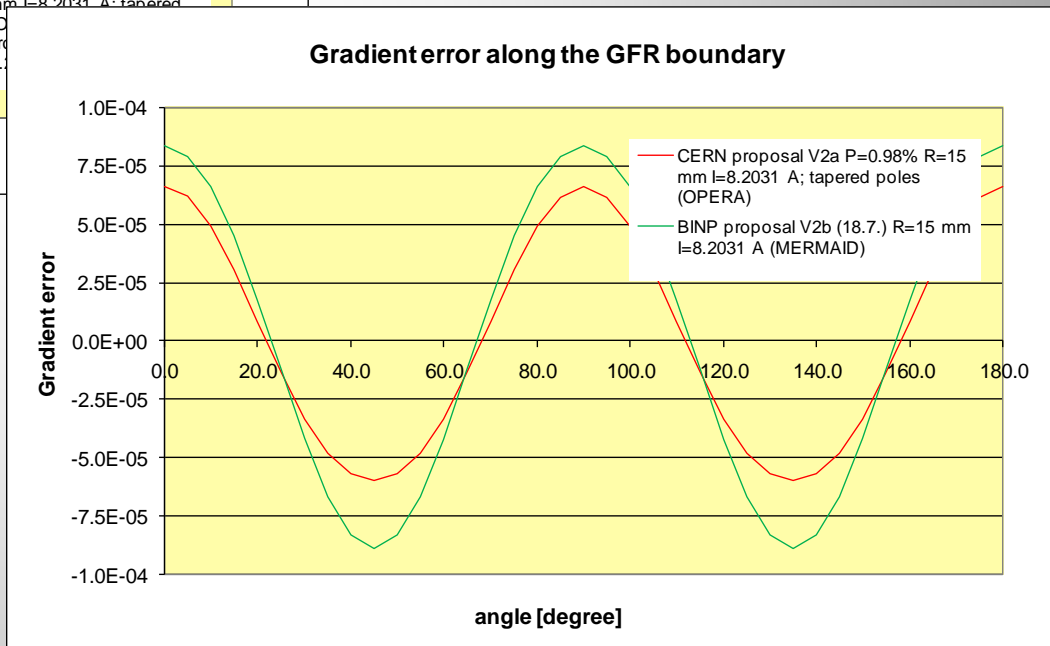
Field error in a quadrupole

$$\varepsilon = \frac{B(x, y)}{B'(0,0)\sqrt{x^2 + y^2}} - 1$$



Gradient homogeneity along the x-axis

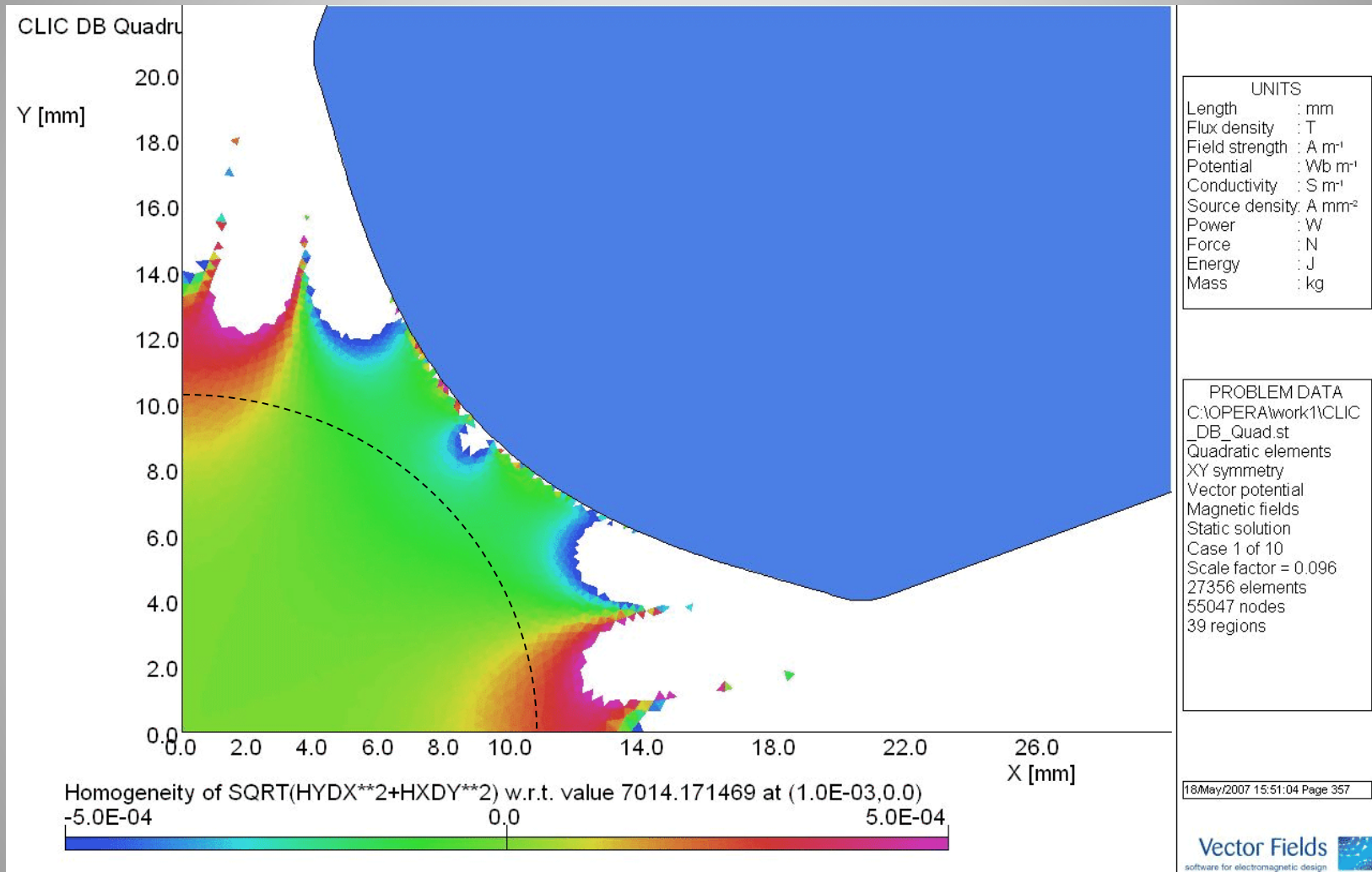
$$\frac{\Delta B'}{B'_0} = \frac{B'(x, y)}{B'(0,0)} - 1 \quad \frac{\Delta B'}{B'_0} \leq 0.1\%$$



Gradient homogeneity along circular GFR



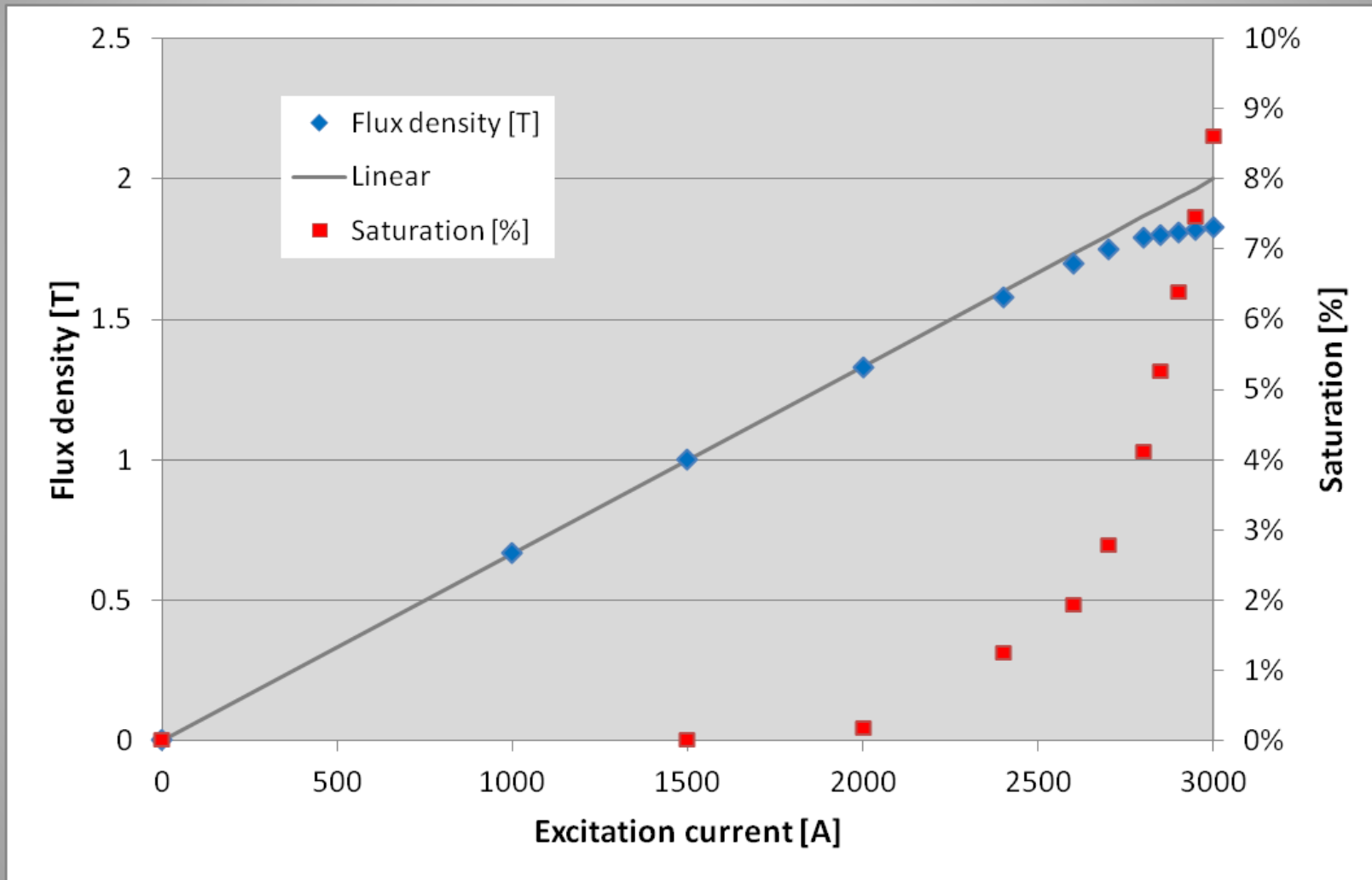
Saturation and field quality



Field quality varies with field strength due to saturation



Saturation





Field analysis

Picking up from lecture 1

$$B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{R_{ref}} \right)^{n-1}$$

and introducing **dimensionless normalized multipole coefficients**

$$b_n = \frac{B_n}{B_N} 10^4 \quad \text{and} \quad a_n = \frac{A_n}{B_N} 10^4$$

with B_N being the fundamental field of a magnet: $B_{N(\text{dipole})} = B_1$; $B_{N(\text{quad})} = B_2$; ...

we can describe each magnet by its ideal fundamental field and higher order harmonic distortions:

$$B_y(z) + iB_x(z) = \underbrace{\frac{B_N}{10^4}}_{\text{Fundamental field}} \underbrace{\sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{z}{R_{ref}} \right)^{n-1}}_{\text{Harmonic distortions}}$$

Fundamental field

Harmonic distortions



Field analysis

These **normalized multipole coefficients** are good indicators to assess the field quality of a magnet i.e. to describe the deviations of the actual field from the ideal one.

Normal dipole: $\vec{B}_{id}(x, y) = B_1 \vec{j}$

$$B_y(z) + iB_x(z) = B_1 + \frac{B_1}{10^4} \left[ia_1 + (b_2 + ia_2) \left(\frac{z}{r_0} \right) + (b_3 + ia_3) \left(\frac{z}{r_0} \right)^2 + (b_4 + ia_4) \left(\frac{z}{r_0} \right)^3 + \dots \right]$$

$$b_2 = \frac{B_2}{B_1} 10^4 \quad b_3 = \frac{B_3}{B_1} 10^4 \quad a_1 = \frac{A_1}{B_1} 10^4 \quad a_2 = \frac{A_2}{B_1} 10^4 \quad \dots$$

Normal quadrupole: $\vec{B}_{id}(x, y) = B_2 [x\vec{j} + y\vec{i}] \frac{1}{r_0}$

$$B_y(z) + iB_x(z) = B_2 \frac{z}{r_0} + \frac{B_2}{10^4} \left[ia_2 \left(\frac{z}{r_0} \right) + (b_3 + ia_3) \left(\frac{z}{r_0} \right)^2 + (b_4 + ia_4) \left(\frac{z}{r_0} \right)^3 + \dots \right]$$

$$b_3 = \frac{B_3}{B_2} 10^4 \quad b_4 = \frac{B_4}{B_2} 10^4 \quad a_2 = \frac{A_2}{B_2} 10^4 \quad \dots$$



Field analysis

The field quality of a magnet can be also described by:

- Homogeneity plot:

- difference between the actual field B and the ideal field B_{id} , normalized by the ideal field B_{id}

$$\frac{\Delta B}{B} = \frac{B(x, y) - B_{id}(x, y)}{B_{id}(x, y)}$$

- can be expressed by multipole coefficients: for a dipole with $B_{y,id}(x) = B_1$

$$B_y(x) = B_1 + \frac{B_1}{10^4} \left[b_2 \left(\frac{x}{r_0} \right) + b_3 \left(\frac{x}{r_0} \right)^2 + b_4 \left(\frac{x}{r_0} \right)^3 + \dots \right]$$

$$\frac{\Delta B}{B}(x) = \frac{1}{10^4} \left[b_2 \left(\frac{x}{r_0} \right) + b_3 \left(\frac{x}{r_0} \right)^2 + b_4 \left(\frac{x}{r_0} \right)^3 + \dots \right]$$

- Harmonic distortion factor F_d :

$$F_d(r_0) = \sum_{n=1; n \neq N}^K \sqrt{b_n^2(r_0) + a_n^2(r_0)}$$

Note: For good field quality, F_d should be a few units in 10^{-4}



Field analysis

Multipole errors can be divided into two families:

‘**Allowed**’ multipoles are design intrinsic and result from the finite size of the poles

$$n = N(2m + 1)$$

n : order of multipole component

N : order of the fundamental field

m : integer number ($m \geq 1$)

fully symmetric dipole

allowed: b_3, b_5, b_7, b_9 , etc.

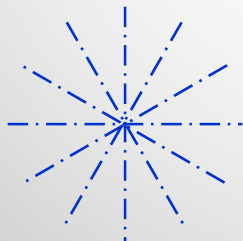
non-allowed: all others



fully symmetric sextupole

allowed: b_9, b_{15}, b_{21} , etc.

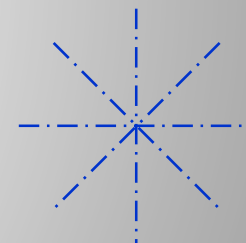
non-allowed: all others



fully symmetric quadrupole

allowed: $b_6, b_{10}, b_{14}, b_{18}$, etc.

non-allowed: all others

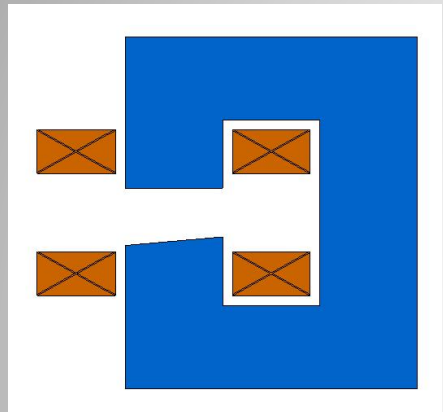


‘**Non-allowed**’ multipoles result from a violation of symmetry and indicate a fabrication or assembly error

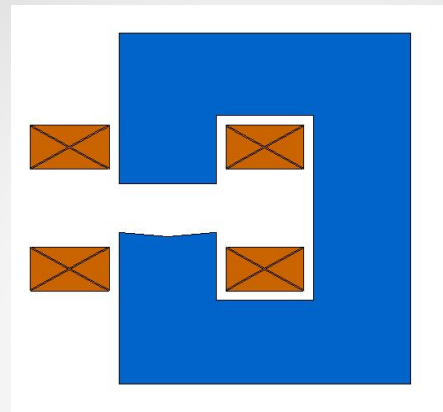


Asymmetries

Asymmetries generating 'non-allowed' harmonics

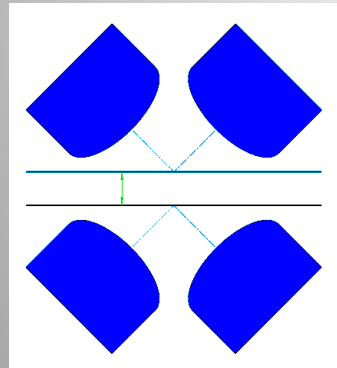


$n = 2, 4, 6, \dots$

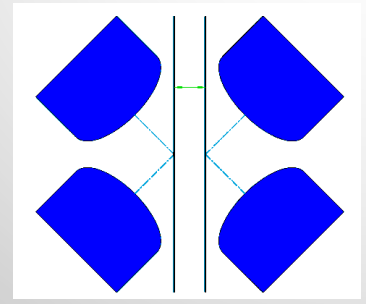


$n = 3, 6, 9, \dots$

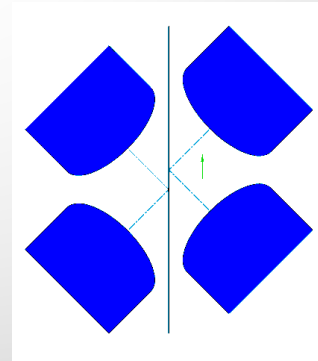
Comprehensive studies about the influence of manufacturing errors on the field quality have been done by [K. Halbach](#).



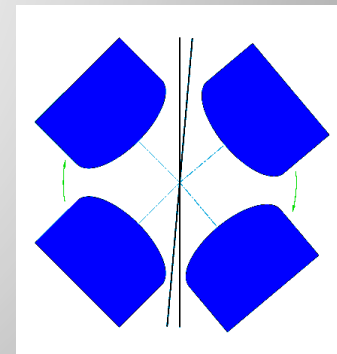
$n = 4$ (neg.)



$n = 4$ (pos.)



$n = 3$



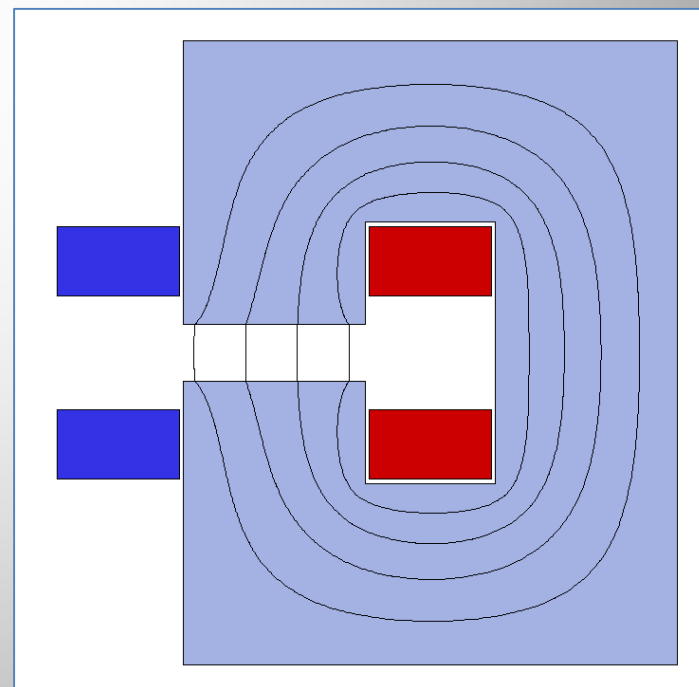
$n = 2, 3$

These errors can seriously affect machine behaviour and must be controlled!



Asymmetry in a C-magnet

- C-magnet: one-fold symmetry
- Since $NI = \oint \vec{H} \cdot d\vec{l} = \text{const.}$ the contribution to the integral in the iron has different path lengths
- Finite (low) permeability will create lower B on the outside of the gap than on the inside
- Generates ‘forbidden’ harmonics with $n = 2, 4, 6, \dots$ changing with saturation
- Quadrupole term resulting in a gradient around 0.1% across the pole

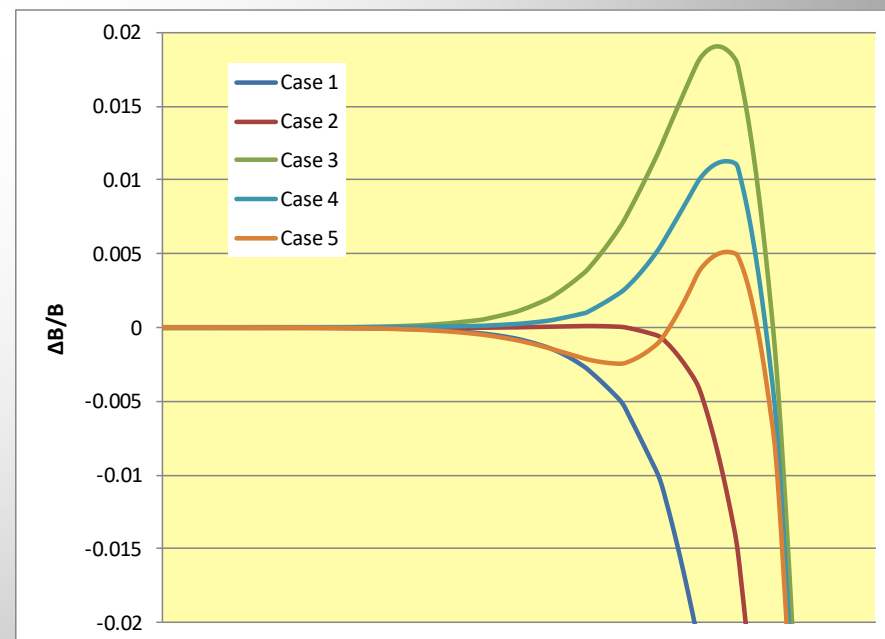
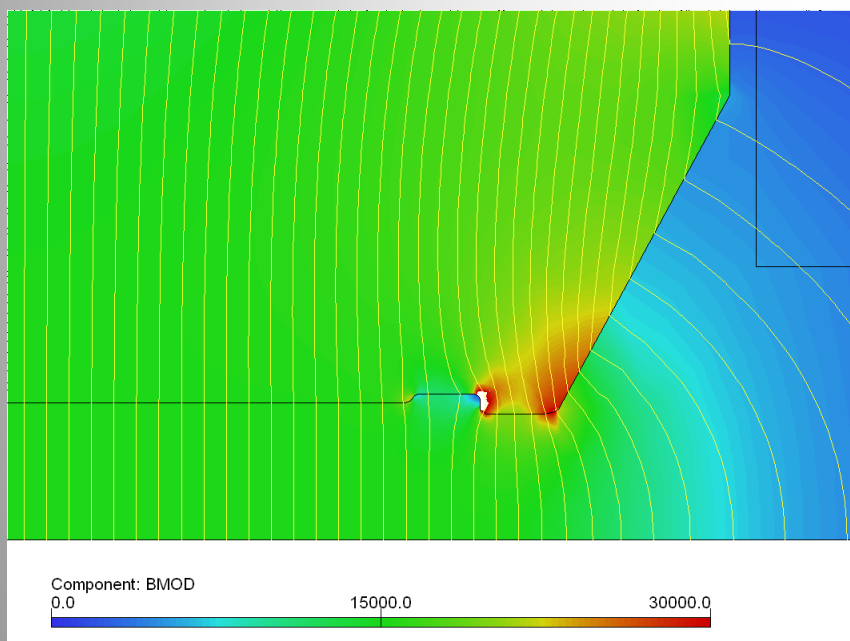




Pole optimization

‘**Shimming**’ (often done by ‘try-and-error’) can improve the field homogeneity

1. Add material on the pole edges: field will rise and then fall
2. Remove some material: curve will flatten
3. Round off corners: takes away saturation peak on edges
4. Pole tapering: reduces pole root saturation -> **Rogowsky profile**



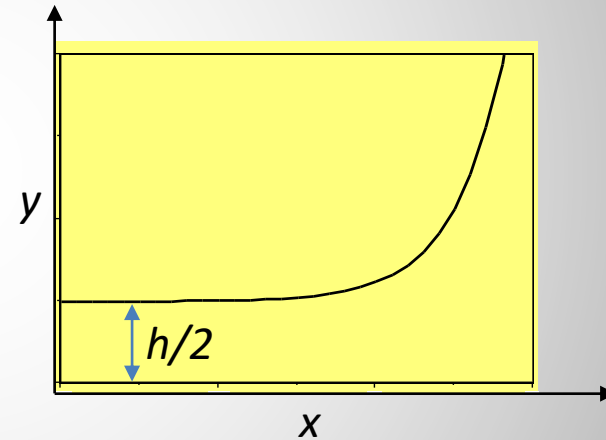
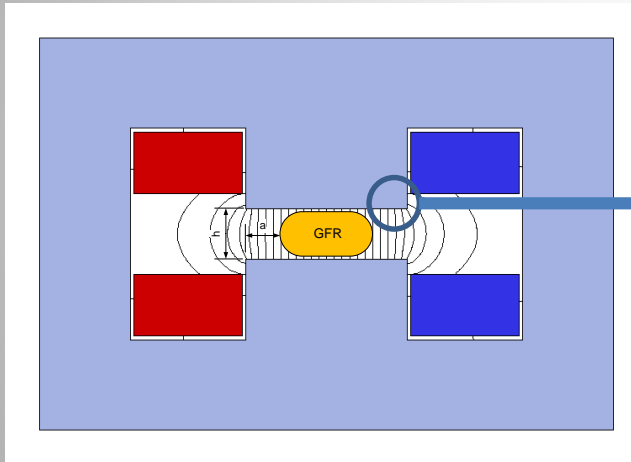


Rogowsky roll-off

The ‘Rogowsky’ profile provides the maximum rate of increase in gap with a monotonic decrease in flux density at the surface, i.e. no saturation at the pole edges!

The edge profile is shaped according to:

$$y = \frac{h}{2} + \left(\frac{h}{\pi}\right) \exp\left(\left(\frac{x\pi}{h}\right) - 1\right)$$



For an **optimized** pole:

$$x_{\text{optimized}} = 2 \frac{a}{h} = -0.14 \ln \frac{\Delta B}{B_0} - 0.25$$

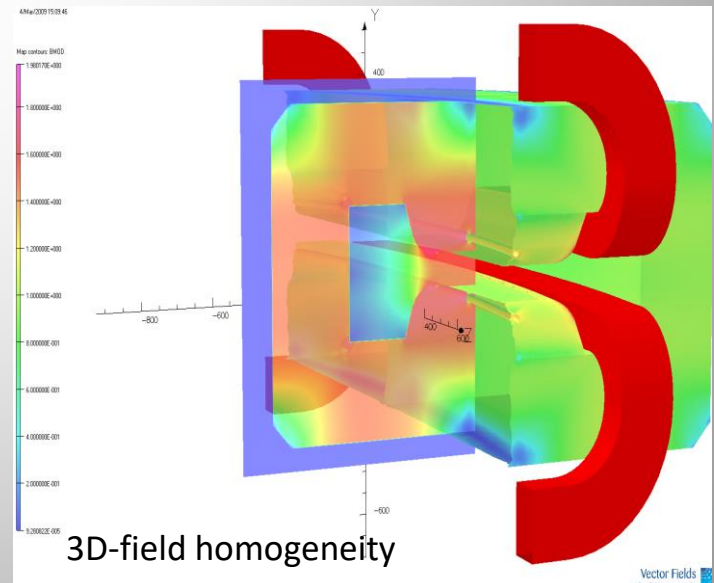
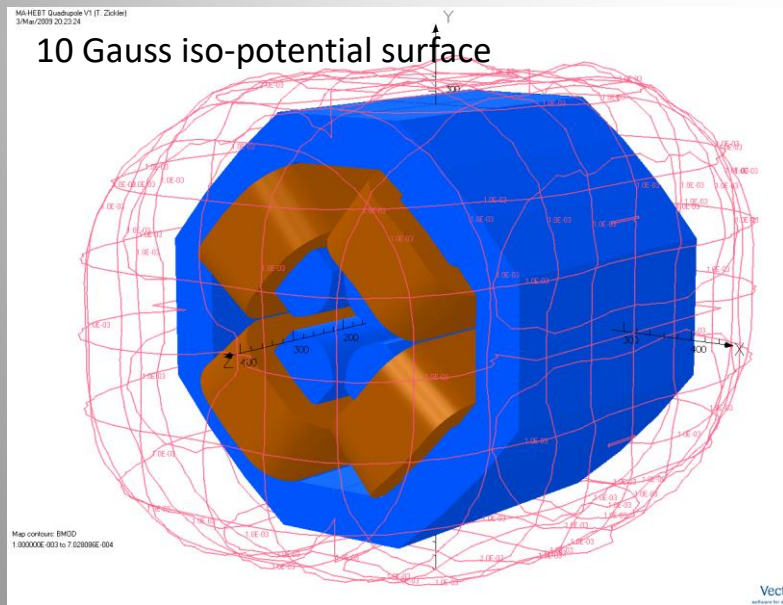
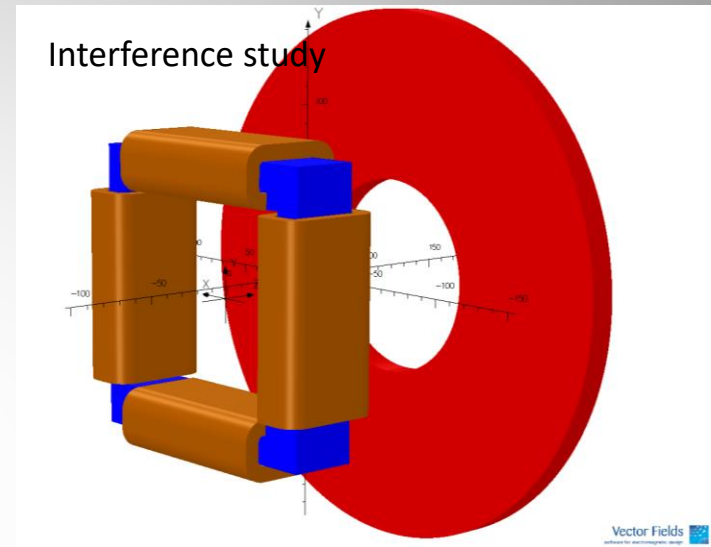
- x : pole overhang normalized to the gap
- a : pole overhang: excess pole beyond the edge of the good field region to reach the required field uniformity
- h : magnet gap



3D Design

Becomes necessary to study:

- the longitudinal field distribution
- end effects in the yoke
- end effects from coils
- magnets where the aperture is large compared to the length
- spacial field distribution
- particle motion in electro-magnetic fields

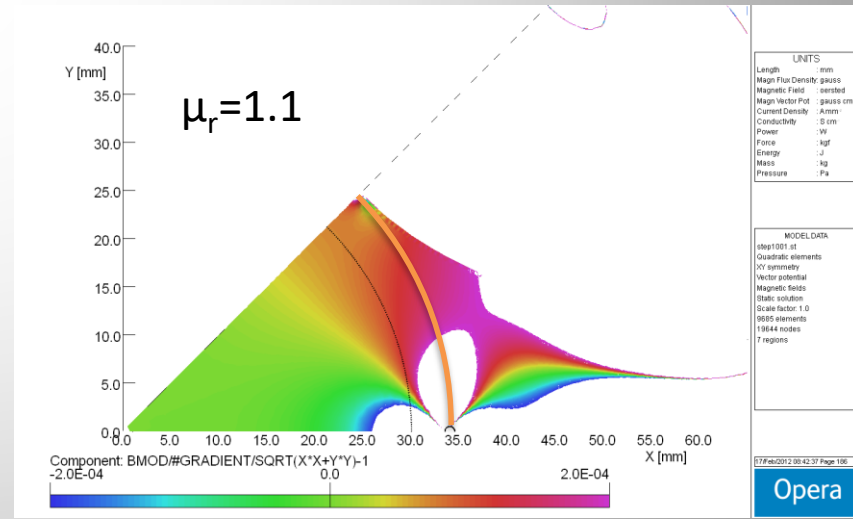
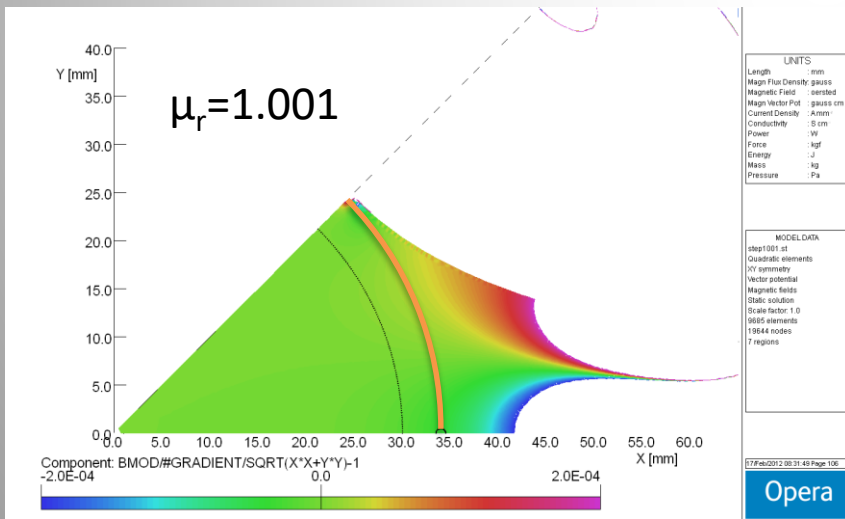
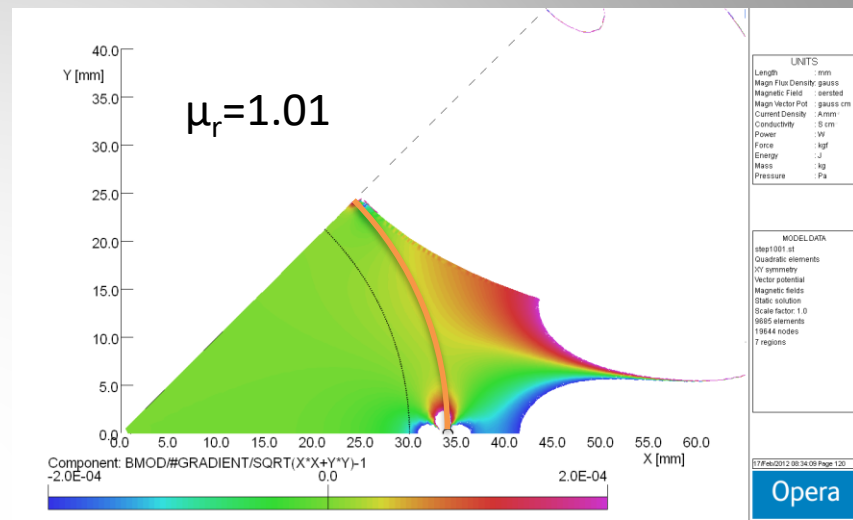




Case 1: A material problem

Welding seam on stainless-steel vacuum chamber:

- GFR radius: 30 mm
- Chamber radius: 35 mm
- Welding seam diameter: 1 mm
- Rel. permeability of 316 LN: < 1.001

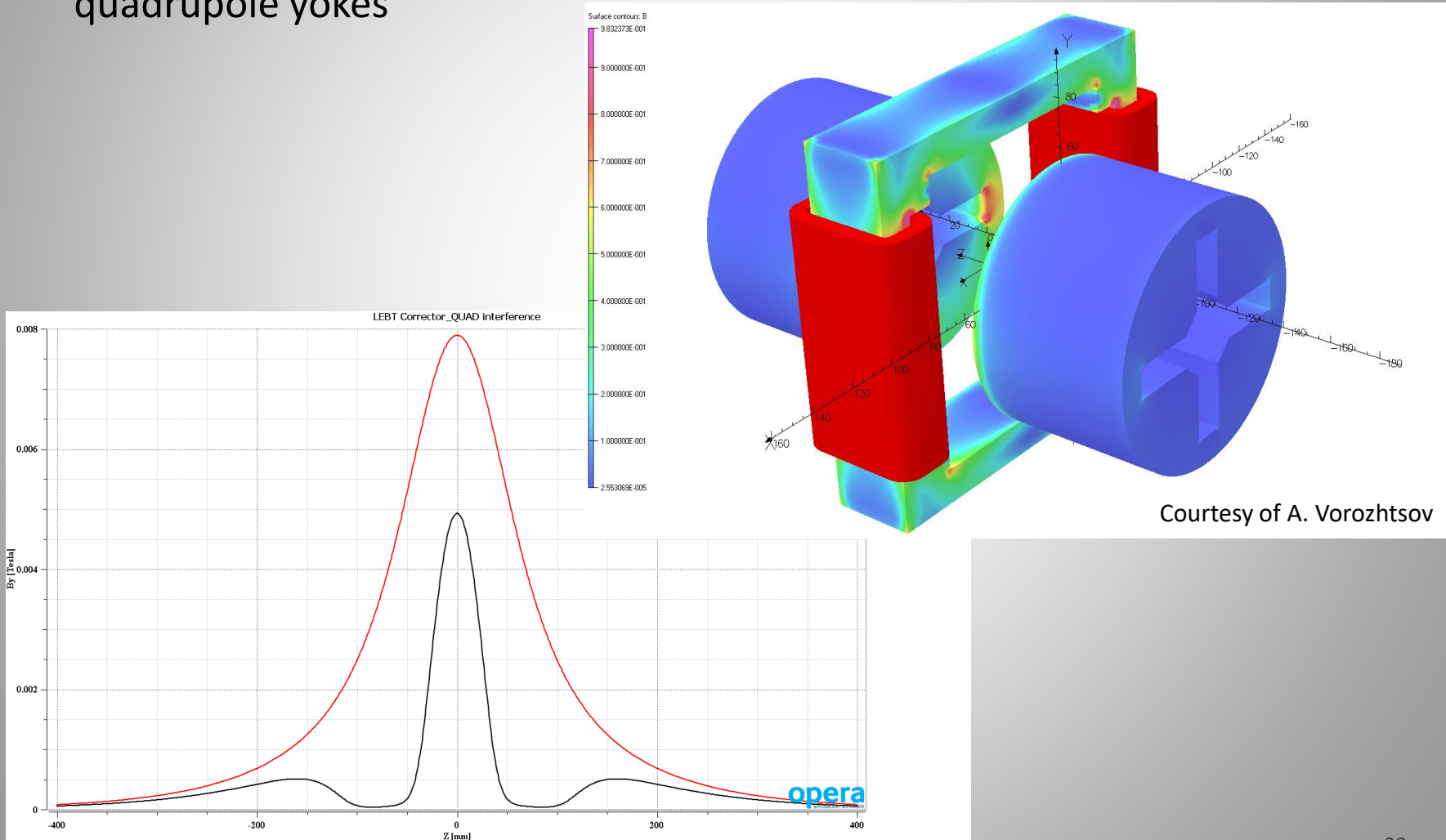


A **small** distortion can **significantly** influence the field quality in the GFR!



Case 2: An interference problem

Significant attenuation of the corrector field due to the close presence of two quadrupole yokes

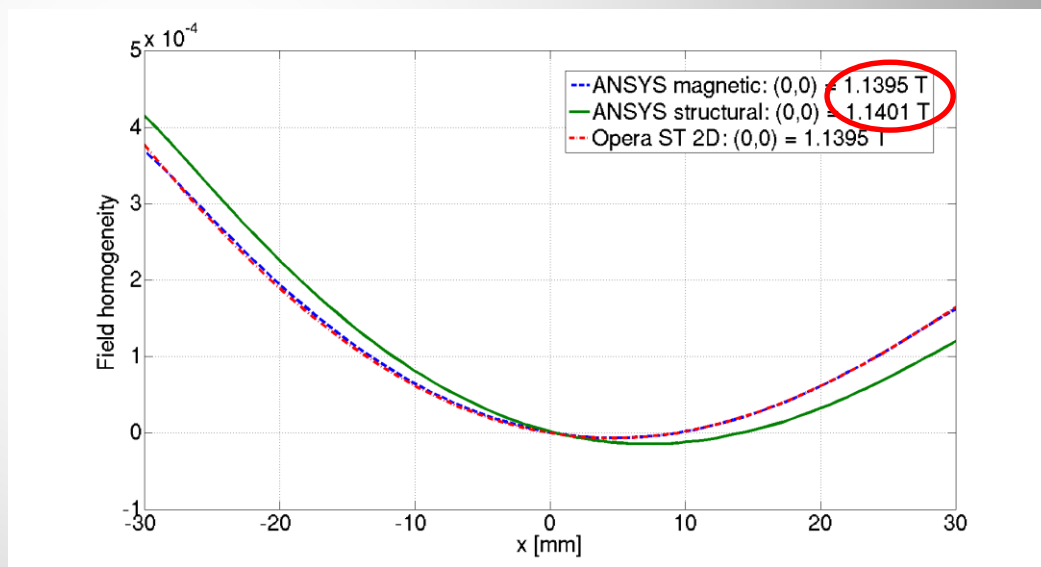
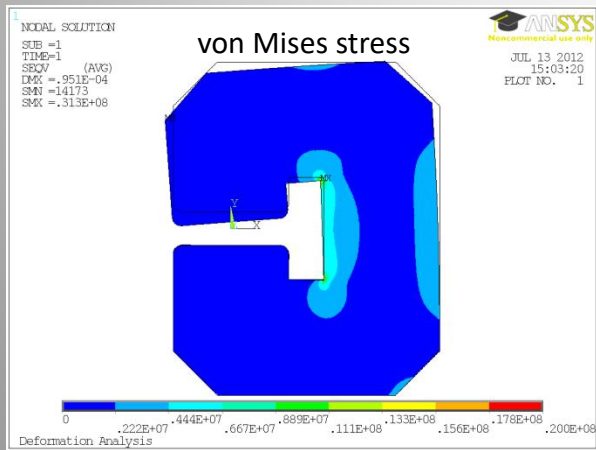
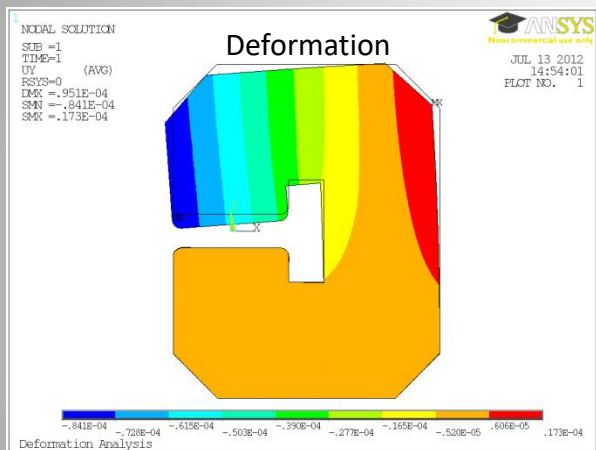


Courtesy of A. Vorozhtsov



Case 3: Mechanical deformation

- Mechanical deformation due to magnetic pressure can influence the field homogeneity
- Multi-physics models can help to quantify the effect



Field homogeneity calculated for the center line of the magnet with ANSYS magnetic, ANSYS structural + magnetic, and Opera ST 2D



Limitations of numerical calculation

Advantages

- predict behaviour without having the physical object
- for relatively simple cases they are fast and inexpensive

Limitations

- **multi-physics model:** including all couplings (thermal, mechanical) and phenomena (magnetostriction, magneto-resistivity ...) that *may* be relevant is very complex and expensive
- **off-nominal geometry:** random assembly errors can dominate field distribution and quality; often, a large number of degrees-of-freedom and the resulting combinatorial explosion makes Monte Carlo prediction costly
- **material properties uncertainty :** inhomogeneous properties cannot practically be measured throughout volume; even homogeneous materials can be measured only within 2-5% typical accuracy
- **numerical errors:** e.g. singularities in re-entrant corners, boundary location of open regions may spoil results; special techniques (special corner elements, BEM) require special skills and time
- **high cost** of detailed 3D models ($\propto \Delta x^{2\sim 3}$); transient simulations increase computing time significantly

Computer simulation targeting $<10^{-4}$ accuracy are difficult and expensive



Summary

- A large variety of FE-codes with different features exist – the right choice depends of the complexity of the problem
- The FE-models shall be **as simple as possible** and adapted to the problem to reduce computing time
- Numeric computations should be used to **quantify, not to qualify**
- **Benchmarking** the results with measurements is a good practice
- Computer simulations have a lot of advantages, but also their **limitations**