JUAS 2019 – Tutorial 2

$\mu = \mu_0\, \mu_r$

$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am)

$\varepsilon = \varepsilon_0\, \varepsilon_r$

$\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/(Vm)

$c_0 = 2.998 \cdot 10^{8}$ m/s

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1.) S-Parameters

Match the following S-Matrices to the corresponding components

$$S_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \qquad S_3 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \qquad S_4 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Component	**Isolator**	**Circulator**	**Transmission line, length = λ/2**	**3-dB attenuator**
S-matrix				

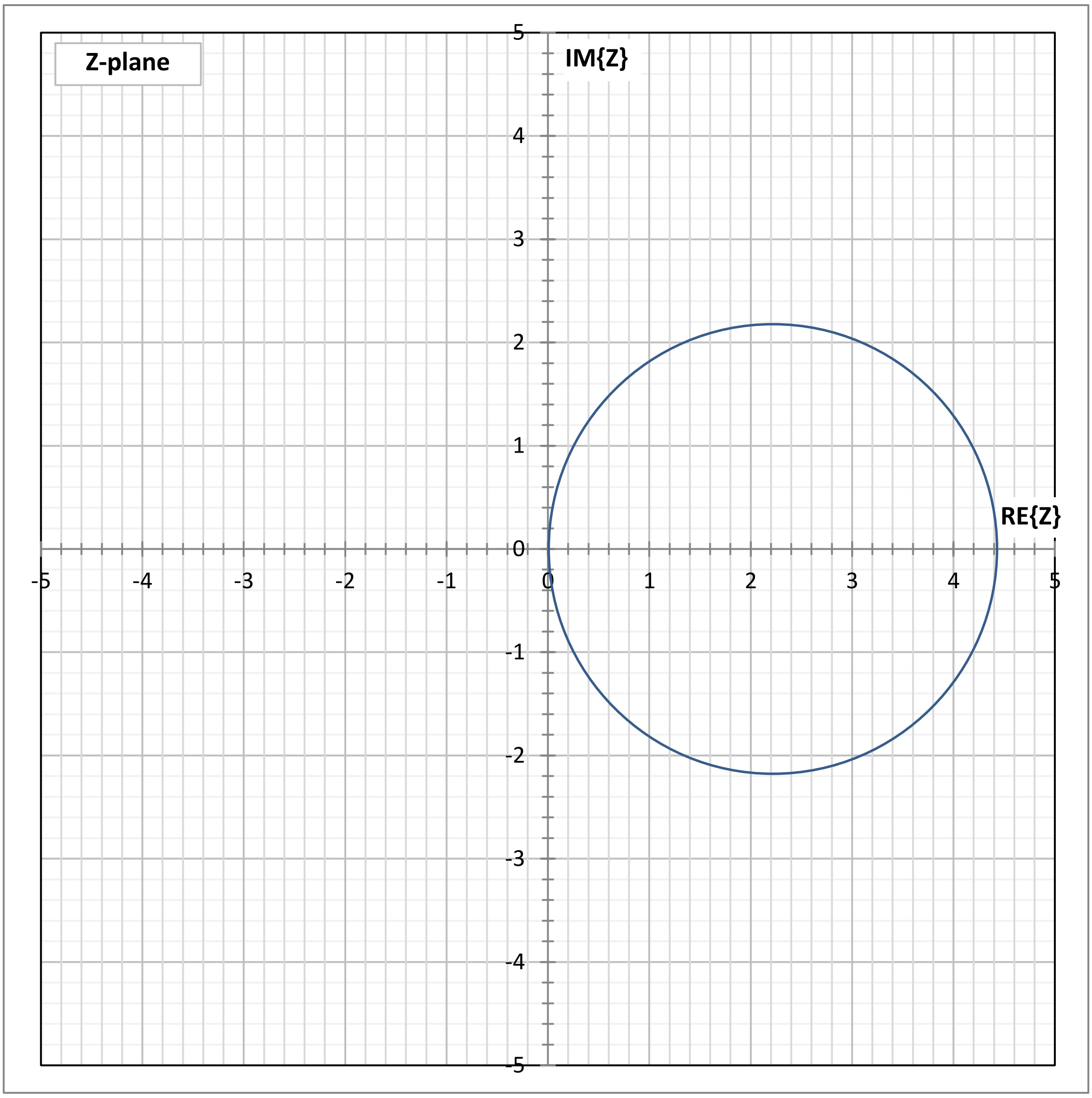
2.) Impedances in the complex plane

Questions:

1. Plot the following impedances in the *Z*-plane, use the plot axes on the next page:

$Z = (3 + 4\,j)\ \Omega$	$\lvert Z\rvert = 2$, $\arg(Z) = \pi/4$	Z = short circuit
$Z = 2\ \Omega$	$\lvert Z\rvert = 1$, $\arg(Z) = -\pi/2$	$Y = Z^{-1} = (0.16 + 0.12j)\ \Omega^{-1}$
$Z = (1 - 4\,j)\ \Omega$	$\lvert Z\rvert = 5$, $\arg(Z) = 53°$	

2. Qualitatively, how would an inductor look like, plotted from DC to some arbitrary frequency, in the *Z*-plane? Hint: $Z_L = j\omega L$
3. How would a capacitor look like? Hint: $Z_C = 1/(j\omega C)$
4. The input impedance of a RLC circuit has been plotted in the *Z*-plane (blue circle). Mark the points in the diagram describing:
    a. Impedance at the resonant frequency
    b. DC impedance
    c. 3-dB bandwidth
    d. Impedance at $f \rightarrow \infty$


Z-plane
IM{Z}
RE{Z}
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
-4
-3
-2
-1
0
1
2
3
4
5

Important Points:

Short Circuit

Γ = -1, *z* = 0

Open Circuit

Γ = +1, *z* → ∞

Matched Load

Γ = 0, *z* = 1

On the circle Γ = 1:

lossless element

Upper half:

”inductive” =

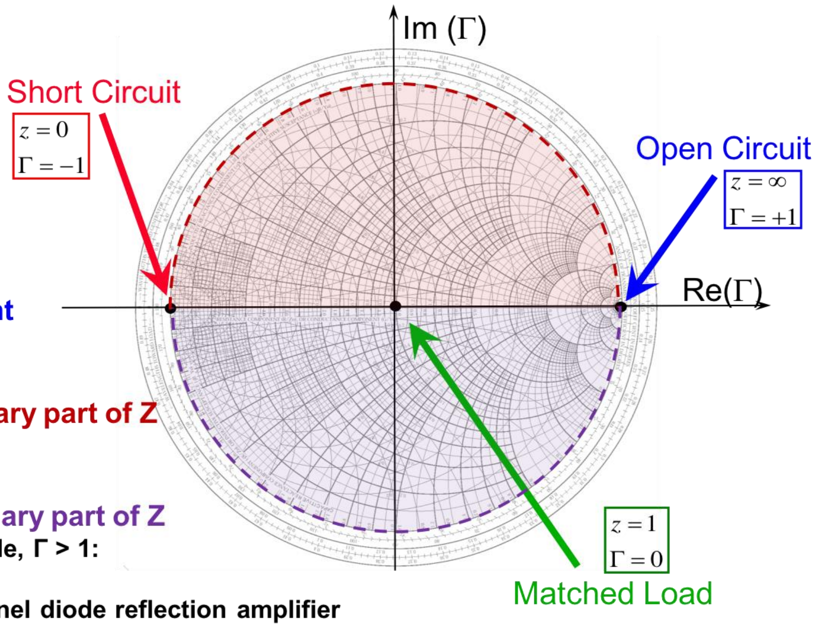
positive imaginary part of Z

Lower half:

”capacitive” =

negative imaginary part of Z

- **Outside the circle, Γ > 1:**
  active element,
  for instance tunnel diode reflection amplifier



3.1.) Smith Chart (1)

1. Mark the reflection factors *Γ* of points A to F in the Smith Chart and find approximate values for the corresponding (normalized) impedances z:

Point	Reflection factor *Γ*	Normalized impedance *z*
A	1 \0°	
B	1 \45°	
C	1 \90°	
D	1 \180°	
E	1 \-90°	
F	0.5	

3.2) Smith Chart (2)

1. Plot the following **normalized** impedances *z* into the Smith Chart:

Point	a	b	c	d
z	0.6 + j0	0.6 - j0.6	0.6 - j0.8	0.6 – j1.0

2. Plot the following impedances *Z* into the Smith Chart, assuming a reference impedance of Z_0 = 50 Ω

Point	A	B	C	D
Z	50 + j0	20 – j15	10 + j25	0 – j50

3.3.) Impedances in the complex plane and in the Smith chart

Plot the following impedances as points (marks)

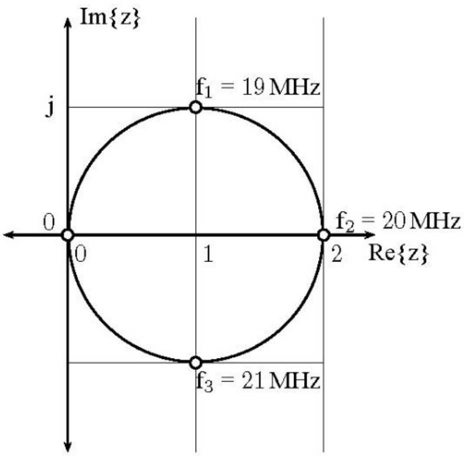
- in a "normal" Cartesian coordinate system (complex plane)
- as reflection factors in the *Smith* chart (the reflection factor coordinates are given for convenience)

X_{rect}	X_{polar}	Γ_{rect}	Γ_{polar}
0.05	$0.05 \angle 0^0$	-0.904	$0.904 \angle 180^0$
0.5	$0.5 \angle 0^0$	-0.333	$0.333 \angle 180^0$
1	$1.0 \angle 0^0$	0	0
2	$2.0 \angle 0^0$	0.333	$0.333 \angle 0^0$
20	$20 \angle 0^0$	0.904	$0.904 \angle 0^0$
0.8	$0.8 \angle 0^0$	-0.111	$0.111 \angle 180^0$
0.8 + j0.6	$1.00 \angle 36.9^0$	0 + j0.333	$0.333 \angle 90^0$
0.8 + j1.0	$1.28 \angle 51.3^0$	0.159 + j0.472	$0.459 \angle 72.3^0$
0.8 + j1.5	$1.70 \angle 61.9^0$	0.344 + j0.546	$0.645 \angle 57.8^0$
0.8 + j2.0	$2.15 \angle 68.2^0$	0.502 + j0.552	$0.747 \angle 47.7^0$
0.8 – j0.6	$1.00 \angle -36.9^0$	0 - j0.333	$0.333 \angle -90^0$

Convince yourself with a few examples that $\Gamma(1/X) = -\Gamma(X)$

3.4) Smith Chart (3)

The locus of impedance of a parallel RLC resonant circuit is given in the complex *z*-plane (*z*-pane = normalized *Z*-plane, normalization to 50 Ω; *z* = *Z* / 50 Ω).



Questions:

1. Transform this locus of impedance into the Smith Chart
2. Mark the resonance frequency, both, in the *z*-plane and in the Smith Chart.
3. Mark the 3-dB points (for the unloaded Q), both, in the *z*-plane and in the Smith Chart.

5.) Various questions

1. What is the difference between a *Stripline* and a *Microstripline*?
2. Name 3 disadvantages of *Microstriplines* compared to *Striplines*.
3. A RF signal needs to be guided from a power amplifier on the surface to the cavity of a particle accelerator in the tunnel below ground.
   The distance is l = 100 m. The signal has parameters: f = 50 MHz, P = 100 kW
   a. Would you use waveguide or coaxial transmission line? Why?
   b. What would you use if the signal would have a frequency of 500 MHz? Why?
4. Why are some accelerator cavities (for frequencies in the MHz range) loaded with ferrite? Explain how the resonant frequency of those cavities can be tuned without moving parts.
5. After deploying a new accelerating cavity, a RF-engineer starts pounding on it with a hammer. What is he doing?

6.) Scaling laws

A cavity shall be scaled from existing designs for a frequency f_x = 318.32 MHz and C_x = 10 pF.
There are three test designs, with the following parameters:

Cavity	f_{res} / MHz	C / pF	Q	Diameter / mm
A	100	7.957	10000	600
B	500	3.18	5000	200
C	3000	1.061	2000	25

Questions:

1. Which cavity is suitable as reference design?
2. Calculate the diameter of the new design.
3. Calculate the expected Q factor of the new design, provided it will be build out of the same material as the reference design.

7.) Thermal expansion and scaling laws

An accelerator cavity heats up under high RF power load.
The cavity used is constructed from a material having a:

thermal expansion coefficient: $\Delta l/l$ = 20e-6/^{0}C (per degree Centigrade)
thermal resistivity coefficient: $\Delta\rho/\rho$ = 4e-3/^{0}C (per degree Centigrade)

At room temperature the cavities resonance frequency is f_1 = 100 MHz, and has a 3-dB bandwidth of BW_1 = 100 kHz.
Under RF power the cavity temperature increases by 100 ^{0}C (subscripts 2 apply).

Questions:

Determine

- the ratio λ_2/λ_1
- the ratio L_2/L_1
- the ratio C_2/C_1
- the ratio Q_2/Q_1 (hint: the skin depth δ is proportional to $\sqrt{\rho/f}$
- the resonance frequency f_2 under load
- and the 3-dB bandwidth BW_2 of the resonance under load