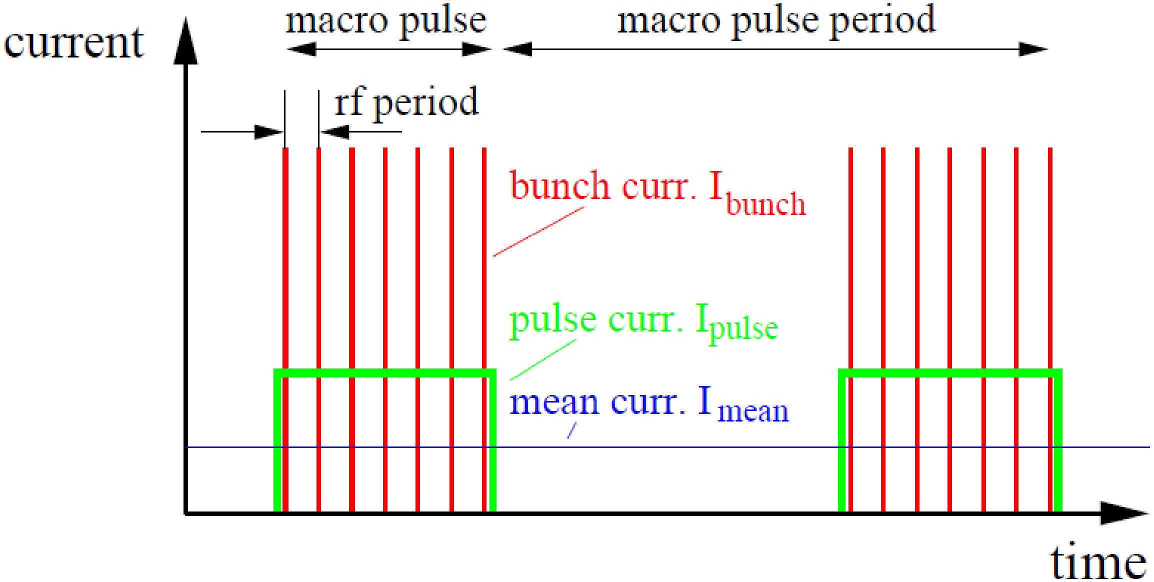
Beam Structure of a pulsed LINAC

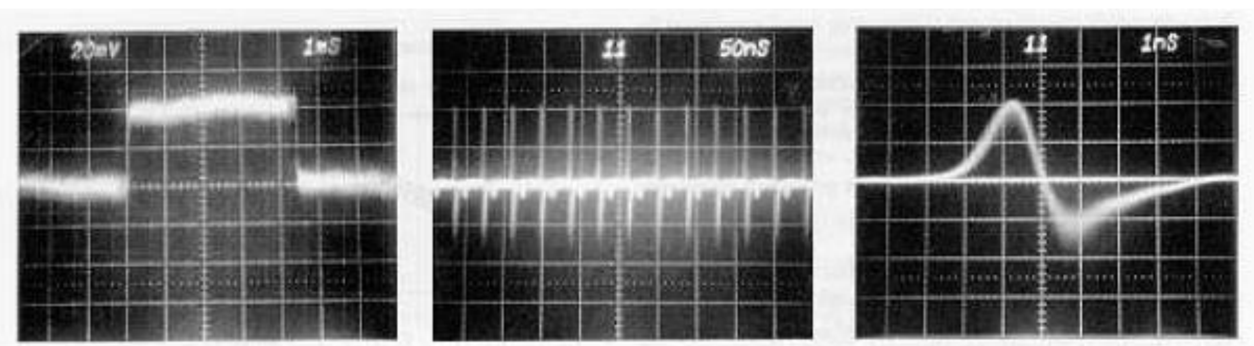
Pulsed LINACs and cyclotrons used for injection to synchrotrons with $t_{pulse} \approx 100\ \mu s$:



One distinguish between:

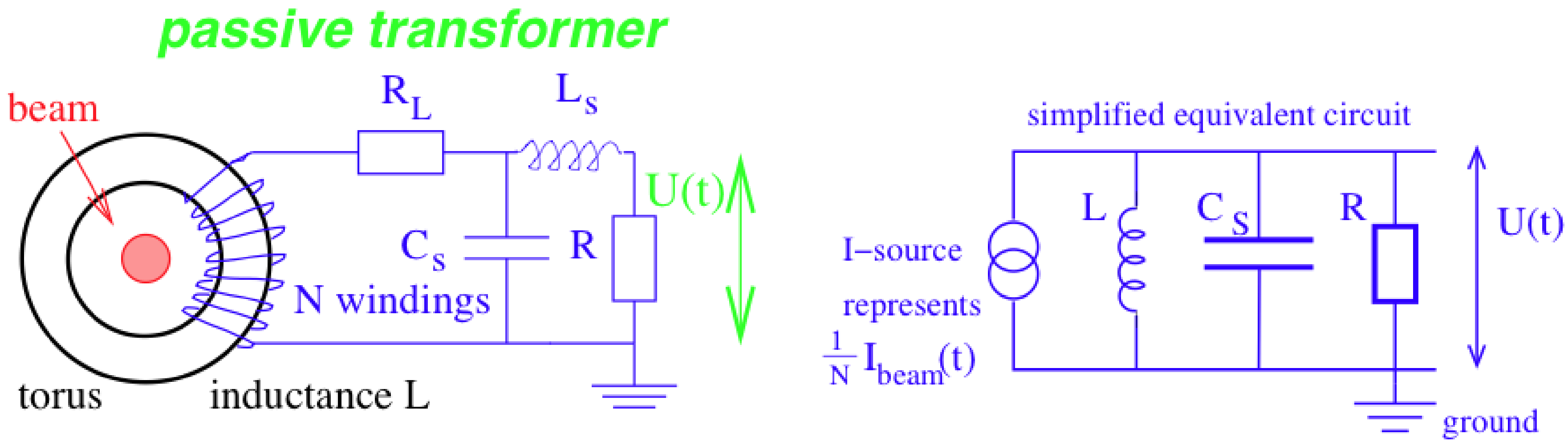
- Mean current I_{mean}
  - → long time average in [A]
- Pulse current I_{pulse}
  - → during the macro pulse in [A]
- Bunch current I_{bunch}
  - → during the bunch in [C/bunch] or [particles/bunch]

Remark: Van-de-Graaff (ele-static):
→ no bunch structure



Example:
Pulse and bunch structure at GSI LINAC:

Simplified electrical circuit of a passively loaded transformer:



A voltages is measured: $\boldsymbol{U = R \cdot I_{sec} = R / N \cdot I_{beam} \equiv S \cdot I_{beam}}$

with ***S* sensitivity [V/A],** equivalent to transfer function or transfer impedance ***Z***.

Equivalent circuit for analysis of sensitivity and bandwidth

(disregarding the loss resistivity R_L)

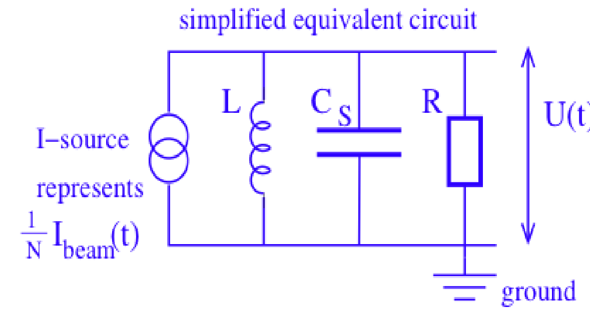
Time domain description:

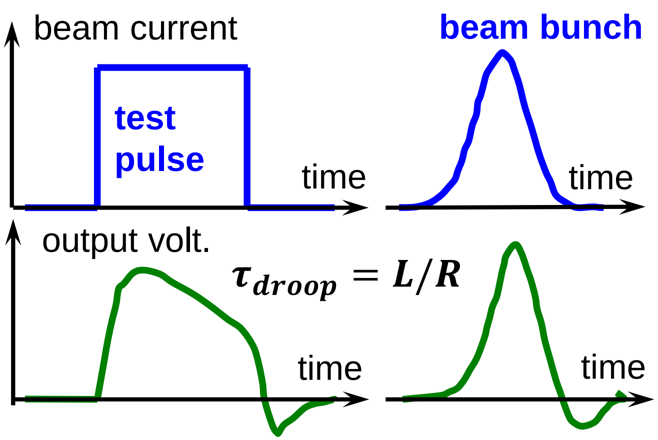
Droop time: $\tau_{droop} = 1/(2\pi f_{low}) = L/R$

Rise time: $\tau_{rise} = 1/(2\pi f_{high}) = 1/RC_S$ (ideal without cables)

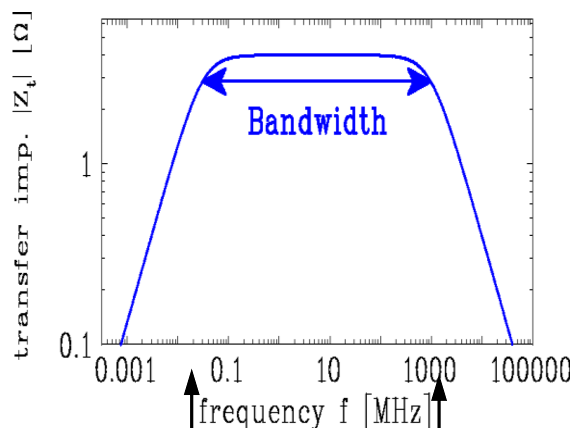
Rise time: $\tau_{rise} = 1/(2\pi f_{high}) = \sqrt{L_S C_s}$ (with cables)

R_L: loss resistivity, R: for measuring.





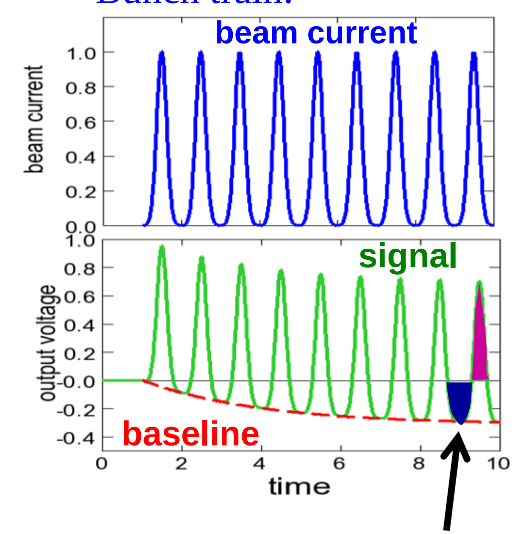
$$\tau_{rise} = \sqrt{L \cdot C_S}$$



$2\pi f_{low} = R/L$

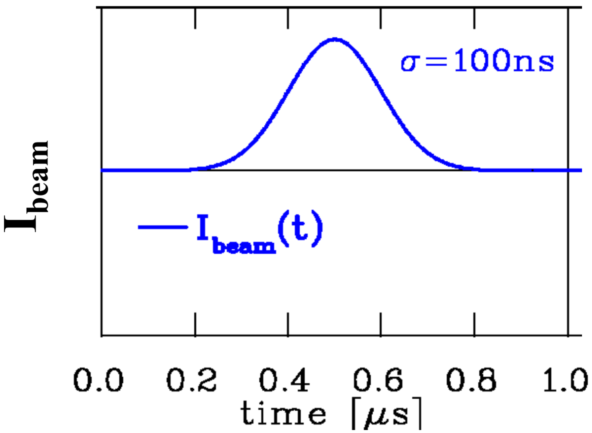
$2\pi f_{high} = 1/RC_S$

Bunch train:



Baseline: $U_{base} \propto 1 - \exp(-t/\tau_{droop})$
positive & negative areas are equal

Time domain: **Recording of a voltage as a function of time:**



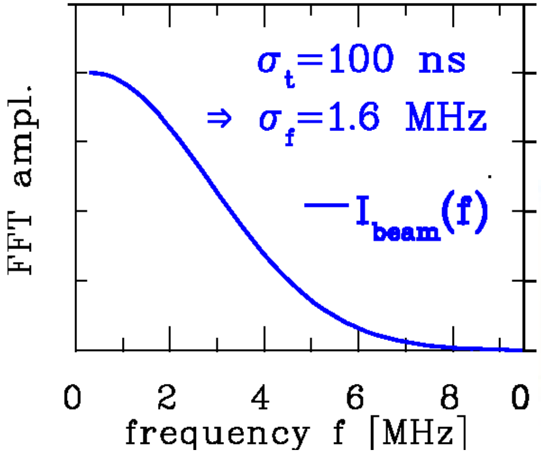
Instrument:

Oscilloscope

Fourier Transformation:

$$\tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

Frequency domain: **Displaying of a voltage as a function of frequency:**



Instrument:

Spectrum Analyzer

Fourier Transformation

of time domain data

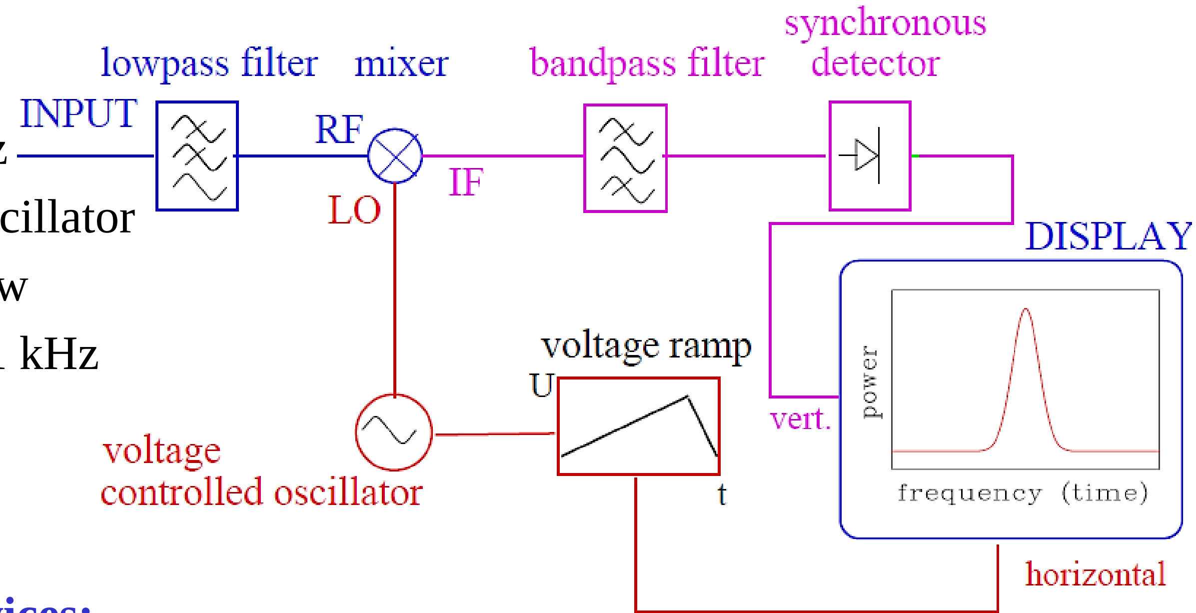
<u>Care:</u> Contains amplitude

***<u>and</u>* phase**

The spectrum analyzer determines the frequency spectrum of a signal:

Steps for an analog device:

- low pass filter, typ. $f < 3$ GHz
- mixing with scanning local oscillator
- difference frequency by narrow band pass filter, width typ. ≈ 1 kHz
- rectification
- scan duration typ. ≈ 1 s



Digital analyzer as modern devices:

- low pass filter, typ. $f < 3$ GHz
- down mixing typ. 0 .... 100 MHz
- digitalization with 100 MSa/s high resolution 16 bit ADC
- calculation of FFT and versatile display possibilities

Assume a beam with 1 ms macro-pulse length at a LINAC

The current should be measured by a current transformer

The maximum droop should be 3% within 1 ms .

Calculate the required droop time constant and the lower cut-off frequency f_{low} !

Result: $U(t) = U_0 \cdot e^{-t/\tau_{droop}}$ for t = 1 ms $\frac{U(t)}{U_0} = e^{-1\,\mathrm{ms}/\tau_{droop}} = 0.97 \Rightarrow \tau_{droop} = 32$ ms

cut-off frequency: $f_{low} = \frac{1}{2\pi\tau_{droop}} = 4.9$ Hz

Core size: r_i = 30 mm, r_o = 60 mm, length l = 40 mm

Permeability of the core: $\mu_r = 10^5$ ($\mu_0 = 4\pi\cdot 10^{-7}$ Vs/Am)

Passive transformer with R = 1 kΩ termination, R_L = 10 Ω loss resistivity

Calculate the number of winding for the given droop and the sensitivity [V/A]!

Hint: 1. Calculate the required inductance L:

2. The inductance of a core with N windings is:

$$L = \frac{\mu_0 \mu_r}{2\pi} \cdot l \cdot N^2 \cdot \ln \frac{r_o}{r_i}$$

Result: Inductance via $\tau_{droop} = \frac{L}{R+R_L} \Leftrightarrow L = 33$ Hy $\Rightarrow N = \sqrt{\frac{2\pi L}{\mu\mu_0 l \ln r_i/r_o}} = 244$

Sensitivity $S = \frac{U}{I_{beam}} = \frac{R}{N} \approx 4$ V/A

General Noise Sources

Any electronics is accompanied with noise due to:

- **Thermal noise** as given by the thermal movement of electrons described by Maxwell-Boltzmann distribution

  Within **resistive matter**

  average movement cancels $U_{mean} = < U > = 0$

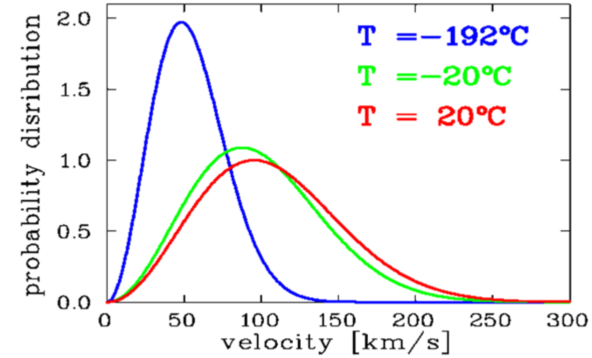
  but standard deviation remains:

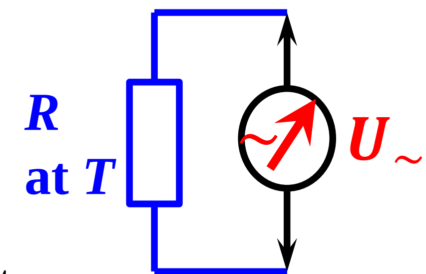
  $$U_{eff} = \sqrt{< U^2 >} = \sqrt{4k_B \cdot T \cdot R \cdot \Delta f}$$

  this is **white noise** i.e. no frequency dependence, no dc part

  k_B Boltzmann constant, T temperature, R resistivity, Δf bandwidth

Example: Maxwell-Boltzmann distribution of a **free** electron gas





- **Shot noise** as given by the fluctuations of finite amount of electrons for most electronics not important due to large amount of electrons
- **Flicker noise or '1/f - noise'** as given by trapping of electrons in matter **pink noise** due to a frequency , 'corner-frequency' $f_c \approx$ 3 kHz i.e. 1/f-noise ≠ thermal noise

Disturbance by Electro-Magnetic Interference EMI:

Not noise but pick-up of electro-magnetic waves from the environment leads unwanted signal deformation and depends on the surrounding

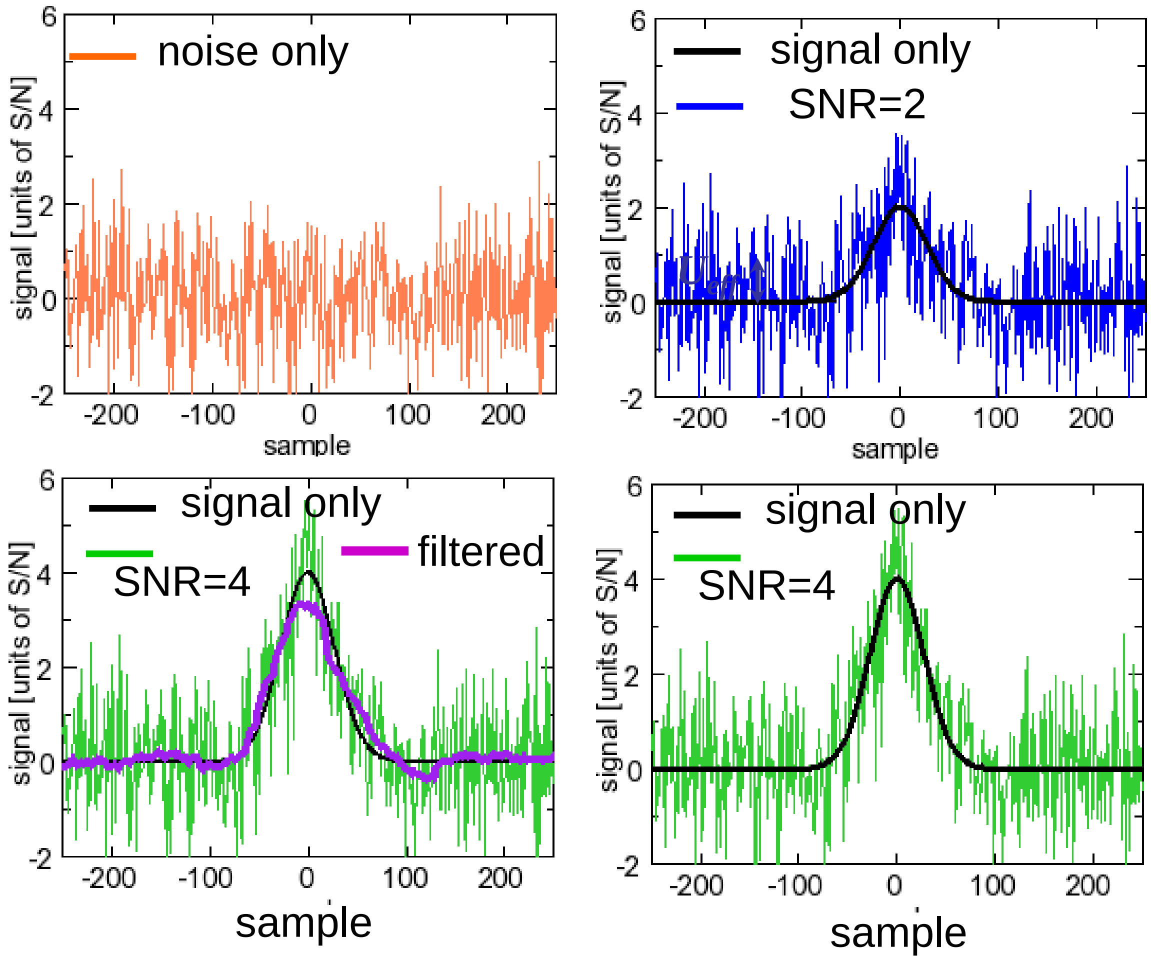
Maxwell-Boltzmann distribution: $\frac{1}{N} \cdot \frac{dN}{dv} = \frac{4}{\sqrt{\pi}} \left(\frac{m}{3k_B T}\right)^{3/2} \cdot v^2 \cdot \exp\left(-\frac{mv^2}{2k_B T}\right)$

Signal to Noise Consideration

The minimum noise is given thermal noise: $U_{eff} = (4k_B T \cdot R \cdot f)^{1/2}$

R is the input Ohmic resistor → low noise amplifier at input stage (realistic 2-3 x U_{eff})

f is the bandwidth of the signal chain



Simulation:

- Broadband 'white' noise
- For Signal-to-Noise = 2 peak just visible
- For Signal-to-Noise = 4 peak well visible
- Filtering improves ***SNR*** here: n=51 tap moving average ⇔ bandwidth restriction

$$y_j^{new} = \frac{1}{n} \cdot \sum_{i=j-n/2}^{i=j+n/2} y_i$$

Exercise #2: Transformers for a pulsed LINAC 1/3

Assume a beam with 1 ms macro-pulse length at a LINAC

The current should be measured by a current transformer

The maximum droop should be 3% within 1ms.

Core size: $\boldsymbol{r_i}$ = 30mm, $\boldsymbol{r_o}$ = 60 mm, length $\boldsymbol{l}$ = 40 mm

Permeability of the core: $\boldsymbol{\mu_r} = 10^5$ ($\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am)

Passive transformer with $\boldsymbol{R}$ = 1 kΩ termination, $\boldsymbol{R_L}$ = 10 Ω loss resistivity

The upper cut-off frequency is $\boldsymbol{f_{high}}$ = 100 kHz, the lower $\boldsymbol{f_{low}}$ = 100 Hz, $\boldsymbol{R}$ is at $\boldsymbol{T}$=300 K

Use $\boldsymbol{U_{noise} = (4k_B \cdot T \cdot R \cdot \Delta f)^{1/2}}$ and $\boldsymbol{k_B} = 1.4 \cdot 10^{-23}$ J/K, bandwidth $\boldsymbol{\Delta f = f_{high} - f_{low}}$

Calculate the thermal noise contribution!

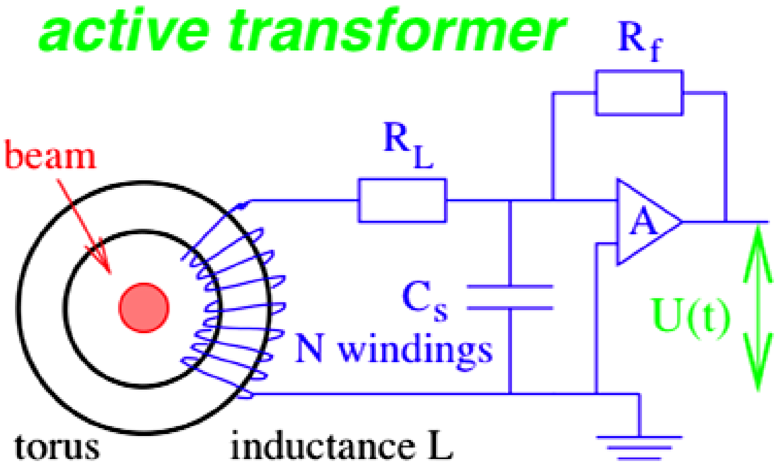
Calculate the threshold concerning the beam current for a signal-to-noise ratio of ***SNR*** = 1!

Result: Thermal noise $U_{noise} = \sqrt{4k_BT(R+R_L)\Delta f} = 1.3\ \mu\text{V}$

Beam current for ***S/N***=1 : Sensitivity $I_{beam} = \frac{1}{S} \cdot U_{noise} = 0.32\ \mu\text{A}$

Active Transformer or Alternating Current Transformer ACT:
uses a trans-impedance amplifier (I/U converter) to $\boldsymbol{R \approx 0}$ Ω load impedance i.e. a current sink
+ compensation feedback
⇒ longer droop time τ_{droop}

Application: measurement of longer t > 10 μs e.g. at pulsed LINACs



The input resistor is for an op-amp: $R_f/A \ll R_L$

$\Rightarrow \tau_{droop} = L/(R_f/A + R_L) \simeq L/R_L$

Droop time constant can be up to 1 s!

The feedback resistor is also used for range switching.

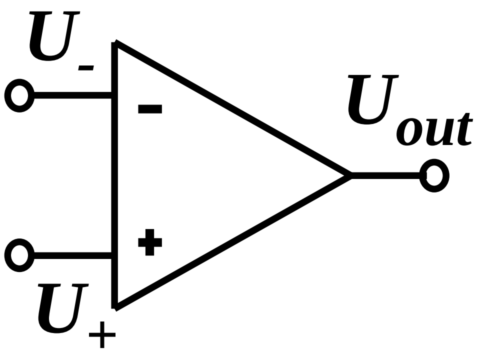
An additional active feedback loop is used to compensate the droop.

Operational Amplifier: Chips used as a simple equivalent circuit, but has complex built-in electronics

Without feedback: $U_{out} = g_{ol} \cdot (U_{+} - U_{-})$

Open loop gain: $g_{ol} \approx 10^4$

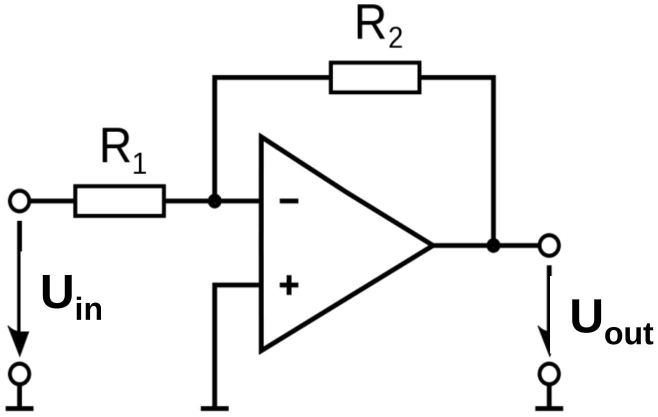
With feedback: compensation of both inputs

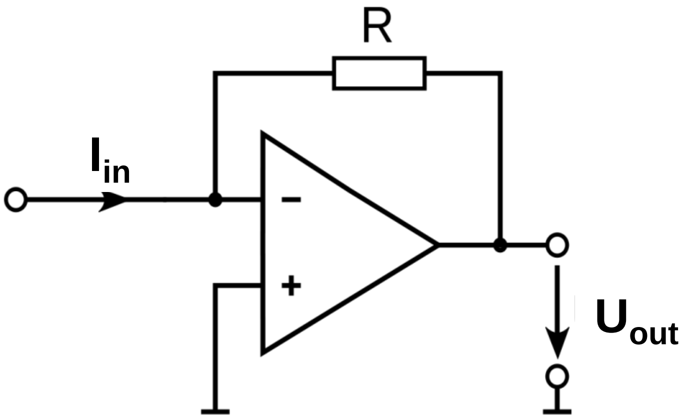


Examples of applications:

- **Inverting voltage amplifier**: $U_{out} = g \cdot U_{in} = -R_2 / R_1 \cdot U_{in}$
- **Current-to-voltage converter:** $U_{out} = - R \cdot I_{in}$

i.e. feedback leads current sink at inverting input



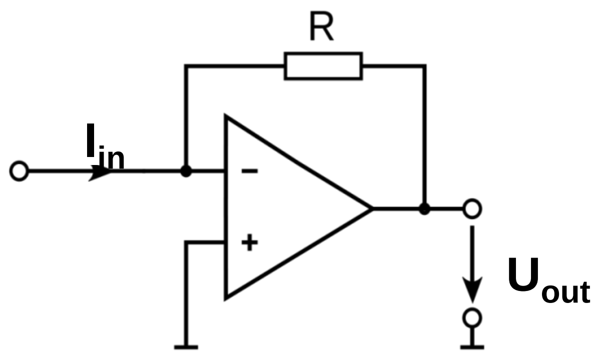


Excurse: Transimpedance Amplifier = I/U Converter

Current-to-voltage converter: $U_a = -R \cdot I_e \equiv Z_t \cdot I_e$

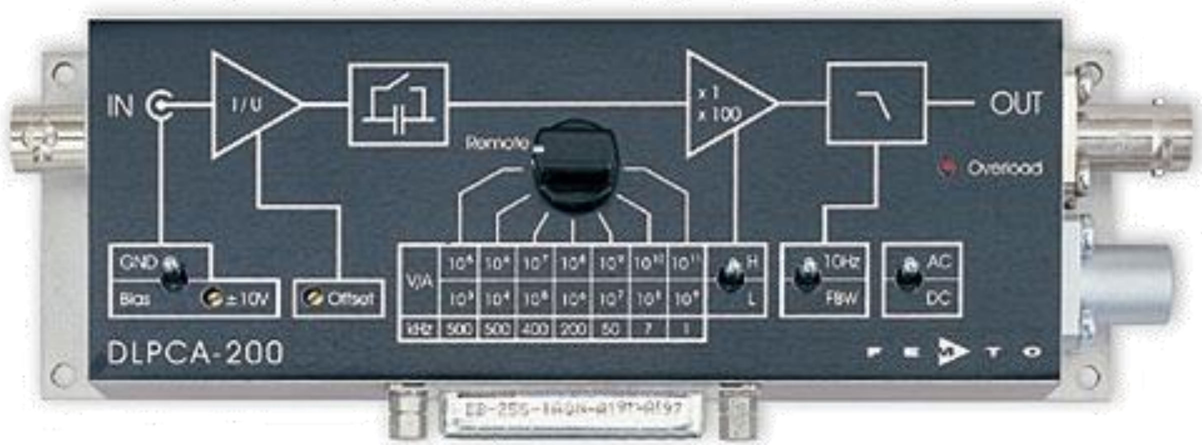
i.e. feedback leads current sink at inverting input

Z_t [V/A]	10^5	10^7	10^9	10^{11}
Full scale	100µA	1µA	10nA	100pA
Bandwidth f_{cut} [kHz]	500	400	50	1.1
Risetime t_{rise} 10 to 90%	0.7µs	0.9µs	7µs	300µs
Equivalent noise Noise/full-scale (rel.) Rel.to thermal noise	10nA 10^{-4} ≈ 1	450pA $2\cdot10^{-4}$ ≈ 1	3.7pA $3\cdot10^{-4}$ ≈ 1.6	0.8pA 10^{-3} ≈ 10



Counteraction between
- Current resolution
- Time resolution

$$t_{rise} \approx \frac{1}{3\cdot f_{cut}}$$



Exercise #2: Transformers for a pulsed LINAC 2/3

As for the previous example: Assume a beam with 1 ms macro-pulse length at a LINAC

The current should be measured by current transformer

Core size: $\boldsymbol{r_i}$ = 30mm, $\boldsymbol{r_o}$ = 60 mm, length $\boldsymbol{l}$ = 40 mm

Permeability of the core: $\boldsymbol{\mu_r} = 10^5$ ($\mu_0 = 4\pi\cdot 10^{-7}$ Vs/Am)

The maximum droop should be 3% within 1 ms

The upper cut-off frequency is 100 kHz, the resistor is at $\boldsymbol{T}$ = 300 K

Active transformer with open-loop ampl. $\boldsymbol{A} = 10^6$ and $\boldsymbol{R_f}$ = 1 MΩ feedback, $\boldsymbol{R_L}$ = 10 Ω

Calculate the inductance for the given droop time!

Calculate the number of winding and the sensitivity!

Calculate the threshold concerning the beam current for a signal-to-noise ratio of ***SNR*** = 1!

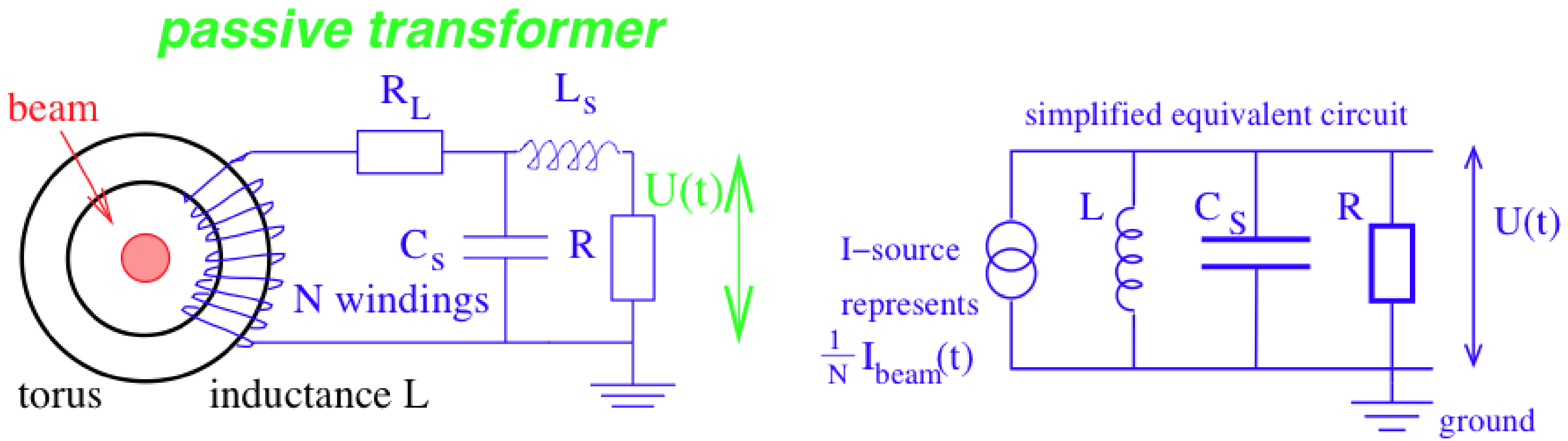
Result: Inductance via $\tau_{droop} = \frac{L}{R_L}$ due to $\frac{R_f}{A} \ll R_L \Longrightarrow L = 0.32\,Hy$ & $N = \sqrt{\frac{2\pi L}{\mu\mu_0 l \ln r_i/r_o}} = 24$

Sensitivity $S = \frac{R_f}{N} \approx 4.2 \cdot 10^4$ V/A

Noise: $U_{noise} = \sqrt{4k_B T(R_f/A + R_L)\Delta f} = 0.14$ µV & $I_{noise} = \frac{A}{R_f} \cdot U_{noise} = 0.14$ µA

Minimum beam current: $I_{beam} = \frac{I_{noise}}{N} = 6$ nA $\rightarrow$ unrealistically low due further noise

Simplified electrical circuit of a passively loaded transformer:



A voltages is measured: $\boldsymbol{U = R \cdot I_{sec} = R / N \cdot I_{beam} \equiv S \cdot I_{beam}}$

with ***S*** **sensitivity [V/A],** equivalent to transfer function or transfer impedance ***Z***.

Equivalent circuit for analysis of sensitivity and bandwidth

(disregarding the loss resistivity R_L)

Assume a beam with **1 μs pulse length behind a synchrotron** to be measured by a transformer

Core size: r_i = 30mm, r_o = 60 mm, length l = 40mm

The maximum droop should be **3% within 1 μs**

Permeability of the core: $\mu_r = 10^3$ due to $\mu_r \propto 1/f$ for f > 100 kHz ($\mu_0 = 4\pi\cdot 10^{-7}$ Vs/Am)

The upper cut-off frequency is **100 MHz**, the resistor is at T=300 K

Fast passive transformer with **R = 50 Ω** termination

Calculate the droop time and the lower cut-off frequency!

Calculate the inductance for the given droop time!

Calculate the number of winding and the sensitivity!

Calculate the threshold concerning the beam current for a signal-to-noise ratio of ***SNR*** = 1!

Result: $U(t) = U_0 \cdot e^{-t/\tau_{droop}}$ for t = 1 μs $\Rightarrow \frac{U(t)}{U_0} = e^{-1\,\mu s/\tau_{droop}} = 0.97 \Rightarrow \tau_{droop} = 33\ \mu s$

cut-off frequency: $f_{low} = \frac{1}{2\pi\tau_{droop}} = 4.9$ kHz

Inductance via $\tau_{droop} = \frac{L}{R+R_L} \Longrightarrow L = 1.97\ Hy\ \&\ N = \sqrt{\frac{2\pi L}{\mu\mu_0 l \ln r_i/r_o}} = 19 \approx 20$

Sensitivity $S = \frac{R}{N} \approx 2.5$ V/A

Noise: $U_{noise} = \sqrt{4k_B T(R+R_L)\Delta f} = 10\ \mu V \Rightarrow$ min. $I_{beam} = \frac{1}{S}\cdot U_{noise} = 4\ \mu A$

Criteria:

1. The output voltage is $U \propto 1/N \Rightarrow$ low number of windings for large signal.
2. For a low droop, a large inductance L is required due to $\tau_{droop} = L/R$:

   $L \propto N^2$ and $L \propto \mu_r$ ($\mu_r \approx 10^5$ for amorphous alloy).
3. For a large bandwidth the integrating capacitance C_s should be low $\tau_{rise} = \sqrt{L_s C_s}$.

Depending on applications the behavior is influenced by external elements:

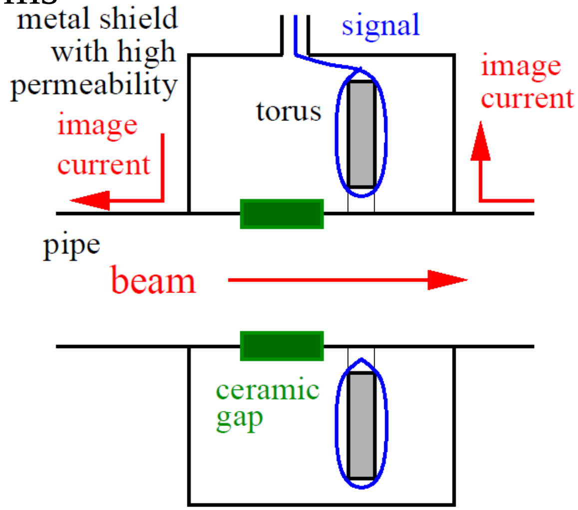
- **Passive transformer:** $R = 50\ \Omega$, $\tau_{rise} \approx 1$ ns for short pulses.

  Application: Transfer between synchrotrons : 100 ns < t_{pulse} < 10 µs
- **Active transformer:** Current sink by I/U-converter, $\tau_{droop} \approx 1$ s for long pulses.

  Application: macro-pulses at LINACs : 100 µs < t_{pulse} < 10 ms

General:

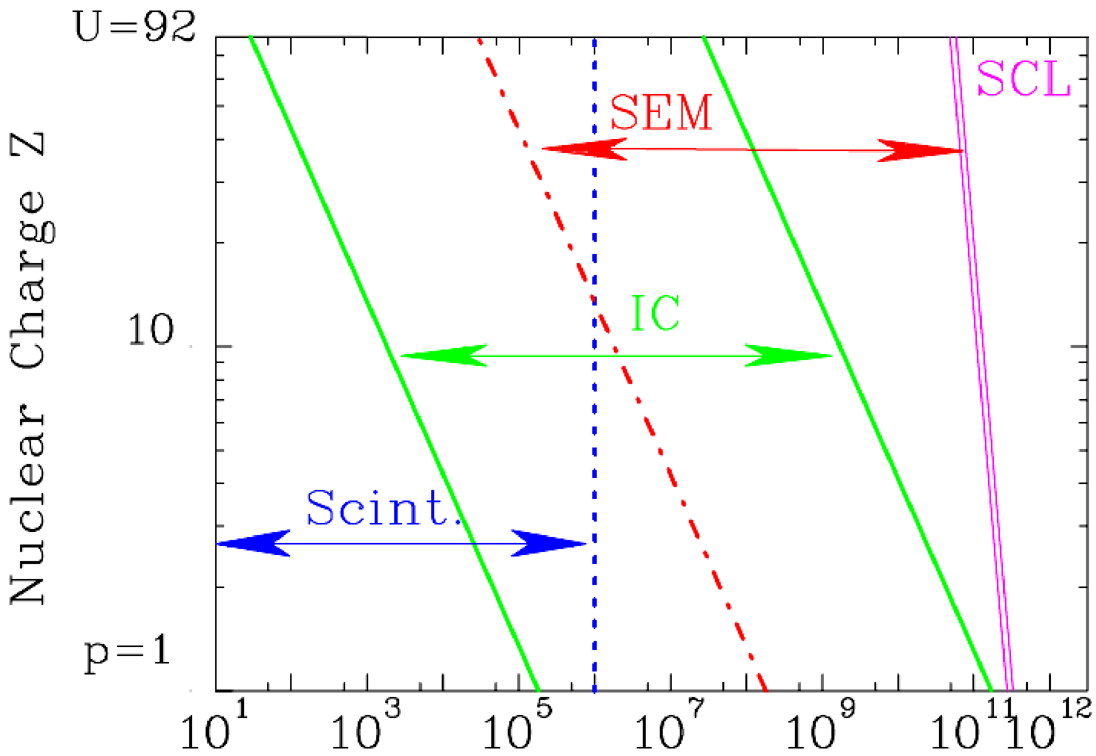
- The beam pipe has to be intersected to prevent the flow of the image current through the torus.
- The torus is made of 25 µm isolated flat ribbon spiraled to get a torus of ≈15 mm thickness, to have large electrical resistivity.
- Additional winding for calibration with current source.



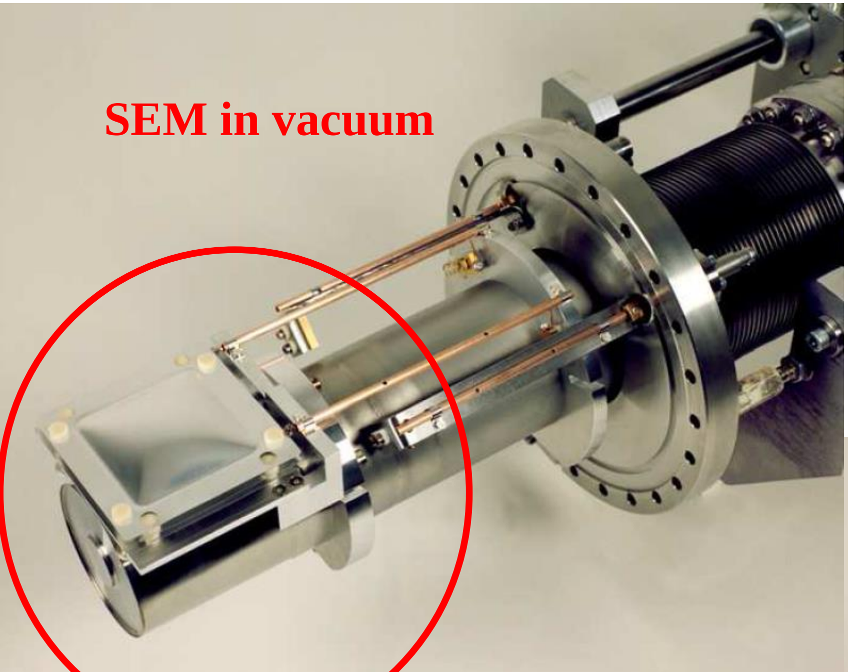
Slow extraction from synchrotron: lower current compared to LINAC, but higher energies and larger range $R \gg 1$ cm.

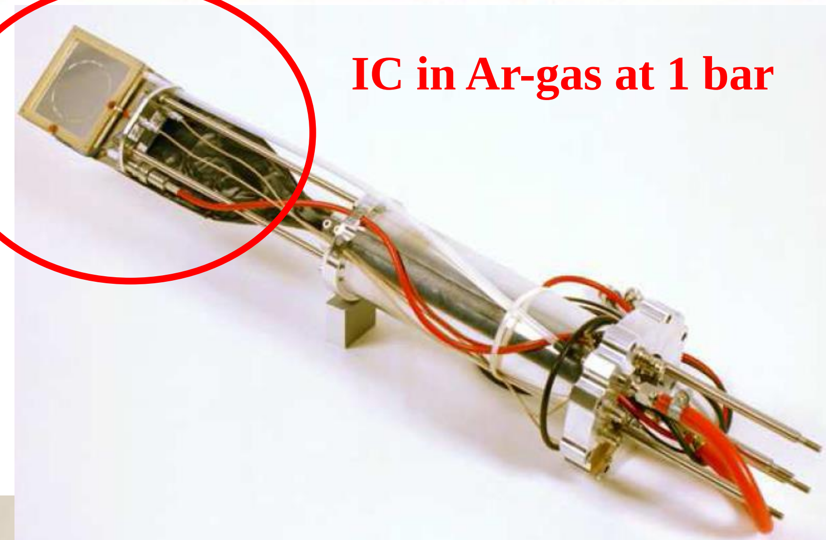
- **Particle counting:**
  max: $r \simeq 10^6$ 1/s
- **Energy loss in gas (IC):**
  min: $I_{sec} \approx 1$ pA
  max: $I_{sec} \approx 1$ µA
- **Sec. e– emission:**
  min: $I_{sec} \approx 1$ pA
- **Max. synch. filling:**
  Space Charge Limit (SCL).

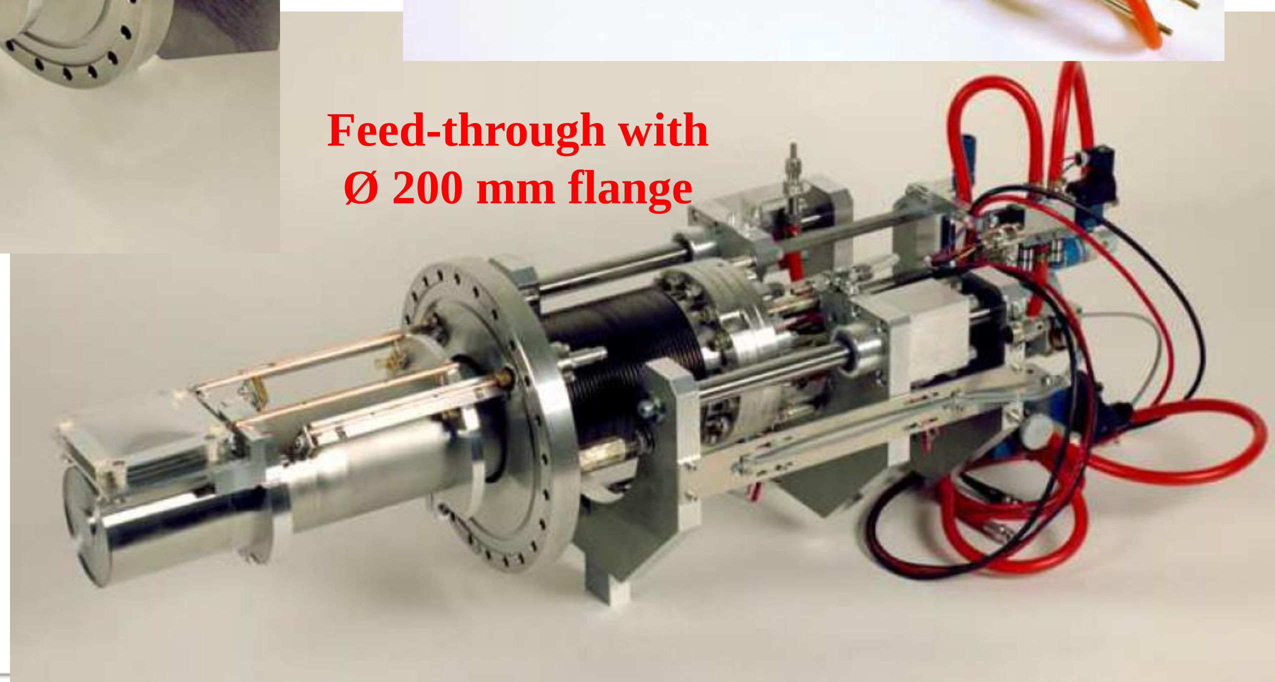
Particle detector technologies for ions of 1 GeV/u, A = 1 cm²:



Example: GSI Installation for SEM and iC



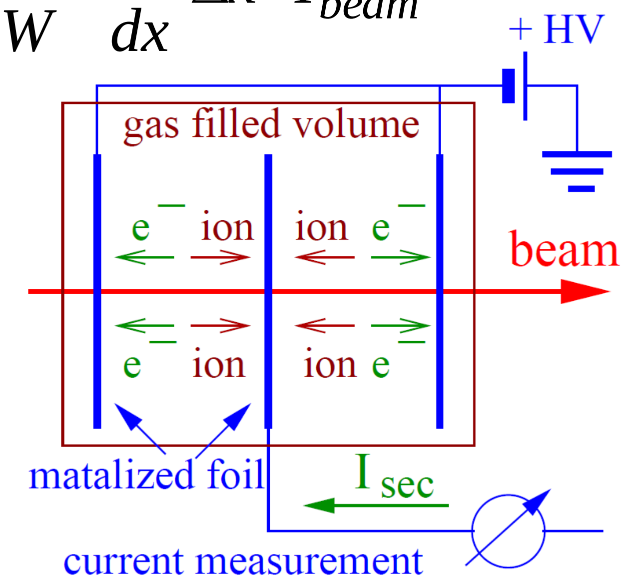




Ionization Chamber (IC): Electron Ion Pairs

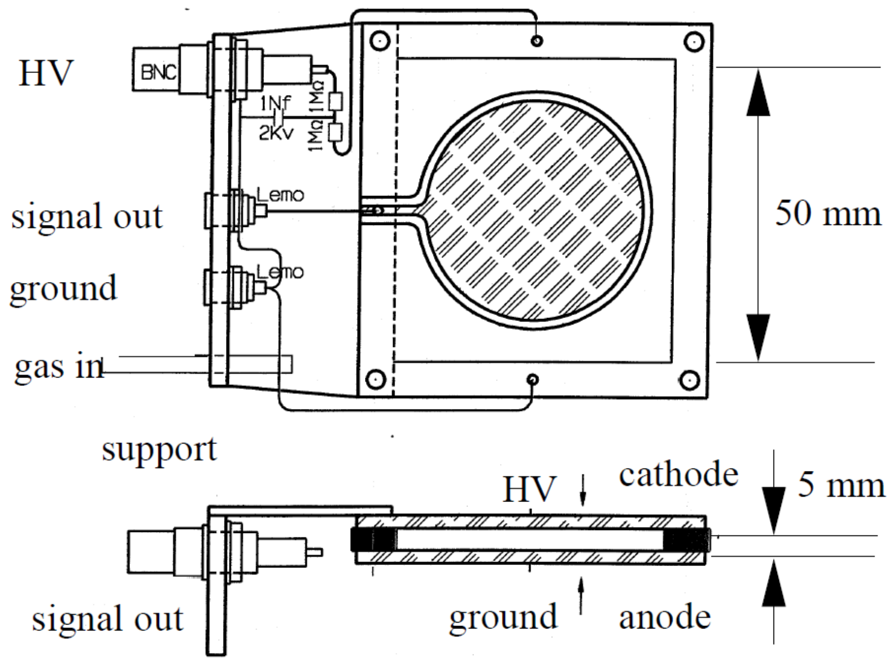
Energy loss of charged particles in gases → electron-ion pairs → low current meas.

$$I_{sec} = \frac{1}{W} \cdot \frac{dE}{dx} \Delta x \cdot I_{beam}$$



Example: GSI type

active surface	64×64 mm^2
active length Δx	5 mm
electrode material	1.5 μm Mylar
coating	100 μg/cm^2 silver
gas (flowing)	80 % Ar + 20 %CO_2
pressure	1 bar
voltage	500 ... 2000 V



W is average energy for one e$^-$-ion pair:

Gas	ioni. pot. [eV]	W-value [eV]
He	24.5	42.7
O_2	12.5	32.2
Ar	15.7	26.3
CH_4	14.5	29.1
CO_2	13.7	33.0

Exercise #3: Slow Extraction Current Measurement 1/2

Assume a beam of $\boldsymbol{N_i} = 1.25 \cdot 10^{12}$ protons extracted from a synchrotron within $\boldsymbol{t} = 1$ s

The current should be measured by an ionization chamber of 0.5cm length filled with Ar

The average energy to create a ion-electron pair is $\boldsymbol{W} = 26.3$ eV

The energy loss is $\boldsymbol{dE/dx} = 2.58$ keV/cm

Calculate the ion-electron pairs per proton and the secondary current within the IC!

The energy loss dE/dx of a proton is $\Delta E_p = \frac{dE}{dx} \cdot \Delta x = 1290\ \text{eV}$

With the average energy for one ion electron pair W, the number of pair is $N_p = \frac{\Delta E_p}{W} = 49$

For the extraction ions N_i per second the amount is $N_{tot} = N_i \cdot N_p / t = 6.1 \cdot 10^{13} \frac{1}{s}$

or a current of $I_e = e \cdot N_{tot} = 9.8$ µA .

The energy loss (described by Bethe-Bloch equation) scales with $\boldsymbol{dE/dx \propto Z_p^2}$

Assume a beam of $\boldsymbol{N_i} = 1.8 \cdot 10^{10}$ Uranium with $\boldsymbol{Z_p}$=92 extracted within 1 s

Calculate the ion-electron pair per proton and the secondary current within the IC!

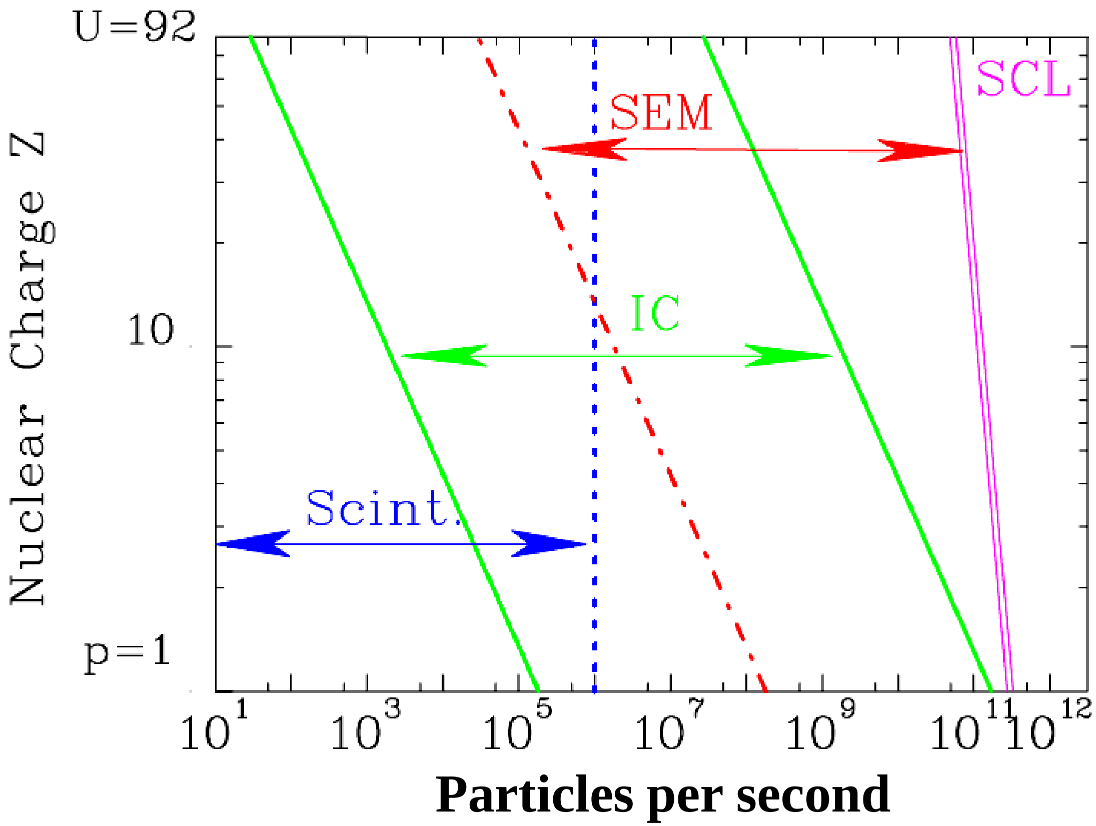
The energy loss per Uranium is $\Delta E_U = (92)^2 \cdot \Delta E_p = 8464 \cdot \Delta E_p = 10.9\ \text{MeV}$

The same calculation as above leads to $I_e = 1.2\ \text{mA}$ i.e. saturation of the IC and a SEM is better!

Slow extraction from synchrotron: lower current compared to LINAC, but higher energies and larger range ***R*** >> 1 cm.

- **Particle counting:**
  max: $r \simeq 10^6$ 1/s
- **Energy loss in gas (IC):**
  min: $I_{sec} \approx 1$ pA
  max: $I_{sec} \approx 1$ µA
- **Sec. e– emission:**
  min: $I_{sec} \approx 1$ pA
- **Max. synch. filling:**
  Space Charge Limit (SCL).

Particle detector technologies for ions of 1 GeV/u, A = 1 cm²:



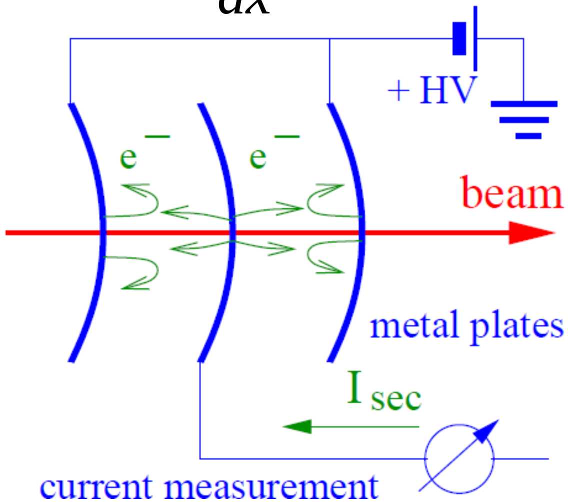
Secondary Electron Monitor (SEM): Electrons from Surface

For higher intensities SEMs are used.

Due to the energy loss, secondary e⁻ are emitted from a metal surface.

The amount of secondary e⁻ is proportional to the energy loss

$$I_{sec} = Y \cdot \frac{dE}{dx} \cdot I_{beam}$$



Example: GSI SEM type

material	pure Al (≃99.5%)
# of electrodes	3
active surface	80 × 80 mm^2
distance	5 mm
voltage	100 V

Advantage for Al: good mechanical properties.

Disadvantage: Surface effect!

e.g. decrease of yield *Y* due to radiation

⇒ Ti foils for a permanent insertion.

It is a ***surface*** effect:

→ Sensitive to cleaning procedure

→ Possible surface modification by radiation

Sometimes they are installed permanently in front of an experiment.

Exercise #3: Slow Extraction Current Measurement 2/2

Assume a beam of $N_i = 1.25\cdot10^{12}$ protons extracted from a synchrotron within 1 s

The current should be measured by an SEM

For Aluminium plates the secondary electron yield is $Y = 27.4\ e^-/(\text{MeV/mg/cm}^2)$

The energy loss (frequently used units!) is $dE/\rho dx = 1.77\ \text{keV/(mg/cm}^2)$

Calculate the electrons per proton and the secondary current emitted by the SEM!

The secondary electrons for one proton is a proton is $I_p = Y \cdot \frac{dE}{\rho dx} = 0..048\ e^-/\text{ion}$

this corresponds for an extraction N_i proton within t = 1s to a current of $I_e = N_i \cdot I_p = 9.7$ nA

The energy loss (described by Bethe-Bloch equation) scales with $dE/dx \propto Z_p^2$

Assume a beam of $N_i = 1.8\cdot10^{10}$ Uranium with Z_p=92 extracted within 1 s

Calculate the electron per Uranium and the secondary current by the SEM!

The scaling of the energy loss is $\Delta E_U = (92)^2 \cdot \Delta E_p$

with the number of Uranium per t = 1 s it is $I_e = N_i \cdot I_p = 1.2\ \mu\text{A}$

Summary for Beam Profile

Different techniques are suited for different beam parameters:

e⁻-beam: typically Ø 0.3 to 3 mm, **protons:** typically Ø 3 to 30 mm

Intercepting ↔ non-intercepting methods

Direct observation of electrodynamics processes:

- Synchrotron radiation monitor: non-destructive, only for e⁻-beams, complex
- OTR screen: nearly non-destructive, large relativistic γ needed, e⁻-beams mainly

Detection of secondary photons, electrons or ions:

- Scintillation screen: destructive, large signal, simple, all beams
- Ionization profile monitor: non-destructive, expensive, limited resolution, for protons
- Residual fluorescence monitor: non-destructive, limited signal strength, for protons

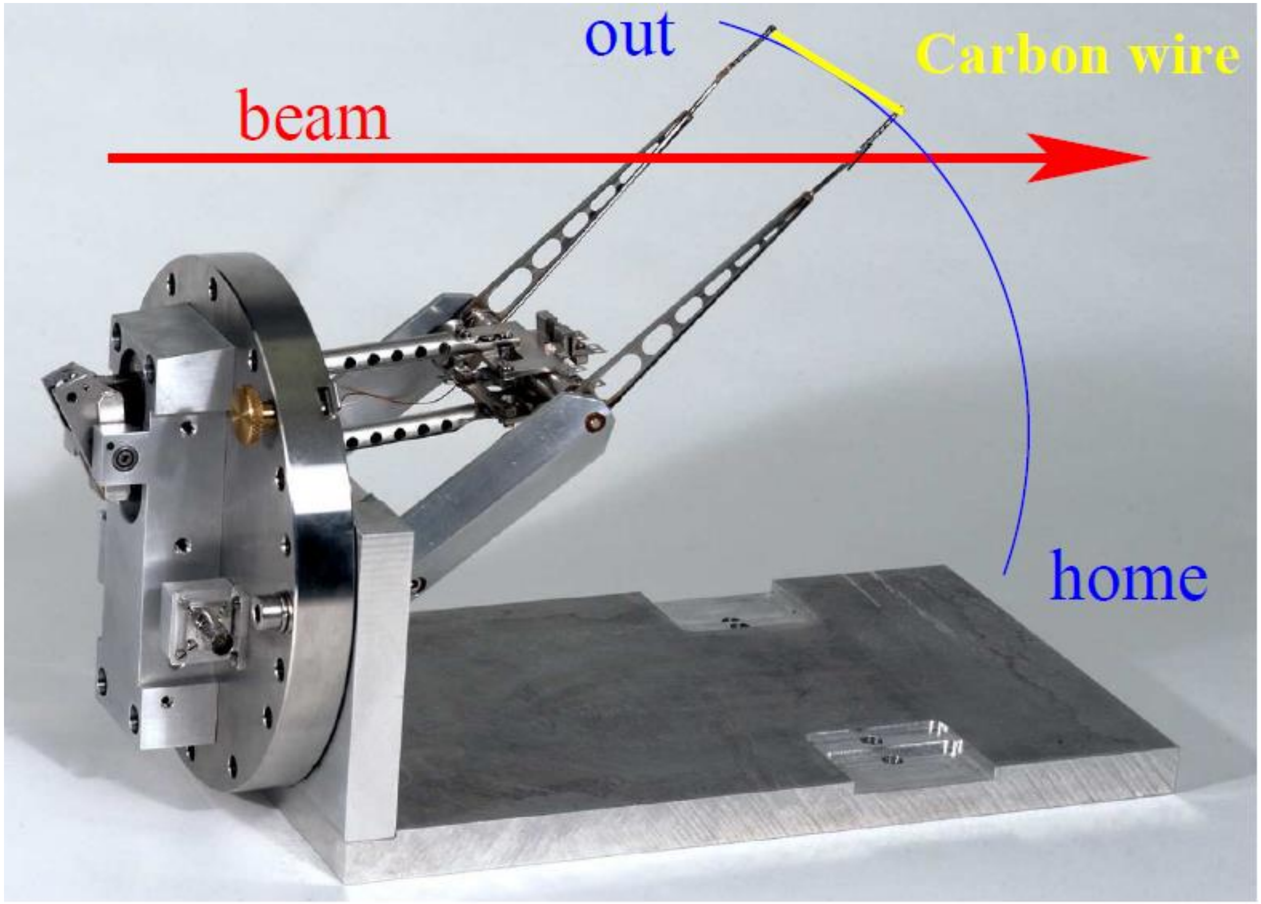
Wire based electronic methods:

- SEM-grid: partly destructive, large signal and dynamic range, limited resolution
- Wire scanner: partly destructive, large signal and dynamics, high resolution, slow scan.
- MWPC-grid: internal amplification, for low current proton-beam.

Exercise #6: Flying Wire for transverse Profile in a Synchrotron

Instead of several wires, ***one*** wire is scanned though the beam.

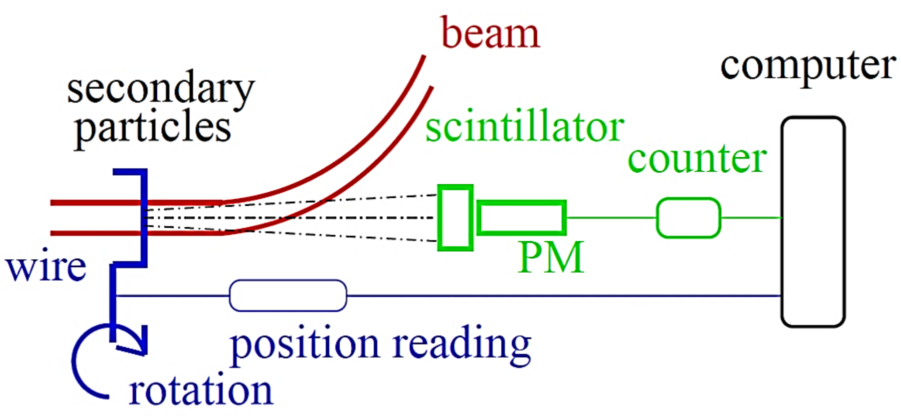
Fast pendulum scanner for synchrotrons; sometimes it is called ***'flying wire'***:



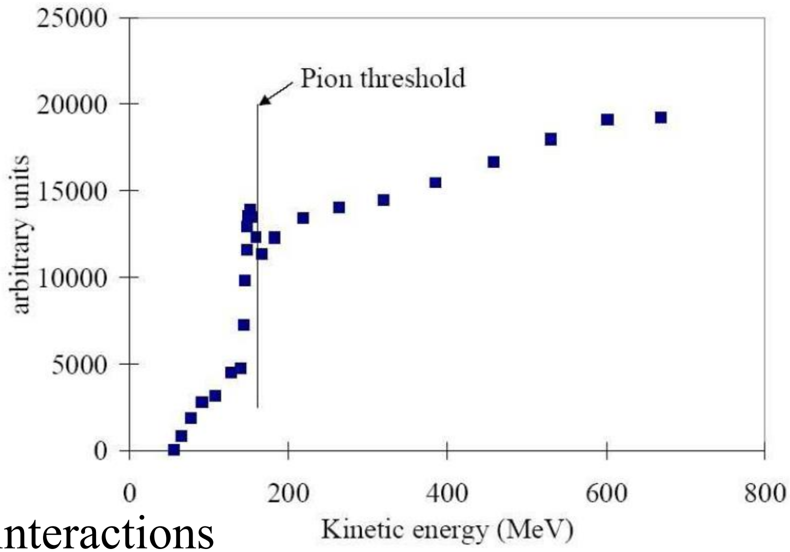
Material: carbon or SiC → low Z-material for low energy loss and high temperature.

Thickness: down to 10 μm → high resolution.

Detection: Either the secondary current (like SEM-grid) or
high energy secondary particles (like beam loss monitor)
flying wire: only sec. particle detection due to induced current by movement.



Proton impact on scanner at CERN-PS Booster:



Secondary particles:

Proton beam → hadrons (π, n, p...) from nuclear interactions

Electron beam → Bremsstrahlung photons.

Assume a beam of 10^{12} protons at 1GeV stored in a synchrotron.

The beam size 10 x 10 mm^2 with ρ_{beam}=const (for simplification), the revolution time is 1.0 µs.

The wire is made of Carbon with 50 x 50 µm^2 and scanned with v =10 m/s.

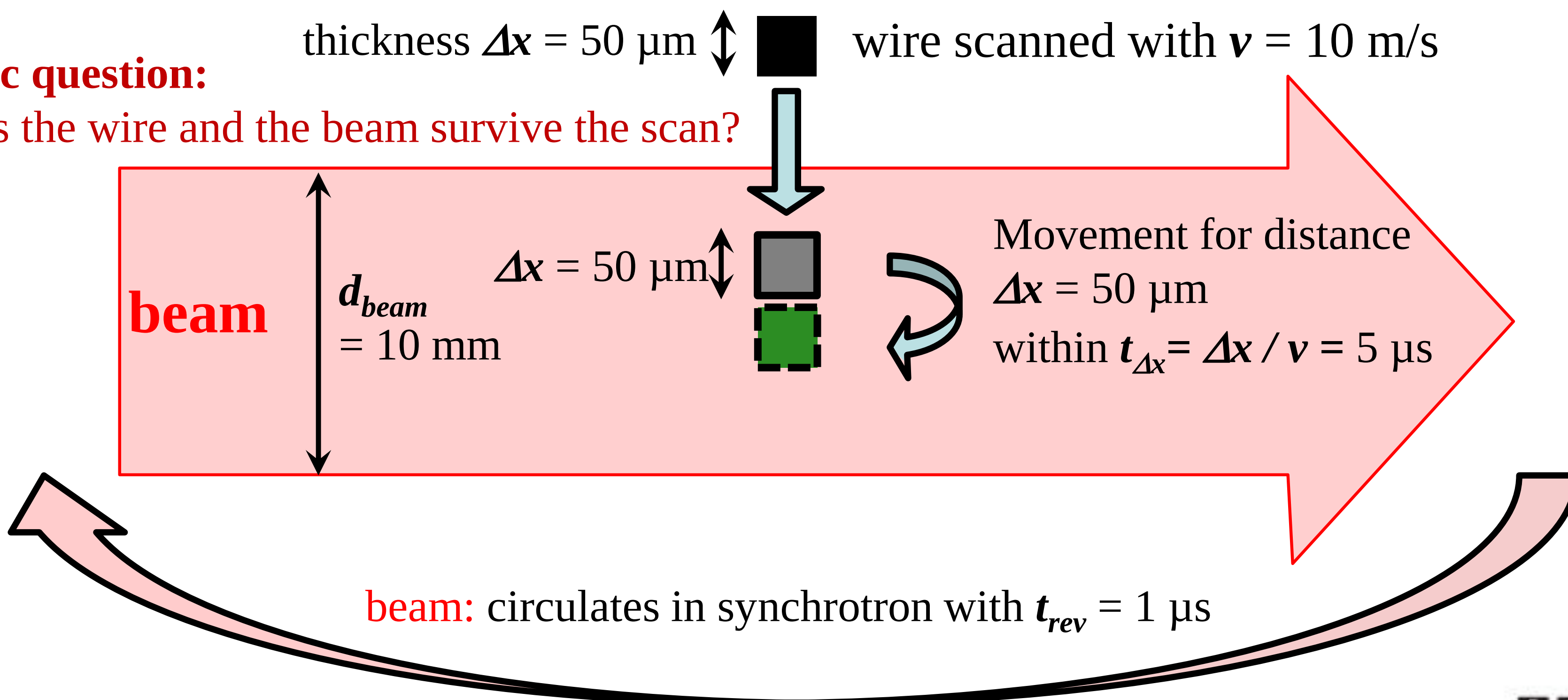
[Carbon is light material of ρ=2.2 g/cm^3 density therefore low stopping power.]

The energy loss is dE/dx= 4.29 MeV/cm.

Assumptions: rectangular beam and wire, no betatron oscillations.

Basic question:

Does the wire and the beam survive the scan?



Exercise #6: Transverse profile by flying wire scanner 1/3

Assume a beam of 10^{12} protons at 1GeV stored in a synchrotron.

The beam size 10x10 mm with $\boldsymbol{\rho_{beam}}$=const (for simplification), the revolution time is 1.0 μs.

The wire is made of Carbon with 50x50 μm and scanned with **v**=10 m/s.

[Carbon is light material of $\boldsymbol{\rho}$=2.2 g/cm^3 density therefore low stopping power.]

The energy loss is ***dE/ dx*** = 4.29 MeV/cm.

Calculate the relative energy loss per passage through the wire!

Calculate the average energy loss during the scan! How many passages an ion does in average?

Is this device nearly 'non-destructive' ⇔ Are the particles lost?

Result: Energy loss for one passage: $\boldsymbol{\Delta}E_{pass} = \frac{dE}{dx} \cdot \boldsymbol{\Delta}x = 0.021\ \text{MeV}$

time to move by one wire thickness of $\boldsymbol{\Delta x}$ = 50 μm: $t_{\Delta x} = \frac{\Delta x}{v} = 5.0\ \mu s$

average numbers of passages: $N_{pass} = \frac{t_{\Delta x}}{t_{rev}} = 5$, total time for the scan: $t_{tot} = \frac{d_{beam}}{v} = 1\ \text{ms}$

Proton's energy loss: $\boldsymbol{\Delta}E = N_{pass}\boldsymbol{\Delta}E_{pass} = 0.1\ \text{MeV}$

$\Rightarrow \frac{\Delta E}{E_{kin}} = 10^{-4}$ i.e. lower than typical longitudinal acceptance i.e. beam survives

Exercise #6: Transverse profile by flying wire scanner 2/3

Assume a beam of 10^{12} protons at 1GeV stored in a synchrotron.

The beam size 10x10 mm with ρ_{beam}=const (for simplification), the revolution time is 1.0 µs.

The wire is made of Carbon with 50x50 µm and scanned with v=10 m/s.

[Carbon is light material of ρ=2.2 g/cm^3 density therefore low stopping power.]

The energy loss is dE/dx= 4.29 MeV/cm.

Role of thumb: The maximum power rate to prevent for destruction is about 1 W/mm.

Is the wire destroyed?

Result: Energy loss for one passage: $\Delta E_{pass} = 0.021\ \text{MeV}$

average numbers of passages: $N_{pass} = \frac{t_{\Delta x}}{t_{rev}} = 5$

Total energy $W = e \cdot \Delta E_{pass} \cdot N_{pass} \cdot N_{stored} = 0.017\ \text{J}$, total power $P = \frac{W}{t_{tot}} = 17\ \text{W}$

power per mm $P_{mm} = \frac{P}{10\ mm} = 1.7\ \frac{\text{W}}{\text{mm}}$ i.e. on the border

$\Rightarrow$ more realistic calculations required

Exercise #6: Transverse profile by flying wire scanner 3/3

Assume a beam of **10^9 Uranium** ions at 1GeV/u stored in a synchrotron.

The beam size 10x10 mm with ρ_{beam}=const (for simplification), the revolution time is 1.0 μs.

The wire is made of Carbon with 50x50 μm and scanned with ***v***=10 m/s.

The energy loss is ***dE/ dx*= 3.6x10^4 MeV/cm** (instead of 4.3 MeV/cm for protons).

Why is the energy loss so much larger?

Are the particles loss? Is the wire destroyed? Repeat the same calculation for this case!

Result: Energy loss for one passage:
i.e. for Uranium after one passage
out for longitudinal acceptance
$\Rightarrow$ beam is lost during scan!

	p	U
ΔE_{pass} [MeV]	0.021	178
$\Delta E_{pass}/E_{kin}$	$2.1 \cdot 10^{-5}$	$7.0 \cdot 10^{-4}$

Result: Energy deposition in the wire
is significantly higher than destruction level
$\Rightarrow$ the wire is destroyed!

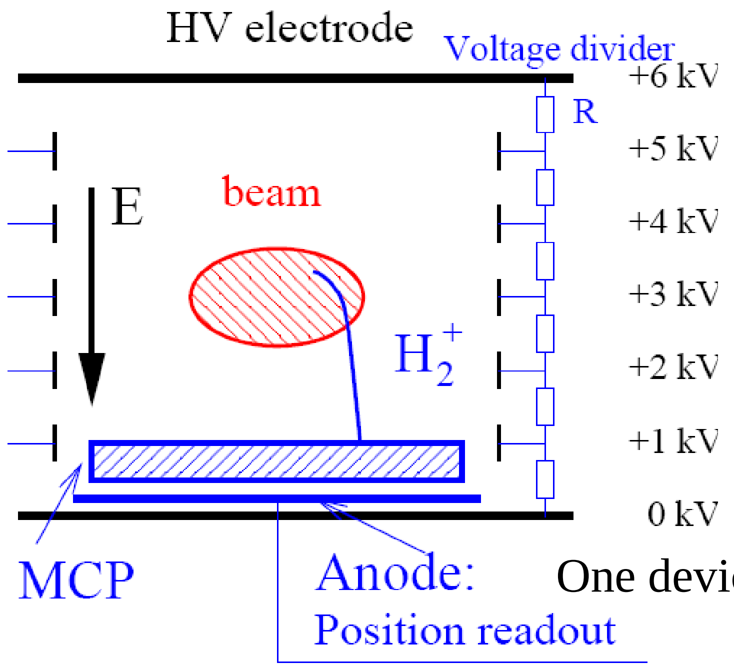
	p	U
W [J]	0.017	0.142
P [W]	17	142
P_{mm} [W/mm]	1.7	14.2

What happens for not fully stripped ions (i.e. ions with some bound electrons)?

What is an appropriate method of profile determination for the Uranium case?

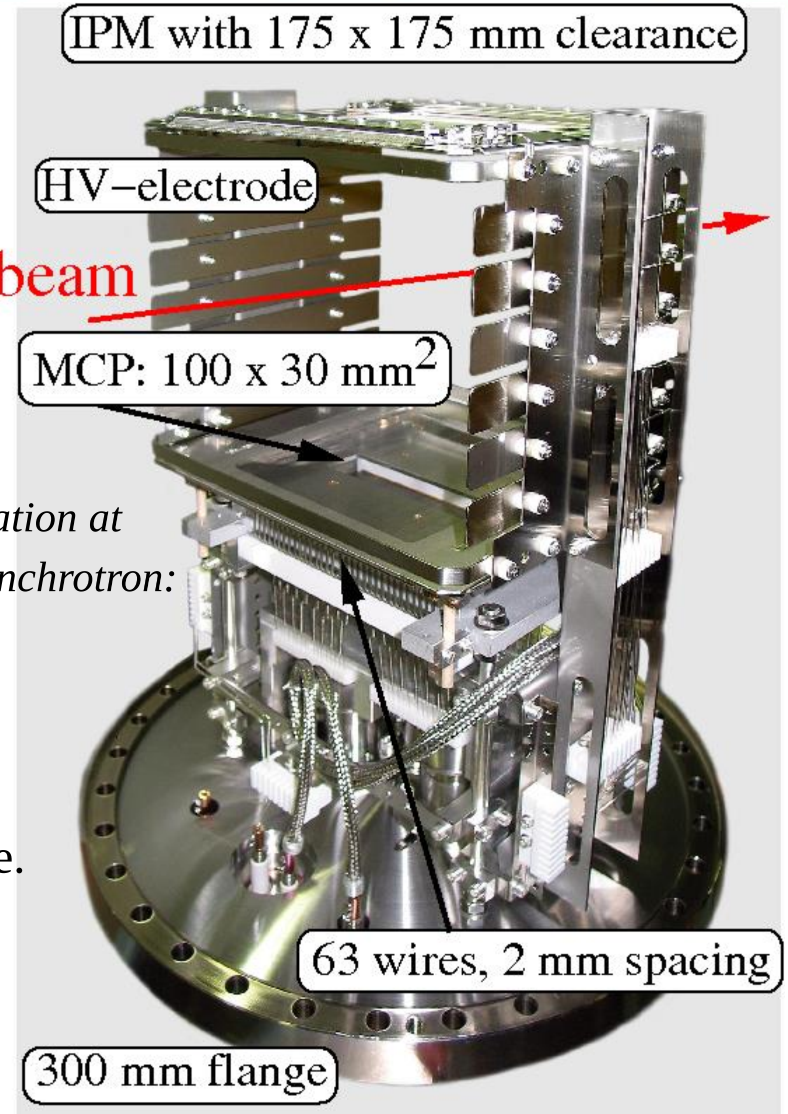
Non-destructive device for proton synchrotron:

- beam ionizes the residual gas by electronic stopping
- gas ions or e⁻ accelerated by E -field ≈1 kV/cm
- spatial resolved single particle detection



One device per plane.

Realization at GSI synchrotron:



Typical vacuum pressure:

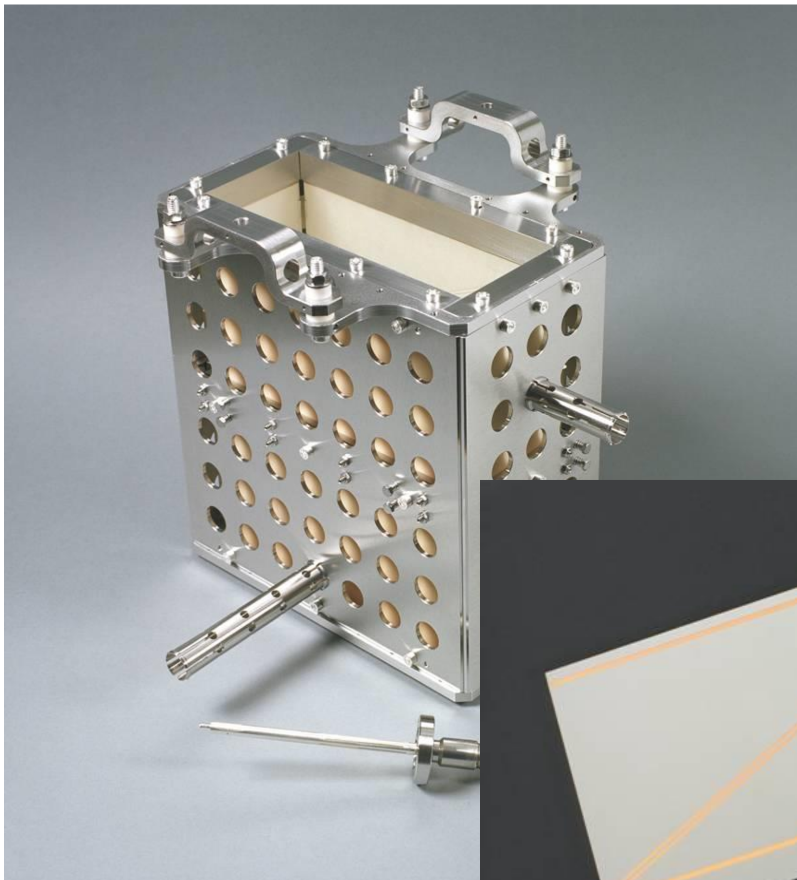
Transfer line: $10^{-8} - 10^{-6}$ mbar (N_2)

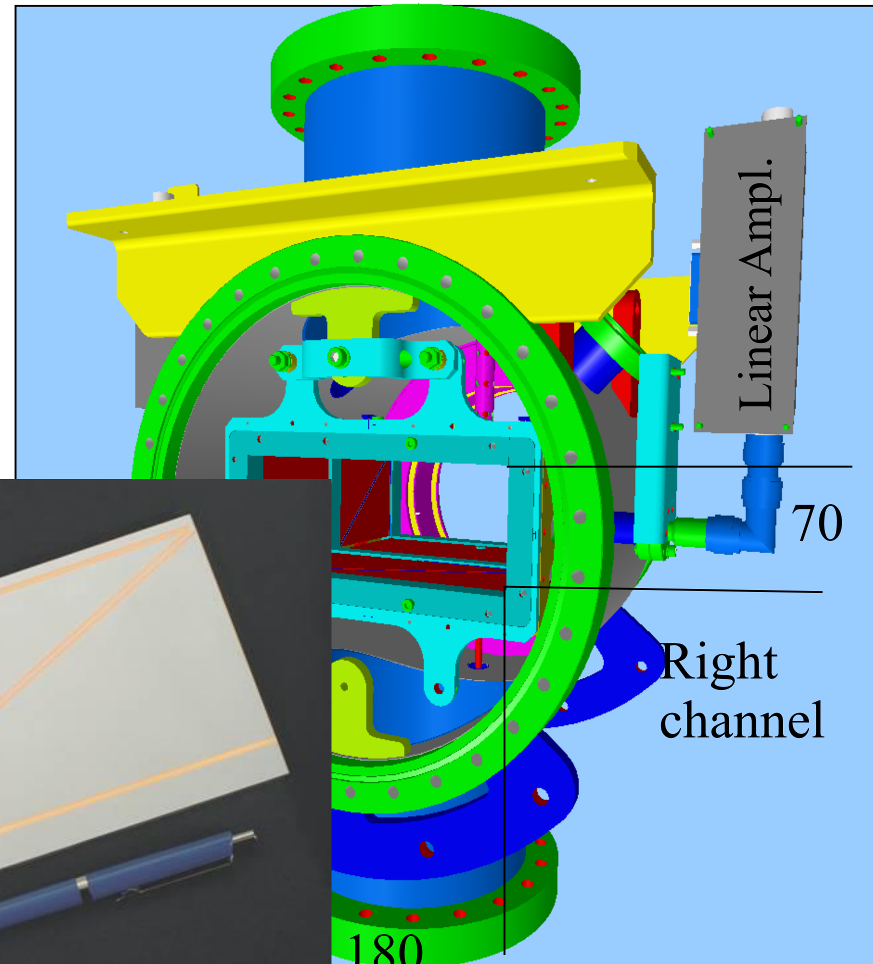
Synchrotron: $10^{-11} - 10^{-9}$ mbar (H_2).

Technical Realization of a Shoe-Box BPM

Technical realization at HIT synchrotron of 46 m length for 7 MeV/u$\rightarrow$ 440 MeV/u

BPM clearance: 180x70 mm^2, standard beam pipe diameter: 200 mm.



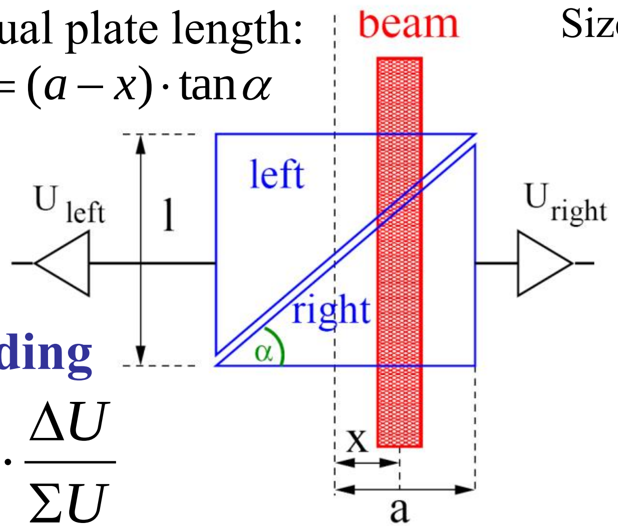


Frequency range: 1 MHz < f_{rf} < 10 MHz $\Rightarrow$ bunch-length >> BPM length.

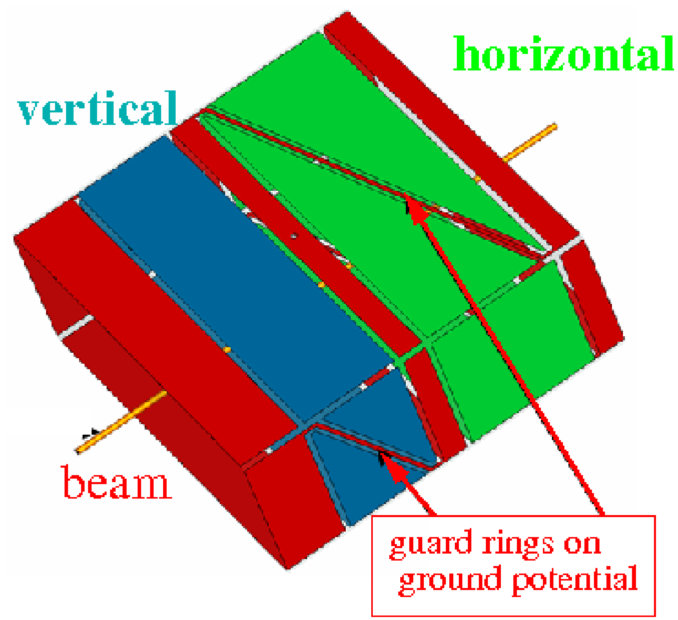
Signal is proportional to actual plate length:

$$l_{right} = (a+x)\cdot\tan\alpha, \quad l_{left} = (a-x)\cdot\tan\alpha$$

$$\Rightarrow \quad x = a\cdot\frac{l_{right} - l_{left}}{l_{right} + l_{left}}$$

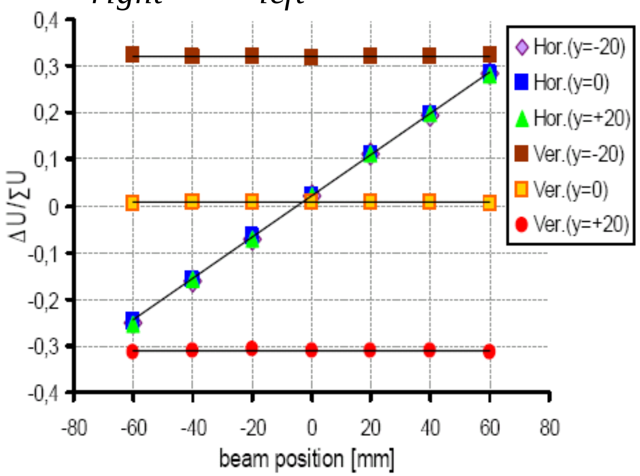


Size: 200x70 mm^2



In ideal case: linear reading

$$x = a\cdot\frac{U_{right} - U_{left}}{U_{right} + U_{left}} \equiv a\cdot\frac{\Delta U}{\Sigma U}$$



Shoe-box BPM:

Advantage: Very linear, low frequency dependence
i.e. position sensitivity ***S*** is constant

Disadvantage: Large size, complex mechanics
high capacitance

Example of Transfer Impedance for Proton Synchrotron

The high-pass characteristic for typical synchrotron BPM:

$$U_{im}(\omega)=Z_t(\omega)\cdot I_{beam}(\omega)$$

$$|Z_t| = \frac{A}{2\pi a}\cdot\frac{1}{\beta c}\cdot\frac{1}{C}\cdot\frac{\omega/\omega_{cut}}{\sqrt{1+\omega^2/\omega^2_{cut}}}$$

$$\varphi = \arctan(\omega_{cut}/\omega)$$

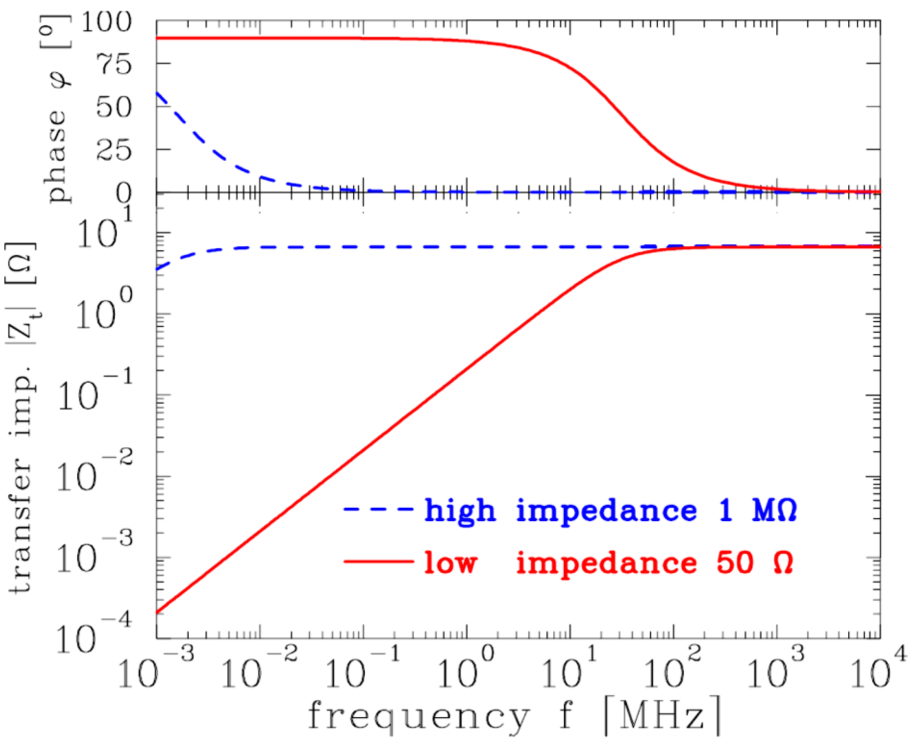
Parameter for shoe-box BPM:

C=100pF, l=10cm, β=50%

$f_{cut}=\omega/2\pi=(2\pi RC)^{-1}$

for R=50 Ω $\Rightarrow f_{cut}$= 32 MHz

for R=1 MΩ $\Rightarrow f_{cut}$= 1.6 kHz



Large signal strength → **high impedance**

Smooth signal transmission → **50 Ω**

Exercise #8: Signal Estimation for a broad-band BPM 1/2

Assume a shoe-box BPM of length $\boldsymbol{l}$=20 cm, radius $\boldsymbol{a}$=10 cm and capacitance $\boldsymbol{C}$=100 pF
The beam velocity is $\boldsymbol{\beta}$= 0.5 and the bunch length is $\boldsymbol{\sigma_t}$= 100 ns.

Assume a position linear sensitivity of $\boldsymbol{S=1/a}$ i.e. $\boldsymbol{U_\Delta / U_\Sigma = x/a}$.

Calculate the transfer impedance for half-cylindrical plate and termination of $\boldsymbol{R}$ = 1 MΩ!

Calculate the sum voltage $\boldsymbol{U_\Sigma}$ for $\boldsymbol{I_{beam}}$ = 1 A !

Calculate the different voltage for $\boldsymbol{x}$ = 1 mm displacement!

Result: Transfer impedance for R= 1 MΩ: $Z_t = \frac{l}{\beta c C} \cdot \frac{A}{\pi a} = \frac{1}{\beta c C} = 6.7\ \Omega$ with $A = \pi a l$

Sum voltage for $\boldsymbol{I_{beam}}$ = 1 A: $\boldsymbol{U_\Sigma = 2\, Z_t \cdot I_{beam}}$ = 13.3 V

Difference voltage for $\boldsymbol{x}$ = 1 mm: $\boldsymbol{U_\Delta = \frac{x}{a} Z_t \cdot I_{beam}}$ = 67 mV

What are the corresponding values for a termination with $\boldsymbol{R}$ = 50 Ω?

The transfer impedance is not constant, what does it mean? Use $\boldsymbol{Z_t(50\Omega)=Z_t(1M\Omega)/20}$.

Result: Sum voltage for $\boldsymbol{I_{beam}}$ = 1 A: $\boldsymbol{U_\Sigma = 2\, Z_t\,(50\cdot\Omega) \cdot I_{beam}}$ = 670 mV

Difference voltage for $\boldsymbol{x}$ = 1 mm: $\boldsymbol{U_\Delta = \frac{x}{a} Z_t\,(50\cdot\Omega) \cdot I_{beam}}$ = 3.3 mV

Signal Shape for capacitive BPMs: differentiated ↔ proportional

Depending on the frequency range ***and*** termination the signal looks different:

➢ ***High frequency range $\omega >> \omega_{cut}$:***

$$Z_t \propto \frac{i\omega/\omega_{cut}}{1+i\omega/\omega_{cut}} \to 1 \Rightarrow U_{im}(t) = \frac{1}{C}\cdot\frac{1}{\beta c}\cdot\frac{A}{2\pi a}\cdot I_{beam}(t)$$

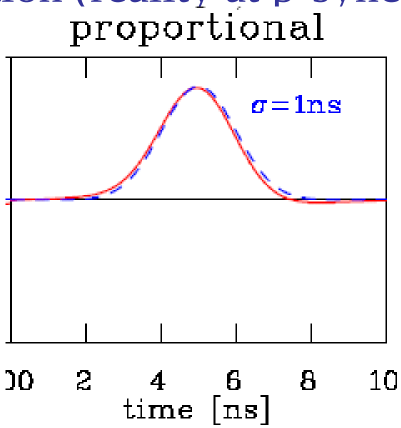
⇒ **direct image** of the bunch. Signal strength $\boldsymbol{Z_t \propto A/C}$ i.e. nearly independent on length

➢ ***Low frequency range $\omega << \omega_{cut}$:***

$$Z_t \propto \frac{i\omega/\omega_{cut}}{1+i\omega/\omega_{cut}} \to i\frac{\omega}{\omega_{cut}} \quad \Rightarrow \quad U_{im}(t) = R\cdot\frac{A}{\beta c\cdot 2\pi a}\cdot i\omega I_{beam}(t) = R\cdot\frac{A}{\beta c\cdot 2\pi a}\cdot\frac{dI_{beam}}{dt}$$

⇒ **derivative** of bunch, single strength $\boldsymbol{Z_t \propto A}$, i.e. (nearly) independent on $\boldsymbol{C}$

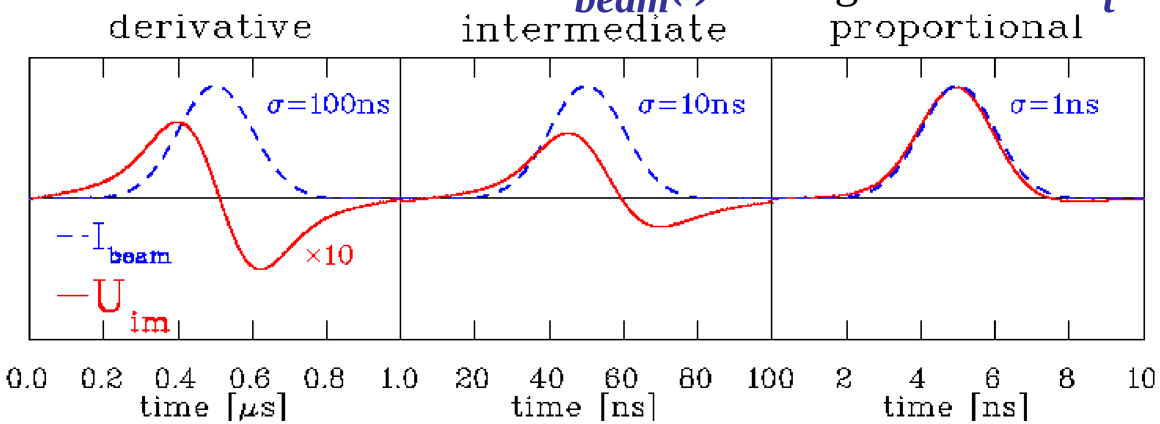
Example from synchrotron BPM with 50 Ω termination (reality at p-synchrotron : σ>>1 ns):



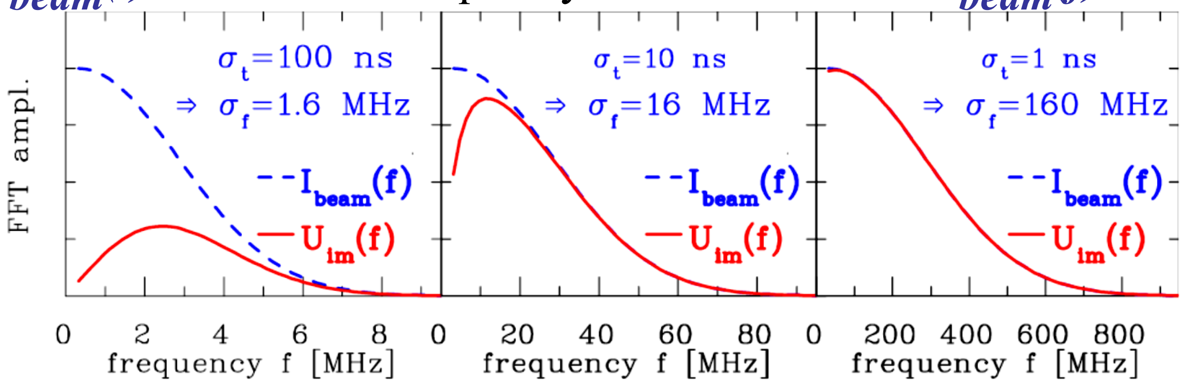
Calculation of Signal Shape (here single bunch)

The transfer impedance is used in frequency domain! The following is performed:

1. Start: Time domain Gaussian function $I_{beam}(t)$ having a width of σ_t



2. FFT of $I_{beam}(t)$ leads to the frequency domain Gaussian $I_{beam}(f)$ with $\sigma_f=(2\pi\sigma_t)^{-1}$



3. Multiplication with $Z_t(f)$ with f_{cut}=32 MHz leads to $U_{im}(f)=Z_t(f)\cdot I_{beam}(f)$

4. Inverse FFT leads to $U_{im}(t)$

Exercise #8: Signal Estimation for a broad-band BPM 2/2

Assume a shoe-box BPM of length $\boldsymbol{l}$=20 cm, radius $\boldsymbol{a}$=10 cm and capacitance $\boldsymbol{C}$=100 pF
The beam velocity is $\boldsymbol{\beta}$= 0.5 and the bunch length is $\boldsymbol{\sigma_t}$= 100 ns.

Assume a position linear sensitivity of $\boldsymbol{S=1/a}$ i.e. $\boldsymbol{U_\Delta/U_\Sigma = x/a}$.
Compare the signal strength to the thermal noise (using the theoretical minimum):

The thermal noise voltage at $\boldsymbol{T}$=300 K is given by

$\boldsymbol{U_{noise} = (4k_B \cdot T \cdot R \cdot \Delta f)^{1/2}}$ and $\boldsymbol{k_B}$=$1.4 \cdot 10^{-23}$ J/K within the bandwidth $\boldsymbol{\Delta f = 100}$ **MHz !**
Calculate $\boldsymbol{U_{eff}}$ for $\boldsymbol{R}$=1 MΩ and $\boldsymbol{R}$=50 Ω termination !
Result: Thermal noise for $\boldsymbol{R}$ = 1 MΩ: $U_{noise} = \sqrt{4k_B TR\Delta f} = 1.3\text{ mV}$

Thermal noise for $\boldsymbol{R}$ = 50 Ω: $U_{noise} = \sqrt{4k_B TR\Delta f} = 9.2\ \mu\text{V}$

A displacement of $\boldsymbol{x}$ = 1 mm should be detected.

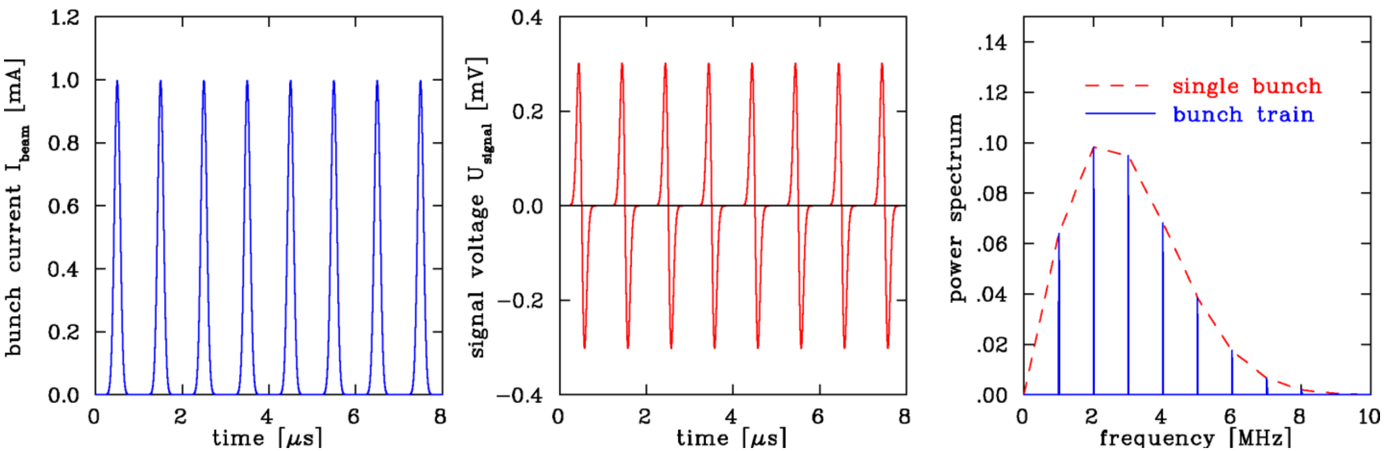
What is the minimum beam current $\boldsymbol{I_{beam}}$ to achieve a Signal-to-Noise Ratio S/N=2 !
Result: For $\boldsymbol{R}$ = 1 MΩ & $\boldsymbol{U_\Delta = 2\ U_{noise}(1M\Omega)}$: $I_{beam} = \frac{a}{x} \cdot \frac{1}{Z_t(1M\Omega)} \cdot U_{noise} = 40\text{ mA}$
For $\boldsymbol{R}$ = 50 Ω & $\boldsymbol{U_\Delta = 2\ U_{noise}(50\Omega)}$: $I_{beam} = \frac{a}{x} \cdot \frac{1}{Z_t(50\Omega)} \cdot U_{noise} = 5.5\text{ mA}$

(not realistic because BPM from a low pass filter f cut (1MΩ)=1.6 kHz: realistic $\boldsymbol{I_{beam}}$ = 5mA)
How can the position resolution be improved significantly? What is the physical reason?

Voltage Spectrum for the BPM and Train of Bunches, R=50 Ω

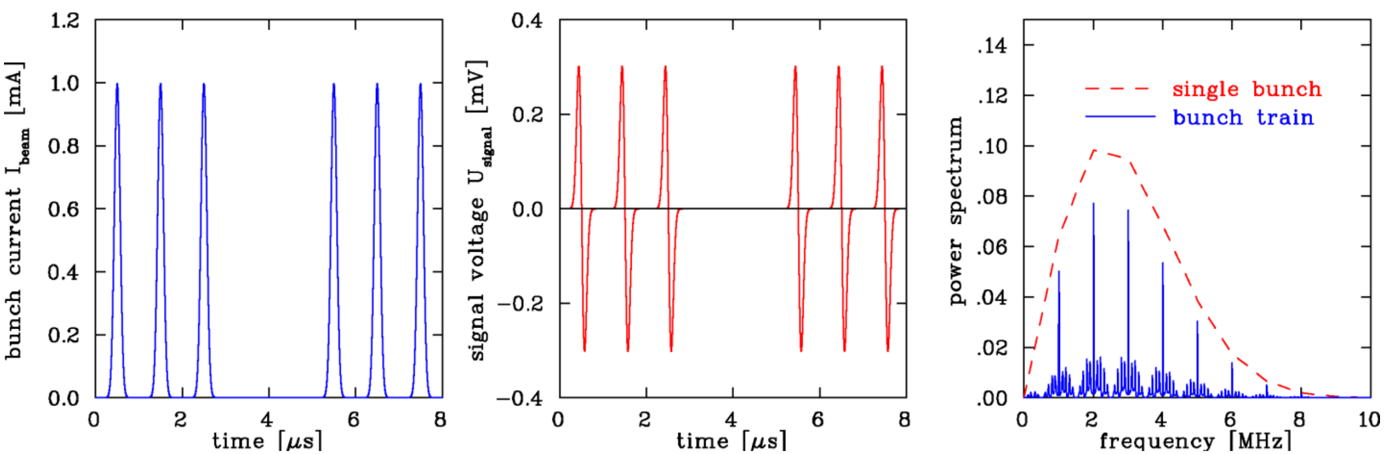
What is the voltage spectrum for the case of a **bunch train** with σ_t=100 ns and f_{acc}=1 MHz ?

How is the spectrum modified if only part of the buckets are filled?



The spectrum consists of line separated by $f_{acc.}$

The envelope is given by the voltage signal of a single bunch.



Side bands appear

$\Rightarrow$ the power at f_{acc} is reduced.

Voltage Spectrum for the BPM and Train of Bunches, R=1 MΩ

What is the spectrum and the signal shape for a termination with R=1 MΩ
Sketch and discuss the signal voltage for the case of a bunch train with σ_t=100 ns !

The cut-off frequency is

$f_{cut}=1/(2\pi RC)$=1.6 kHz

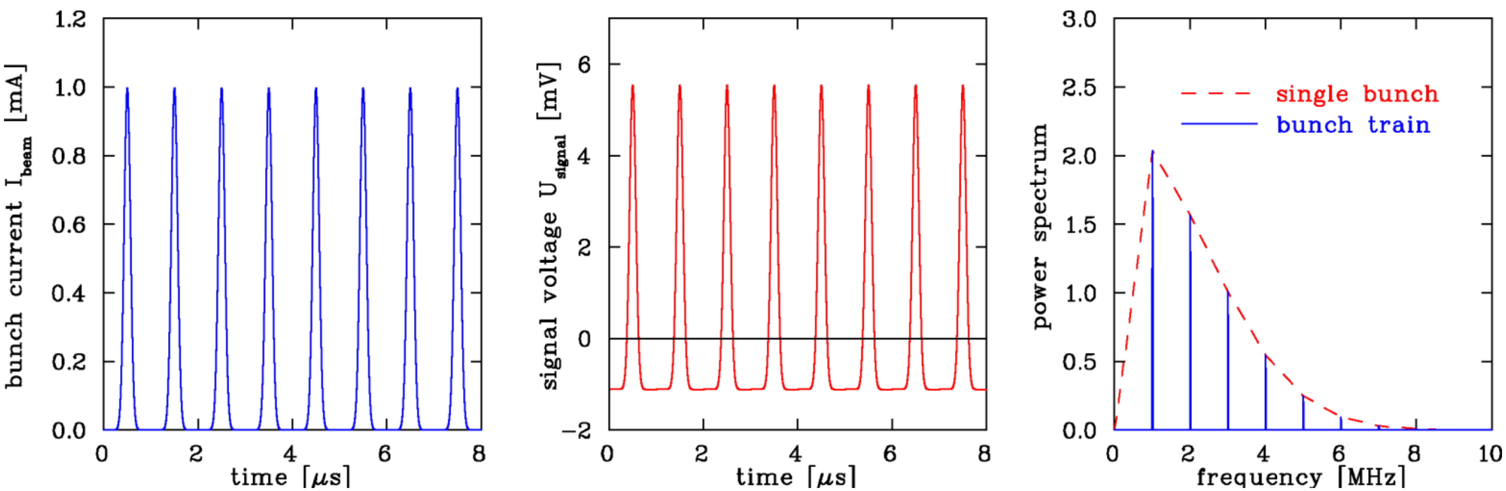
⇒ the proportional shape is recorded

Signal strength for long bunches is $U_{signal}=Z_t(f>f_{cut})\cdot I_{beam}$=7 mV

A baseline shift occur i.e. no dc-transmission

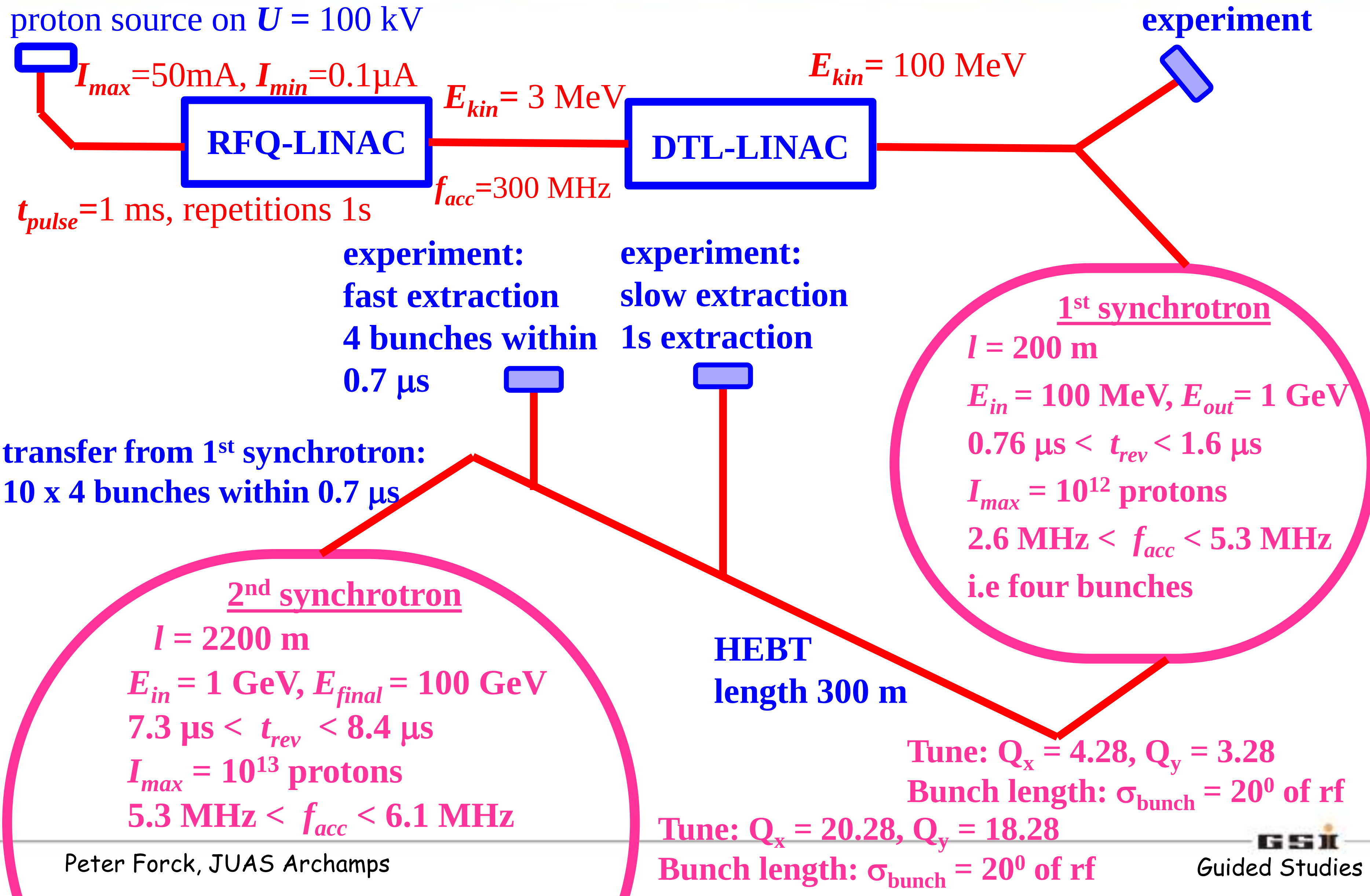
Reason for this choice: larger signal *independent* on bunch length

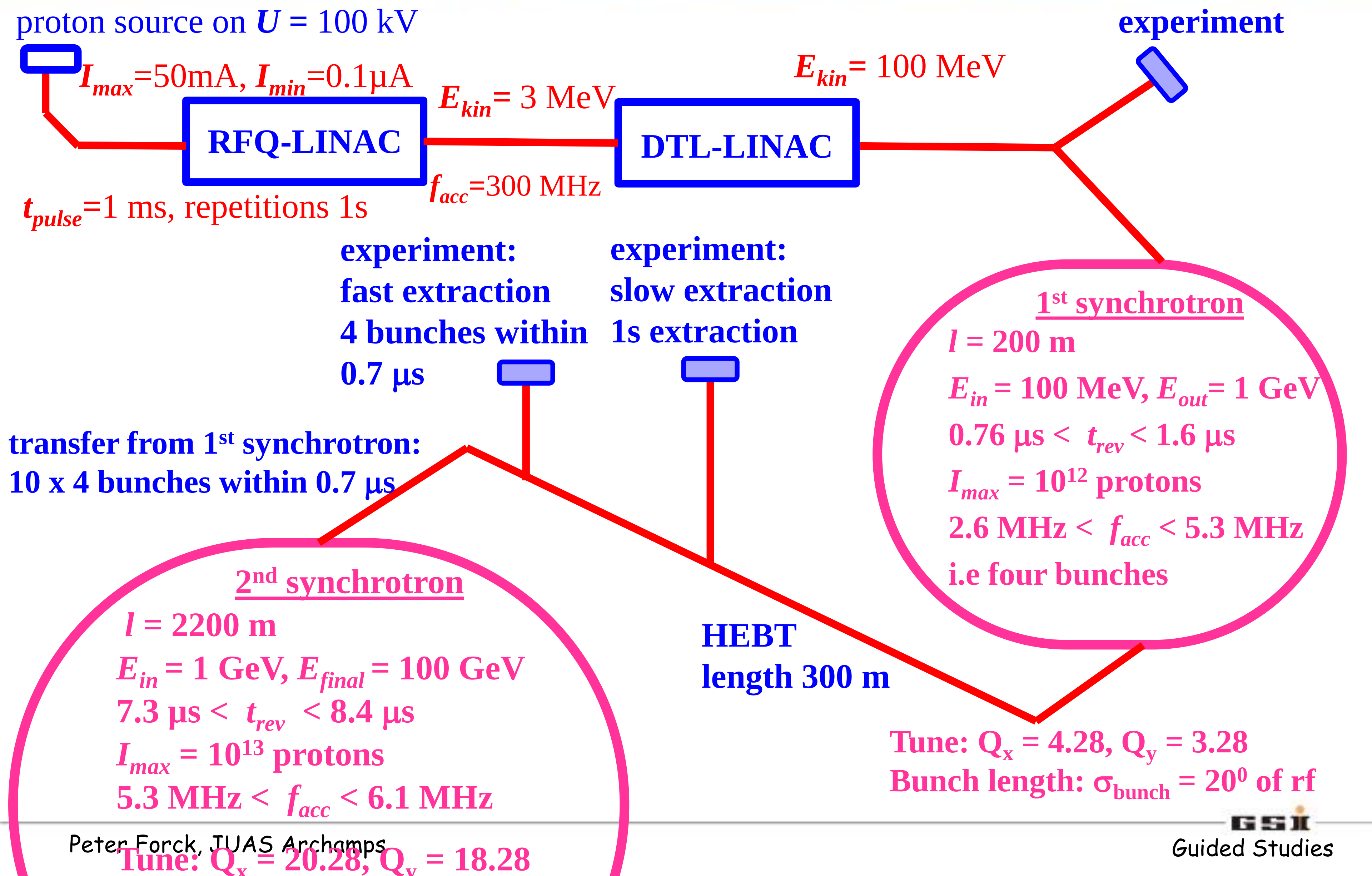
However: larger thermal noise due to $U_{eff}=(4k_B\cdot T\cdot \Delta f\cdot R)^{1/2}$



2.2.4 What is the voltage for the case of a single bunch with σ_t=1, 10 and 100 ns and a current of max. value I_{beam}=1mA? (start with σ_t=1ns, other case only estimation)
Assume a value of the transfer impedance | $Z_t(f>f_{cut})$ |=7 Ω above f_{cut}=32 MHz.

- For short bunches σ_t=1 ns $\rightarrow \sigma_f = 1/2\pi\sigma_t$ = 160 MHz
  i.e. main component above f_{cut} ⇒ proportional shape
  ⇒ $U_{signal} = Z_t(f>f_{cut}) \cdot I_{beam}$ = 7 mV for the σ_t=1 ns case
- For long bunches with σ_t=100 ns $\rightarrow \sigma_f = 1/2\pi\sigma_t$ = 1.6 MHz
  i.e. all frequencies below f_{cut} ⇒ derivative shape
  ⇒ $U_{signal} \approx$ 0.3 mV for the σ_t=100 ns case (i.e. a factor 23 lower!)
- For the σ_t=10 ns the signal must be calculated:
  ⇒ $U_{signal} \approx$ 3 mV for the σ_t=10 ns case


proton source on U = 100 kV
I_{max}=50mA, I_{min}=0.1μA
t_{pulse}=1 ms, repetitions 1s
RFQ-LINAC
E_{kin}= 3 MeV
f_{acc}=300 MHz
DTL-LINAC
E_{kin}= 100 MeV
experiment
experiment:
fast extraction
4 bunches within
0.7 μs
experiment:
slow extraction
1s extraction
1st synchrotron
l = 200 m
E_{in} = 100 MeV, E_{out}= 1 GeV
0.76 μs < t_{rev} < 1.6 μs
I_{max} = 10^{12} protons
2.6 MHz < f_{acc} < 5.3 MHz
i.e four bunches
transfer from 1st synchrotron:
10 x 4 bunches within 0.7 μs
2nd synchrotron
l = 2200 m
E_{in} = 1 GeV, E_{final} = 100 GeV
7.3 μs < t_{rev} < 8.4 μs
I_{max} = 10^{13} protons
5.3 MHz < f_{acc} < 6.1 MHz
HEBT
length 300 m
Tune: Q_x = 4.28, Q_y = 3.28
Bunch length: σ_{bunch} = 20^0 of rf
Tune: Q_x = 20.28, Q_y = 18.28
Bunch length: σ_{bunch} = 20^0 of rf


proton source on U = 100 kV
I_{max}=50mA, I_{min}=0.1µA
t_{pulse}=1 ms, repetitions 1s
RFQ-LINAC
E_{kin}= 3 MeV
f_{acc}=300 MHz
DTL-LINAC
E_{kin}= 100 MeV
experiment
experiment:
fast extraction
4 bunches within
0.7 µs
experiment:
slow extraction
1s extraction
1st synchrotron
l = 200 m
E_{in} = 100 MeV, E_{out}= 1 GeV
0.76 µs < t_{rev} < 1.6 µs
I_{max} = 10^{12} protons
2.6 MHz < f_{acc} < 5.3 MHz
i.e four bunches
transfer from 1st synchrotron:
10 x 4 bunches within 0.7 µs
2nd synchrotron
l = 2200 m
E_{in} = 1 GeV, E_{final} = 100 GeV
7.3 µs < t_{rev} < 8.4 µs
I_{max} = 10^{13} protons
5.3 MHz < f_{acc} < 6.1 MHz
Tune: Q_x = 20.28, Q_y = 18.28
HEBT
length 300 m
Tune: Q_x = 4.28, Q_y = 3.28
Bunch length: σ_{bunch} = 20^0 of rf

Exercise #6: Transverse profile by flying wire scanner

Assume a beam of 10^{12} protons at 1GeV stored in a synchrotron.
The beam size 10x10 mm with ρ_{beam}=const (for simplification), the revolution time is 1.0 μs.
The wire is made of Carbon with 50x50 μm and scanned with v=10 m/s.
[Carbon is light material of ρ=2.2 g/cm^3 density therefore low stopping power.]
The energy loss is dE/dx= 4.29 MeV/cm.

Calculate the relative energy loss per passage through the wire!

Calculate the average energy loss during the scan! How many passages an ion does in average?
Is this device nearly 'non-destructive' ⇔ Are the particles lost?

The maximum power rate to prevent for destruction is 1 W/mm.

Is the wire destroyed?

Repeat the same calculation assuming 10^9 stored Uranium ions!

The energy loss is now dE/dx=35640 MeV/cm.

Why is the energy loss so much larger?

Are the particles loss? Is the wire destroyed?

What happens for not fully stripped ions (i.e. ions with some bound electrons)?
What is an appropriate method of profile determination for the Uranium case?