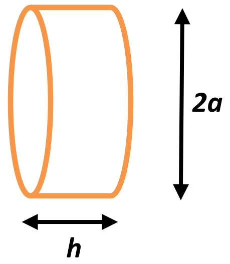
JUAS 2018 – RF Exam

$\mu = \mu_0\, \mu_r$
$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am)
$\varepsilon = \varepsilon_0\, \varepsilon_r$
$\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/(Vm)
$c_0 = 3 \cdot 10^{8}$ m/s

Name: ________________________________ Points: ______ of 20 (25 with bonus points)
Utilities: JUAS RF Course 2018 lecture script, personal notes,
pocket calculator, ruler, compass, and your brain!
(No cell- or smartphone, no iPad, laptop, or wireless devices, no text books or any other tools!!!)
Please compute and write your results clear and readable, if appropriate on a separate sheet of paper. Any unreadable parts are considered as wrong.



1. "Pillbox" Cavity (6½ points)

A simple cylindrical "pillbox" cavity has to be prototyped in air. The beam-pipe ports are neglected.

a) The cavity will be driven by a 400 MHz transmitter to accelerate a proton beam.
What is the fundamental mode of the cavity, used for accelerating particles? (½ point)
Which is the (inner) diameter of the pillbox? (½ point)
Fundamental mode: TM_{010} or E_{010}

$$d = 2\,a = 2 \cdot 0.383\lambda = 0.766 \frac{c_0}{f_{TM010}} = 0.574\,m$$

b) The closest – unwanted – higher-order mode (HOM) should stay approximately 100 MHz away from the fundamental mode.
What is the frequency of this HOM? (½ point)
Of which type is that mode? (½ point)
Which is the height (length) the pillbox cavity has to be manufactured to achieve this goal?
(Hint: Make use of mode chart 1 for cylindrical cavities) (½ point)
$f_{HOM} = 400\,MHz + 100\,MHz = 500\,MHz$
The wavelength of this 1st higher-order mode is: $\lambda_{TE111} = \frac{c_0}{f_{TE111}} = 0.6\,m$
From the mode chart follows at: $\frac{\lambda_{TE111}}{2a} = \frac{c_0}{2af_{TE111}} = 1.05\ (\sim 1.0)$
the 1st higher-order mode is of H_{111} or TE_{111} type
From the mode chart we see at $\frac{\lambda_0}{2a} = 1.05$ follows $\frac{h}{2a} = 0.66$, which results in $h = 0.38\,m$

c) What is the "R-over-Q" of the fundamental mode? (½ point)
(without taking the transit time factor into account)

$$\frac{R}{Q} = 128 \frac{\sin^2\left(1.2024\, {}^{h}/_{a}\right)}{{}^{h}/_{a}} = 96.76$$

Can we use the approximate formula to compute the R-over-Q? (½ point)

$$\frac{R}{Q} \approx 185 \frac{h}{a} = 244.6$$

The approximation cannot be used!

To manufacture the cavity, three different materials are available at the workshop:
stainless steel ($\sigma_{SS} = 1.32 \cdot 10^6\ {}^{S}/_{m}$), brass ($\sigma_{Br} = 15.9 \cdot 10^6\ {}^{S}/_{m}$),
or copper ($\sigma_{Cu} = 58.5 \cdot 10^6\ {}^{S}/_{m}$).
The shop prefers to use brass for the prototype.
Can that material be used to reach $Q_0 > 10000$? (½ point)
What is the unloaded-Q value of the fundamental mode, if the cavity is manufactured from brass? (½ point)

$$\delta_{Br} = \sqrt{\frac{2}{\omega_{TM010}\sigma_{Br}\,\mu}} = \sqrt{\frac{2}{2\,\pi\, f_{TM010}\ \sigma_{Br}\ \mu_0}} = 6.31\ \mu m$$

$$Q_{TM010Br} = \frac{a}{\delta_{Br}}\left[1 + \frac{a}{h}\right]^{-1} = 25880$$

Yes, brass can be used as material to achieve an unloaded $Q_0 > 10000$

d) What is the unloaded-Q value of the closest higher-order mode? (½ point)

$$Q_{TE111Br} = 0.206 \frac{\lambda_{TE111}}{\delta_{Br}} \frac{\left[1 + 0.73\left(\frac{2a}{h}\right)^2\right]^{3/2}}{1 + 0.22\left(\frac{2a}{h}\right)^2 + 0.51\left(\frac{2a}{h}\right)^3} = 26130$$

e) Determine the lumped elements R, L, and C of the equivalent parallel R-L-C circuit. (1½ points)

$$R = \frac{R}{Q} Q_{TM010Br} = 2.5\ M\Omega$$

$$L = \frac{{}^{R}/_{Q}}{2\,\pi\, f_{TM010}} = 38.5\ nH$$

$$C = \frac{1}{{}^{R}/_{Q}\ 2\,\pi\, f_{TM010}} = 4.11\ pF$$

2. Smith chart (3½+3 points)

a) Indicate points **A1**…**A6** for a given complex reflection factor *Γ* in the Smith chart, assuming a reference impedance Z_0 = 50 Ω.
From the Smith chart, determine the missing *Z*, and complete the table. (1 point)
(Use the provided Smith chart)

Point no.	**A1**	**A2**	**A3**	**A4**	**A5**	**A6**
Γ	**1 ∠0°**	**1 ∠53°**	**1 ∠180°**	**0.5**	**0.45 ∠63°**	**0.71 ∠135°**
***Z* / Ω**	∞	J100	0	150	50+j50	10+j20

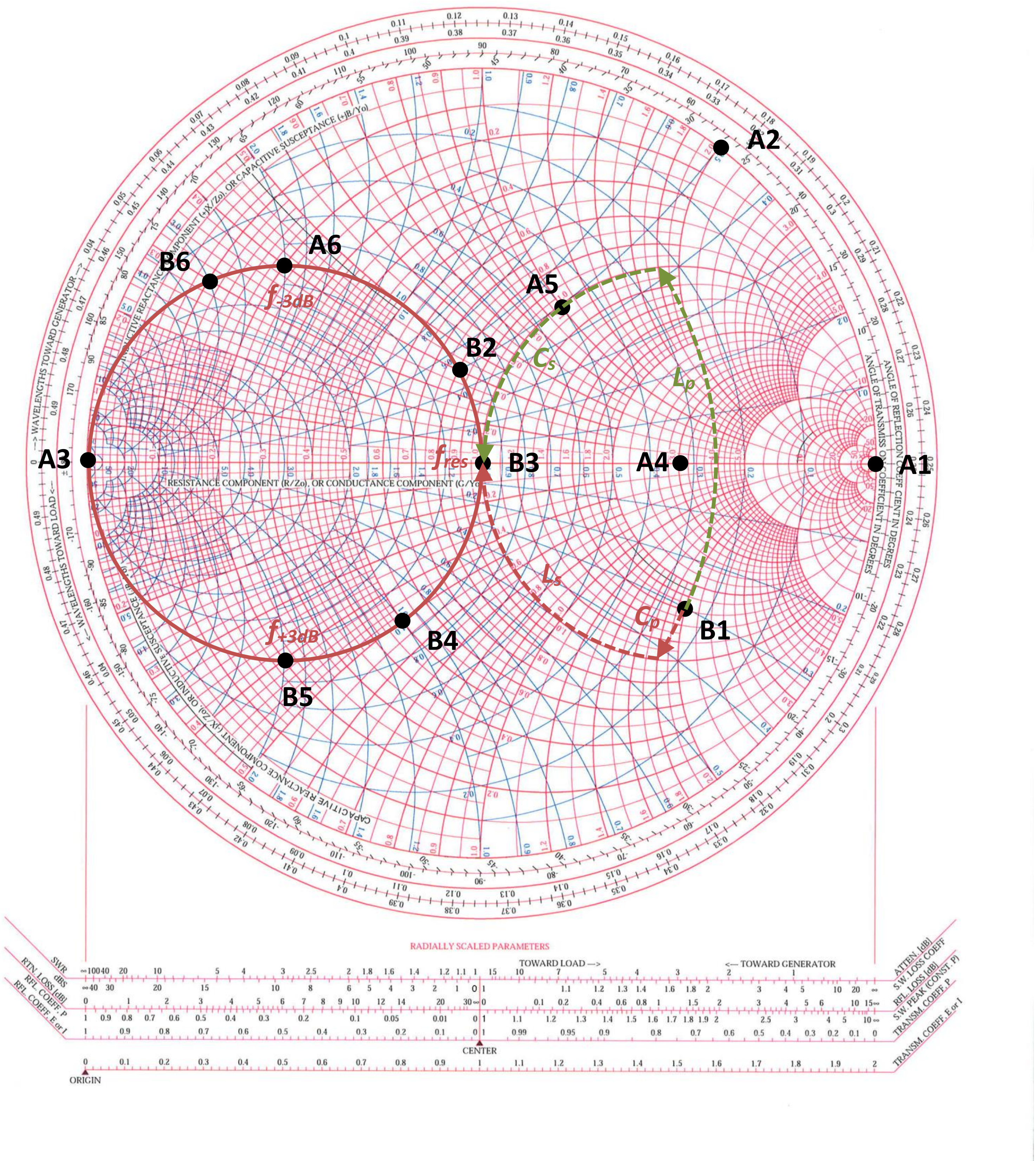
b) Indicate points **B1**…**B6** for a given complex impedance in the Smith chart, assuming a reference impedance Z_0 = 50 Ω.
From the Smith chart, determine the missing *Γ* and complete the table. (1 point)
(Use the provided Smith chart)

Point no.	**B1**	**B2**	**B3**	**B4**	**B5**	**B6**
***Z* / Ω**	**80 - j 100**	**40 + j 20**	**50**	**25 - j 25**	**10 - j 20**	**5 + j 15**
Γ	0.64 ∠-36°	0.24 ∠104°	0	0.45 ∠-117°	0.71 ∠-135°	0.83 ∠146°

c) The points **A3**, **A6**, **B2**, **B3**, **B4**, **B5** and **B6** belong to the locus of *Γ* of a resonant mode of a cavity.
Sketch the locus of that resonance! (½ point)
Indicate the point of the resonance frequency, and the points which characterize the loaded Q-value (Q_L)! (½ point)
B3: f_{res}, A6: f_{-3dB}, B5: f_{+3dB}
What kind of coupling indicates the locus? (½ point)
The resonance is in critical coupling.

a) **Bonus:**
Point **B1** indicates the complex input impedance of a gain stage operating at f = 200 MHz.
Indicate a lossless matching circuit as graph in the Smith chart. (1 point)
(different solutions are possible)
Sketch the circuit diagram of that matching network. (1 point)
Evaluate the component values of the matching circuit! (1 point)
$C_p = 2.0\,pF$ and $L_s = 70.0\,nH$ or $L_p = 54.3\,nH$ and $C_s = 9.0\,pF$

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



3. Fill in all missing fields in the tables below (4 points)

a) (1 point)

Voltage ratio	Power ratio	dB
3.1623	10	10
10	100	20
100	10000	40

b) (1 point)

dBm	RMS voltage	milli Watt
0	223.6 mV	1
+30	7.07 V	1000
-60	223.6 μV	1 e-6
20	2.236 V	100

c) Parts of the S-matrix of an ideal attenuator are given.
Fill the missing matrix elements. (½ point)
What is the nominal attenuation value in dB written on the component? (½ point)

$$[S] = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}$$

20 dB attenuator

d) An amplifier is perfectly matched at input and output, i.e. input and output impedance are 50 Ω. It has a gain of 40 dB and no reverse transmission. (input: port 1, output: port 2)
(1 point)

$$[S] = \begin{bmatrix} 0 & 0 \\ 100 & 0 \end{bmatrix}$$

4. Modes in Waveguides (3+2 Points)

Consider a rectangular cross-section waveguide (empty, i.e. air or vacuum) with a longer side (width) of 100 mm length and a shorter side (height) of 50 mm.

a) Determine the cut-off frequencies of the first 3 modes. (1 point)

calculate: $f_{mn} = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

or look up the mode-chart: $\frac{b}{a} = 0.5$ and determine f from $\frac{\lambda_0}{a} = \frac{1}{a f \sqrt{\varepsilon\mu}} = 2.0 \text{ } or \text{ } 1.0$

$f_{10} = 1.5 \text{ } GHz$ (TE_{10}), $f_{01} = f_{20} = 3.0 \text{ } GHz$ (TE_{01}, TE_{20})

b) Now the waveguide is completely filled with a dielectric material of $\varepsilon_r = 2.25$. What are the cut-off frequencies in this case? (1 point)

$f_{10} = 1.0 \text{ } GHz$ (TE_{10}), $f_{01} = f_{20} = 2.0 \text{ } GHz$ (TE_{01}, TE_{20})

c) The empty waveguide is now "transformed" into a rectangular resonator with the length 200 mm by adding shorts at each end.
Determine the frequencies of the first 4 modes of this resonator. (1 point)

calculate: $f_{mnp} = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$

under consideration of the separation condition: $k_x^2 + k_y^2 + k_z^2 = k_0^2 \varepsilon_r \mu_r$

with: $k_0 = \frac{2\pi}{\lambda_0}$, $k_x = \frac{n\pi}{a}$, $k_y = \frac{m\pi}{b}$, $k_x = \frac{p\pi}{c}$

or more simple find the resonant frequencies in the mode chart at $\frac{a}{b} = 2$ and $\frac{a}{c} = 2$

$f_{101} = 1.68 \text{ } GHz$ (TE_{101}), $f_{102} = 2.12 \text{ } GHz$ (TE_{102}), $f_{103} = 2.7 \text{ } GHz$ (TE_{103}),
$f_{011} = f_{201} = 3.09 \text{ } GHz$ (TE_{011}, TE_{201})

Bonus:

Now we consider an empty waveguide with circular cross-section and a diameter of 100 mm.

a) What are the first 3 modes in this case? (cut-off frequency and type of mode). (1 point)

compute $f_{TMmn} = \frac{j_{mn}}{2\pi a\sqrt{\varepsilon\mu}}$, $f_{TEmn} = \frac{j'_{mn}}{2\pi a\sqrt{\varepsilon\mu}}$

or from $\lambda_{TE11} = 3.412a$ follows $f_{TE11} = \frac{1}{3.412a\sqrt{\varepsilon\mu}}$ the fundamental mode, and the higher modes from the mode chart:

$f_{TE11} = 1.76 \text{ } GHz$, $f_{TM01} = 2.29 \text{ } GHz$, $f_{TE21} = 2.91 \text{ } GHz$

b) Also, this waveguide is converted into a pillbox like cavity, with a height of 80 mm by adding conductive end plates.
What are the first 4 resonances? (type and frequency) (1 point)

compute $f_{TMmnp} = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{p}{h}\right)^2 + \left(\frac{j_{mn}}{\pi a}\right)^2}$, $f_{TEmnp} = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{p}{h}\right)^2 + \left(\frac{j'_{mn}}{\pi a}\right)^2}$

or from $\lambda_{TM010} = 2.61a$ follows $f_{TM010} = \frac{1}{2.61a\sqrt{\varepsilon\mu}}$ the fundamental mode, and the higher modes from the mode chart at $\frac{h}{2a} = 0.8$:

$f_{TM010} = 2.29 \text{ } GHz$, $f_{TE111} = 2.57 \text{ } GHz$, $f_{TM011} = 2.99 \text{ } GHz$, $f_{TE211} = 3.46 \text{ } GHz$

5. Multiple choice (3 points)

Tick ***one*** correct answer like this: ☒.

1. Why is the gap in cavities designed to accelerate beams of low beta ($\beta = v/c$) rather short compared to high beta cavities? (¼ point)
   - ○ To prevent voltage breakdown
   - ☒ To obtain a reasonable transit time factor
   - ○ To save space

2. Examples of TEM transmission lines are: (¼ point)
   - ○ Waveguides operating below cut-off frequency
   - ☒ Coaxial cables
   - ○ Resonant cavities with input and output coupler

3. A vector network analyser is used to (¼ point)
   - ○ Analyse signals in the frequency domain
   - ☒ To characterize the S-parameters of an RF element (DUT = device under test)
   - ○ Measure and calibrate signals from the internet communication structure.

4. For a "H"(or "TE") waveguide mode, the following is true: (¼ point)
   - ○ Its magnetic field has only transverse components
   - ○ Its magnetic field has transverse and longitudinal components
   - ☒ Its electric field has only transverse components

5. A critically coupled cavity is impedance matched to the RF power amplifier and absorbs the available source power. The value for the loaded Q is: (¼ point)
   - ☒ $Q_L = Q_{ext}$
   - ○ $Q_L = 2 \times Q_0$
   - ○ $Q_L = Q_0$

6. A coaxial line is filled homogeneously with a dielectric material, e.g. PTFE ("Teflon"). The time delay of a signal passing a coaxial line of same physical length, but filled with air is (¼ point)
   - ○ identical
   - ☒ shorter
   - ○ longer

7. TEM stands for (¼ point)
   - ○ Transient Electro-Magnetics
   - ☒ Transverse Electro-Magnetic mode
   - ○ Turbo Electric Motor

   - ○

8. For a cylindrical ("pillbox") cavity, the eigen-frequencies are independent of the cavity height *h* dimension: (¼ point)

- ○ For any eigen-mode the resonance frequency depends on height *h* and radius *a*
- ○ True only for the TEM mode
- ☒ True only for TM_{010} modes

9. What is true for 2-conductor transmission-lines (TEM)? (¼ point)
   - ☒ Ideal for broadband (down to DC) signal transmission.
   - ○ The signal transmission is based on “modes”.
   - ○ Low losses at very high frequencies, therefore ideal for high power RF transmission.

10. In order to measure the S-parameters of some N-port device, all unused ports (connections) have to be (¼ point)
    - ○ Left open
    - ○ Shorted
    - ☒ Terminated in their characteristic impedance

11. The Smith chart is a conformal mapping or the complex impedance plane into the plane of the complex reflection coefficient. What is true: (¼ point)
    - ○ Straight lines convert into logarithmic spirals
    - ☒ Generalized circles convert to generalized circles
    - ○ The left half of the complex impedance plane is mapped inside the Smith chart.

12. In a RF cavity used for beam acceleration, the transit time factor is related to: (¼ point)
    - ○ The time is takes for the energy to transfer from the electric to the magnetic field.
    - ☒ The time variation of the accelerating field during the bunch passage.
    - ○ The time it takes for the bunch to travel through the cavity.