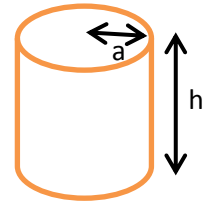


JUAS 2019 – Exercise Solution

$$\begin{aligned}\mu &= \mu_0 \mu_r \\ \mu_0 &= 4\pi \cdot 10^{-7} \text{ Vs/(Am)} \\ \varepsilon &= \varepsilon_0 \varepsilon_r \\ \varepsilon_0 &= 8.854 \cdot 10^{-12} \text{ As/(Vm)} \\ c_0 &= 2.998 \cdot 10^8 \text{ m/s}\end{aligned}$$

1.) Cavities

Design a pillbox cavity. The $E_{010} = TM_{010}$ mode shall resonate at $f_{res} = 2.95$ GHz. The aspect ratio shall be $a/h = 0.5$.



1. What is the radius a of the cavity?

$$a = 0.383 \lambda = \frac{0.383 c}{f} = 38.92 \text{ mm}$$

see script page 47

2. What is the height h of the cavity?

$$h = 2 a = 77.85 \text{ mm}$$

3. The 3 dB bandwidth of the unloaded resonance shall be 150 kHz, how big does the unloaded Q_0 of the cavity need to be?

$$Q_0 = \frac{f_{res}}{f_{3dB}} = 19667$$

see script page 72

4. What is the maximum tolerable surface resistance ρ of the cavity walls to get $Q_0 = 20\,000$.

$$\delta = \frac{a}{Q_0} \left[1 + \frac{a}{h} \right]^{-1} = 1.23 \text{ } \mu\text{m} = \sqrt{\frac{2}{\omega \sigma \mu}}$$

see script page 49

$$\sigma = \frac{2}{\delta^2 2 \pi f_{res} \mu_0} = 51 \cdot 10^6 \text{ S/m}$$

see script page 84

$$\rho = \frac{1}{\sigma} = 19.6 \cdot 10^{-9} \text{ } \Omega\text{m}$$

Note: the resistivity ρ is inverse proportional to the conductivity σ

5. What is the R/Q for this cavity geometry?

$$\frac{R}{Q} = 128 \frac{\sin^2(1.2024 h/a)}{h/a} = 28.9$$

see script page 50

Note: We cannot use the approximate equation as the argument $x \approx 2.4$ in $\sin(x)$ has a large value and therefore $\sin(x) \neq x$

Keep in mind, the R/Q is purely defined by the geometry of the cavity, while the unloaded Q_0 depends on the losses defined by the material properties.

6. Derive the equivalent circuit parameters R, L and C for the cavity with $Q_0 = 20\,000$ and $R/Q = 30$

$$\frac{R}{Q} = \omega_{res} L = \frac{1}{\omega_{res} C}$$

$$L = \frac{R/Q}{2\pi f_{res}} = 1.62 \text{ nH}$$

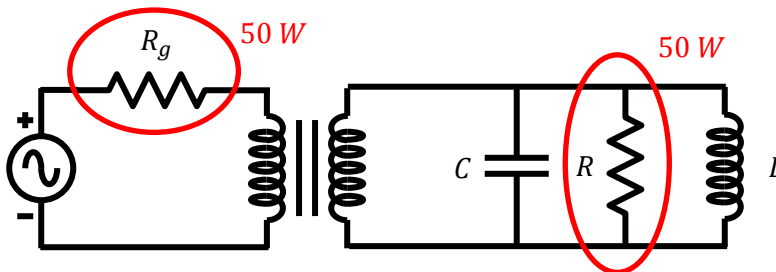
$$C = \frac{1}{2\pi f_{res} R/Q} = 1.8 \text{ pF}$$

$$R = \frac{R}{Q} Q_0 = 600 \text{ k}\Omega$$

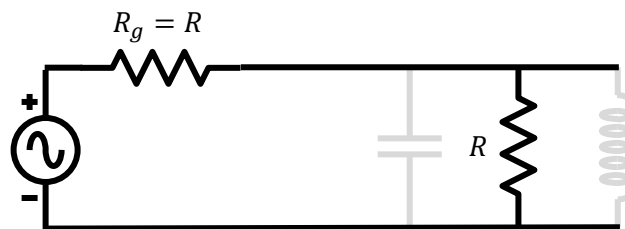
see script page 67

7. The cavity is critically coupled to an RF power amplifier and driven by 50 W of input power on its resonant frequency. The loaded Q_L is 10 000. What is the stored energy W_{cav} in the cavity?

Equivalent circuit:



At critical coupling at resonance this simplifies to:



The transformed source impedance of the generator is equal to the shunt impedance of the resonator $R_g = R$, therefore the total impedance is $R/2$, and the power is $P = 2 \cdot 50 \text{ W} = 100 \text{ W}$.

$$W_{cav} = \frac{Q_L P}{2\pi f_{res}} = 54 \mu\text{Ws}$$

or

$$W_{cav} = \frac{1}{2} C \hat{V}^2 = 54 \mu\text{Ws} \quad \text{with: } \hat{V} = \sqrt{2PR}$$

see script page 67

8. Determine the peak gap voltage V_{gap} ?

Note: Two ways to compute the gap voltage, both using the equations on page 67:

$$V_{gap} = V = \sqrt{P \frac{2R}{2}} = 5.47 \text{ kV}$$

Note: The cavity is critically coupled, i.e. the transformed source impedance (see schematic page 66) equals the cavity resistance R. As both are parallel the total resistance is R/2

$$V_{gap} = V_C = \sqrt{\frac{2W_{cav}}{C}} = 5.47 \text{ kV}$$

see script page 67

9. Operating the cavity in air, is "Kilpatrick" voltage breakdown a problem?

gap = $h = 78 \text{ mm}$, $V_{gap} = 5.5 \text{ kV}$, seems to be no problem

see script page 102

Note: The gap with is $h \approx 78 \text{ mm}$, and the voltage is $\sim 5.5 \text{ kV}$, operating in air.

The graphics shows this operating point is safe, no breakdowns to be expected

2.) Smith chart

Given in the table below are several impedances Z_N or reflection coefficients Γ_N , measured at specific frequencies.

They have been normalized already with the characteristic impedance $Z_C = 50 \Omega$ in the following way:

$$Z_N = Z/Z_C$$

Point no.	P₁	P₂	P₃	P₄
Z_N or Γ_N	$Z_N = 1$	$Z_N = 0.5 + 0.5j$	$ \Gamma_N = 0.45$ $\text{arc}(\Gamma_N) = -117^\circ$	$Z_N = 0$
f [GHz]	3.000	2.997	3.003	0.01

Point no.	P₅	P₆	P₇	P₈
Z_N or Γ_N	$Z_N = 0.2 + 0.4j$	$Z_N = 0.2 - 0.4j$	$ \Gamma_N = 1$ $\text{arc}(\Gamma_N) = 0^\circ$	$ \Gamma_N = 0.49$ $\text{arc}(\Gamma_N) = 52^\circ$
f [GHz]	2.994	3.006	-	-

1. Mark all the points in the attached smith chart (including each point no.)

Use your compass and the rulers at the bottom of the smith chart for the polar coordinates

see script page 118 ff

2. Design a matching circuit. The point P_2 in the smith chart shall be matched to $Z_c = 50 \Omega$. Use only **one 50 Ω transmission line** and **one series capacitor**. Draw the solution in the smith chart. Numerical answers are not needed.

see script page 125 and 127

3. Draw a circle through the points P1 – P6. This is the result of a S_{11} measurement of a microwave cavity with a network analyser.

What is the resonant frequency f_{res} of the cavity? (Look it up in the table)

see script page 140

Note: At resonance the impedance is purely real (no imaginary components)

$$f_{res} = 3 \text{ GHz (P}_1\text{)}$$

4. What is the unloaded Q_0 of the cavity?

$$Q_L = \frac{f_{res}}{f_{3dB}} = \frac{3 \text{ GHz}}{(3.006 - 2.994) \text{ GHz}} = 250$$

$$Q_0 = 2 Q_L = 500$$

see script page 140

Note: At critical coupling the unloaded Q_0 is twice the loaded

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES

