JUAS 2019 – Tutorial 1

$\mu = \mu_0\, \mu_r$
$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am)
$\varepsilon = \varepsilon_0\, \varepsilon_r$
$\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/(Vm)
$c_0 = 2.998 \cdot 10^{8}$ m/s

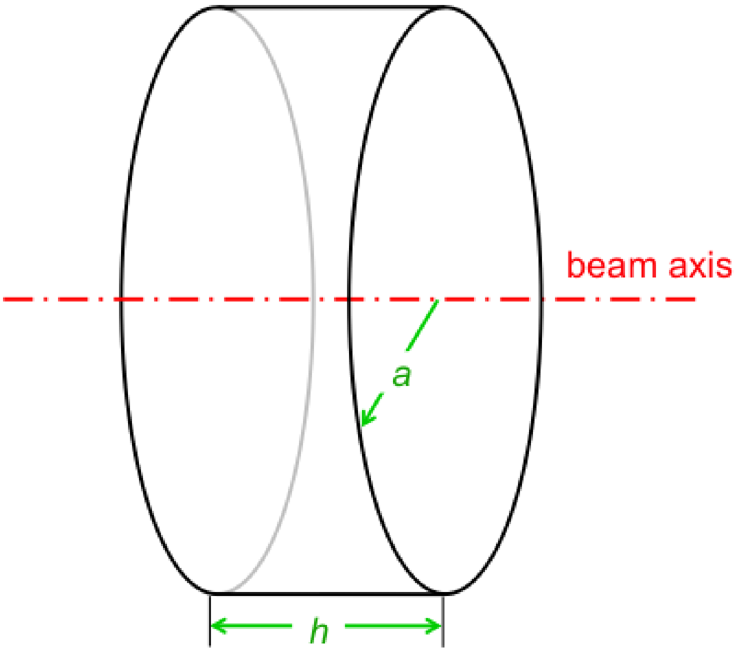
F. Caspers, M. Bozzolan, M. Wendt

1) Design of a pillbox cavity

Problem: Design a simple "Pillbox" cavity with the following parameters

Frequency:	$f = 299.98$ MHz ($\lambda = 1.00$ m)
Wall material:	Copper (equivalent skin depth $\delta = 3.8$ µm)
Axial length:	$h = 0.2$ m

For this example, we ignore beam ports, i.e. vacuum chamber stubs required for the beam passage, so that all analytical formulas describing the pillbox cavity apply.



Questions:

1. Find from the analytical formulas:
   - Cavity radius a

     $$a = \frac{c_0}{2.61\, f} = 0.383\ m$$

     see script page 72
   - Cavity quality factor Q

     $$Q = \frac{a}{\delta} \frac{1}{1 + {}^{a}/_{h}} = 34416$$

     see script page 72

     The approximation cannot be used as ${}^{a}/_{h} = 0.526 > 0.5$

     The skin depth can also be computed based on the conductivity of copper:

     $\sigma_{Cu} = 5.8 \cdot 10^{7}\ S/m$

$$\delta = \sqrt{\frac{2}{2\pi f \sigma_{Cu} \mu_0}} = 3.82\ \mu m$$

see script page 68

- "geometry factor", also known as "characteristic impedance" *R/Q*

$$\frac{R}{Q} = \frac{4\eta}{\chi_{01}^3 \pi J_1^2(\chi_{01})} \frac{\sin^2\left(\frac{\chi_{01}}{2}\frac{h}{a}\right)}{\frac{h}{a}}$$

with: $\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\mu_0^2 \varepsilon_0^2} = 377\ \Omega$ (characteristic impedance of the free space)

$\chi_{01} = 2.4048$ (1st zero of the Bessel function of 0th order)

$J_1(\chi_{01}) = 0.5192$

Follows:

$$\frac{R}{Q} = 185 \frac{\sin^2\left(1.2024\frac{h}{a}\right)}{\frac{h}{a}} = 84.61$$

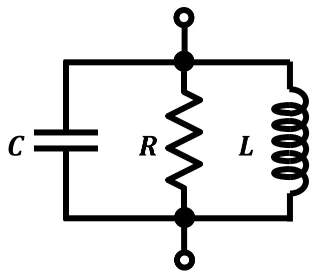
The approximation for small x cannot be applied: $\sin(x) = x$ for $x \ll 1$

see script page 73

Is the cavity completely determined?

Yes, 3 parameters are given, therefore the cavity is fully defined.

2. Find the equivalent circuit of the cavity.



$$R = \frac{R}{Q} Q = 2.91\ M\Omega$$

$$C = \frac{1}{2\pi f\ {}^{R}/_{Q}} = 6.27\ pF$$

$$L = \frac{{}^{R}/_{Q}}{2\pi f} = 44.92\ nH$$

see script pages 48, 49 and 52

3. Calculate the 3-dB bandwidth of the intrinsic (not connector to any generator) cavity.

$$\Delta f = \frac{f}{Q} = 8.711\ kHz$$

see script page 55

4. Calculate the necessary RF power (RMS) for a gap peak voltage of $\hat{V} = 100\ kV$, assuming critical coupling.

$$P = \frac{\hat{V}^2}{2\,R} = 1.717\ kW$$

see script page 67

5. The cavity is critically coupled, fed by an amplifier, designed for a load impedance of 50 Ω. Determine:

   - The peak voltage at the amplifier output.
     $v_{peak} = \sqrt{2Z_0P} = 414\,V$
     see script page 52
   - The necessary transformer ratio k of the input coupler.
     $k = \sqrt{\frac{R}{Z_0}} = 241$
     see script page 52

2.) Multiple choice questions

1. How will the resonant frequency f_{res} of the E_{010} (TM_{010}) mode of a pill box cavity change if height of the cavity is doubled? (check 1)

- ○ The f_{res} decreases by a factor 2.
- ○ The f_{res} decreases by a factor $\sqrt{2}$.
- ○ The f_{res} increases by a factor 2.
- ○ The f_{res} increases by a factor $\sqrt{2}$.
- ☒ The f_{res} will not change.

2. A critically coupled aluminum pill-box cavity is driven by an RF generator. The same pill-box cavity is now made out of copper, again with the generator operating at critical coupling, such that the gap voltage remains the same. $\sigma_{Al} = 3.8 \cdot 10^7\,S/m$, $\sigma_{Cu} = 5.8 \cdot 10^7\,S/m$. What happens with the dissipated power in the cavity? (check 1)

- ☒ The power dissipation decreases
- ○ The power dissipation increases
- ○ The power dissipation will not change

3. Calculate the thickness of a copper wall of 5 times the penetrations depth for 50 Hz signals. $\sigma_{Cu} = 5.8 \cdot 10^7\,S/m$, $\mu = \mu_0\mu_r$ with $\mu_0 = 4\pi \cdot 10^{-7}\,Vs/Am$ (check 1)

- ☒ 46.7 mm
- ○ 4.67 mm
- ○ 0.46 mm
- ○ 0.046 mm

4. A rectangular waveguide has a width (long side!) of a = 10 cm. (check 2)

- ○ The mode TE_{10} or H_{10} has a cutoff frequency of 3 GHz.
- ☒ The mode TE_{10} or H_{10} has a cutoff frequency of 1.5 GHz.
- ○ The electric field is parallel to the side with the larger dimension.
- ☒ The electric field is orthogonal to the side with the larger dimension.

5. Which mode is the fundamental mode (lowest cut-off frequency) in a cylindrical waveguide of circular cross-section *without* inner conductor? (check 1)

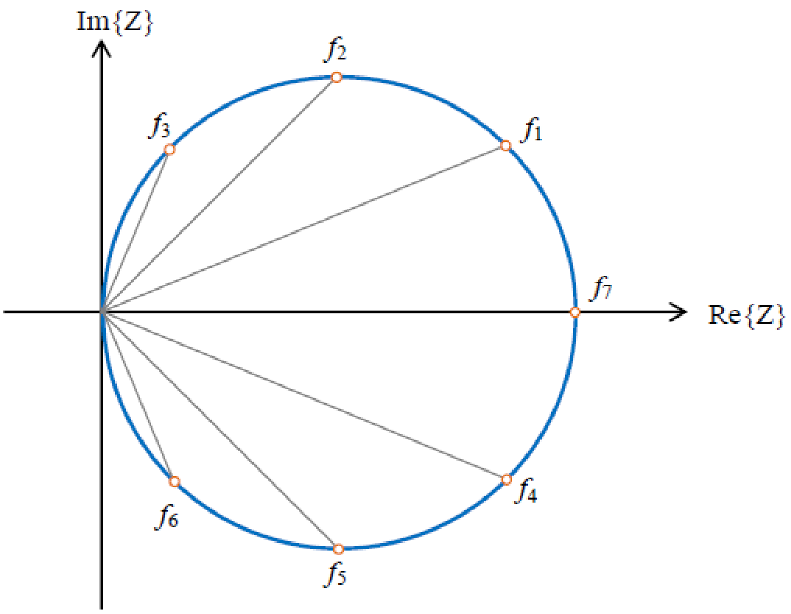
- ○ *TE*
- ○ *TEM*
- ☒ *TM*

6. Adding capacitive loading to a cavity (check 1)
   - ☒ lowers the resonance frequency
   - ○ does not affect the resonance frequency
   - ○ increases the resonance frequency

7. When you cover the antenna of your mobile with your hand, the attenuation caused is in the order of 20 dB. Human tissue is a rather good absorber, so you can neglect reflections for this calculation. How many percent of the mobile's output power stay in the hand? (check 1)
   - ○ 9 %
   - ☒ 99 %
   - ○ 99.9 %
   - ○ 99.99 %

3.) Impedances in the complex plane (2)

The impedance of a resonant circuit is a function of frequency. For a given resonator the impedance was measured at 7 different frequencies, f_1...f_7. The result is shown in the complex Z-plane:



	f_1	f_2	f_3	f_4	f_5	f_6	f_7
f / MHz	105.11	105.05	104.94	105.29	105.35	105.46	105.20
Z / kΩ	$200.0\,e^{j30°}$	$162.6\,e^{j45°}$	$115.0\,e^{j60°}$	$200.0\,e^{-j30°}$	$162.6\,e^{-j45°}$	$115.0\,e^{-j60°}$	$230.0\,e^{j0°}$

Questions:

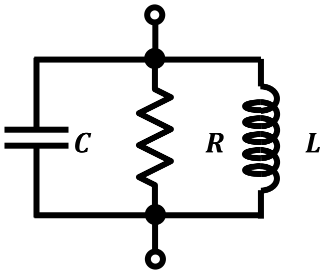
1. Determine the resonant frequency.

   $f_{res} = f_7 = 105.20\ MHz$

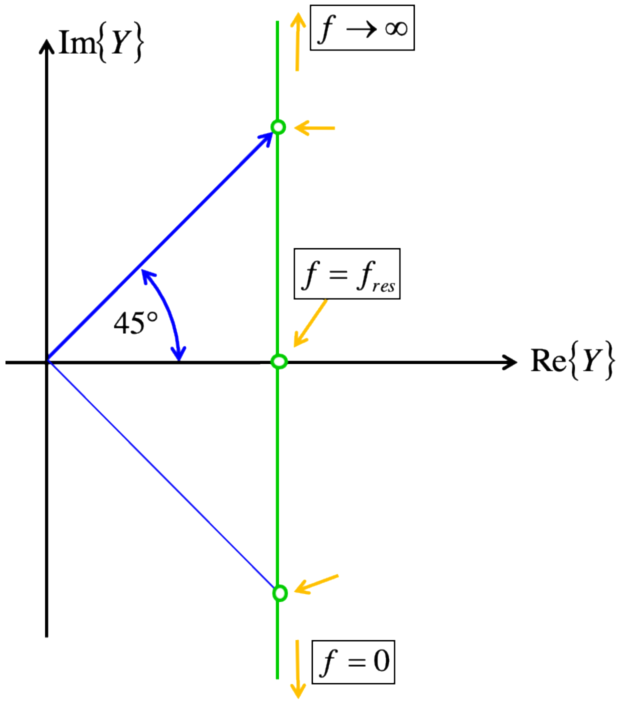
2. Determine the 3-dB bandwidth (*BW*) of this resonator.
   (Hint: The bandwidth of a resonator is defined as the frequency difference between the upper and lower 3-dB frequency points.)
   $BW = f_5 - f_2 = 300\ kHz$

In order to evaluate the properties of a resonator,
it is common to model it as equivalent circuit with lumped RLC elements.

3. Sketch the equivalent circuit for the measured resonator.



4. Determine *R*.
   $R = 230\ \Omega$ (at $f_{res} = f_7$)
5. Draw the locus of admittance of this circuit in the *Y*-plane, and indicate lower and upper 3-dB points.



Straight line parallel to the Im{Y} axes, crossing Re{Y} at 1/230 mS. The 3-dB points are located on the locus as points crossing with lines from the origin under ±45⁰.

see script page 52

6. Determine the Q-value, as well as L and C for this circuit.

$$Q = \frac{f_{res}}{BW} = 350.667$$

$$L = \frac{R}{2\pi f_{res} Q} = 0.992\, nH$$

$$C = \frac{1}{(2\pi f_{res})^2 L} = 2.31\, nF$$

4.) Transmission-lines

Given is a coaxial transmission-line with an inner diameter of the outer conductor of 100 mm, the dielectric is air (so-called "air-line").

Questions:

1. What is the outer diameter of the inner conduction to achieve a characteristic impedance of 50 Ω?

$$Z_0 = \sqrt{\frac{\mu_r}{\varepsilon_r}} 60\Omega \ln\left(\frac{R}{r}\right) = 60\Omega \ln\left(\frac{D}{d}\right)$$

$$d = \frac{D}{\sin\left(\frac{50}{60}\right)} = \frac{D}{2.3} = 43.46\, mm$$

see script page 10

2. With which velocity is a wave travelling in this line?

$$v = c = 2.998 \cdot 10^8 \; m/s$$

3. Specify the capacitance and inductance per meter length of this transmission-line?

$$C' = \frac{1}{vZ_0} = 66.71 \; pF/m$$

$$L' = \frac{Z_0}{v} = 166.78 \; nH/m$$

see script page 10

4. Instead of an air dielectric this transmission line is now homogeneously filled with Teflon ($\varepsilon_r = 2$). Determine the phase velocity, characteristic impedance, as well as capacitance and inductance per meter length?

$$v = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = 2.12 \cdot 10^8 \; m/s$$

$$Z_0 = \frac{60\Omega}{\sqrt{\varepsilon_r}} \ln\left(\frac{D}{d}\right) = 35.36\; \Omega$$

$$C' = \frac{1}{vZ_0} = 133.43 \; pF/m$$

$$L' = \frac{Z_0}{v} = 166.78 \; nH/m$$

5.) Waves of a transmission line Z = 50 Ω

Problem: Convert the circuit-based formats, voltage *V* and current *I* into the equivalent wave-based formats, forward wave *a* and backward wave *b* and vice versa using the relations:

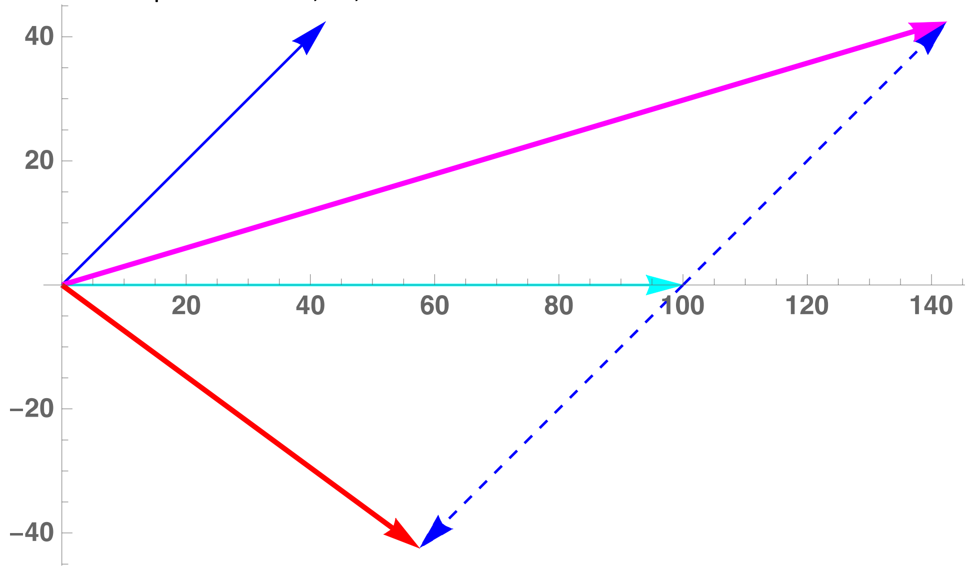
$a = \frac{V + I\,Z}{2}$	$V = a + b$
$b = \frac{V - I\,Z}{2}$	$I\,Z = a - b$

Questions:

1. In a 50 Ω system, a directional coupler measured forward and reflected waves a and b at a certain plane as: a = 100 ∠0° and b = 60 ∠45°.
    - Calculate the corresponding voltage V and current I

      $V = a + b = (100 + 42.43 + j42.43)\ V = (142.43 + j42.43)V = 148.61\ Ve^{j16.59°}$

      $I = \frac{a-b}{Z} = \frac{(100 - 42.43 - j42.43)V}{50\ \Omega} = (1.15 - j0.849)\ A = 1.43\ Ae^{-j36.39°}$
    - Sketch the "phasors" of V, I Z, a and b.

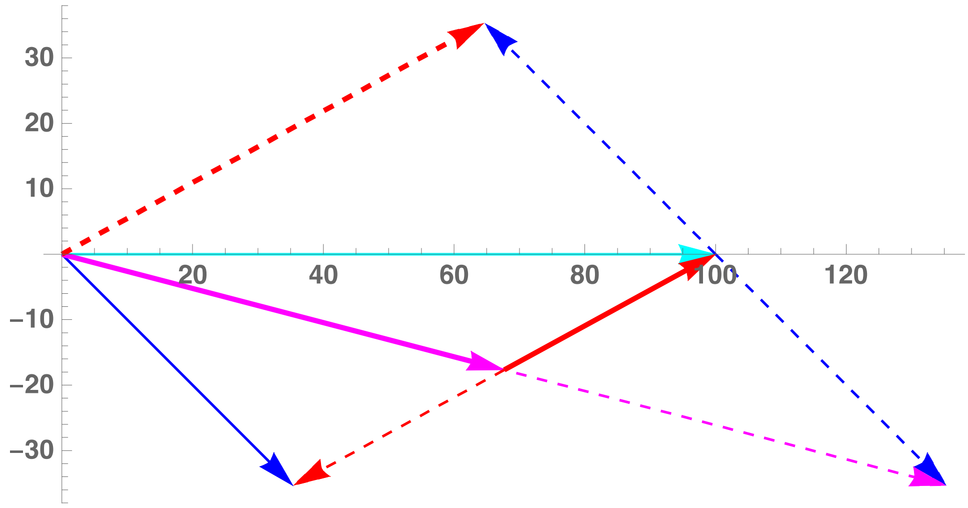


2. At some plane in the 50 Ω system, a voltage of V = 100 ∠0° V and a current of I = 1.0 ∠-45° A are measured.
    - Calculate the corresponding forward and backward waves a and b.

      $a = \frac{V + IZ}{2} = \frac{100\ V + (0.7 - j0.7)A\ 50\ \Omega}{2} = (67.68 - j17.68)\ = 69.95\ Ve^{-j14.6°}$

      $b = \frac{V - IZ}{2} = \frac{100\ V - (0.7 - j0.7)A\ 50\ \Omega}{2} = (32.32 - j17.68)\ = 36.84\ Ve^{j28.68°}$

- Sketch the “phasors” of V, I Z, a and b.



6.) “Pillbox” cavity characteristics

The following data was measured on a “pillbox” cavity”:

Inductance:	*L* = 15.915 nH
Capacitance:	*C* = 1.5915 pF
3-dB bandwidth:	*BW* = 50 kHz

Questions:

Determine

- the frequency at resonance
  $$f_{res} = \frac{1}{2\pi\sqrt{LC}} = 1\ GHz$$
  see script page 49
- the characteristic impedance *R/Q*
  $$\frac{R}{Q} = \sqrt{\frac{L}{C}} = 100$$
  see script page 52
- the quality factor *Q*
  $$Q = \frac{f_{res}}{\Delta f} = 20000$$
  see script page 51 and 55
- the time constant τ
  $$\tau = \frac{Q}{\pi f_{res}} = 6.37\ \mu s$$
  see script page 62
- the peak induced voltage immediately after the passage of a short particle bunch with charge $q = 15.916 \cdot 10^{-9}\ As$

$v_{step} = \left|0 - \frac{q}{C}\right| = 10\ kV$
see script page 64

- the remnant cavity voltage 10 µs after the passage of the bunch
$v_{end} = v_{step} e^{-\frac{t}{\tau}} = 2.08\ kV$
see script page 64

7.) Gap-width optimization of a cavity

The following parameters of a 100 MHz cavity have been evaluated by a numerical simulation software as function of the gap-width g:

characteristic impedance R/Q and quality factor Q

The cavity is connected to an amplifier delivering 1 kW of RF power.
The beam has a relative velocity of $\beta = 0.15$.

Questions:

Calculate for each gap-width:

- shunt impedance R
$R = \frac{R}{Q} Q$
- intrinsic cavity voltage V_{cav} for 100 kW power
$V_{cav} = \sqrt{2RP}$
see script page 52 and 67
- angle θ of the beam passage through the gap
Velocity v of the beam through the cavity gap g:
$v = \beta = \frac{g}{t}$
Time t through the gap:
$t = \frac{g}{\beta c} = \frac{g}{\beta \lambda f}$
The total phase angle accounts for $\theta = 2\pi$
The phase θ is given by multiplying both sides of the equation by $2\pi/f$ (remember: $ft = 1$)
$2\pi f t = 2\pi \frac{gf}{\beta \lambda f} \Rightarrow 2\pi = \theta = 2\pi \frac{g}{\beta \lambda} = \frac{2\pi f g}{\beta c}$
- transit time factor T
The transit time factor T is related to the phase angle θ:
$\frac{g\omega}{2\beta c} = \frac{2\pi f g}{2\beta c} = \frac{\theta}{2}$
$T = \frac{\sin \theta/2}{\theta/2}$
see script page 95

- beam voltage V_{beam} maximally seen by the beam taking the transit time factor T into account
  $V_{beam} = V_{cav}T$

G [mm]	**R/Q** [Ω]	**Q**	**R** [Ω]	**V_{cav}** [V]	**θ**	**T**	**V_{beam}** [V]
100	100	5000	500000	316228	80.1^0	0.92	291125
200	150	7000	1050000	458258	160.1^0	0.7	323048
300	200	9000	1800000	600000	240.2^0	0.41	247719

8.) Higher-order mode of a cavity

An RF cavity has an unwanted higher-order mode (HOM) at 600 MHz, with shunt impedance *R* = 6 MΩ, a 3-dB bandwidth of *BW* = 15 kHz, and a transit time factor *T* ~1.

The beam consists of very short bunches, following each other at intervals of 20 µs. The circulating beam current is 0.1 A. (Reminder: current *I* = charge per time)

Questions:

Calculate:

- Q, R/Q, and C at the HOM frequency
  $$Q_{HOM} = \frac{f_{HOM}}{\Delta f_{HOM}} = 40000$$
  see script page 55
  $$\frac{R}{Q} = \frac{R_{HOM}}{Q_{HOM}} = 150\ \Omega$$
  $$C_{HOM} = \frac{1}{2\pi f_{HOM}\, R/Q} = 1.77\ pF$$
  see script page 52
- HOM voltage induced by a single bunch
  Assuming a single bunch in the ring:
  $$q_{bunch} = i_{beam} t_{bunch}$$
  $$\Delta V = \frac{q_{bunch}}{C_{HOM}} = 1.13\ MV$$
  see script page 62
- Time constant τ of the cavity
  $$\tau = 2R_{HOM}C_{HOM} = 21.22\ \mu s$$
  see script page 62
- HOM voltage at the arrival of the next bunch
  $$V_{end}e^{-\frac{t}{\tau}} = V_{step} \Rightarrow V_{next} = \Delta V e^{-\frac{t}{\tau}} = \Delta V e^{-\frac{t_{bunch}}{\tau}} = 440.7\ kV$$
  see script page 64
- Total HOM voltage in steady state, after the passage of an infinite number of bunches, assuming the HOM resonance is an exact multiple of the beam revolution frequency and in sync with the beam.
  $$V_{end} = \frac{q_{bunch}}{C_{HOM}(1 - e^{-t_{bunch}/\tau})} = 1.853\ \mu s$$