Transverse Beam Dynamics

JUAS 2019 - Tutorial 3

1 Exercise: Basics of lattice design

Design a FODO cell such that it has: phase advance $\mu = 90$ degrees, a total length of 10 m, and a total bending angle of 5 degrees. What are β_{max}, β_{min}, D_{max}, D_{min}?

2 Exercise: Bump and Orbit Control

Given two kickers located at the two ends of a FODO cell with phase advance 45 degrees (the two kickers are located at L_{cell} distance from each other), compute the strengths of such kickers (in radians) in order to give the beam, initially at $(x_i,\, x'_i) = (0,\, 0)$, an arbitrary offset at the end of the cell while preserving its angle, $\left(x_f,\, x'_f\right) = (x_{\text{arbitrary}},\, 0)$.

3 Exercise: Measurement of Twiss parameters

One of the possible ways to determine experimentally the Twiss parameters at a given point makes use of a so-called quadrupole scan. One can measure the transverse size of the beam in a profile monitor, called Wire Beam Scanner (WBS), located at a distance L downstream a focusing quadrupole, as a function of the normalised gradient in this quadrupole. This allows to compute the emittance of the beam, as well as the β and the α functions at the entrance of the quadrupole.

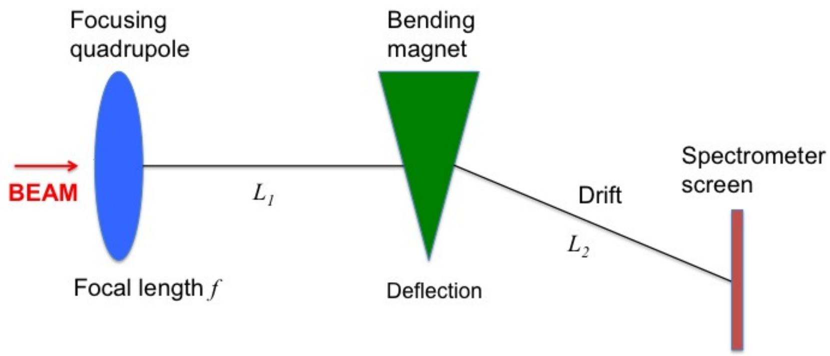
Let's consider a quadrupole Q with a length of $l = 20$ cm. This quadrupole is installed in an electron transport line where the particle momentum is 300 MeV/c. At a distance $L = 10$ m from the quadrupole the transverse beam size is measured with a WBS, for various values of the current I_Q. The maximum value of the quadrupole gradient G is obtained for a current of 100 A, and is $G = 1$ T/m.

Hint: G is proportional to the current. **Advice:** use thin-lens approximation.

1. How does the normalised focusing strength K vary with I_Q?
2. Give the expression Σ_2 as function of α_1, β_1, and γ_1
3. Show that β_2 can be written in the form: $\beta_2 = A_2\,(Kl)^2 + A_1\,(Kl) + A_0$, and express A_0, A_1, and A_2 as a function of L, α_1, β_1, and γ_1.
4. Express the final beam size, σ_2, as a function of Kl, and find its minimum, which will correspond to $(Kl)_{\text{min}}$.
5. How does σ_2 vary with Kl when $|Kl - (Kl)_{\text{min}}| \gg 1/\beta_1$?
6. Deduce the values of α_1, β_1, and γ_1 from the measurement σ_2, as a function of the quadrupole current I_Q.

4 Exercise: The spectrometer line of CTF3

The CTF3 (CLIC Test Facility 3) experiment at CERN consists of a linac that injects very short electron bunches into an isochronous ring. A spectrometer line made of one quadrupole and one bending magnet is located at the end of the linac where the particle momentum is 350 MeV/c. The goal of the spectrometer is to measure the energy before injecting the electrons in the ring. The spectrometer line is sketched on the figure below. It is made of a focusing quadrupole of focal length f, a drift space of length L_1, a bending magnet of deflection angle θ in the horizontal plane, and a drift space of length L_2. We assume that the spectrometer line starts at the quadrupole and ends at the end of the second drift. We neglect the focusing effect of the dipole.



1. If the effective length of the dipole is $l_B = 0.43$ m, what should be the magnetic field (in Tesla) inside the dipole to deflect the electrons by an angle of 35 degrees?

2. Starting from the general horizontal 3×3 transfer matrix of a sector dipole of deflection angle θ, show that the transfer matrix of a dipole in the thin-lens approximation is

$$M_{dipole} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$

   Which approximations are done?

   Hint: A sector dipole has the following 3×3 transfer matrix:

$$M_{\mathrm{dipole}} = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -\frac{\sin\theta}{\rho} & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix}$$

3. In the thin-lens approximation, derive the horizontal extended 3×3 transfer matrix of the spectrometer line. Show that it is:

$$M_{spectro} = \begin{pmatrix} \frac{f-L_1-L_2}{f} & L_1+L_2 & L_2\theta \\ -\frac{1}{f} & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$

4. Assuming $D = D' = 0$ at the entrance of the quadrupole, what is the dispersion and its derivative at the end of the spectrometer line? Give the numerical value of D' at the end of the spectrometer line for the angle of 35 degrees.

5. What is the difference between a periodic lattice and a beam transport lattice (or transfer line) as concerns the betatron function ?

6. Derive the betatron function β_2 at the end of the spectrometer line in terms of L_1, L_2, f and β_1, assuming $\alpha_1 = 0$.

   Hint 1. Remember from the lecture:

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_s = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & SC'+S'C & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0$$

   An alternative way to transport the Twiss parameters is through the σ matrix:

$$\sigma_i = \begin{pmatrix} \beta_i & -\alpha_i \\ -\alpha_i & \gamma_i \end{pmatrix}$$

   This matrix multiplied by the emittance ϵ gives the so-called beam matrix (which has already been introduced during the lecture):

$$\Sigma_i = \begin{pmatrix} \beta_i \epsilon & -\alpha_i \epsilon \\ -\alpha_i \epsilon & \gamma_i \epsilon \end{pmatrix}$$

If σ_1 is the matrix at the entrance of the transfer line, the matrix σ_2 at the exit of the transfer line is given by

$$\sigma_2 = M \sigma_1 M^T$$

where M is the 2×2 transfer matrix of the line extracted from the extended 3×3 transfer matrix (see question 3), and M^T the transpose matrix of M.

Hint 2. For the calculations, write M as $M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$ and replace the values of the matrix elements only at the end.

7. Given the numerical values $L_1 = 1$ m, $L_2 = 2$ m, $\beta_1 = 10$ m, $\alpha_1 = 0$, compute the value of the focal length f such that the betatron function at the end of the spectrometer line is minimum.
8. For an off-momentum particle, compute the deviation from the design orbit? Why is it important to minimise the β function in the spectrometer?

5 Exercise: Transfer matrix of a dipole magnet

- Remember weak focusing:
  $K = \frac{1}{\rho^2}$:

$$M_{\text{Dipole}} = \begin{pmatrix} \cos\left(\sqrt{K}L\right) & \frac{1}{\sqrt{K}} \sin\left(\sqrt{K}L\right) \\ -\sqrt{K} \sin\left(\sqrt{K}L\right) & \cos\left(\sqrt{K}L\right) \end{pmatrix} = \begin{pmatrix} \cos \frac{L}{\rho} & \rho \sin \frac{L}{\rho} \\ -\frac{1}{\rho} \sin \frac{L}{\rho} & \cos \frac{L}{\rho} \end{pmatrix}$$

- Compute the 3×3 matrix of a sector dipole including the dispersion terms.