

# JUAS 2019 – Tutorial 1

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$$\mu = \mu_0 \mu_r$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/(Am)}$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$\varepsilon_0 = 8.854 \cdot 10^{-12} \text{ As/(Vm)}$$

$$c_0 = 2.998 \cdot 10^8 \text{ m/s}$$

## 1) Design of a pillbox cavity

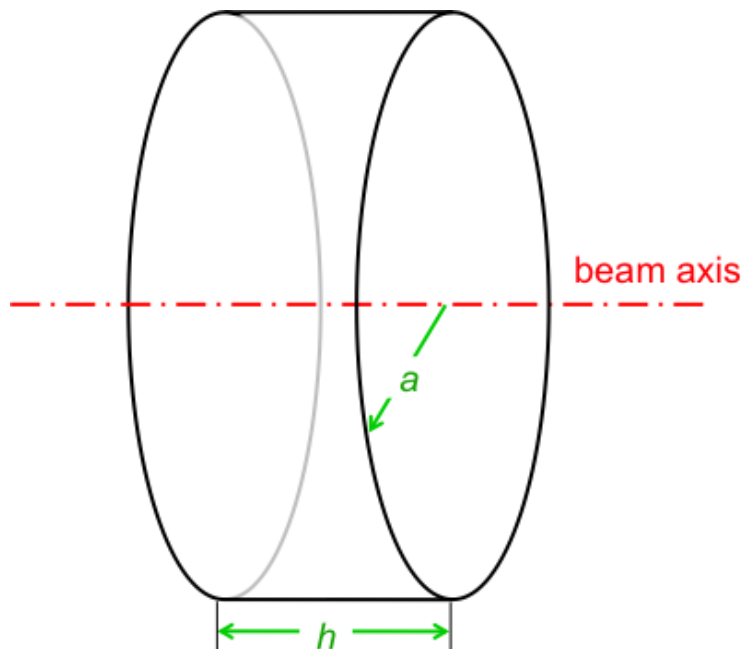
Problem: Design a simple “Pillbox” cavity with the following parameters

Frequency:  $f = 299.98 \text{ MHz}$  ( $\lambda = 1.00 \text{ m}$ )

Wall material: Copper (equivalent skin depth  $\delta = 3.8 \mu\text{m}$ )

Axial length:  $h = 0.2 \text{ m}$

For this example, we ignore beam ports, i.e. vacuum chamber stubs required for the beam passage, so that all analytical formulas describing the pillbox cavity apply.



### Questions:

- Find from the analytical formulas:
  - Cavity radius  $a$
  - $Q$
  - “geometry factor”, also known as “characteristic impedance”  $R/Q$Is the cavity completely determined?
- Find the equivalent circuit of the cavity.
- Calculate the 3-dB bandwidth of the intrinsic (not connector to any generator) cavity.

4. Calculate the necessary RF power (RMS) for a gap peak voltage of  $\hat{V} = 100 \text{ kV}$ , assuming critical coupling.
5. The cavity is critically coupled, fed by an amplifier, designed for a load impedance of  $50 \Omega$ . Determine:
  - The peak voltage at the amplifier output.
  - The necessary transformer ratio  $k$  of the input coupler.

## 2.) Multiple choice questions

1. How will the resonant frequency  $f_{res}$  of the  $E_{010}$  ( $TM_{010}$ ) mode of a pill box cavity change if height of the cavity is doubled? (check 1)
  - The  $f_{res}$  decreases by a factor 2.
  - The  $f_{res}$  decreases by a factor  $\sqrt{2}$ .
  - The  $f_{res}$  increases by a factor 2.
  - The  $f_{res}$  increases by a factor  $\sqrt{2}$ .
  - The  $f_{res}$  will not change.
2. A critically coupled aluminum pill-box cavity is driven by an RF generator. The same pill-box cavity is now made out of copper, again with the generator operating at critical coupling, such that the gap voltage remains the same.  $\sigma_{Al} = 3.8 \cdot 10^7 \text{ S/m}$ ,  $\sigma_{Cu} = 5.8 \cdot 10^7 \text{ S/m}$ . What happens with the dissipated power in the cavity? (check 1)
  - The power dissipation decreases
  - The power dissipation increases
  - The power dissipation will not change
3. Calculate the thickness of a copper wall of 5 times the penetrations depth for 50 Hz signals.  $\sigma_{Cu} = 5.8 \cdot 10^7 \text{ S/m}$ ,  $\mu = \mu_0 \mu_r$  with  $\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$  (check 1)
  - 46.7 mm
  - 4.67 mm
  - 0.46 mm
  - 0.046 mm
4. A rectangular waveguide has a width (long side!) of  $a = 10 \text{ cm}$ . (check 2)
  - The mode  $TE_{10}$  or  $H_{10}$  has a cutoff frequency of 3 GHz.
  - The mode  $TE_{10}$  or  $H_{10}$  has a cutoff frequency of 1.5 GHz.
  - The electric field is parallel to the side with the larger dimension.
  - The electric field is orthogonal to the side with the larger dimension.
5. Which mode is the fundamental mode (lowest cut-off frequency) in a cylindrical waveguide of circular cross-section *without* inner conductor? (check 1)
  - $TE$
  - $TEM$
  - $TM$

6. Adding capacitive loading to a cavity (check 1)

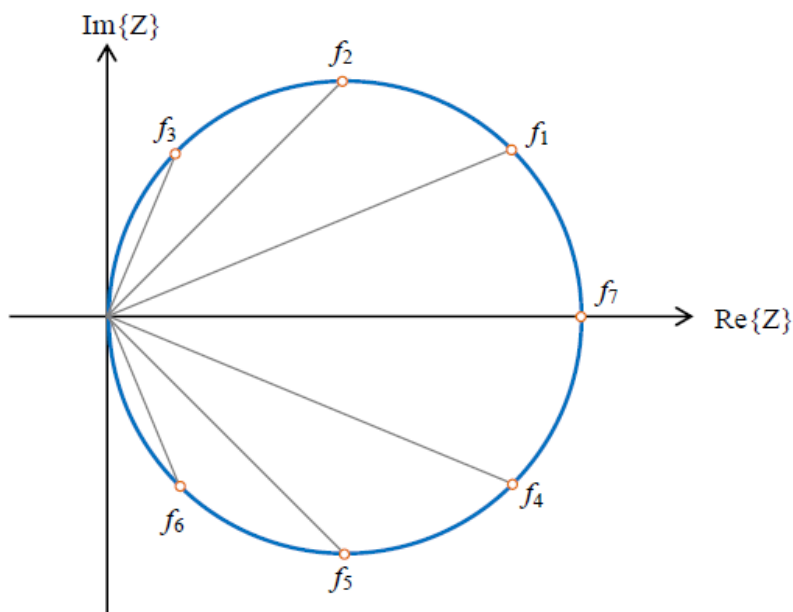
- lowers the resonance frequency
- does not affect the resonance frequency
- increases the resonance frequency

7. When you cover the antenna of your mobile with your hand, the attenuation caused is in the order of 20 dB. Human tissue is a rather good absorber, so you can neglect reflections for this calculation. How many percent of the mobile's output power stay in the hand? (check 1)

- 9 %
- 99 %
- 99.9 %
- 99.99 %

### 3.) Impedances in the complex plane (2)

The impedance of a resonant circuit is a function of frequency. For a given resonator the impedance was measured at 7 different frequencies,  $f_1 \dots f_7$ . The result is shown in the complex  $Z$ -plane:



	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$	$f_6$	$f_7$
$f / \text{MHz}$	105.11	105.05	104.94	105.29	105.35	105.46	105.20
$Z / \text{k}\Omega$	$200.0 e^{j30^\circ}$	$162.6 e^{j45^\circ}$	$115.0 e^{j60^\circ}$	$200.0 e^{-j30^\circ}$	$162.6 e^{-j45^\circ}$	$115.0 e^{-j60^\circ}$	$230.0 e^{j0^\circ}$

#### Questions:

1. Determine the resonant frequency.
2. Determine the 3-dB bandwidth ( $BW$ ) of this resonator.  
(Hint: The bandwidth of a resonator is defined as the frequency difference between the upper and lower 3-dB frequency points.)

In order to evaluate the properties of a resonator, it is common to model it as equivalent circuit with lumped RLC elements.

3. Sketch the equivalent circuit for the measured resonator.
4. Determine  $R$ .
5. Draw the locus of admittance of this circuit in the  $Y$ -plane, and indicate lower and upper 3-dB points
6. Determine the Q-value, as well as  $L$  and  $C$  for this circuit.

#### 4.) Transmission-lines

Given is a coaxial transmission-line with an inner diameter of the outer conductor of 100 mm, the dielectric is air (so-called "air-line").

#### Questions:

1. What is the outer diameter of the inner conductor to achieve a characteristic impedance of  $50 \Omega$ ?
2. With which velocity is a wave travelling in this line?
3. Specify the capacitance and inductance per meter length of this transmission-line?
4. Instead of an air dielectric this transmission line is now homogeneously filled with Teflon ( $\epsilon_r = 2$ ). Determine the phase velocity, characteristic impedance, as well as capacitance and inductance per meter length?

#### 5.) Waves of a transmission line $Z = 50 \Omega$

Problem: Convert the circuit-based formats, voltage  $V$  and current  $I$  into the equivalent wave-based formats, forward wave  $a$  and backward wave  $b$  and vice versa using the relations:

$a = \frac{V + I Z}{2}$	$V = a + b$
$b = \frac{V - I Z}{2}$	$I Z = a - b$

#### Questions:

1. In a  $50 \Omega$  system, a directional coupler measured forward and reflected waves  $a$  and  $b$  at a certain plane as:  $a = 100 \angle 0^\circ$  and  $b = 60 \angle 45^\circ$ .
  - Calculate the corresponding voltage  $V$  and current  $I$
  - Sketch the "phasors" of  $V$ ,  $I Z$ ,  $a$  and  $b$ .

2. At some plane in the  $50 \Omega$  system, a voltage of  $V = 100 \angle 0^\circ$  V and a current of  $I = 1.0 \angle -45^\circ$  A are measured.
- Calculate the corresponding forward and backward waves  $a$  and  $b$ .
  - Sketch the “phasors” of  $V$ ,  $I$ ,  $Z$ ,  $a$  and  $b$ .

## 6.) “Pillbox” cavity characteristics

The following data was measured on a “pillbox” cavity”:

Inductance:	$L = 15.915$ nH
Capacitance:	$C = 1.5915$ pF
3-dB bandwidth:	$BW = 50$ kHz

### Questions:

Determine

- the frequency at resonance
- the characteristic impedance  $R/Q$
- the quality factor  $Q$
- the time constant  $\tau$
- the peak induced voltage immediately after the passage of a short particle bunch with charge  $q = 15.916 \cdot 10^{-9}$  As
- the remnant cavity voltage  $10 \mu\text{s}$  after the passage of the bunch

## 7.) Gap-width optimization of a cavity

The following parameters of a 100 MHz cavity have been evaluated by a numerical simulation software as function of the gap-width  $g$ :

characteristic impedance  $R/Q$  and quality factor  $Q$

The cavity is connected to an amplifier delivering 1 kW of RF power.

The beam has a relative velocity of  $\beta = 0.15$ .

### Questions:

Calculate for each gap-width:

- shunt impedance  $R$
- intrinsic cavity voltage  $V_{cav}$  for 100 kW power
- angle  $\theta$  of the beam passage through the gap
- transit time factor  $T$
- beam voltage  $V_{beam}$  maximally seen by the beam taking the transit time factor  $T$  into account

$G$ [mm]	$R/Q$ [ $\Omega$ ]	$Q$	$R$ [ $\Omega$ ]	$V_{cav}$ [V]	$\theta$	$T$	$V_{beam}$ [V]
100	100	5000					
200	150	7000					
300	200	9000					

## 8.) Higher-order mode of a cavity

An RF cavity has an unwanted higher-order mode (HOM) at 600 MHz, with shunt impedance  $R = 6 \text{ M}\Omega$ , a 3-dB bandwidth of  $BW = 15 \text{ kHz}$ , and a transit time factor  $T \sim 1$ .

The beam consists of very short bunches, following each other at intervals of  $20 \mu\text{s}$ . The circulating beam current is  $0.1 \text{ A}$ . (Reminder: current  $I =$  charge per time)

### Questions:

Calculate:

- $Q$ ,  $R/Q$ , and  $C$  at the HOM frequency
- HOM voltage induced by a single bunch
- Time constant  $\tau$  of the cavity
- HOM voltage at the arrival of the next bunch
- Total HOM voltage in steady state, after the passage of an infinite number of bunches, assuming the HOM resonance is an exact multiple of the beam revolution frequency and in sync with the beam.