Measurement of longitudinal parameter:

- **Definition of longitudinal phase space**
- **Proton LINAC: Determination of mean energy**
- **Determination of longitudinal emittance**
- **Bunch length measurement for non-relativistic beams**
- **Bunch length measurement for relativistic beams**
- **Summary**

Longitudinal ↔ transverse correspondences:

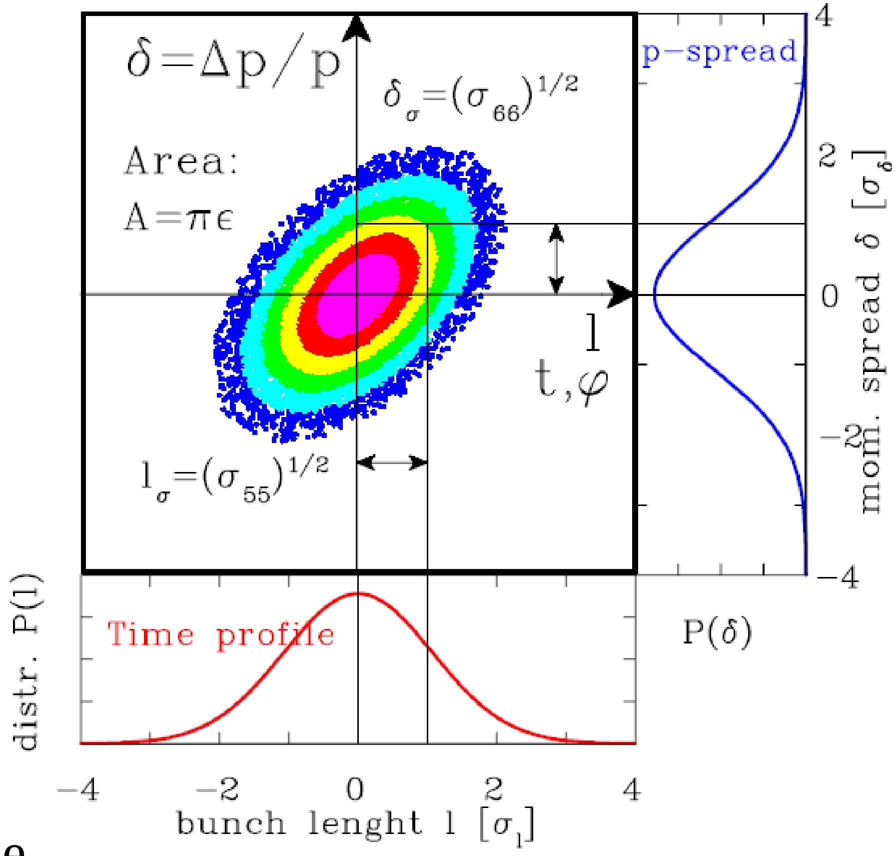
- position relative to rf ↔ transverse center-of-mass
- bunch structure in time ↔ transverse profile in horizontal and vertical direction
- momentum or energy spread ↔ transverse divergence
- longitudinal emittance ↔ transverse emittance.

Measurement of longitudinal Parameters

The longitudinal dynamics is described by the longitudinal emittance as given by:

- Spread of the bunches ***l*** in time, length ***or*** rf-phase.
- Momentum spread $\delta = \Delta p/p$, or energy spread $\Delta W/W$

$$\Rightarrow \quad \varepsilon_{long} = \frac{1}{\pi}\int_A dl \cdot d\delta$$



The normalized value is preserved:

$$\varepsilon_{long}^{norm} = \beta\gamma \cdot \varepsilon_{long}$$

Discussed devices:

- Pick-ups for bunch length and emittance.
- Special detectors (low E_{kin} protons), streak cameras & ele.-optical modulation (e^-)

The Bunch Position measured by a Pick-Up

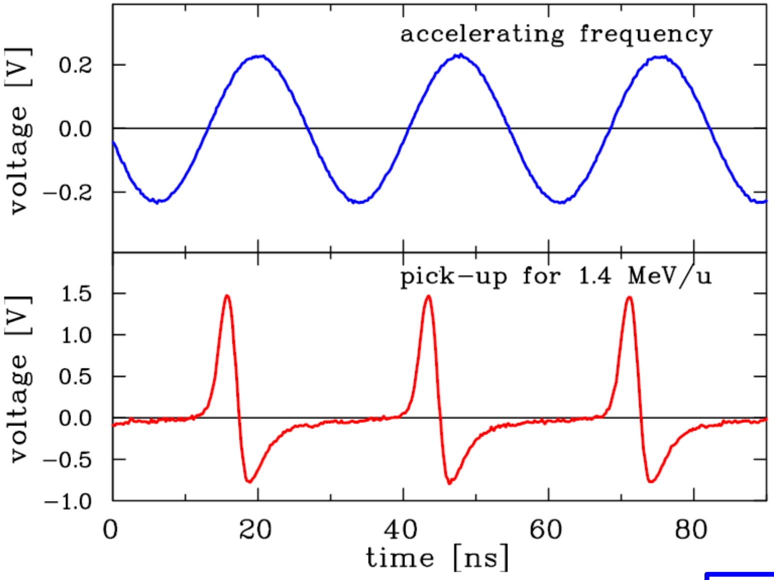
The ***bunch position*** is given relative to the accelerating rf.

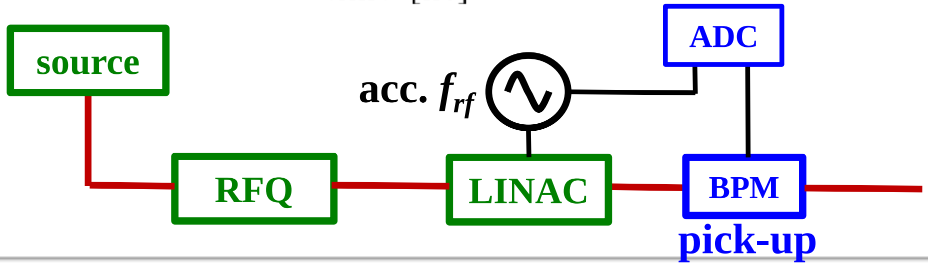
e.g. φ_{ref}=-30^{o} inside a rf cavity

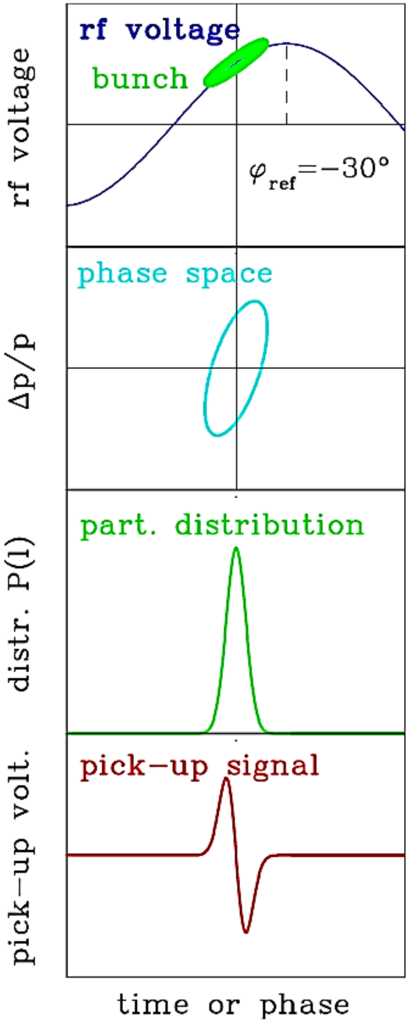
must be well aligned for optimal acceleration

Transverse correspondence: Beam position

Example: Pick-up signal and 36 MHz rf at GSI-LINAC:







Outline:

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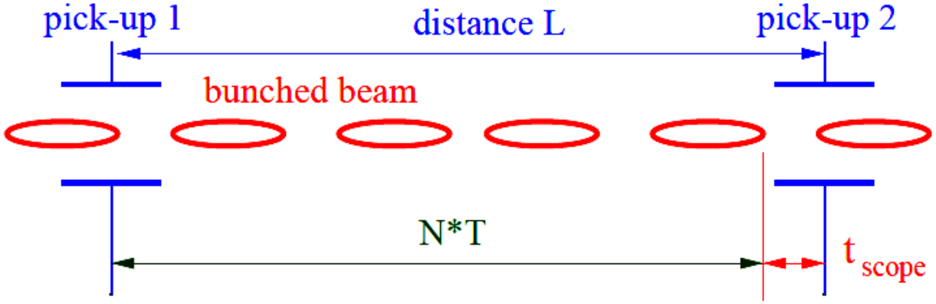
Determination of non-relativistic mean Energy using Pick-Ups

The energy delivered by a LINAC is sensitive to the mechanics, rf-phase and amplitude.

For non-relativistic energies at proton LINACs time-of-flight (TOF) with two pick-ups is used:

$$\beta c = \frac{L}{NT + t_{\mathrm{scope}}}$$

→ the velocity ***β*** is measured.



Example: Time-of-flight signal from two pick-ups at 1.4 MeV/u:

The reading is t_{scope}= 15.82(5) ns with

f_{rf} = 36.136MHz ⇔ T = 27.673ns

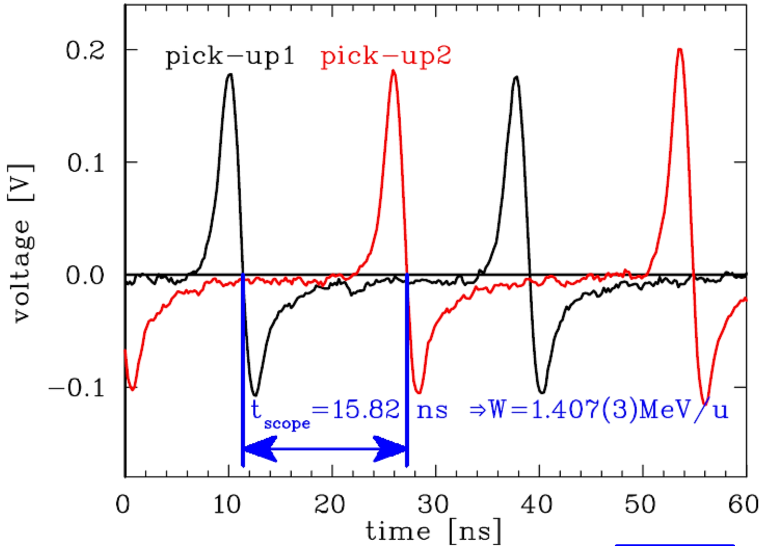
L = 1.629(1) m and N = 3

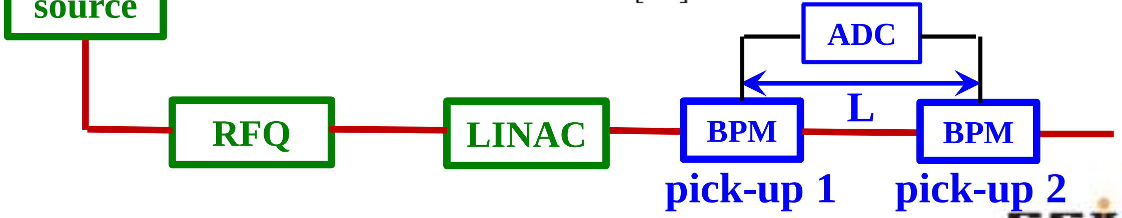
⇒ β = 0.05497(7)

⇔ W=1.407(3) MeV/u

The accuracy is typically 0.1 %

i.e. comparable to ΔW/W





Precision of TOF Measurement for non-relativistic Energy

The precision of TOF is given by the accuracy in time and distance reading:

$$\frac{\Delta\beta}{\beta} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta t}{NT + t_{\mathrm{scope}}}\right)^2}$$

Accuracy of scope reading $\boldsymbol{\Delta t}$ ≈100 ps, uncertainty in distance $\boldsymbol{\Delta L}$ ≈ 1 mm.

Example: GSI-LINAC: L = 3.25 m and f_{rf} = 36 MHz:

Location (LINAC module name)	**unit**	**RFQ**	**IH1**	**IH2**	**AL4**
Output energy W	MeV/u	0.12	0.75	1.4	11.4
Velocity β	%	1.6	4.0	5.5	15.5
Total time-of-flight t_{ToF}	ns	677	271	197	70
Bunch spacing $\beta c/f_{rf}$	cm	13	33	45	129
Resolution $\Delta W/W$	**%**	**0.07**	**0.10**	**0.12**	**0.22**

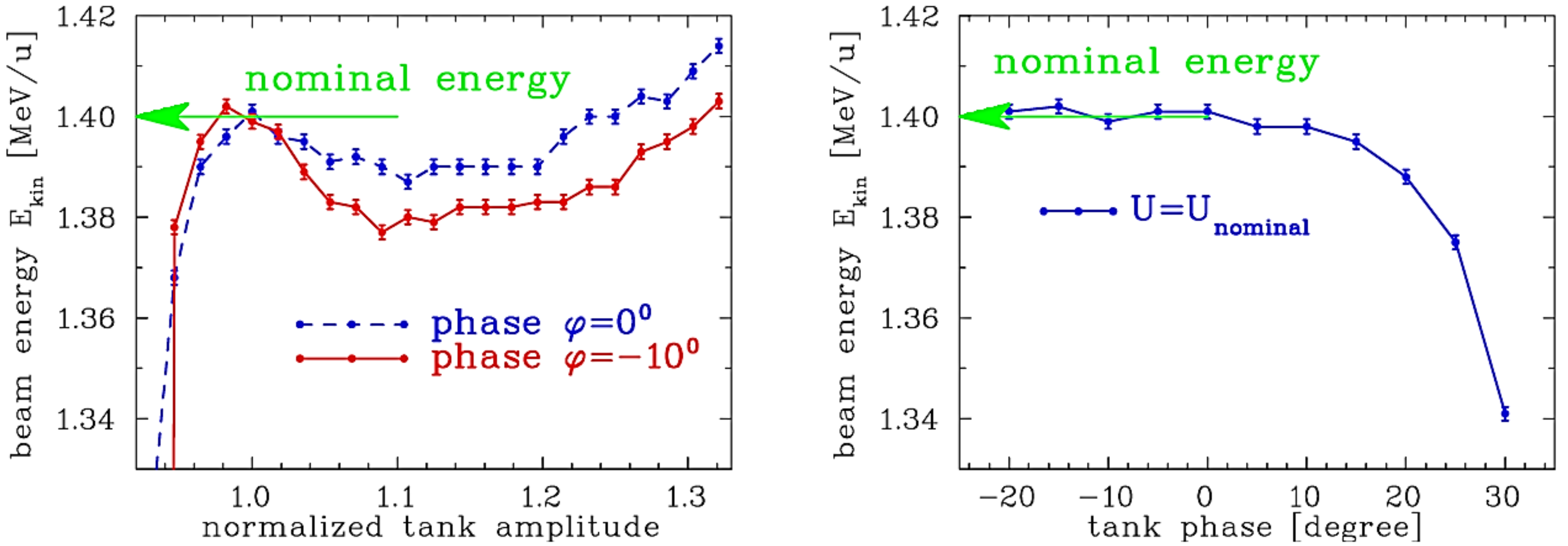
- The accuracy is typically 0.1 % (same order of magnitude as $\Delta W/W$)
- The length has to be matched to the velocity
- Due to the distance of ≈ 3 m, different solutions for the # of bunches N are possible

→ A third pick-up has to be installed closed by, to get an unique solution.

Cavity Alignment using a TOF Measurement

The mean energy is important for the matching between LINAC module.
It depends on phase and amplitude of the rf wave inside the cavities.

Example: Energy at GSI LINAC (nominal energy 1.400 MeV/u):
(distance between pick-ups: ***L*** = 1.97 m ⇒ ***N*** = 4 bunches)



- ***Proton LINACs:*** Amplitude and phase should be carefully aligned by precise TOF
- ***Electron LINACs***: Due to relativistic velocity, TOF is not applicable.

Outline:

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- **Determination of longitudinal emittance**
  LINAC: variation of bunch length
  Synchrotron: Topographic reconstruction
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- **Bunch length measurement for relativistic beams**
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6-dim Phase Space for Accelerators

The particle trajectory is described with the 6-dim vector $x^t = (x, x', y, y', l, \delta)$

For linear beam behavior the 6x6 transport matrix R is used:

Transformation from location s_0 to s_1 for a single particle is:

$$\vec{x}(s_1) = \mathrm{R} \cdot \vec{x}(s_0)$$

$$\vec{x}(s_1) = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & \ldots & \ldots & \overset{=0}{\ldots} & \ldots \\ R_{31} & \ldots & R_{33} & R_{34} & \ldots & \ldots \\ R_{41} & \ldots & R_{43} & R_{44} & \overset{=0}{\ldots} & \ldots \\ R_{51} & \ldots & \ldots & \ldots & R_{55} & R_{56} \\ \overset{=0}{R_{61}} & \ldots & \overset{=0}{\ldots} & \ldots & R_{65} & R_{66} \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix}$$

Envelope i.e. emittance defined by beam matrix:

$$\sigma(s_1) = \mathrm{R} \cdot \sigma(s_0) \cdot \mathrm{R}^T$$

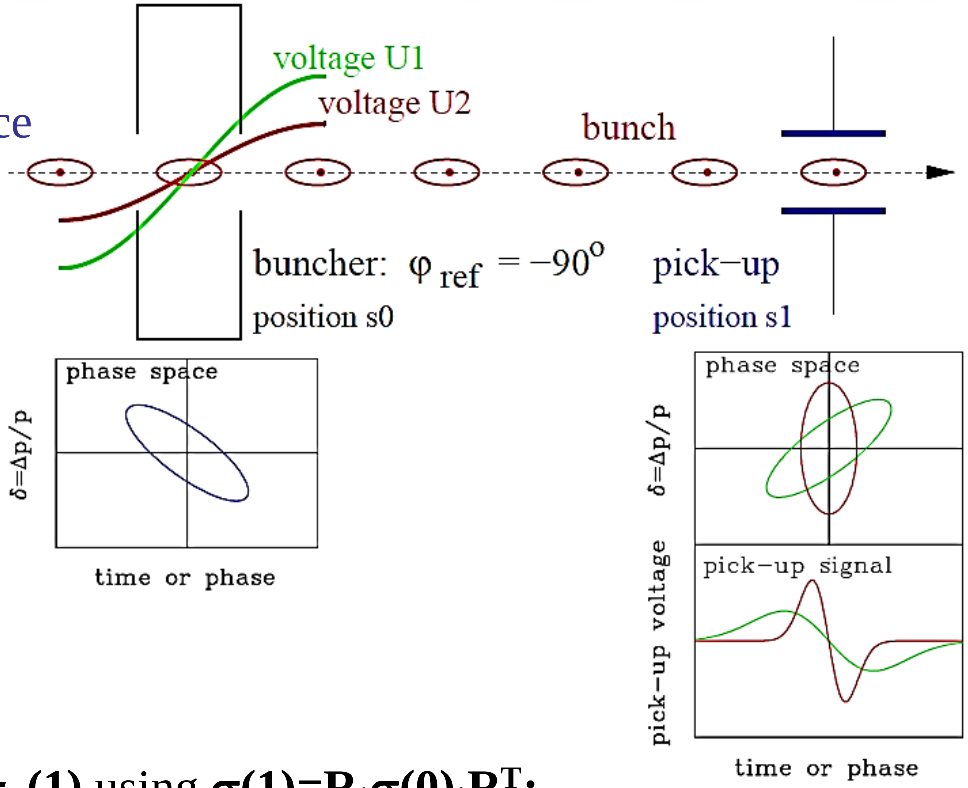
R separates in 3 matrices only **if** the transverse and longitudinal planes do **not** couple,
e.g. no dispersion $D = -R_{16} = 0$

The longitudinal beam matrix σ is **then** a 2 x 2 matrix
with bunch length $l_{rms} = \sqrt{\sigma_{55}}$ & momentum spread $\frac{\Delta p}{p} = \delta_{rms} = \sqrt{\sigma_{66}}$

Longitudinal Emittance by linear Transformation using a Buncher

Longitudinal focusing:

Variation of the bunch shape by a rf-buncher
→ components 5 and 6 from 6-dim phase-space
Transversal corres.: Quadrupole variation



- Transfer matrix of buncher & drift:

$$R_{buncher} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}, \; R_{drift} = \begin{pmatrix} 1 & L/\gamma^2 \\ 0 & 1 \end{pmatrix}$$

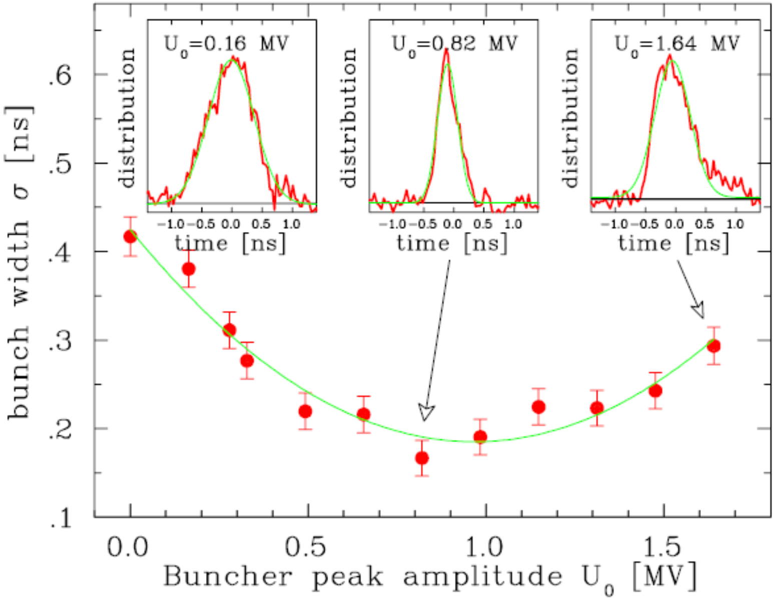
with focal length: $1/f = \frac{2\pi f_{rf}}{Apv^2} \cdot U$

- Variation of buncher amplitude ***U***
  $\Rightarrow$ different bunch width at s_1:
  beam matrix $\Delta t^2_{rms} = \sigma_{55}(1, f)$
- System of redundant linear equations for $\sigma_{ij}(1)$ using $\sigma(1) = R \cdot \sigma(0) \cdot R^T$:

$$\begin{aligned} \sigma_{55}(1, f_1) &= R_{55}^2(f_1) \cdot \sigma_{55}(0) + 2R_{55}(f_1)R_{56}(f_1) \cdot \sigma_{56}(0) + R_{56}^2(f_1) \cdot \sigma_{66}(0) && \text{focusing } f_1 \\ &\vdots \\ \sigma_{55}(1, f_n) &= R_{55}^2(f_n) \cdot \sigma_{55}(0) + 2R_{55}(f_n)R_{56}(f_n) \cdot \sigma_{56}(0) + R_{56}^2(f_n) \cdot \sigma_{66}(0) && \text{focusing } f_n \end{aligned}$$

Result of a longitudinal Emittance Measurement

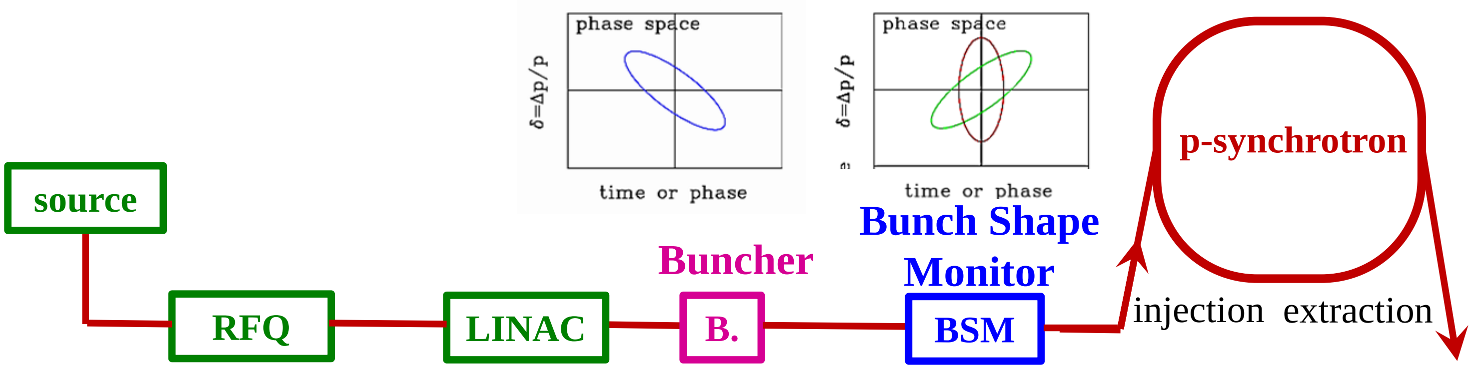
Example GSI LINAC: Voltage variation at buncher for 11.4 MeV/u Ni^{14+} beam, 31 m drift:



- ➢ The structure of short bunches can be determined with special monitor
- ➢ This example: The resolution is better than 50 ps or 2° for 108 MHz
- ➢ Typical bunch length at proton LINACs: $\sigma_{bunch} \approx$ 10 to 300 ps
- ➢ Determination of longitudinal emittance possible

Application for synchrotron injection:

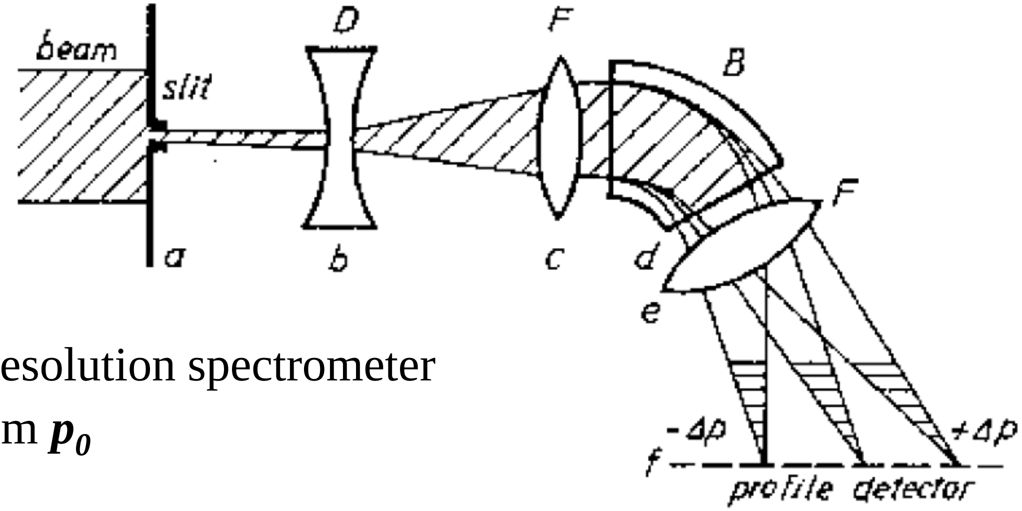
Shaping of longitudinal phase space by buncher
i.e. long bunches ⇔ low momentum spread
to match to the synchrotron long acceptance



Measurement of Energy Spread by magnetic Spectrometer

Transfer line: The mom. spread $\delta = \Delta p/p$ can be determined by a magnetic spectrometer: via dispersion, the momentum is shifted to a spatial distance.

An appropriate optic must b e chosen to separate the transverse and longitudinal parameters

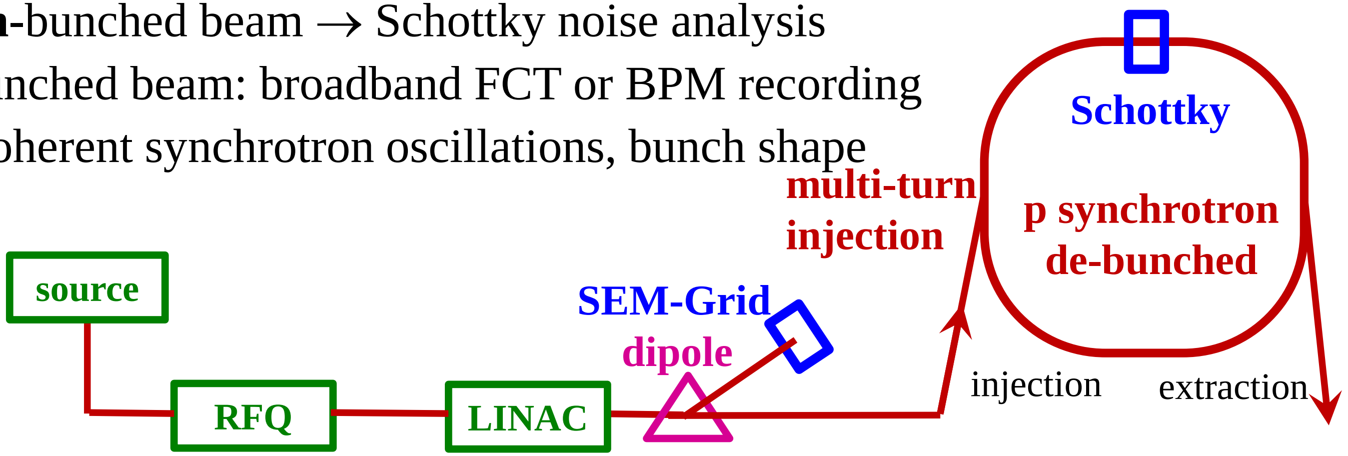


However, a synchrotron is a very high resolution spectrometer

Goal: Measurement of central momentum p_0

and momentum spread $\Delta p / p_0$

- **un**-bunched beam → Schottky noise analysis
- bunched beam: broadband FCT or BPM recording
  coherent synchrotron oscillations, bunch shape


Schottky

p synchrotron de-bunched

multi-turn injection

source

RFQ

LINAC

SEM-Grid

dipole

injection

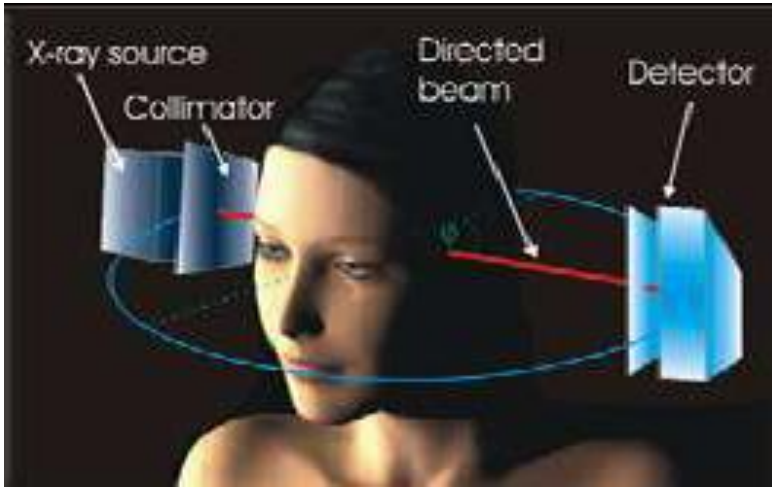
extraction

Longitudinal Emittance using tomographic Reconstruction

Tomography is medical image method

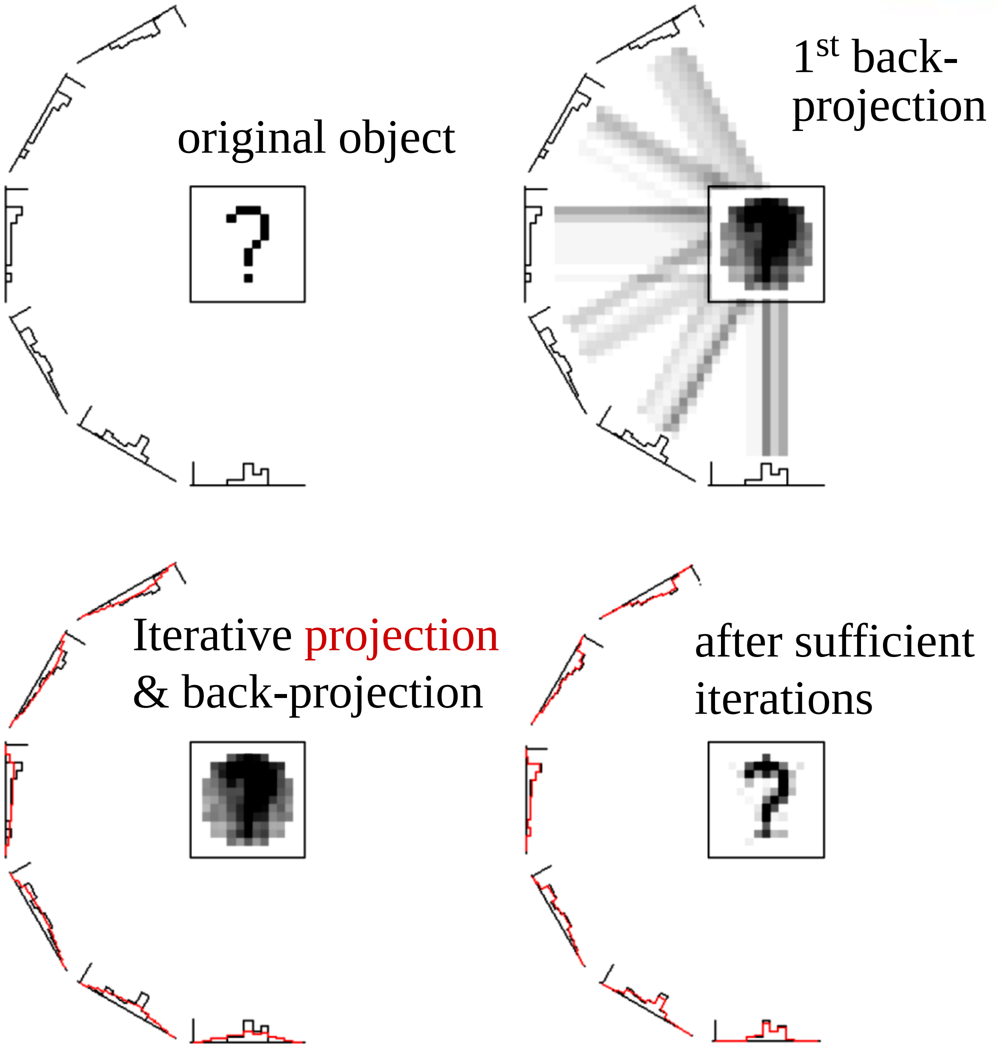
Tomography:

2-dim reconstruction of
sufficient 1-dim projections



Algebraic back projection:

Iterative process by redistributing
the 2-dim image and considering the
differences to the previous iteration step.



original object

1st back-projection

Iterative projection & back-projection

after sufficient iterations

Longitudinal Emittance using tomographic Reconstruction

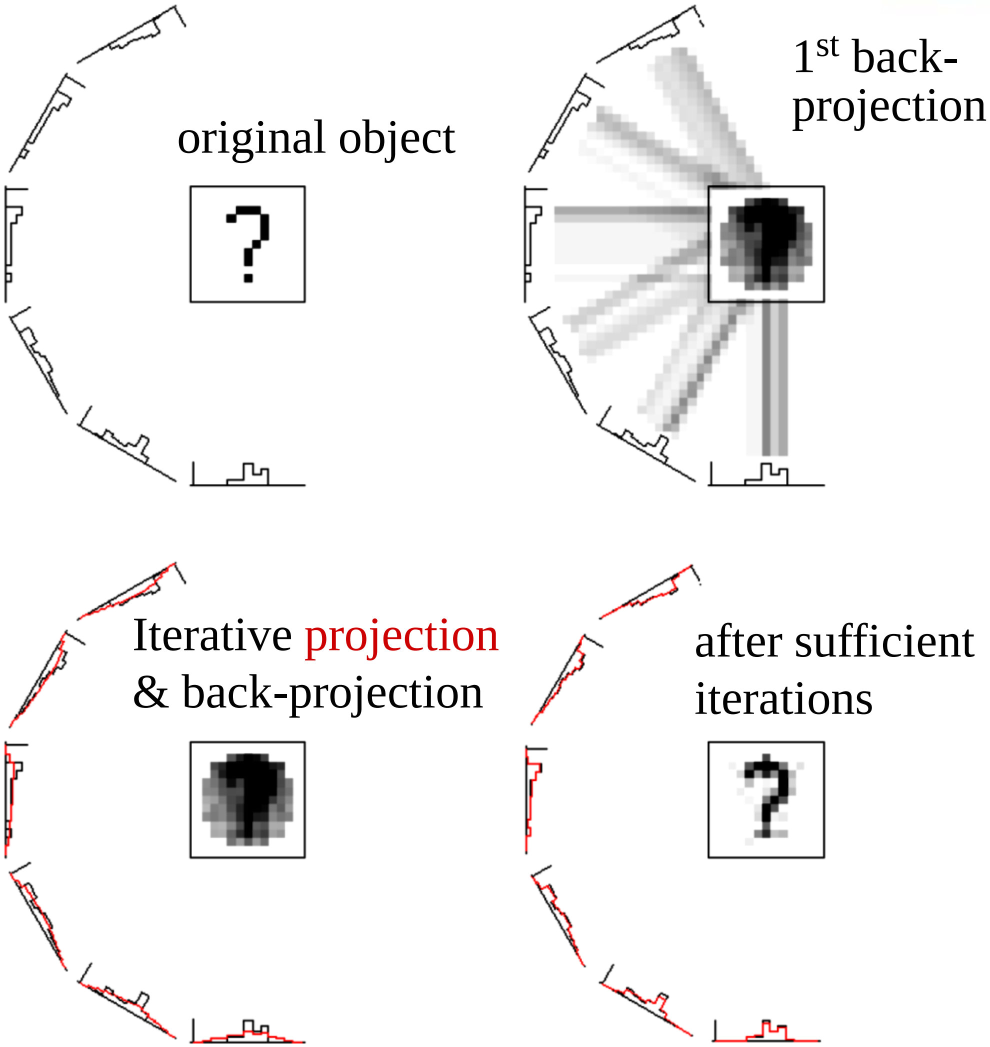
Tomography is medical image method

Tomography:
2-dim reconstruction of
sufficient 1-dim projections

Application at accelerators:
Longitudinal emittance
evolution in synchrotrons.

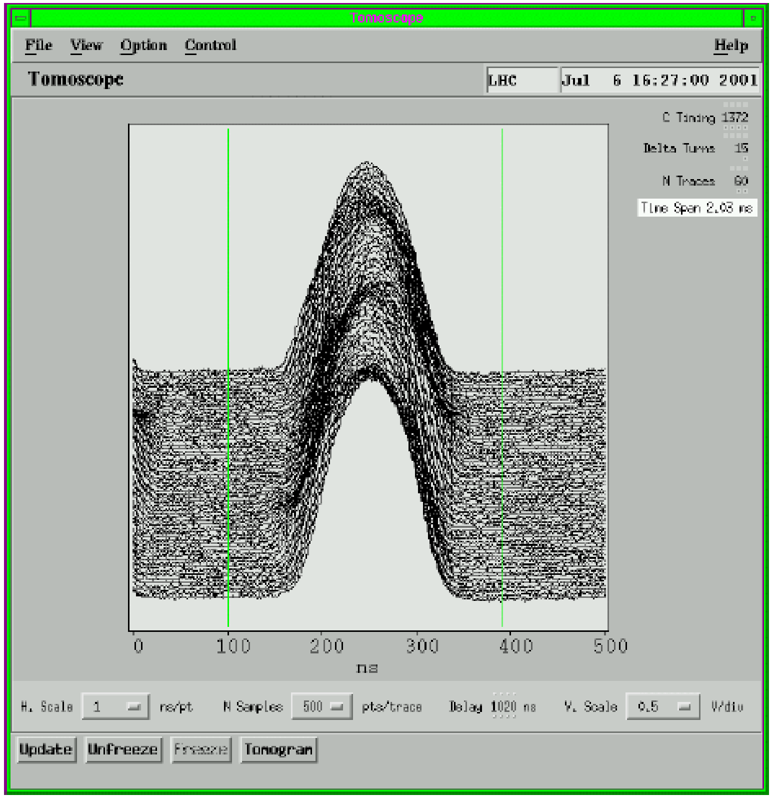
Bunch observation:
Each revolution,
the bunch shape changes a bit
due to synchrotron oscillations.
Fulfilled condition: $f_{synch} << f_{ref}$.

Algebraic back projection:
Iterative process by redistributing
the 2-dim image and considering the
differences to the previous iteration step.



Results of tomographic Reconstruction at a Synchrotron I

Bunches from 500 turns at the CERN PS and the phase space for the first time slice, measured with a wall current monitor:



C Timing 1372

Delta Turns 15

N Traces 60

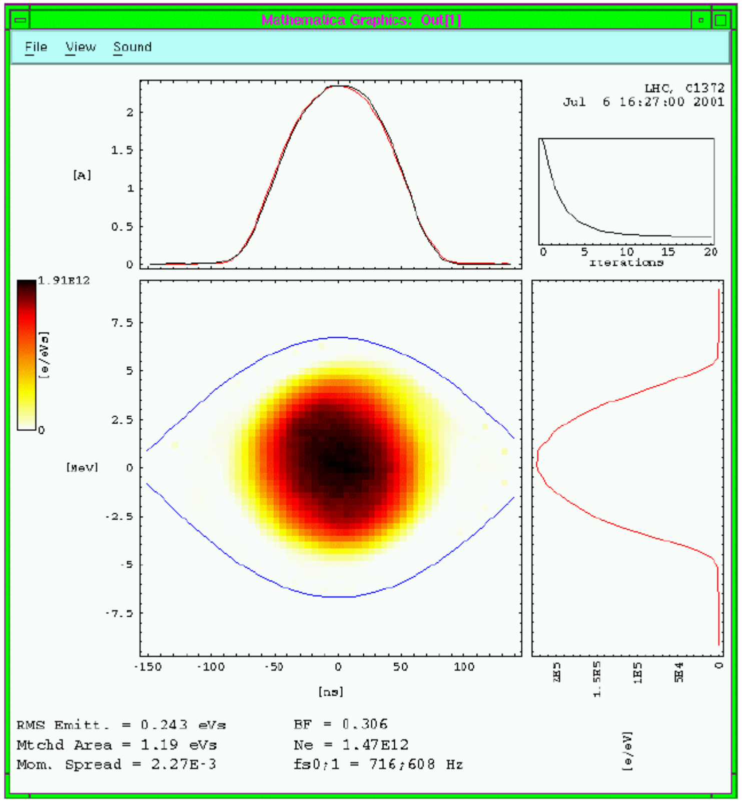
Time Span 2.03 ns

H. Scale 1 ns/pt

N Samples 500 pts/trace

Delay 1020 ns

V. Scale 0.5 V/div



RMS Emitt. = 0.243 eVs

Mtchd Area = 1.19 eVs

Mom. Spread = 2.27E-3

BF = 0.306

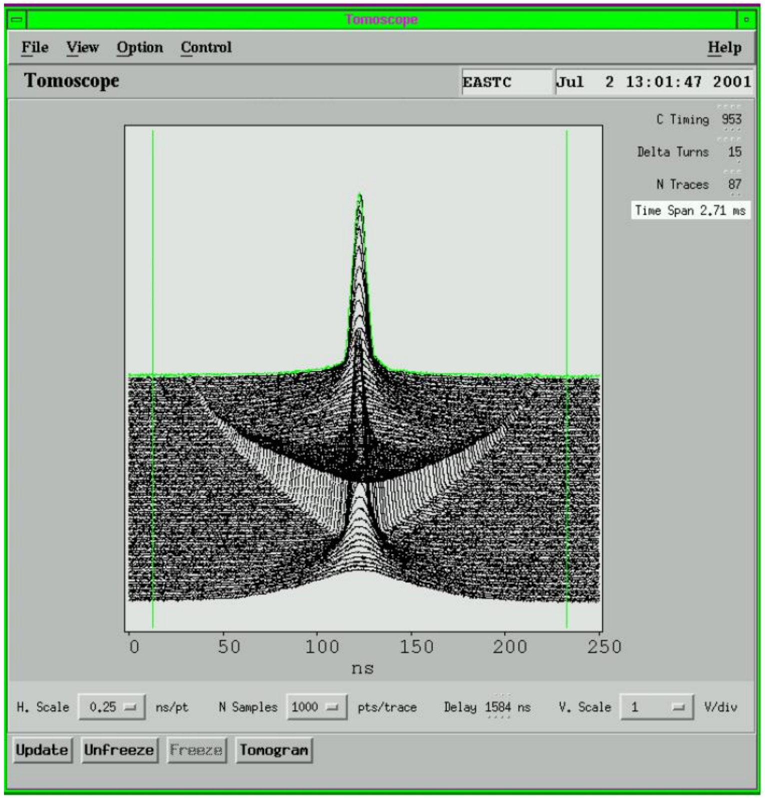
Ne = 1.47E12

fs0;1 = 716;608 Hz

Typical bucket filling. Important knowledge for bunch ’gymnastics’.

Results of tomographic Reconstruction at a Synchrotron II

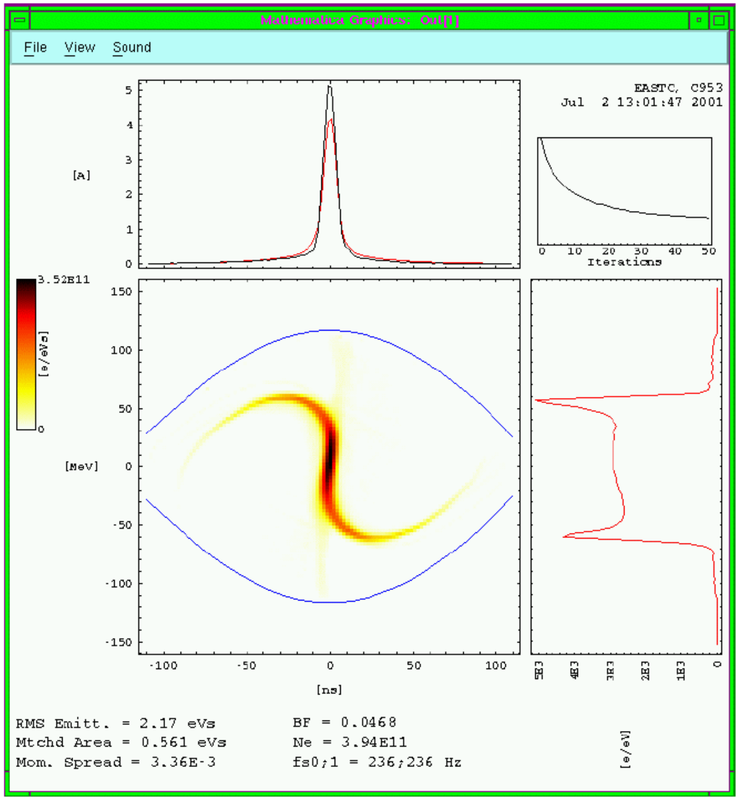
Bunches from 500 turns at the CERN PS and the phase space for the first time slice, measured with a wall current monitor:



EASTC Jul 2 13:01:47 2001

C Timing 953
Delta Turns 15
N Traces 87
Time Span 2.71 ms

H. Scale 0.25 ns/pt
N Samples 1000 pts/trace
Delay 1584 ns
V. Scale 1 V/div



```
EASTC, C953
Jul  2 13:01:47 2001

RMS Emitt. = 2.17 eVs          BF = 0.0468
Mtchd Area = 0.561 eVs         Ne = 3.94E11
Mom. Spread = 3.36E-3          fs0;1 = 236;236 Hz
```

Mismatched bunch shown oscillations and filamentation due to ‘bunch-rotation’.

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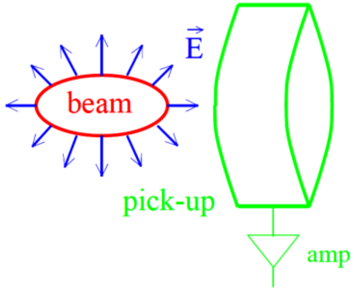
Bunch Structure at low E_{kin}: Not possible with Pick-Ups

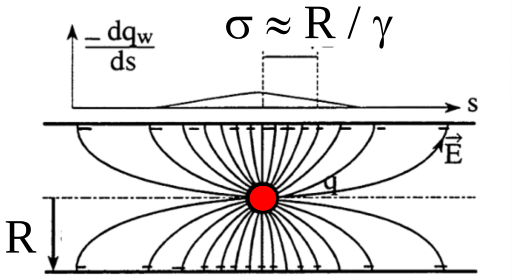
Pick-ups are used for:

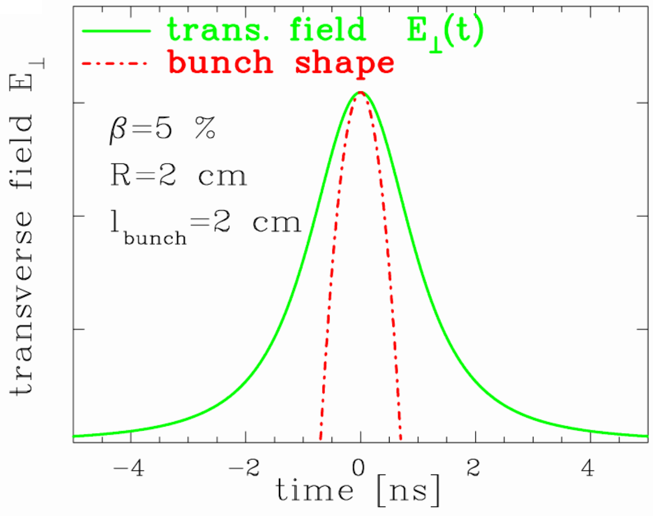
- ➢ precise for bunch-center relative to rf
- ➢ course image of bunch shape

But:

For $\beta \ll 1 \rightarrow$ long. ***E***-field significantly modified:

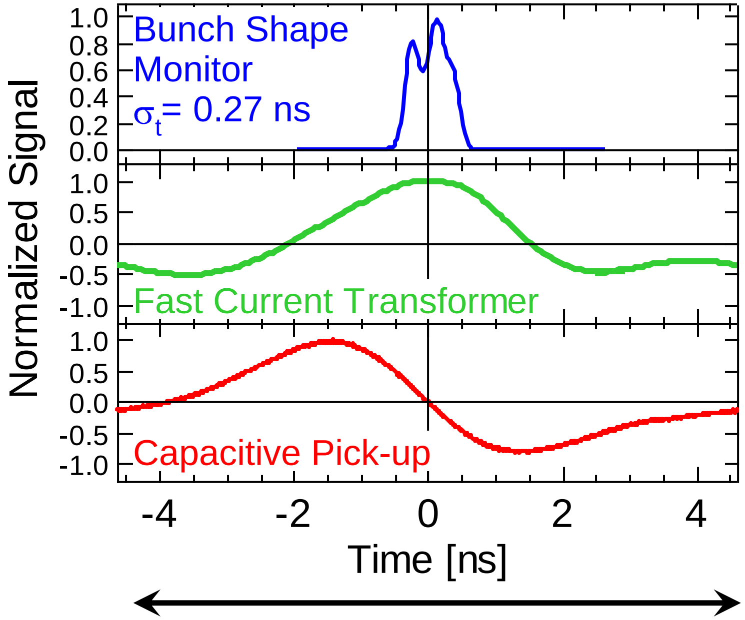






Example: Comparison pick-up – particle counter:

Ar beam of 1.4 MeV/u (***β*** = 5.5%) , f_{rf} = 108 MHz



1/ f_{rf} = 9.2 ns

⇒ the pick-up signal is insensitive to bunch ’fine-structure’

Low Velocity Effect: General Consideration

Lorentz transformation of single point-like charge:

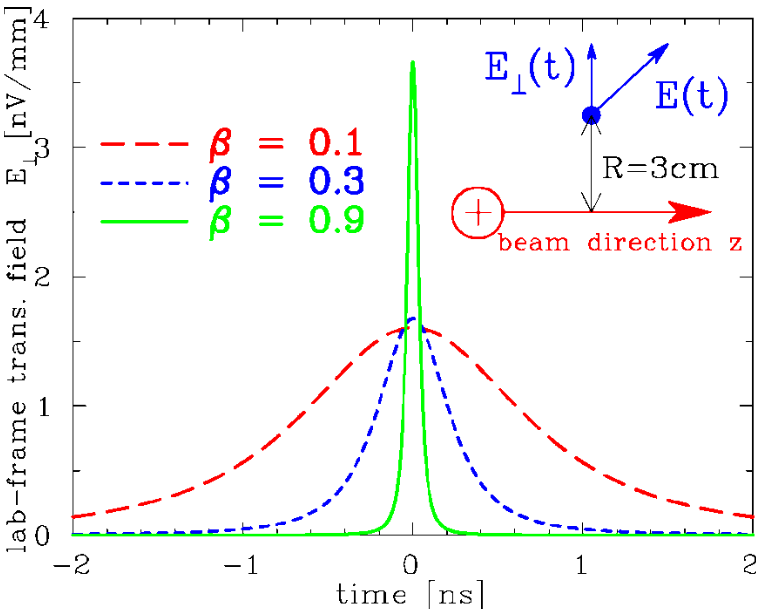
Lorentz boost ***and*** transformation of time: $E_\perp(t) = \gamma \cdot E'_\perp(t')$ and $t \rightarrow t'$

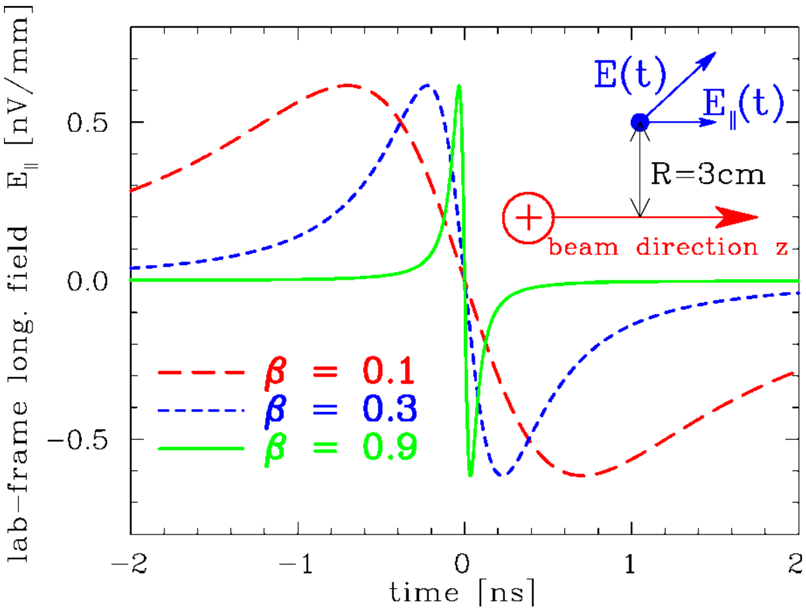
Trans. $\boldsymbol{E}_\perp$ lab.-frame of a point charge:

$$E_\perp(t) = \frac{e}{4\pi\varepsilon_0} \cdot \frac{\gamma R}{\left[R^2 + (\gamma\beta c t)^2\right]^{3/2}}$$

Long. $\boldsymbol{E}_\parallel$ lab.-frame of a point charge:

$$E_\parallel(t) = -\frac{e}{4\pi\varepsilon_0} \cdot \frac{\gamma\beta c t}{\left[R^2 + (\gamma\beta c t)^2\right]^{3/2}}$$





Broadband coaxial Faraday Cups for Bunch Structure

The bunch structure can be observed with cups, having a bandwidth up to several GHz.

Bandwidth and rise time: BW [GHz] = $\mathbf{0.3/t_{rise}}$[ns]

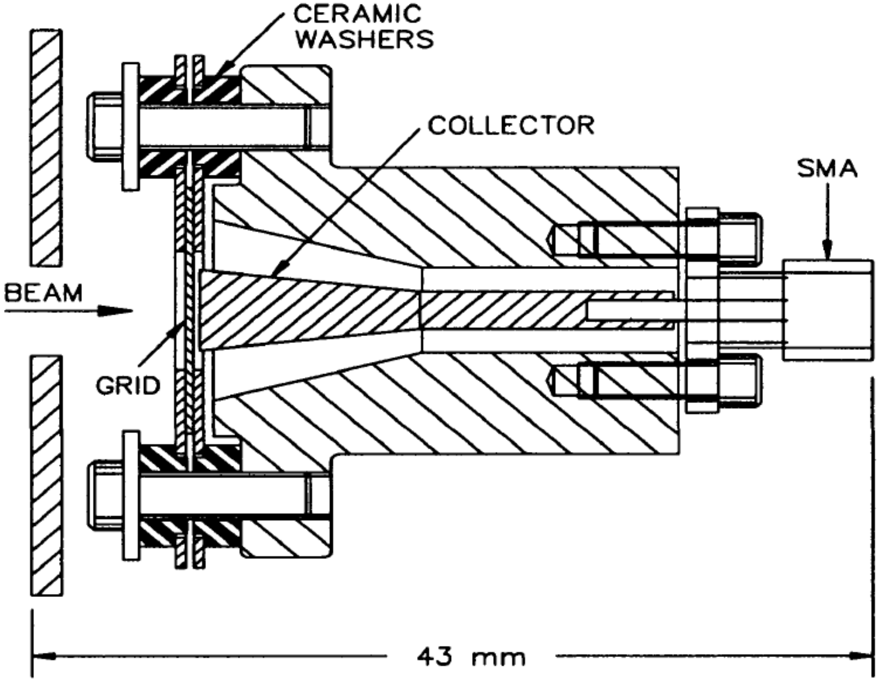
Impedance of a coaxial transmission line:

$$Z_0 = \frac{Z_c}{2\pi} \cdot \ln \frac{r_{\text{shield}}}{r_{\text{coll}}}$$

with $Z_c = \sqrt{\dfrac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$

for vacuum $Z_C = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = 377\,\Omega$

→ impedance matching to prevent for reflections



Voltage reflection: $\rho_V = \dfrac{Z - Z_0}{Z + Z_0}$

Voltage Standing Wave Ratio: $\text{VSWR} = \dfrac{Z}{Z_0} = \dfrac{1+\rho_V}{1-\rho_V}$

$\boldsymbol{Z = Z_0}$: no reflection. $\boldsymbol{Z = 0} \Rightarrow \boldsymbol{\rho_V} = -1$: short circuit. $\boldsymbol{Z} = \infty \Rightarrow \boldsymbol{\rho_V} = 1$: open circuit.

Realization of a Broadband coaxial Faraday Cup

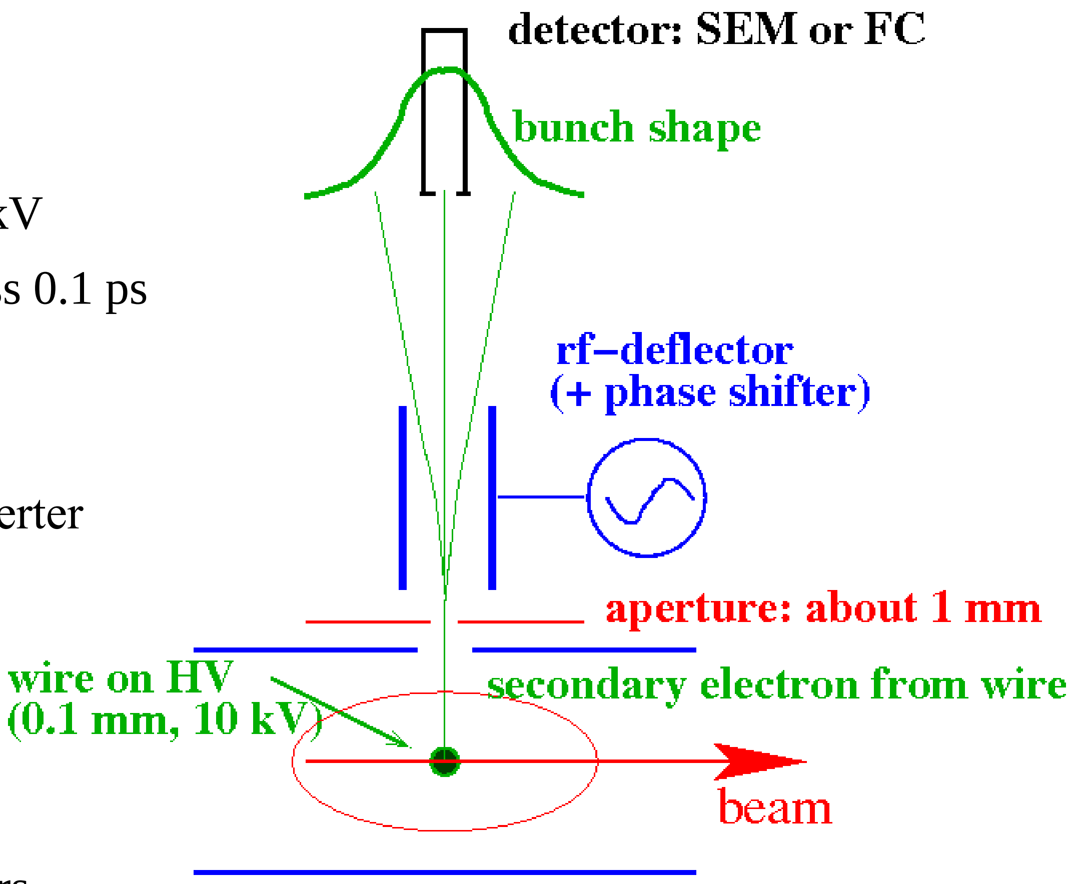
Bunch Structure using secondary Electrons for low E_{kin} Protons

Secondary e^- liberated from a wire carrying the time information.

→ Bunch Shape Monitor (BSM)

Working principle:

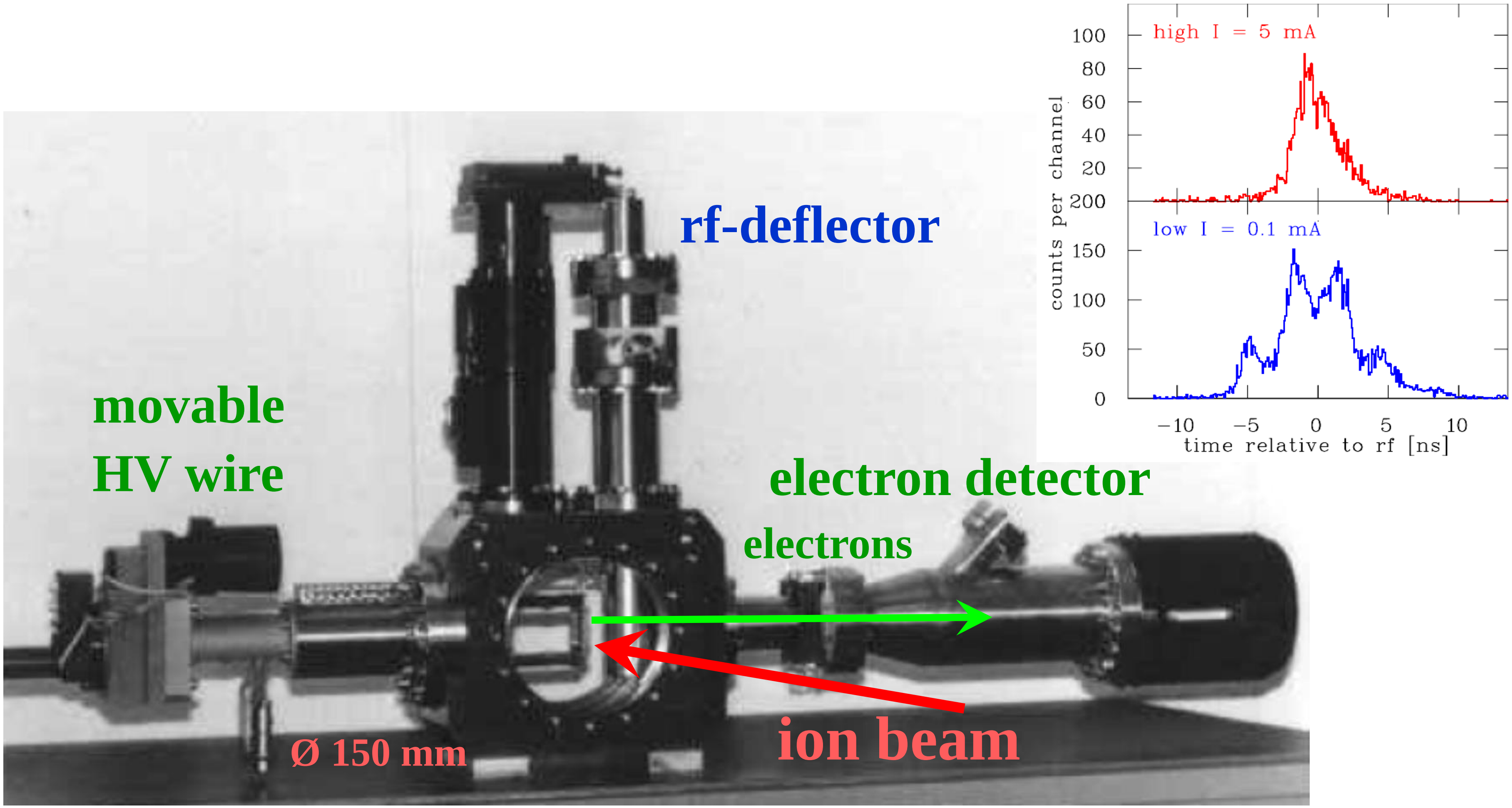
- ➢ insertion of a 0.1 mm wire at ≈ 10 kV
- ➢ emission of secondary e^- within less 0.1 ps
- ➢ secondary e^- are accelerated
- ➢ toward an rf-deflector
- ➢ rf-deflector as 'time-to-space' converter
- ➢ detector with a thin slit
- ➢ slow shift of the phase
- ➢ resolution $\approx 1^o < 10$ ps
- ➢ Measurements are comparable to that obtained with particle detectors.



SEM: secondary electron multiplier

Realization of Bunch Shape Monitor at CERN LINAC2

Example: The bunch shape at 120 keV/u for 120 keV/u:



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  Synchrotron light monitor and electro-optical modulation of a laser beam
- Summary

Excurse: 3rd and 4th Generation Light Sources

3rd Generation Light Sources:

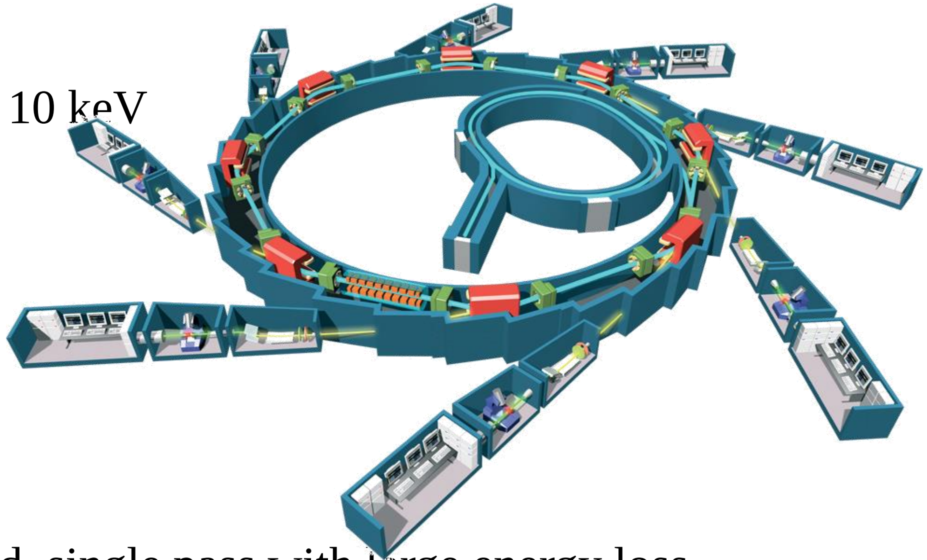
Example: Soleil, Paris, $E_{electron}$= 2.5 GeV, C = 354 m

Synchrotron-based with $E_{electron} \approx$ 1…8 GeV

Light from dipoles, undulators& wigglers, E_γ < 10 keV

Users: biology, chemistry, material science, solid state and atomic physics

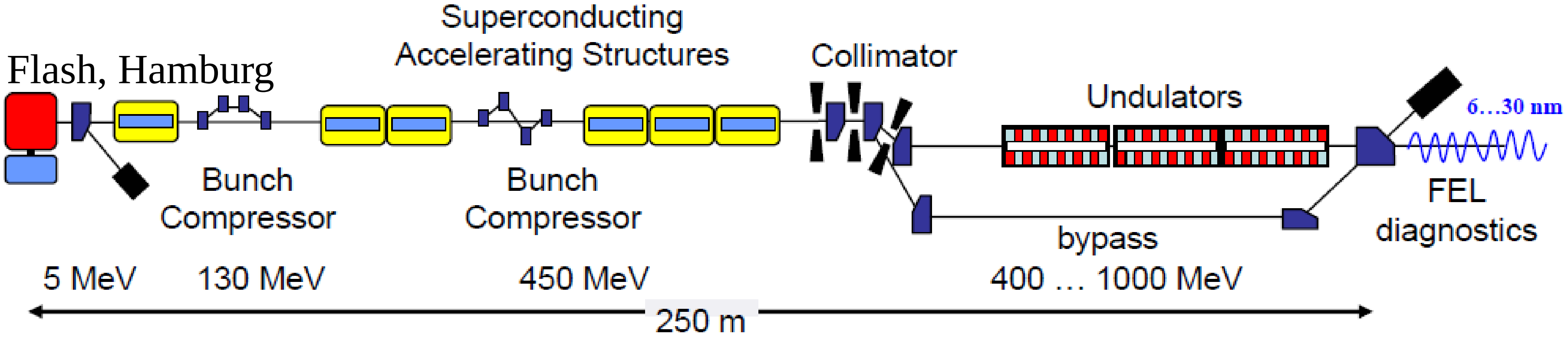
National facilities in many counties, some international facilities.



4th Generation Light Sources: LINAC based, single pass with large energy loss

$E_{electron} \approx$ 1 … 18 GeV, **coherent** light from undulator, E_γ < 1000 keV range, short pulse

Europe: Germany, Italy, Netherlands, Switzerland, **America:** USA**, Asia:** China, Japan …

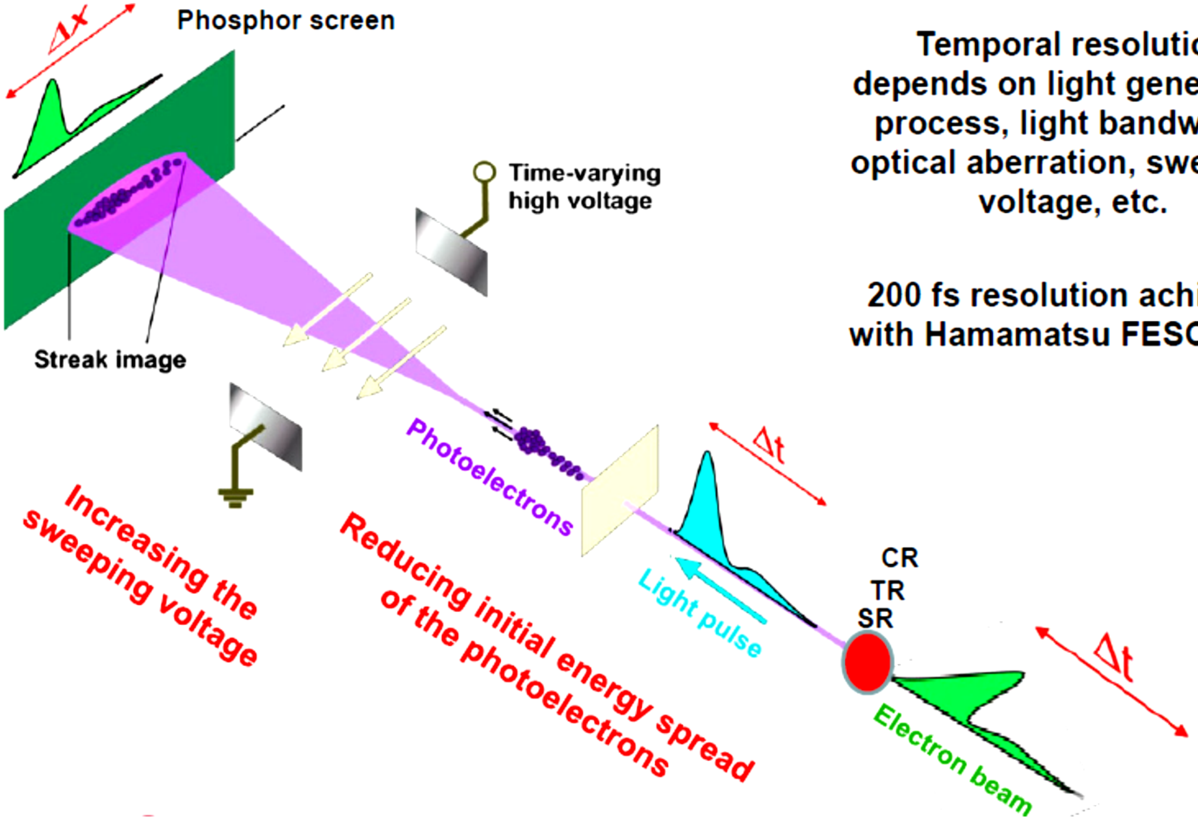


Flash, Hamburg

Bunch Length Measurement for relativistic e^-

Electron bunches are too short (σ_t < 300 ps) to be covered by the bandwidth of pick-ups (f < 1 GHz ⇔ t_{rise} >300 ps) for structure determination.

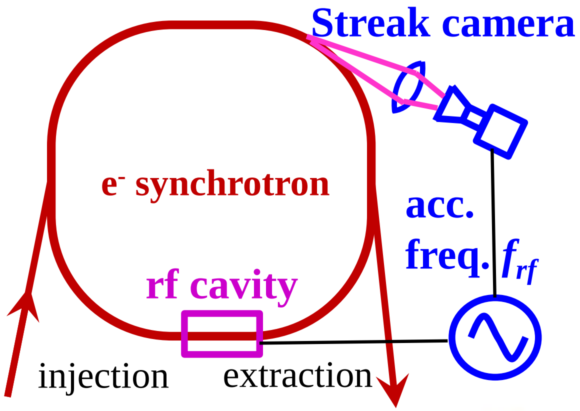
→ Time resolved observation of synchr. light with a streak camera: Resolution ≈ 1 ps.



Temporal resolution depends on light generation process, light bandwidth, optical aberration, sweeping voltage, etc.

200 fs resolution achieved with Hamamatsu FESCA-200

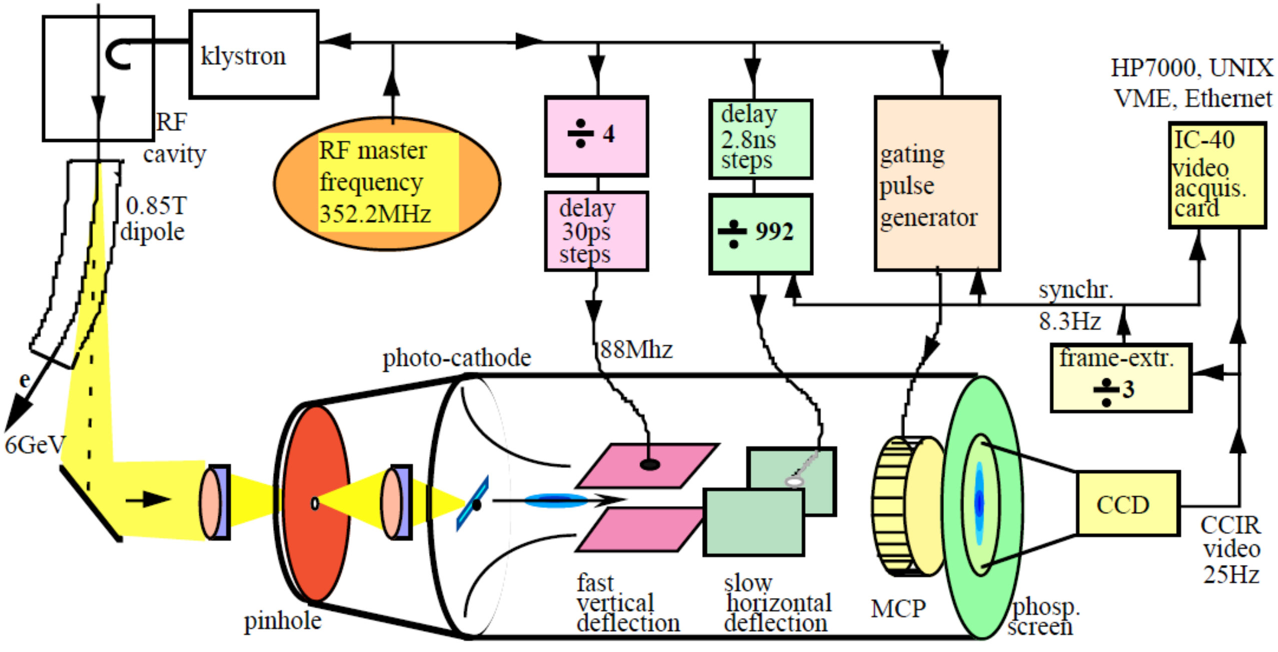
From D. Xiang, IPAC'12

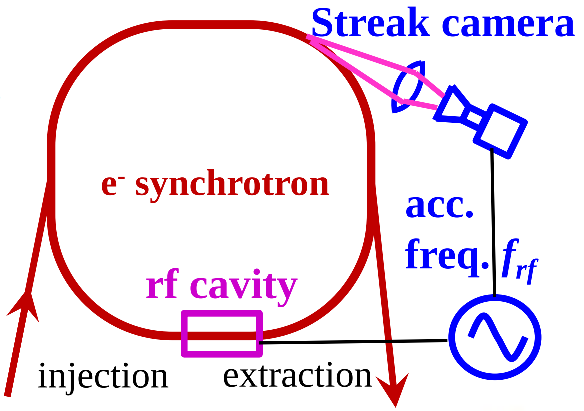


Bunch Length Measurement for relativistic e^-

Electron bunches are too short (σ_t < 300 ps) to be covered by the bandwidth of pick-ups (f < 1 GHz ⇔ t_{rise} >300 ps) for structure determination.

→ Time resolved observation of synchr. light with a streak camera: Resolution ≈ 1 ps.





Technical Realization of Streak Camera

acceleration

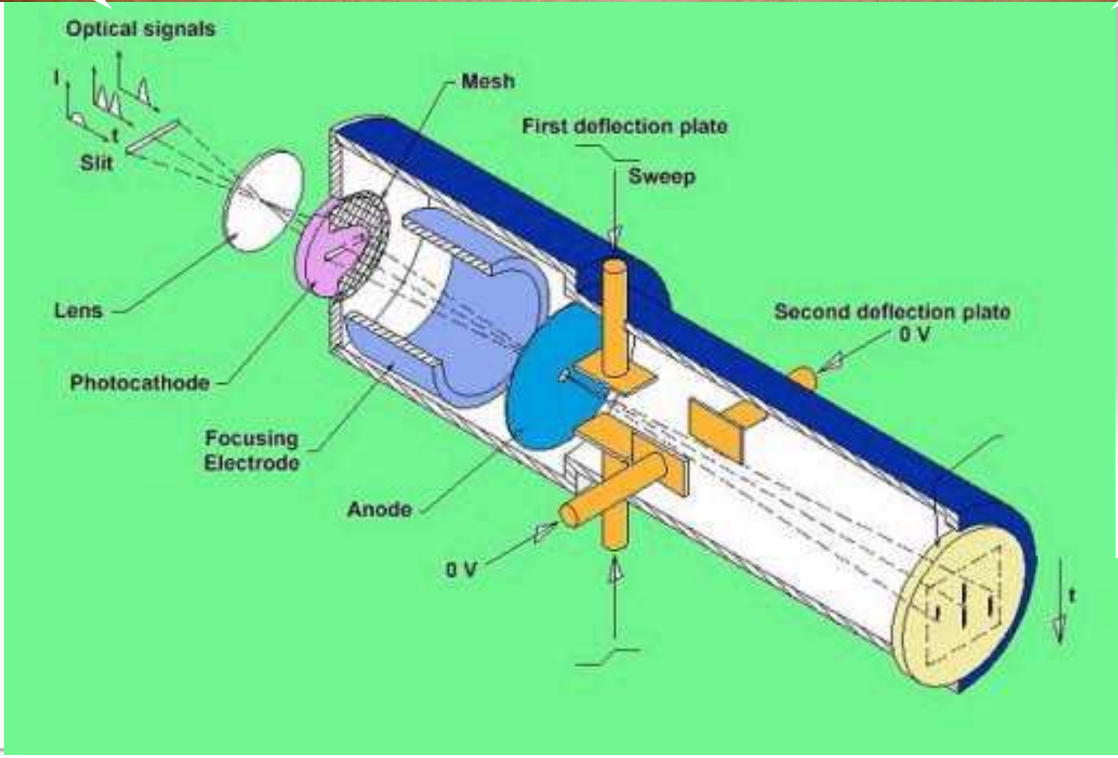
focusing

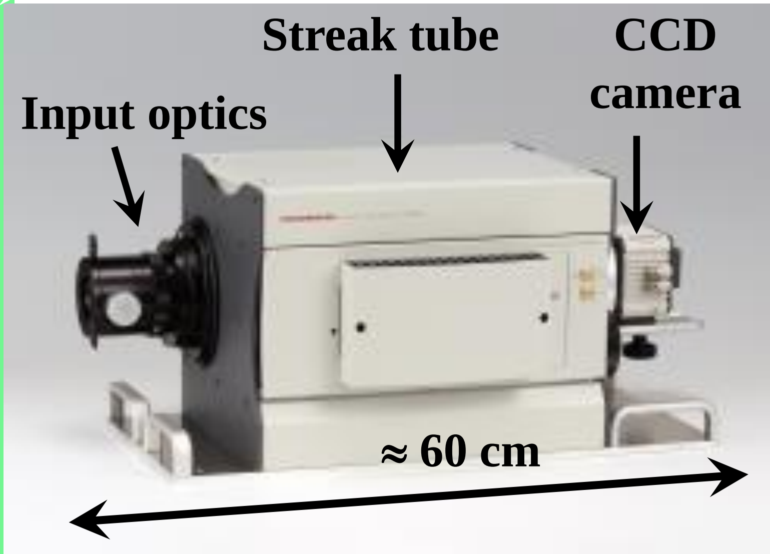
deflection

≈ 30 cm

Hardware of a streak camera

Time resolution down to 0.5 ps:





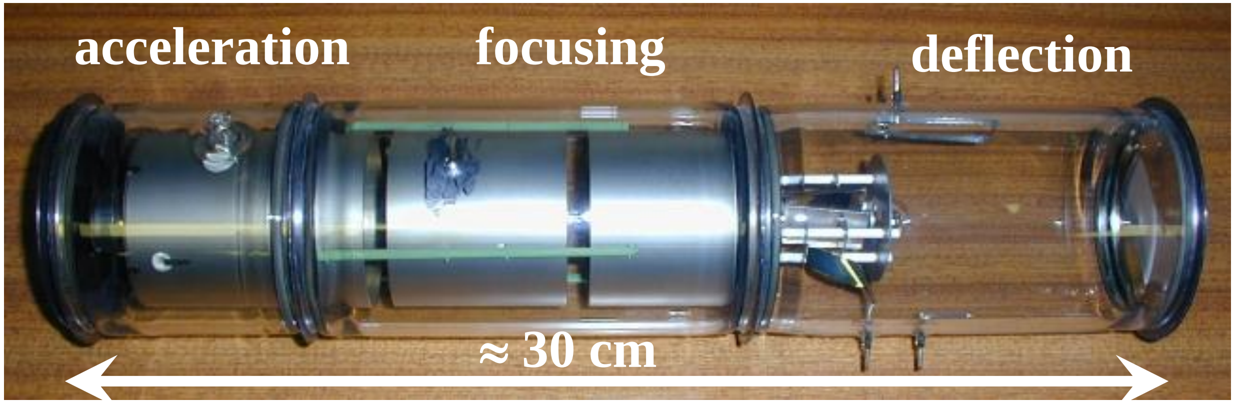
Streak tube

CCD camera

Input optics

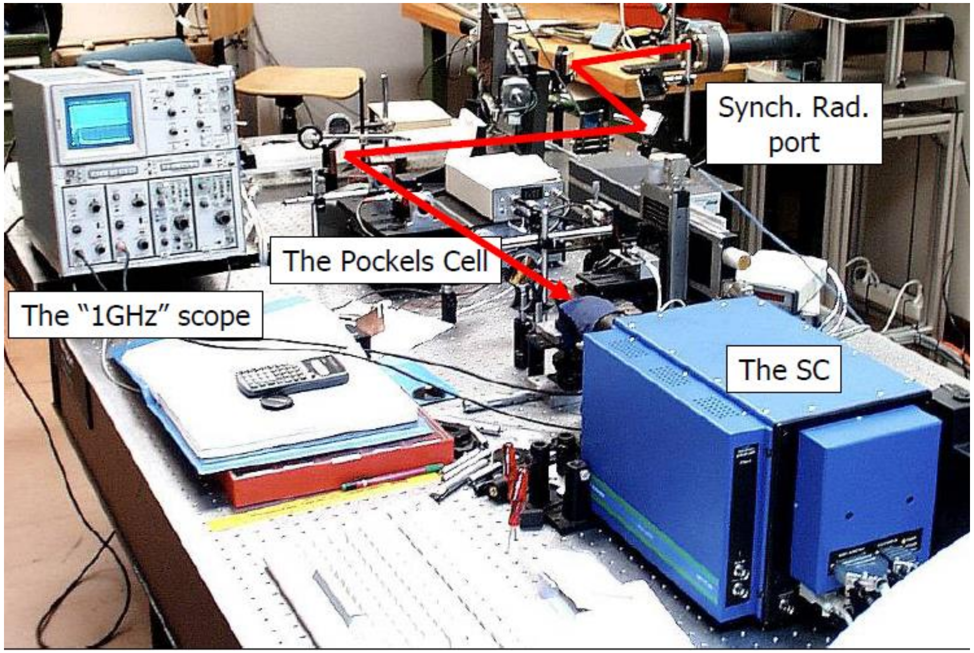
≈ 60 cm

Technical Realization of Streak Camera

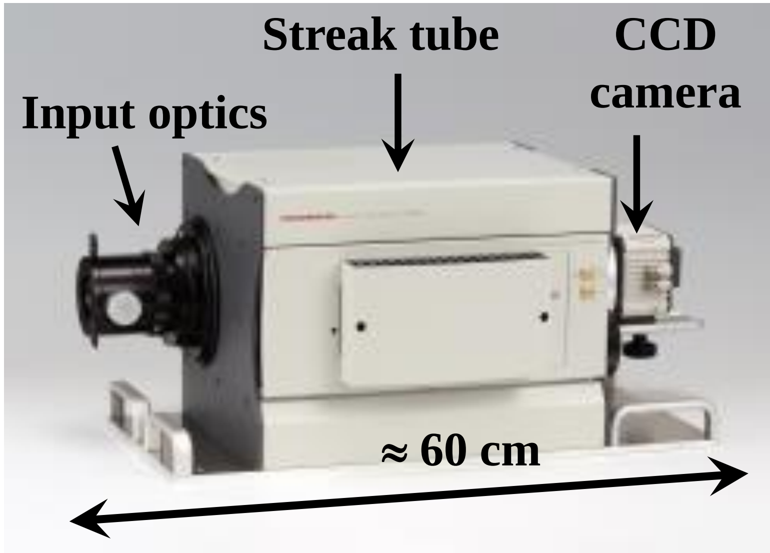


Hardware of a streak camera

Time resolution down to 0.5 ps:



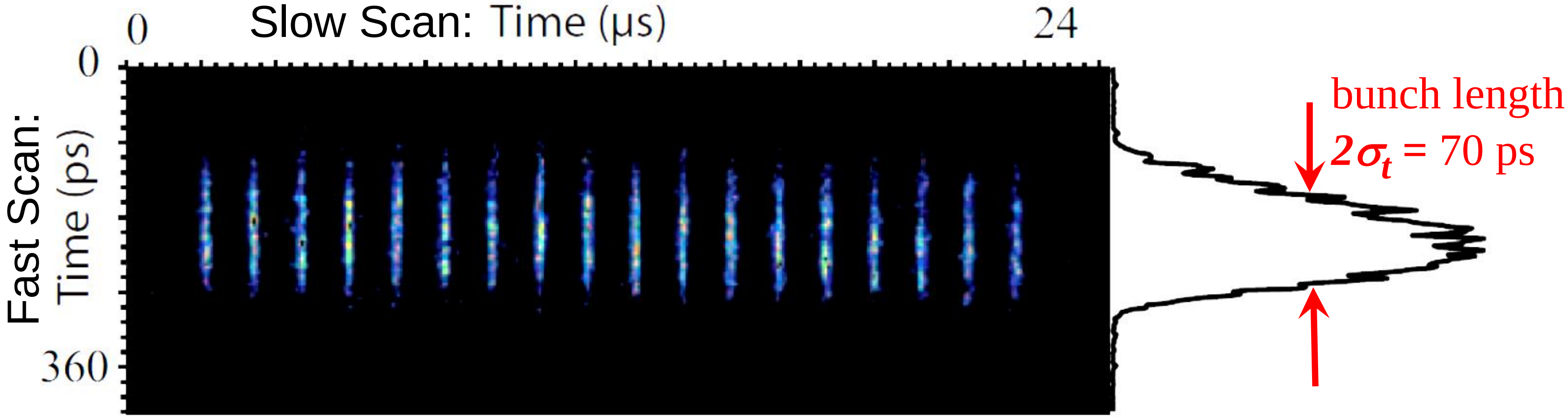
The Streak Camera setup at ELETTRA, Trieste, Italy



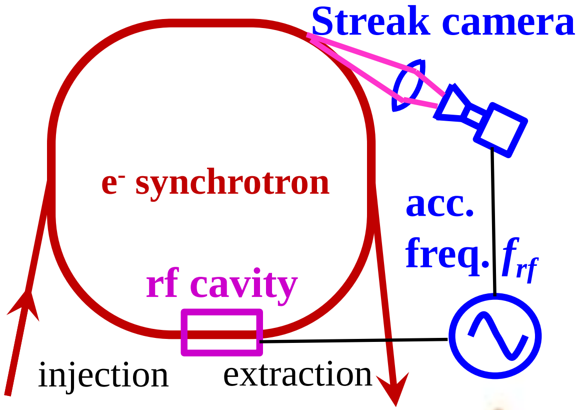
Results of Bunch Length Measurement by a Streak Camera

The streak camera delivers a fast scan in vertical direction (here 360 ps full scale) and a slower scan in horizontal direction (24 μs).

Example: Bunch length at the synchrotron light source SOLEIL for $U_{rf} = 2$ MV for slow direction 24 μs and scaling for fast scan 360 ps: measure $\sigma_t = 35$ ps.



bunch length $2\sigma_t = 70$ ps



Courtesy of M. Labat et al., DIPAC'07

→ conclusion

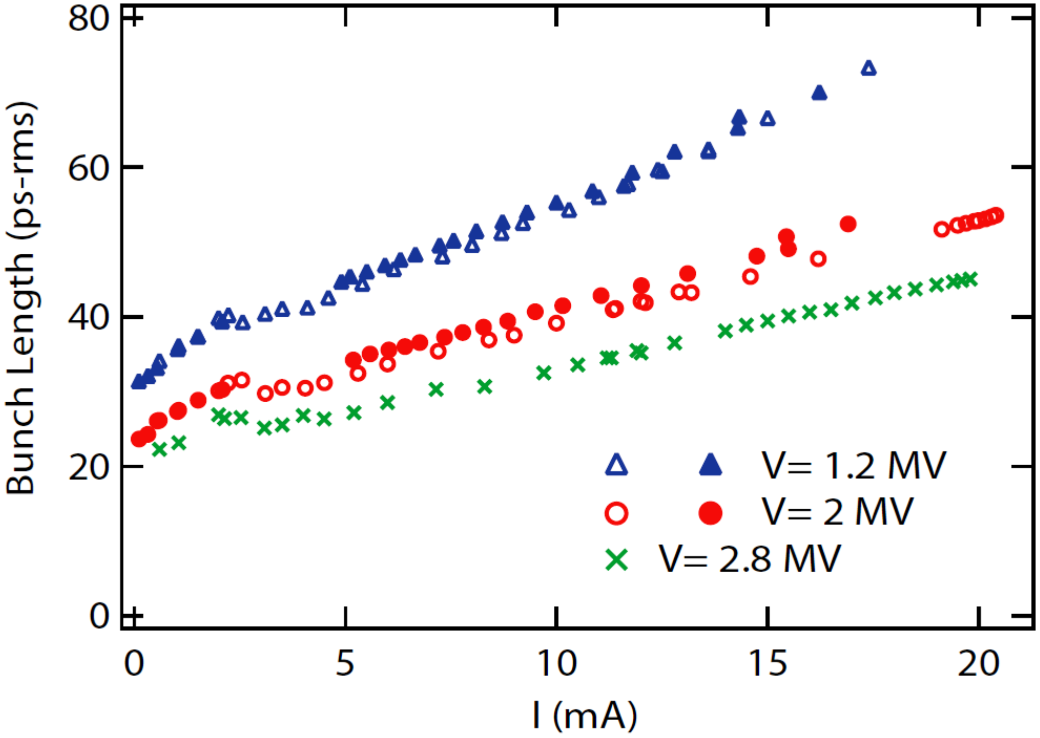
The Importance of Bunch Length by Streak Camera

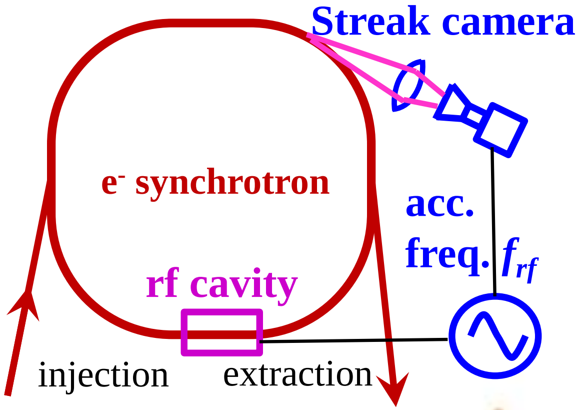
Short bunches are desired by the synchrotron light users for time resolved spectroscopy.

The bunch focusing is changed by the rf-amplitude.

Example: Bunch length σ_t as a function of stored current

(space-charge de-focusing, impedance broadening) for different rf-amplitudes at SOLEIL:





The Artist View of a Streak Camera

FARADAY CUP
1998

Purpose. To recognize and encourage innovative achievements in the field of accelerator beam instrumentation.

Award. The Faraday Cup Award consists of a US$ 5000 prize and a certificate to be presented at the next Beam Instrumentation Workshop. Winners participating in the BIW will be given a $1000 travel allowance.

Eligibility. Nominations are open to contributors of all nations regardless of the geographical location at which the work was done.
The Award goes normally to one person, but may be shared by recipients having contributed to the same accomplishment. It will normally be awarded to scientists in the early stage of their career. Nominations of candidates shall remain active for 2 competitions.

Establishment and support. The Award was established in 1991 with the support of the Beam Instrumentation Workshop Organizing Committee.

Rules. The Faraday Cup shall be awarded for an outstanding contribution to the development of an innovative beam diagnostics instrument of proven workability. The Faraday Cup is only awarded for published contribution and delivered performance -as opposed to theoretical performance. Rules are available on request.

Award Committee. The Beam Instrumentation Workshop Organizing Committee.

Nominations. The nomination package shall include the name of the candidate, relevant publications, a statement outlining his/her personal contribution and that of others, two letters from co-workers familiar with the candidate and his contribution. Two master copies suitable for photocopying of this package must be submitted not later than the 15th of November 1997 to Steven Smith c/o BIW'98 Secretariat, SLAC, Stanford University, Stanford CA 94305-4085, U.S.A..

Bunch length measurement by electro-optical methods

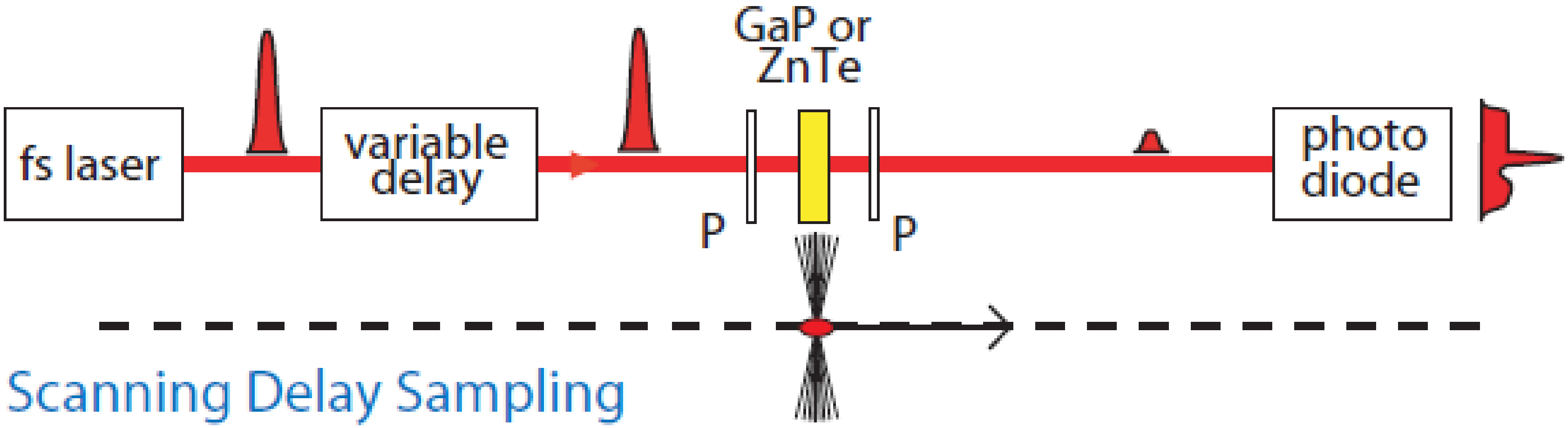
For Free Electron Lasers → bunch length below 1 ps is achieved

- → below resolution of streak camera
- → short laser pulses with $t \approx 10$ **fs** and electro-optical modulator

Electro optical modulator: birefringent, rotation angle depends on external electric field

Relativistic electron bunches: transverse field $E_{\perp,\,lab} = \gamma E_{\perp,\,rest}$ carries the time information

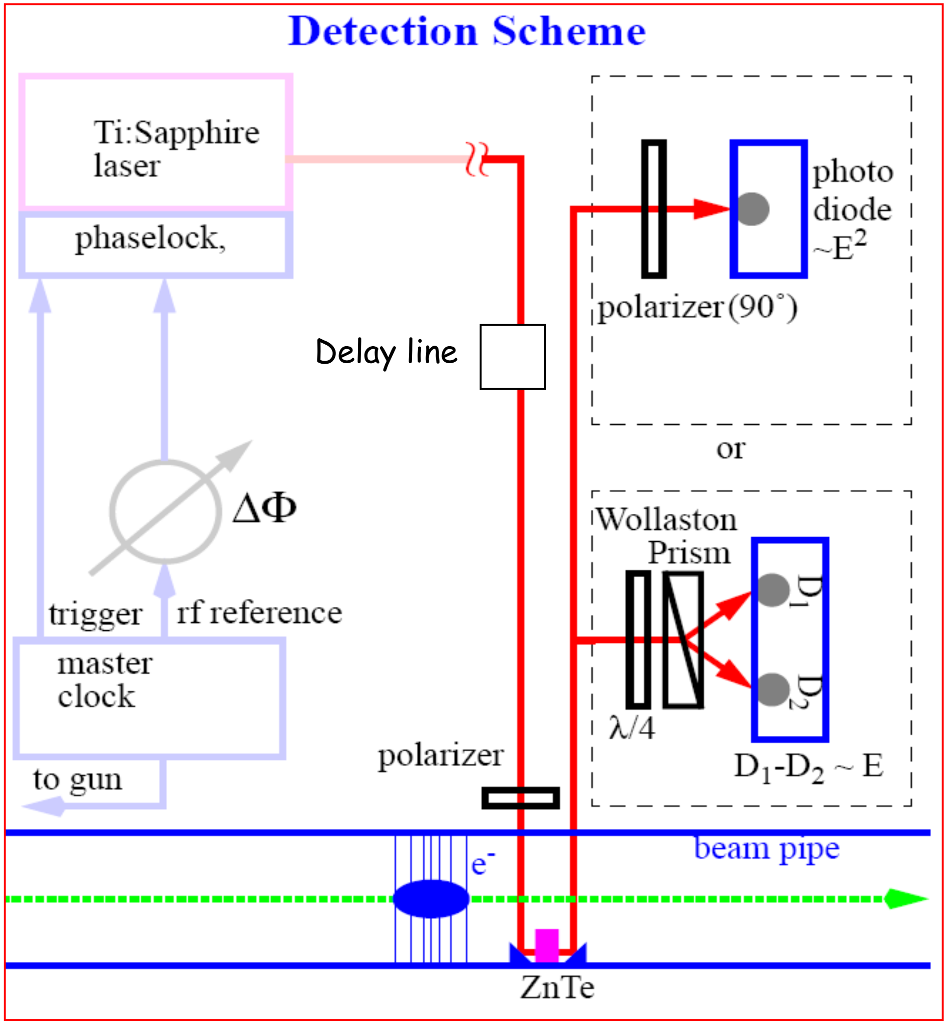
Scanning of delay between bunch and laser → time profile after several pulses.

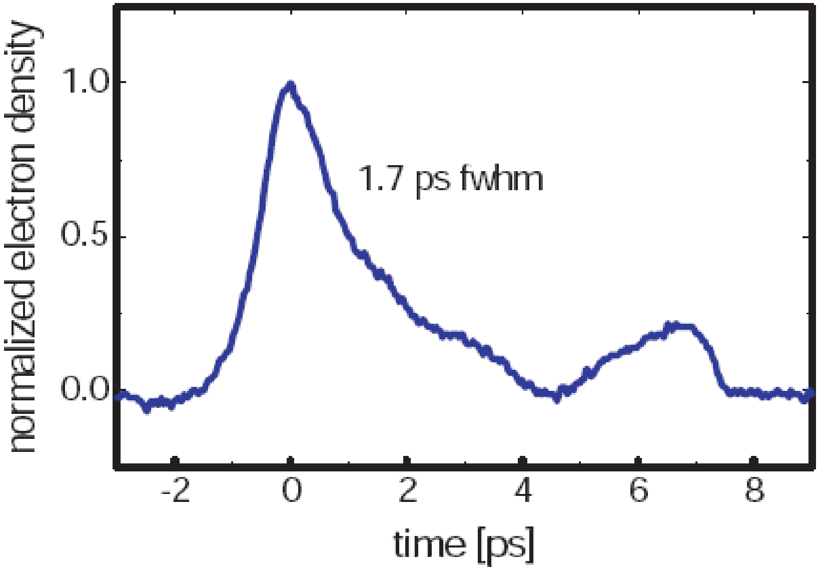


From S.P.Jamison et al., EPAC 2006

Realization of EOS Scanning

Setup of a scanning EOS method



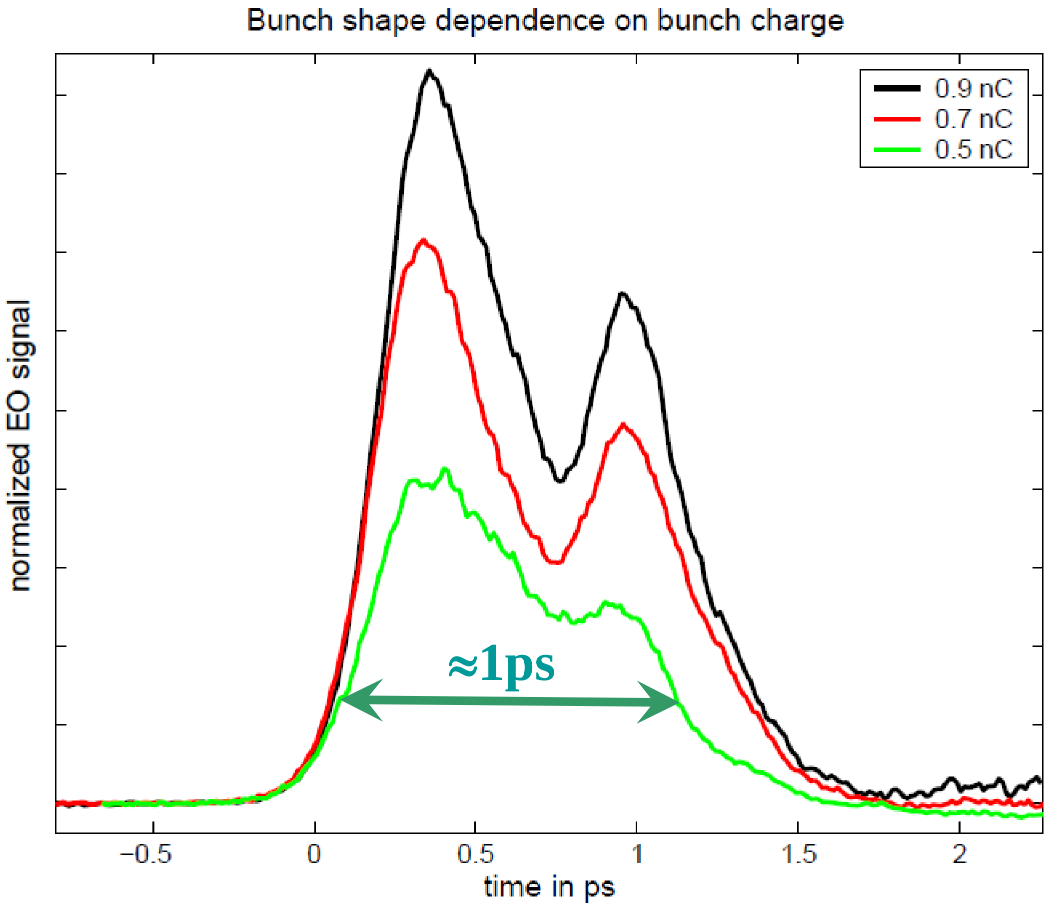


Using 12fs pulses from
Ti:$A_{12}O_3$ laser at 800nm and
ZnTe crystal 0.5mm thick
with a e^- - beam 46MeV of 200pC

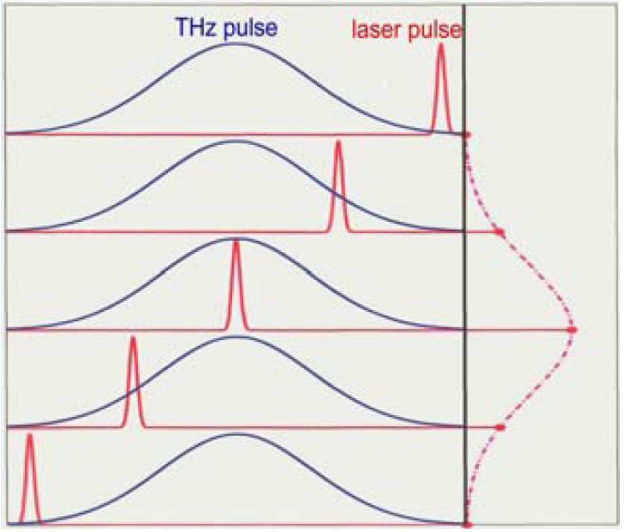
X. Yan *et al*, PRL 85, 3404 (2000)

Measurement of Bunch Shape at FEL-Facility

Example: Bunch length at FEL test facility FLASH



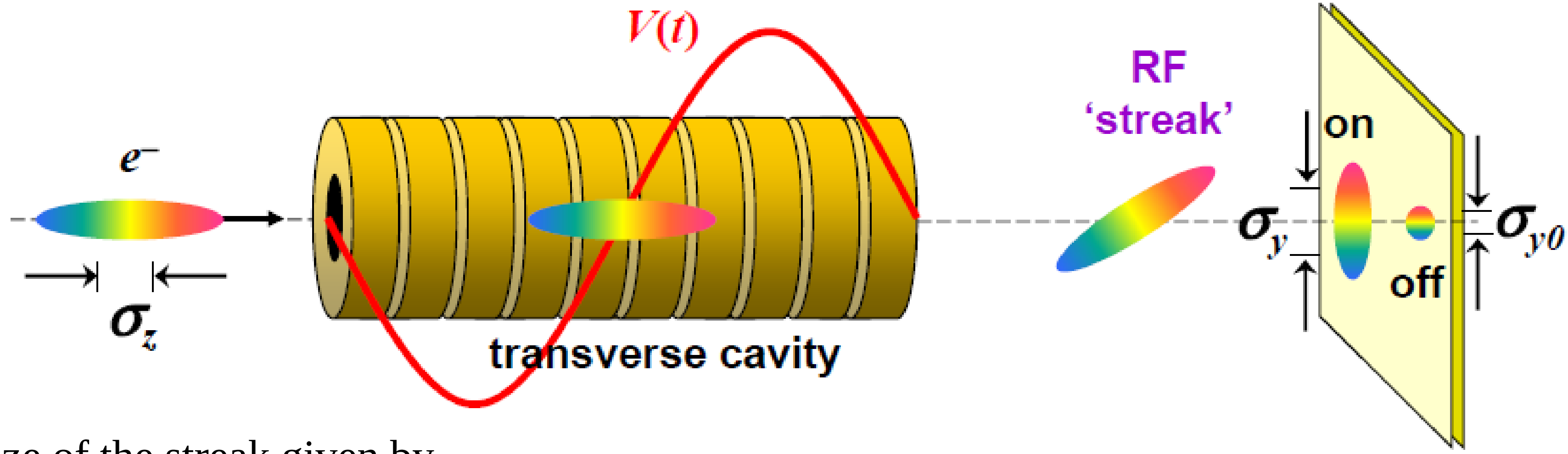
Scanning of the short laser pulse relative to bunch:



Results at FLASH, Hamburg, see B. Steffen et al., FEL Conf. Stanford, p. 549, 2005.

Bunch Length by rf-Deflection: Principle

Transversal deflection of the bunch i.e. time-to-space conversion

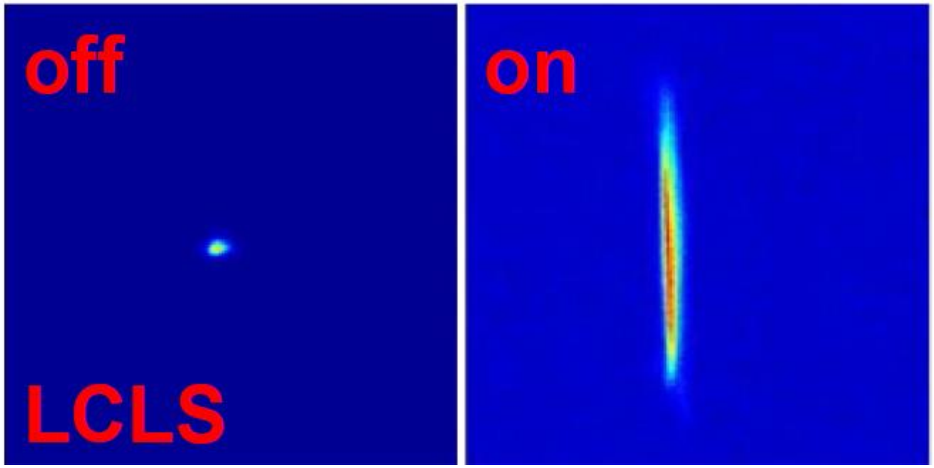


Size of the streak given by

$$\sigma_y = \sqrt{\sigma_{y0}^2 + R_{35} \cdot k \cdot \sigma_z^2}$$

k is determined by the rf-power

$$k = \frac{2\pi e \cdot U_{rf}}{\lambda_{rf} E}$$



From D. Xiang, IPAC'12

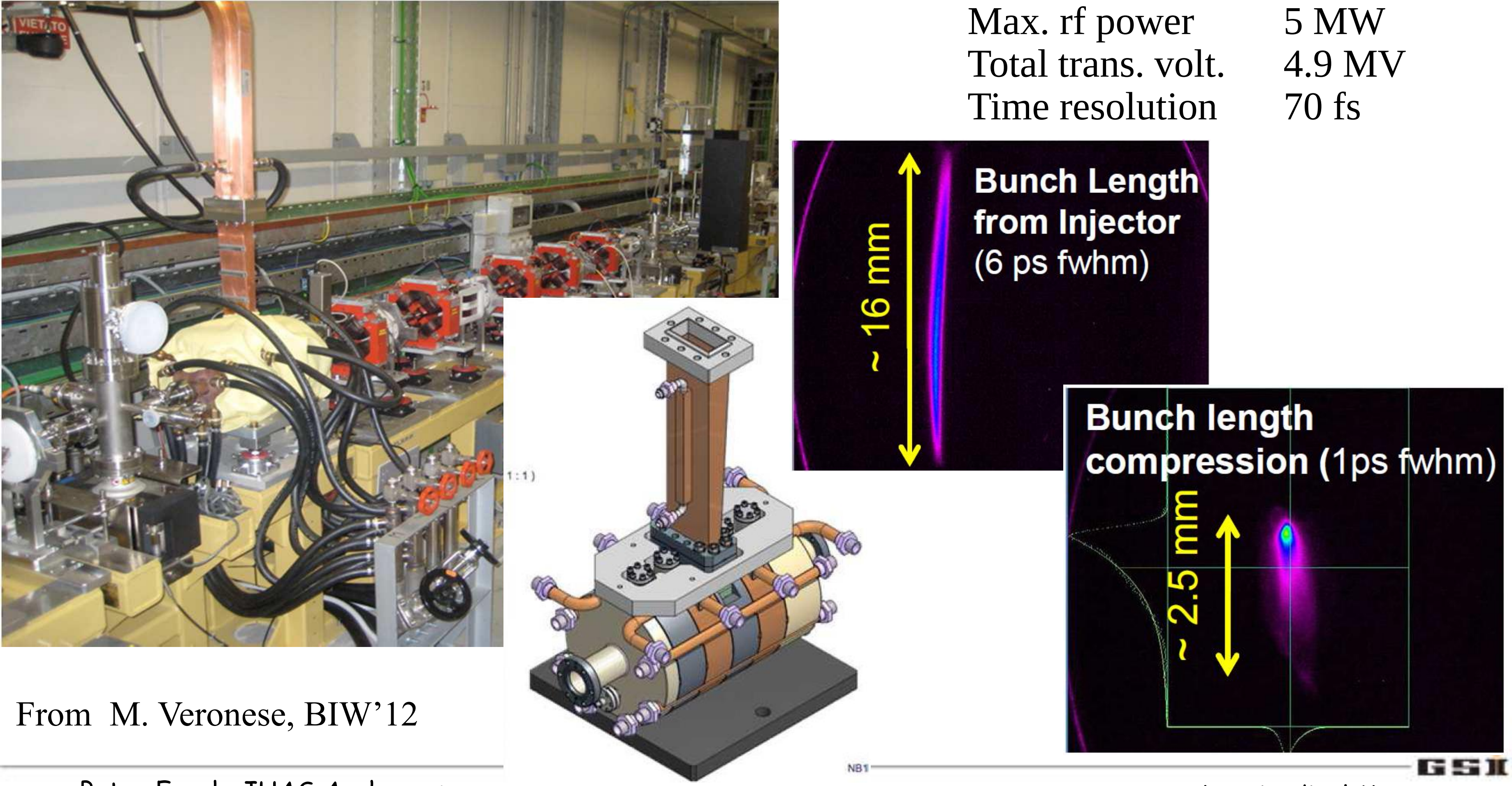
Bunch Length by rf-Deflection: Hardware

Transversal deflection of the bunch

i.e. time-to-space conversion

Example: Cavity at FERMI, Trieste, Italy

Beam energy	320 MeV
Typical beam size	0.2 mm
Length	0.5 m
Frequency	2.998 GHz
Max. rf power	5 MW
Total trans. volt.	4.9 MV
Time resolution	70 fs



From M. Veronese, BIW'12

Summary of longitudinal Measurements

Longitudinal ↔ transverse correspondences:

- position relative to rf ↔ transverse center-of-mass
- bunch structure in time ↔ transverse profile in space
- momentum or energy spread ↔ transverse divergence.

Determination uses:

Broadband pick-ups:
- position relative to rf, mean energy
- emittance at transfer lines or synchrotron via tomography

  assumption: bunches longer than pick-up.

Particle detectors:
- TOF or secondary e^- from wire

  → for non-relativistic proton beams

  reason: ***E***-field does not reflect bunch shape.

Streak cameras:
- time resolved monitoring of synchrotron radiation

  → for relativistic e^--beams, $t_{bunch} < 1$ ns

  reason: too short bunches for rf electronics.

Laser scanning:
- Electro-optical modulation of short laser pulse

  → very high time resolution

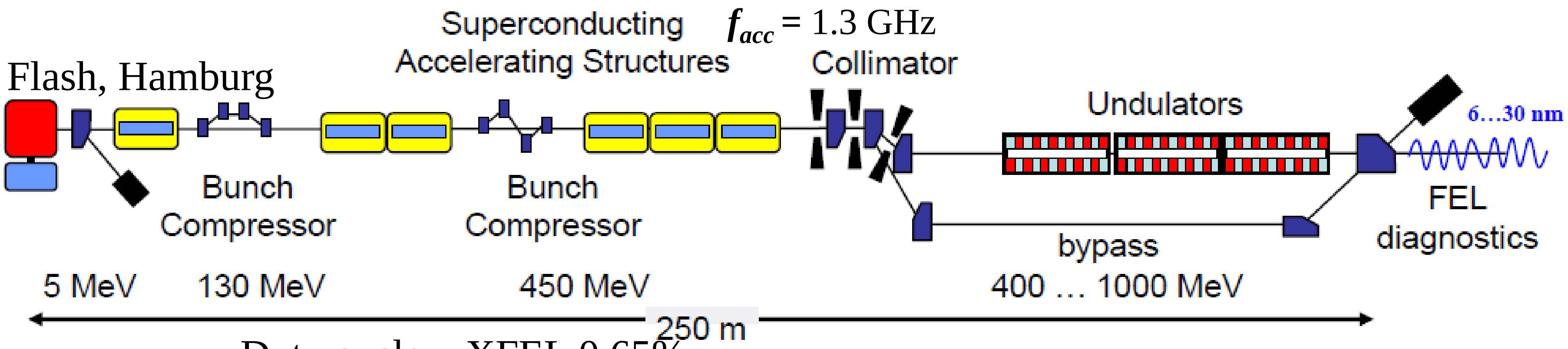
Beam deflection:
- Transverse deflection of primary beam

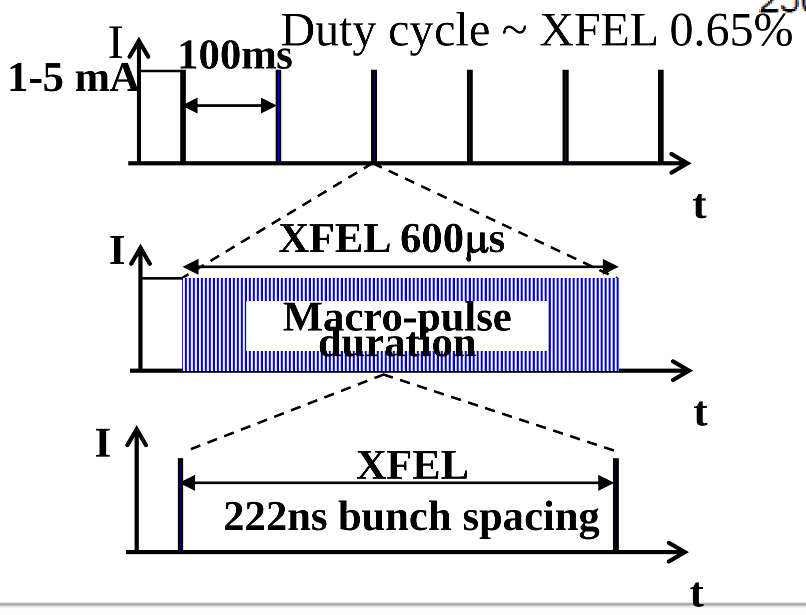
  → very high time resolution, but most expensive 'device'.

Excurse: 4th Generation Light Sources & Beam Delivery

4th Generation Light Sources: LINAC based, single pass with large energy loss

$E_{electron} \approx 1 \ldots 18$ GeV, **coherent** light from undulator, $E_\gamma < 1000$ keV, temporally short pulse





Goal: Short bunches with **high** number of particles

→ short, intense laser pulses for electron generation

Requirement: Position stability ⇒ resolution < 1 μm

Single bunch duration < 1 ps