Beam dynamics for cyclotrons

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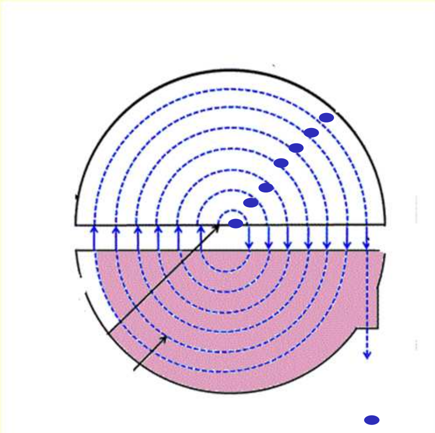
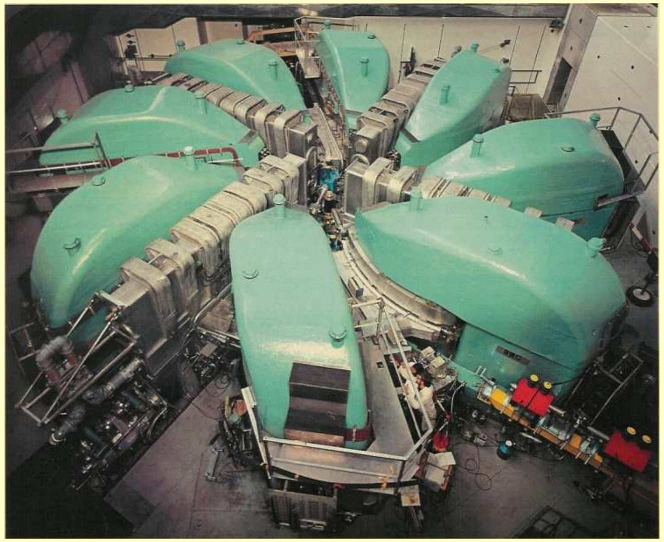




compact cyclotrons

Separated sectors (ring cyclotrons)

Synchrocyclotrons

Fixed energy

Variable energy

Superconducting

Normal conducting

OUTLINE

Chapter 1 : theory

- Principle
- Basic equation
- Longitudinal dynamics
- Transverse dynamics

Chapter 2 :specific problems

- Longitudinal dynamics
- Acceleration
- Injection
- Extraction

Chapter 3 : design

- Design strategy
- Tracking
- Simulations

Chapter 4 :

-Theory vs reality (cost, ,tunes,isochronism,...)

Exemples

-Medical cyclotron

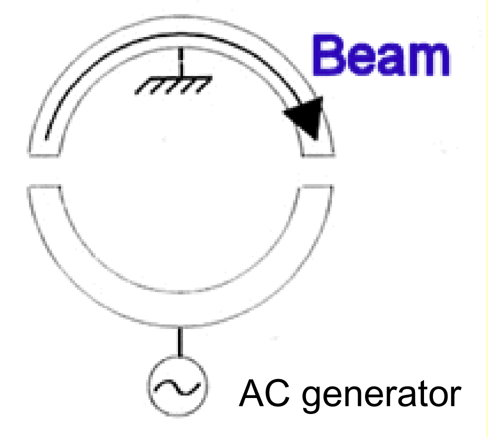
-Reseach facility

CYCLOTRON HISTORY

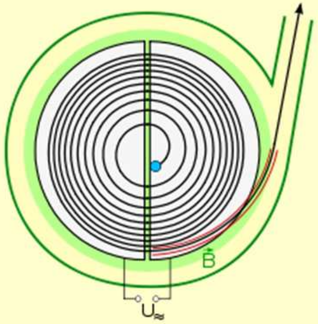
The Inventor, **E. Lawrence**, get the **Nobel** in Physics (1939) (**first nuclear reactions Without alpha source**)

- brilliant idea (E. Lawrence, Berkeley, 1929) : RF accelerating field is technically complex and expensive.

So Let 's use **only 1 RF cavity**, but many times



A device is put into a magnetic field, curving the ion trajectories and only one electrode is used several times.

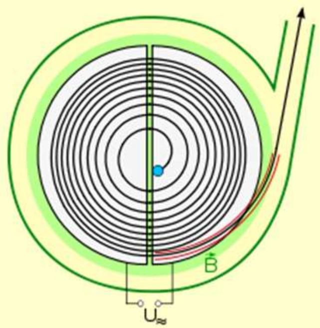


What is a CYCLOTRON ?

•RF accelerator for the ions :

from proton A=1 to Uranium A=238

- Energy range for proton 1MeV -1GeV (γ~1-2)
- Circular machine : CW (and Weak focusing)
- Size Radius=30cm to R=6m
- RF Frequency : 10 MHz -60 MHz



APPLICATIONS : Nuclear physics

(from fundamental to applied research)

: Medical applications

Radio Isotopes production (for PET scan,....)

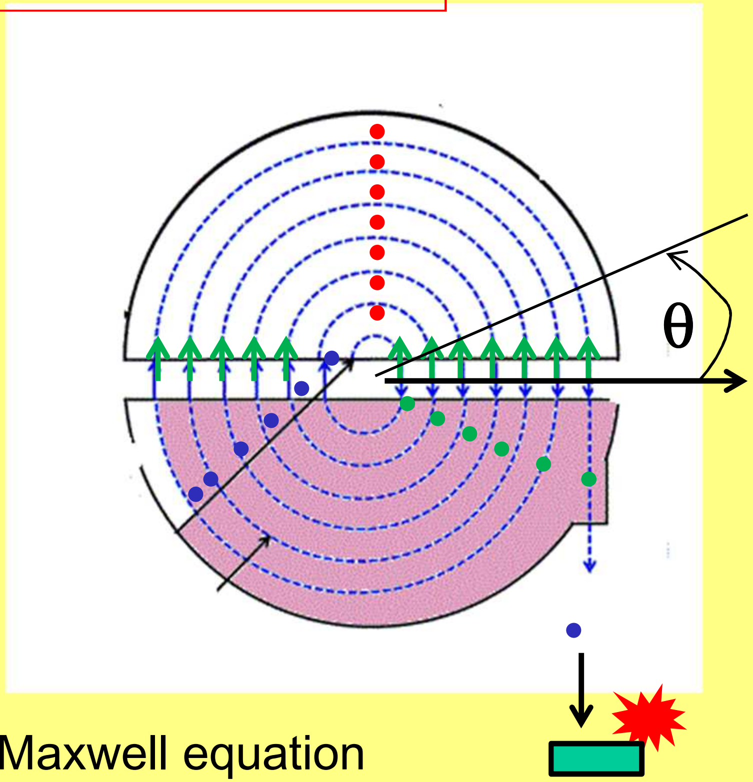
Cancer treatment

Quality : Compact and Cost effective

Usefull concepts for the cyclotrons

$$\boxed{B\rho = \frac{P}{q} = \frac{\gamma\, m.v}{q}}$$

$$\boxed{E_K = (\gamma - 1).mc^2}$$



Maxwell equation

$\nabla \times B = 0$

Cyclotron coordinates

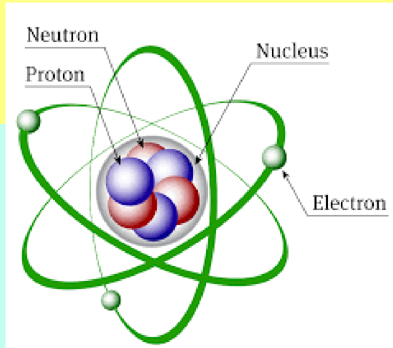
r Radial = horizontal

z Axial = vertical

θ « Azimuth » = cylindrical angle

MeV/A= kinetic energy unit in MeV per nucleon

Ions : $^{A}_{Z}X^{Q}$



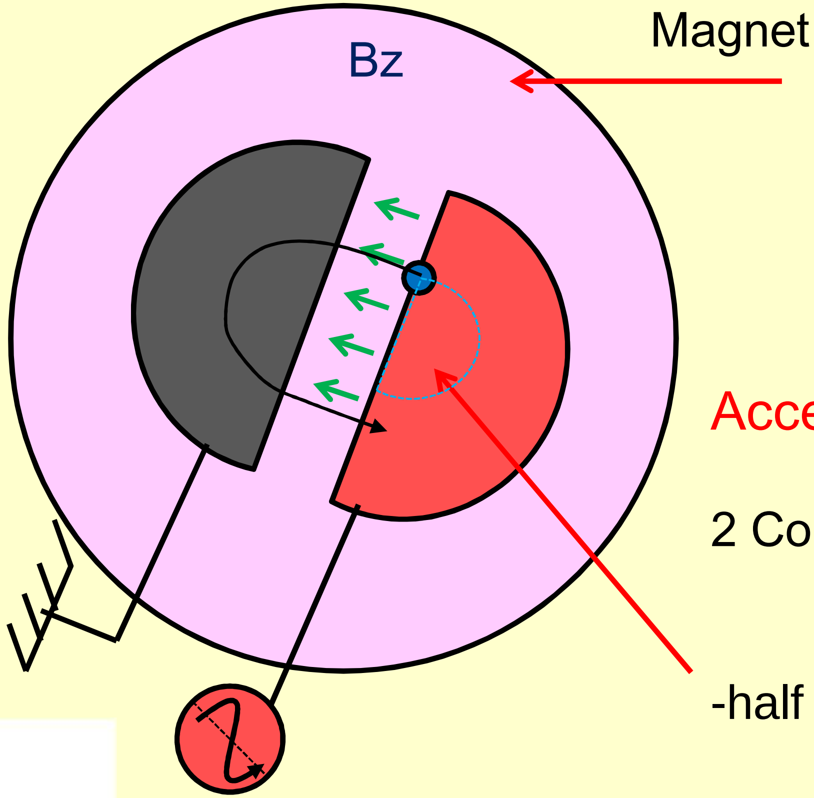
A : nucleons number

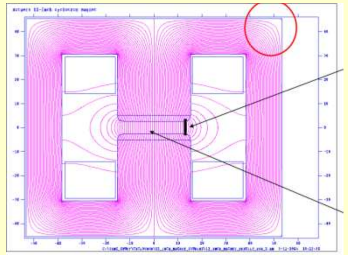
Z: protons number

Q : charge state : 0+,1+,2+,…..

Principle :the hardware

$$\vec{E} = [\,V_1(t) - V_0\,]/d$$
$$\sim \cos(\omega t)$$





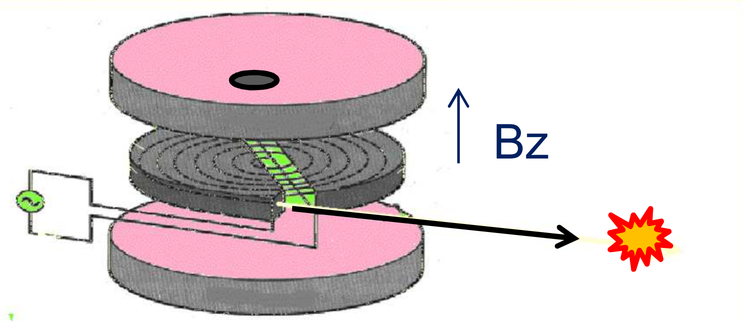
H-type
electro-magnet

Accelerating Dee's

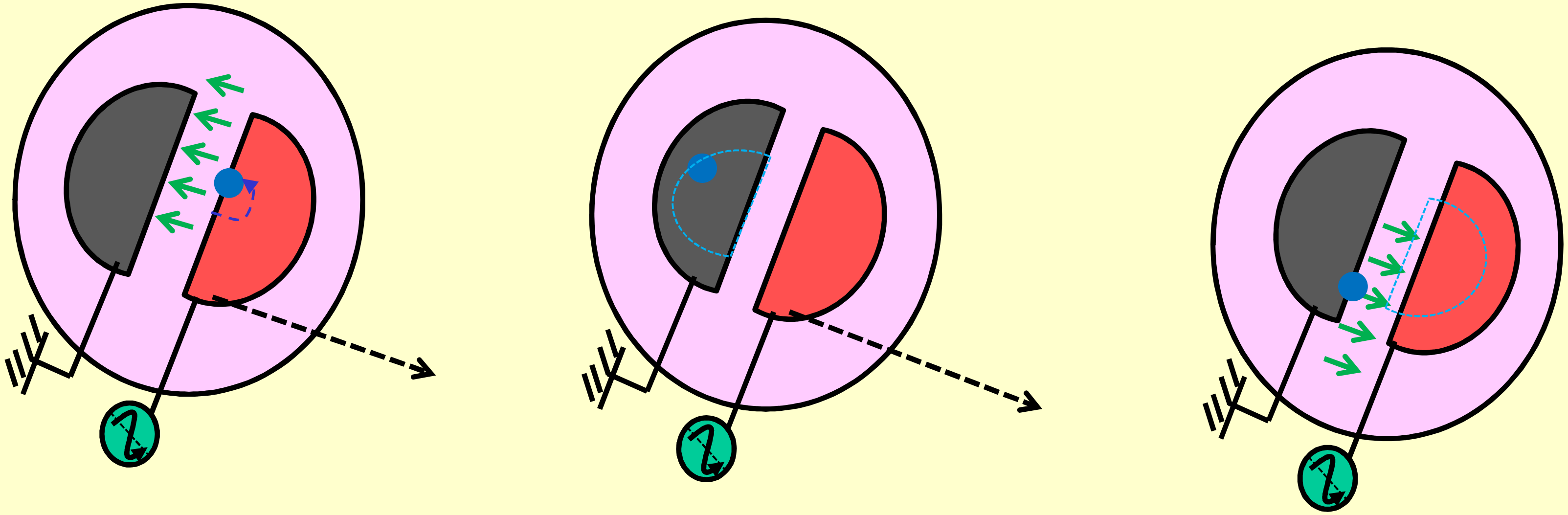
2 Copper boxes
≠ potential

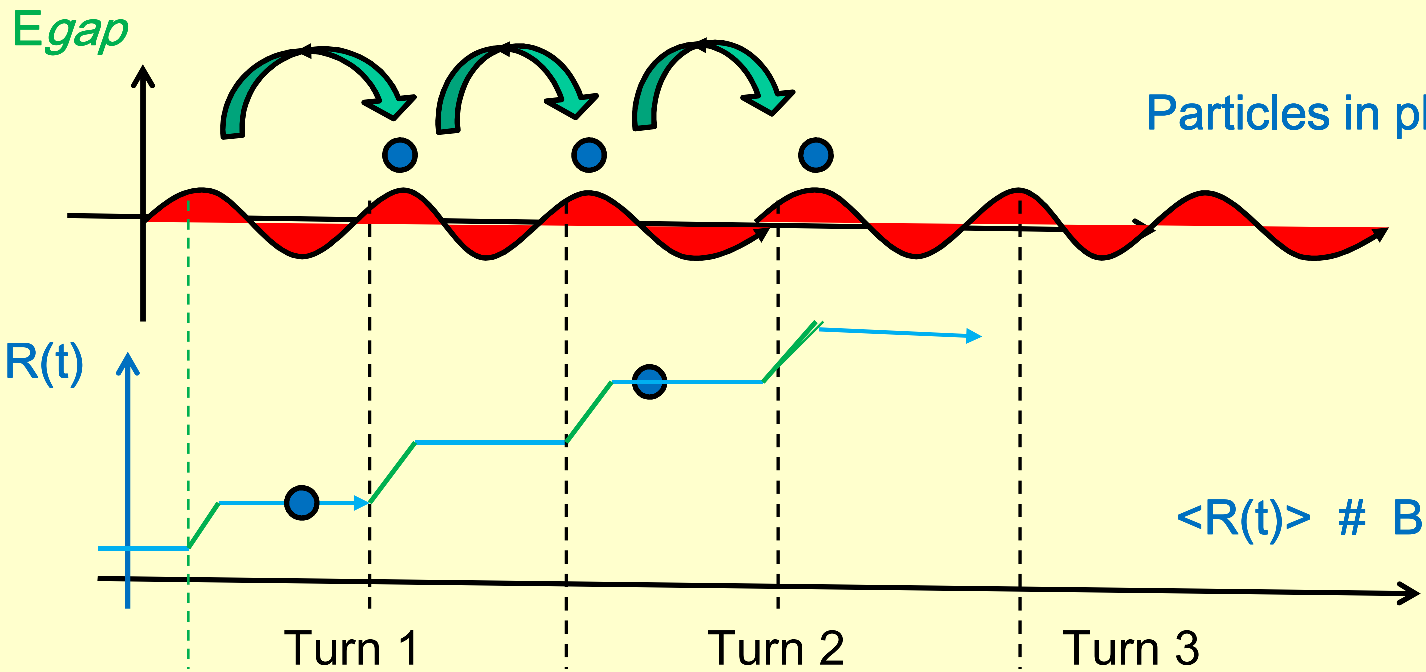
-half is $V_1(t) \sim \cos(\omega t)$

- half is
at the ground potential

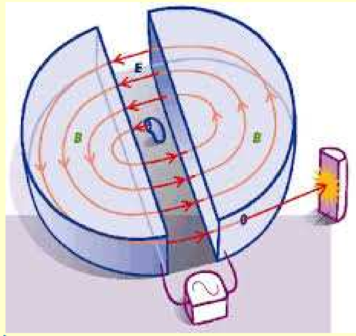


Principle B: the trajectories



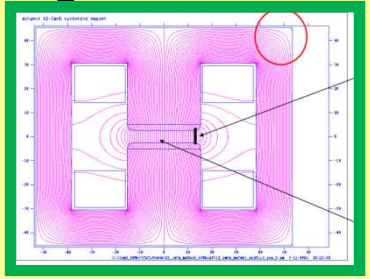


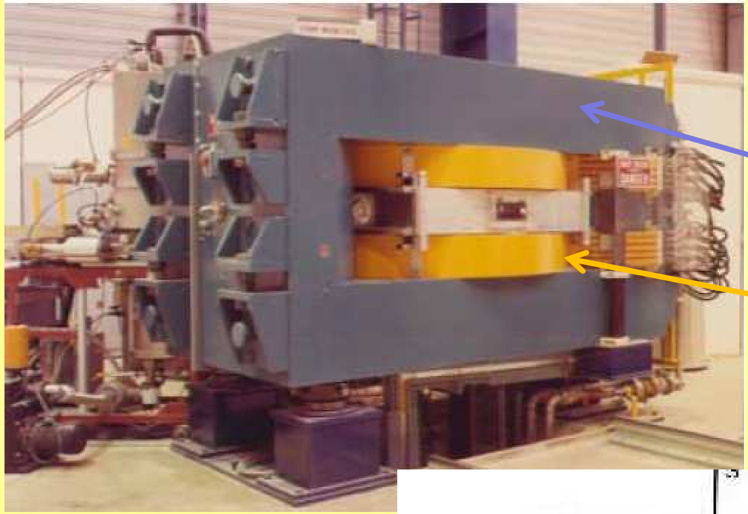
Particles in phase with *RF*

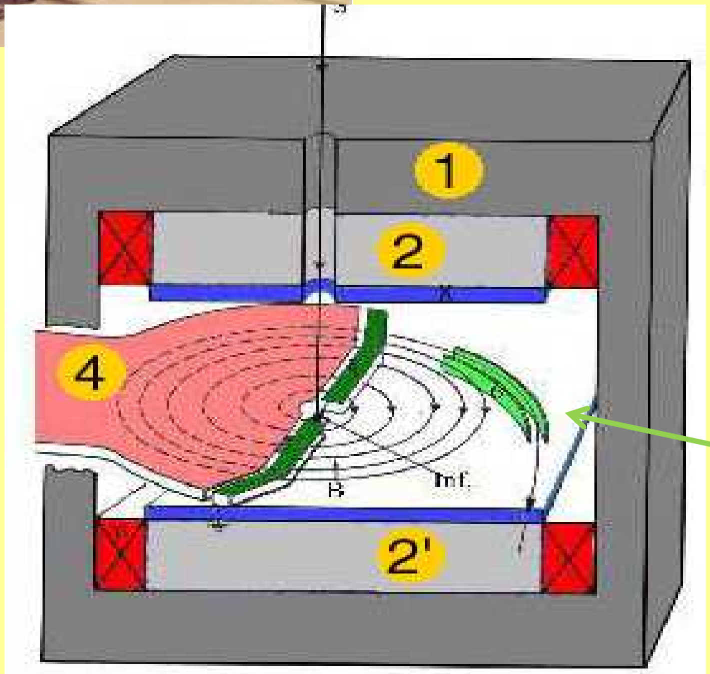


<R(t)> # Bρ / <B>

A compact cyclotron in reality







Magnet (Bz) :

1) Yoke
2) poles
3) coils

RF cavities

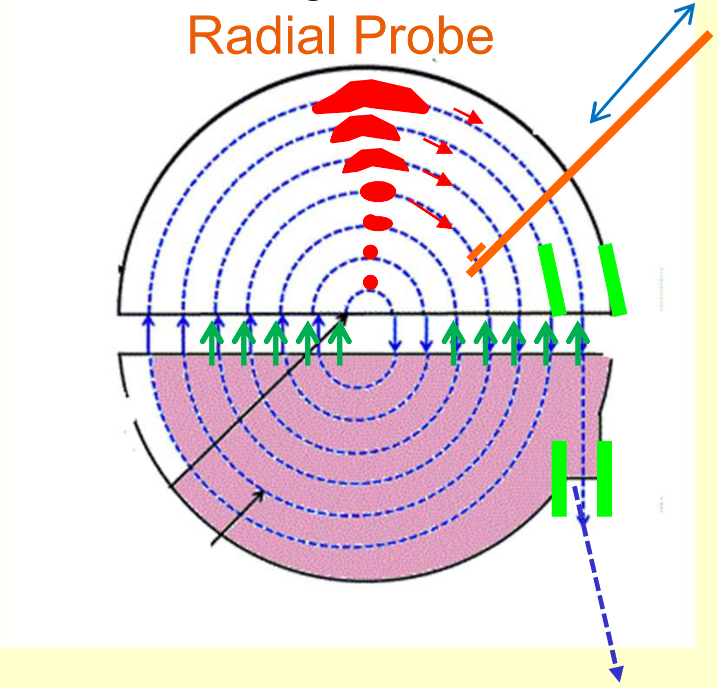
4) Dee

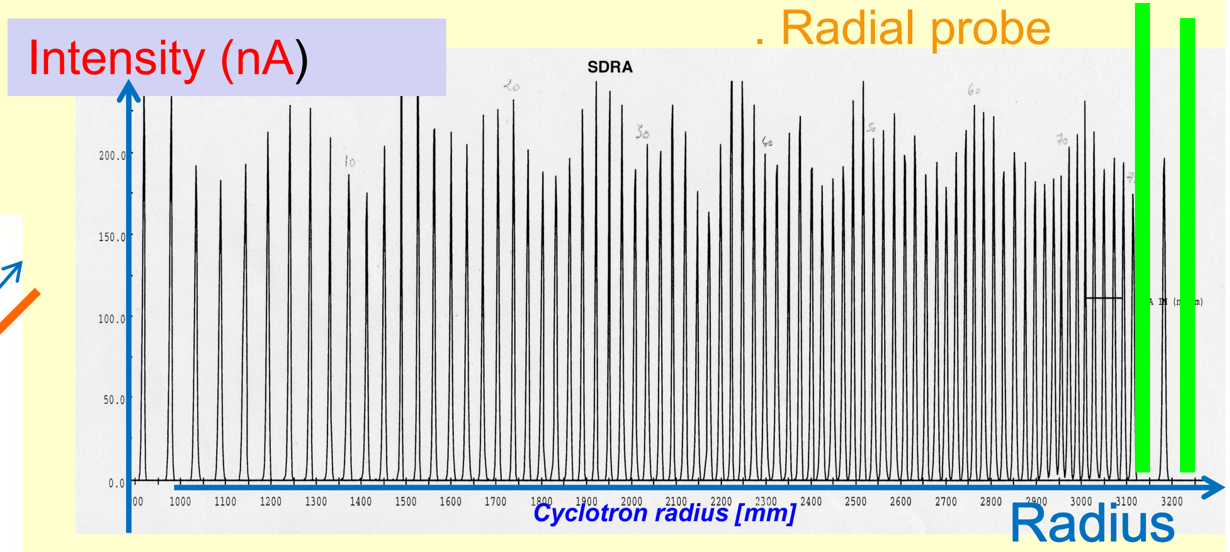
Electrostatic Deflector

Radial probes

useful tool to check the acceleration

Monitoring beam with a
Radial Probe





Radial probe : I = F(Radius)

Radius in the cyclo = f(Turn Number)

Turn separation

: δr =R(turn N) - R(turn N-1)

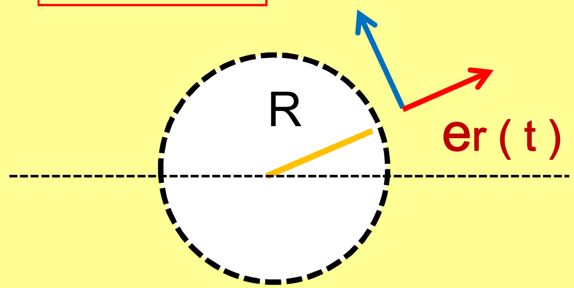
Trajectory in uniform B field

$$\frac{d(\gamma m\vec{v})}{dt} = \vec{F}$$

Let’s consider an ion with a charge ***q*** and a mass ***m*** circulating at a speed $\boldsymbol{v_\theta}$ in a uniform induction field ***B****.=(0,0,Bz)*

The motion equation can be derived from the **Newton’s law** and the **Lorentz force *F*** ***in a cylindrical coordinate system*** (er,eθ,ez):

$$\vec{F} \perp \vec{v}$$



$$\boxed{\frac{d(\vec{v})}{dt} = \vec{a} = \frac{d^2(R.\vec{er})}{dt^2} = -\left[\left\|\vec{v}\right\|^2 / R\right].\vec{e_r}}$$

$$\frac{d\vec{p}}{dt} = \vec{F} = q(\vec{v}\times\vec{B}) = -q v_\theta B_z \vec{e_r}$$

$$\gamma\frac{mv^2}{R} = -q v_\theta B_z$$

$$v_\theta = R\dot{\theta}$$

$$\boxed{R = \frac{\gamma\, mv}{qB_z}}$$

Trajectory in uniform B field

$$R = \frac{\gamma\, mv}{qB_z}$$

$$Frevolution = \frac{\mathrm{v}}{2\pi\, R} = \frac{1}{2\pi}\frac{qB}{\gamma\, m}$$

$$\omega_{rev} = 2\pi\, F_{rev} = \dot{\theta} = \frac{d\theta}{dt} = \frac{\mathrm{v}_\theta}{R} = \frac{qB}{\gamma\, m}$$

$$\boxed{\omega_{rev} = \frac{qB}{\gamma\, m}}$$

Centrifugal force = Magnetic force

$$\gamma\frac{m\mathrm{v}_\theta^2}{R} = q\mathrm{v}_\theta B_z$$

Cyclotrons Tutorials 0

- **Demonstrate that the revolution frequency ($F_{rev} = \omega_{rev}/2\pi$) of an ion in a perpendicular uniform field B_z is**

$$\omega_{rev} = q\,B_z / m\gamma$$

1) **With the Newton-lorentz equation (demo 1)**
2) **With magnetic rigidity (demo 2)**

Nota : Electric field is supposed to be Zero

so $||V||$ =constant and

hence $\gamma = [1-v^2/c^2]^{-1/2} = $ constant

Cyclotrons Tutorials 0

- **Demonstrate that the revolution frequency** $F_{rev} = \omega_{rev}/2\pi$ **of an ion in a perpendicular uniform field** B_z **is**

$$\omega_{rev} = q\, B_z / m\gamma$$

Demo N°1 : newton equation

$d\,p/dt = q\,(v \times B)$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ 0 & 0 & V\theta \\ 0 & B_z & 0 \end{vmatrix}$$

$m\gamma\,(d\,v/dt) = -\,q\,v\theta\,Bz\,er$

$m\gamma\,(-\,v^2/R)\,er = -\,q\,v\theta\,Bz\,er$

$R = q\,v\theta\,Bz\,/\,m\gamma$

$$\boxed{Frevolution = \frac{v}{2\pi\,R} = \frac{1}{2\pi}\frac{qB}{\gamma\,m}}$$

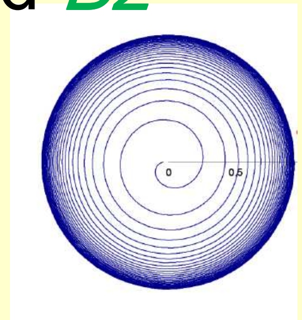
Demo N°2 : Bρ formula

$$\boxed{R = \frac{B\rho}{B_z} = \frac{\gamma\,mv}{qB_z}}$$

$$\boxed{Frevolution = \frac{v}{2\pi\,R} = \frac{1}{2\pi}\frac{qB}{\gamma\,m}}$$

$$\omega_{rev} = 2\pi\,F_{rev} = q\,B_z / m\,\gamma$$

Let's accelerate ions, in a constant vertical field *Bz*



The Radius evolves with P/q :

$$R(t) = \frac{\gamma\, m\mathrm{v}}{qB_z} = \frac{B\rho}{B_z}$$

For *non relativistic* ions (low energy) ⇨ $\gamma \sim 1$

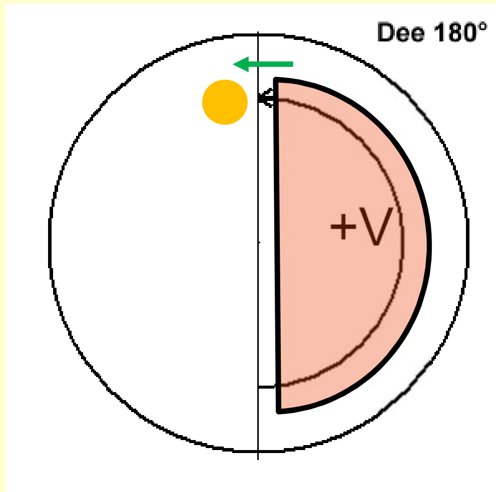
In this domain, if *Bz* =const ⇨ ω=const

$$\omega_{rev} = \frac{qB_z}{\gamma m} \approx \quad const$$

same ΔT for each Turn

So, it is easy to synchronize an Accelerating cavity (RF)

having a "D" shape, with accelerated ions



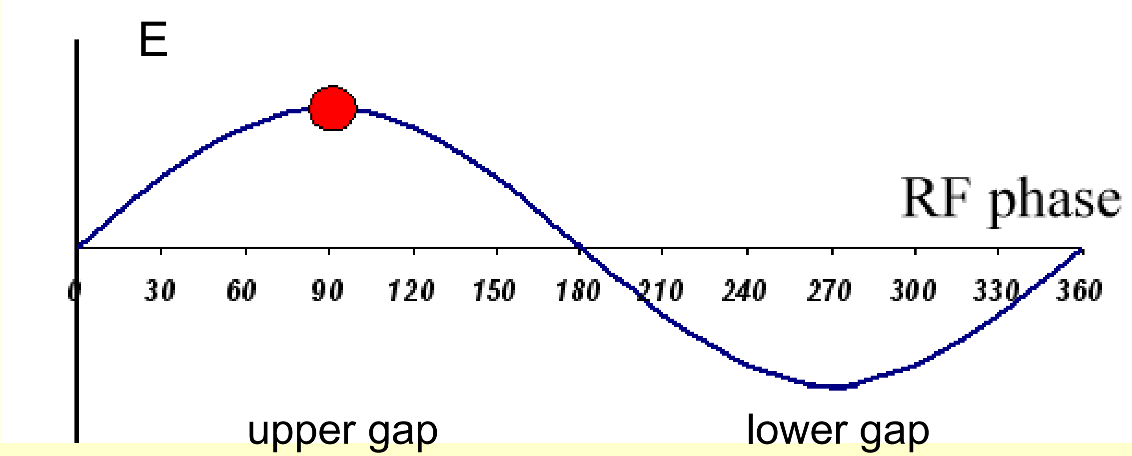
$$V = V_0 \cos(\omega_{RF}\, t)$$

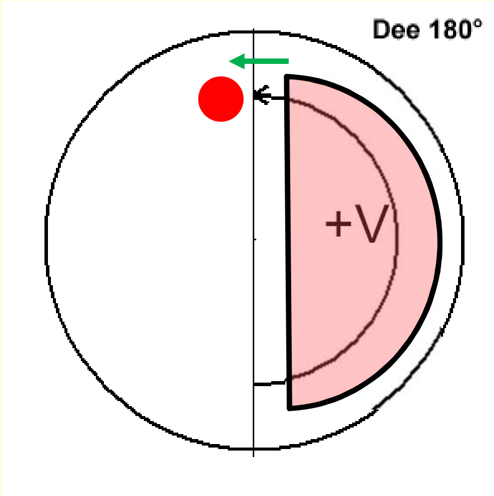
$$\omega_{RF} = \mathbf{h}\ \ \omega_{rev}$$

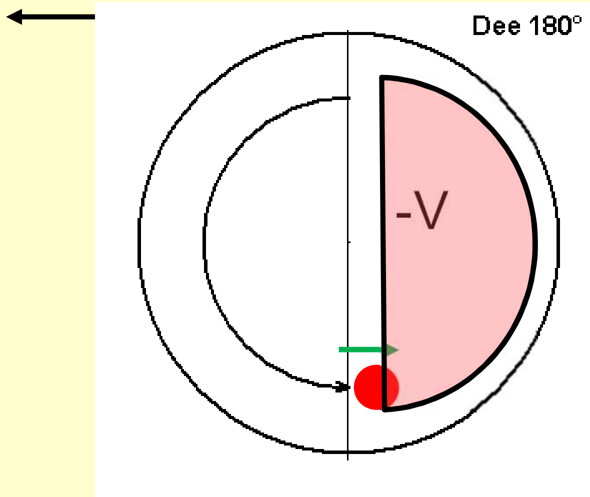
h = 1, 2, 3, … called the RF harmonic number (integer)

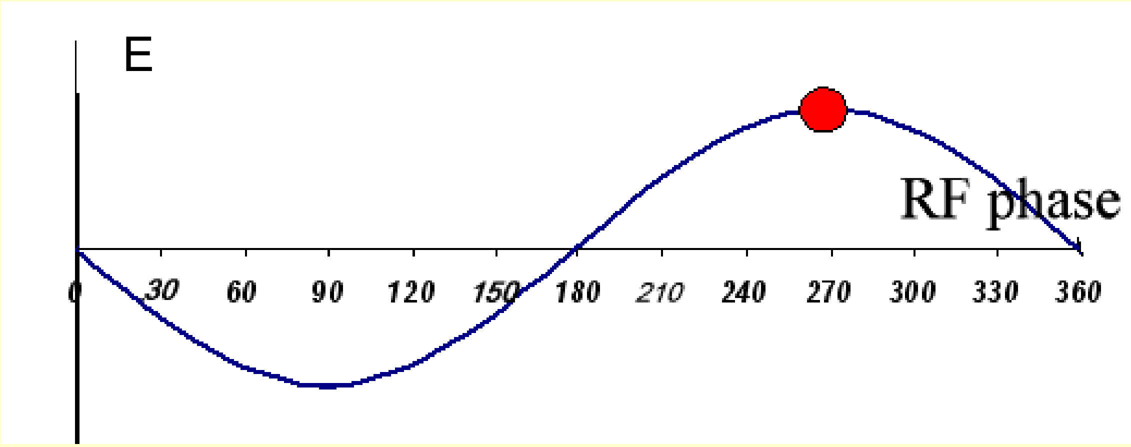
Harmonic number h=FRF/Frevolution

h = 1 *1 bunch by turn* $\omega_{rf} = \mathbf{h}\ \omega_{rev}$





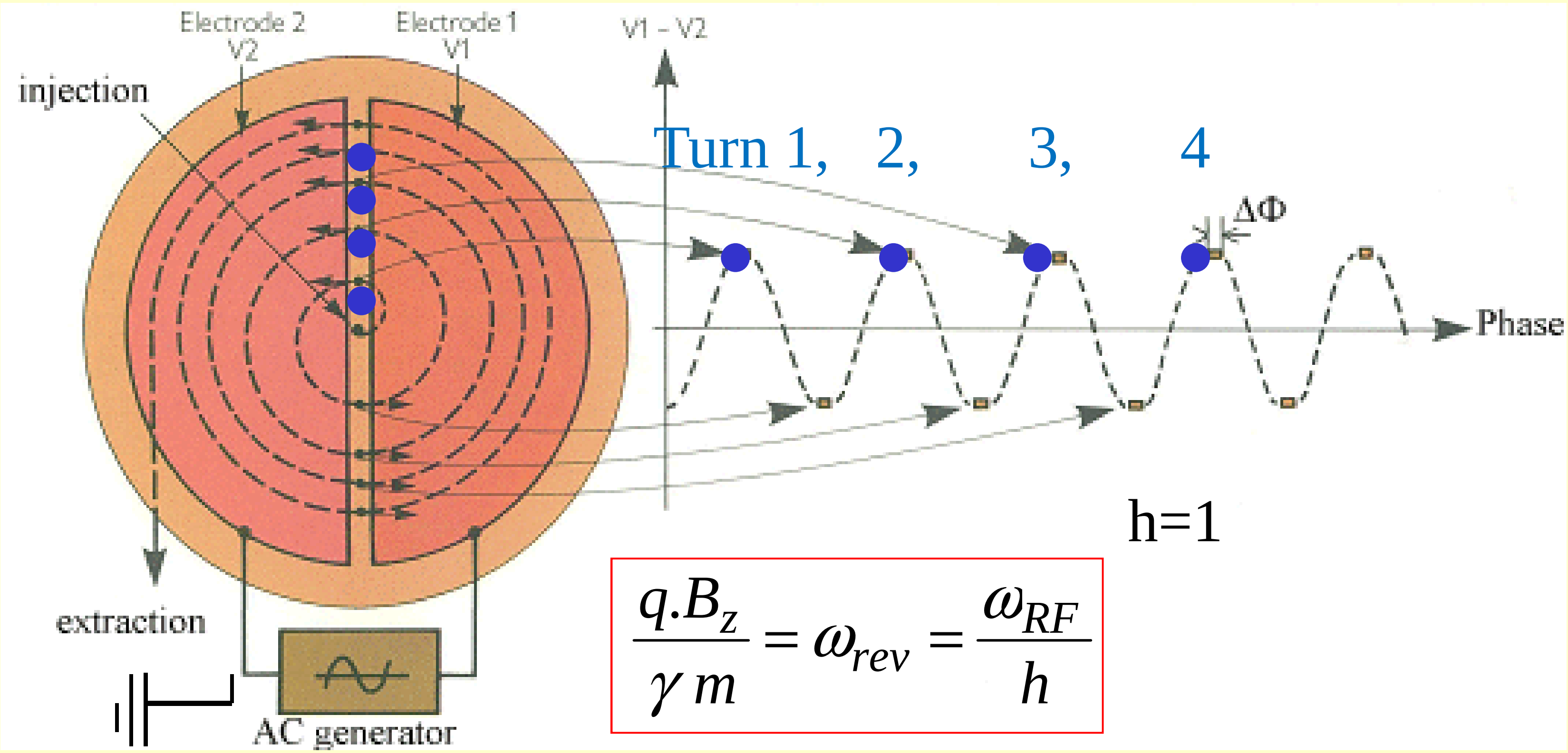




<u>Isochronism condition</u>: The particle takes the same amount of time to travel one turn : (constant revolution frequency ω_{rev} =constant

and with $\omega_{rf} = h\ \omega_{rev}$, the particle is synchronous with the RF wave.

In other words, the particle arrives always at the same RF phase in the middle of the accelerating gap.



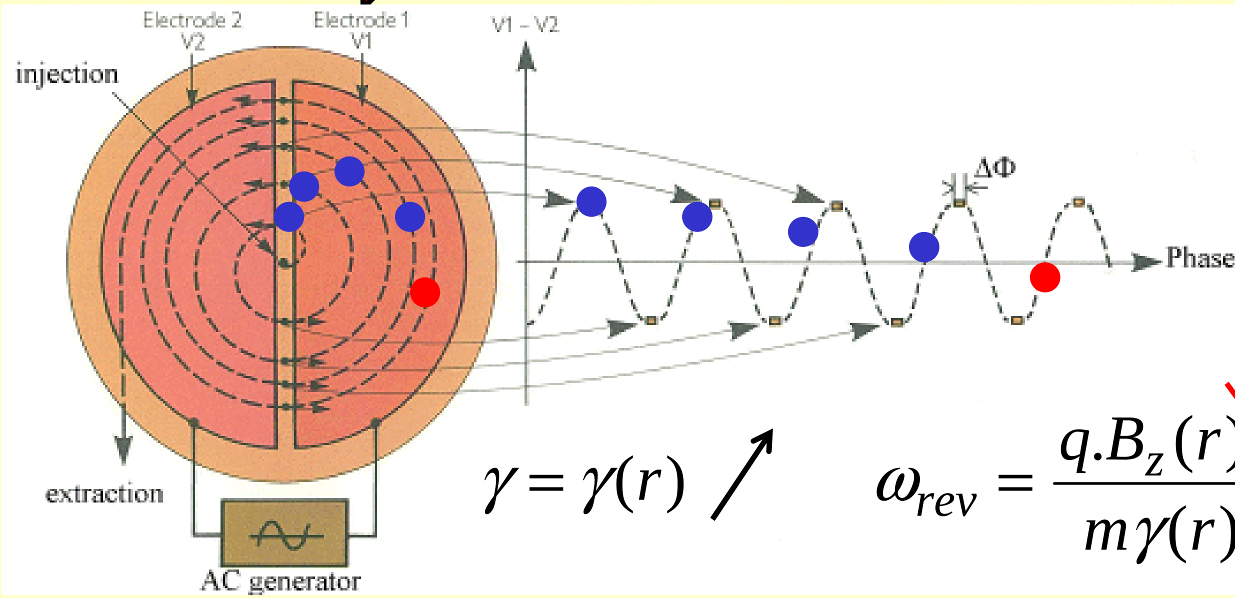
$$\boxed{\frac{q.B_z}{\gamma\, m} = \omega_{rev} = \frac{\omega_{RF}}{h}}$$

Longitudinals with relativistic particles

With ***Bz*** = constant , relativistic γ increases AND ω_{rev} decreases

$$\boxed{\omega_{rev} = \frac{qB_0}{\gamma\, m}}$$

Isochronism condition not fullfilled



$$\gamma = \gamma(r) \nearrow \qquad \omega_{rev} = \frac{q.B_z(r)}{m\gamma(r)} \searrow$$

$$\boxed{\omega_{rev} = \frac{q.B_z(r)}{\gamma(r)\, m}}$$

Isochronism condition fullfilled i

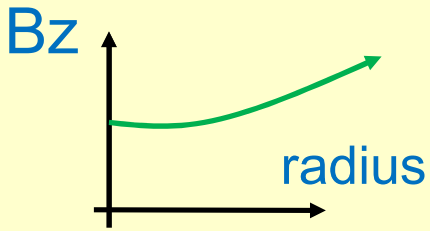
Bz(r) / γ(r) =CONSTANT

longitudinal Dynamics in the cyclotrons

$$\boxed{\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m} = const}$$

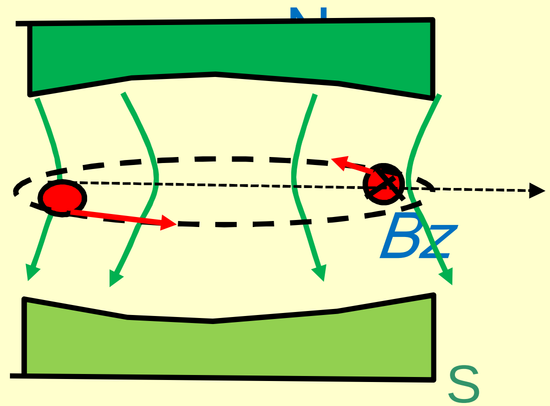
Isochronism condition

Bz =Bz(radius) ~ γ



pole gap

gap=F(Radius)



=f(Radius) obtained by pole gap shaping

The distance between magnet pole (gap) evolves with Radius : Bz~ 1/gap

We will show that that isochronism

have a bad consequence on vertical oscillations

Cyclotrons Tutorials 1

•**An isochronous cyclotron uses a RF cavity at 60 MHz at the RF harmonic h=3**

a. Compute the time needed to perform one turn T_{rev} for the accelerated ions.

b. Compute the average field B_z needed to accelerate a proton beam (in a non relativistic approximation)

Cyclotrons Tutorials 1

•**An isochronous cyclotron uses a RF cavity at 60 MHz at the RF harmonic h=3**

a. Compute the time needed to perform one turn for the accelerated ions.
b. Compute the average field B needed to accelerate proton

in a non relativistic approximation

Answer ***a*** ***Revolution freq=60/3 = 20 Mhz*** $\Delta T=1/20.\ 10^{-6}\ s=$ ***50ns***

.

b $\omega = qB/\gamma m = \omega_{RF}/h$ *and we have* γ close to 1

proton mass~ $1.6\ 10^{-27}$ kg // proton charge~ $1.6\ 10^{-19}$ C

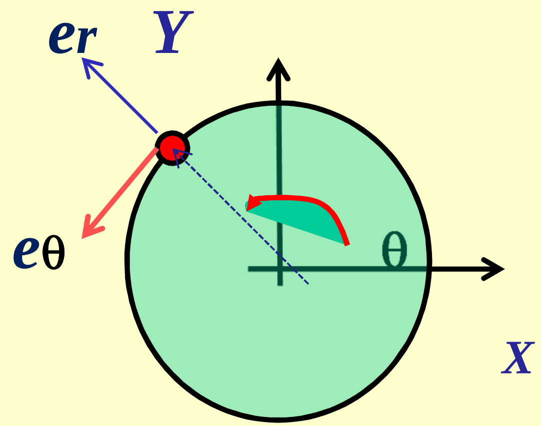
Frf **=60 MHz=** $\omega_{RF}/2\pi$

$Bz = m_p/q\ .\ 2\pi F_{RF}/h = 10^{-8}.\ 10^{6}\ 20\ .2\pi .= 1.26$ *Tesla*

Cyclotrons Tutorials 2

•**Demonstrate than in a uniform circular motion , the radial acceleration is**

$$a_r = |V^2 / R| .$$



Nota : ***You can use parametric equations*** **:**

$$X(t) = R\cos(\omega t)$$
$$Y(t) = R\sin(\omega t)$$

Then compute the velocity and the acceleration.

Demonstrate that the acceleration is radial

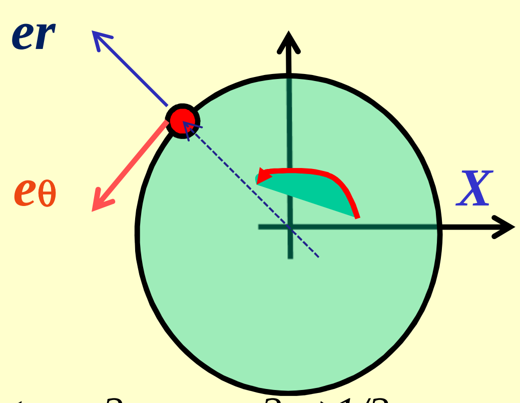
Nota : $\omega t = \theta$ $\omega = d\theta/dt$

Cyclotrons Tutorials 2

uniform circular motion

$$X(t) = R\cos(\omega t)$$
$$Y(t) = R\sin(\omega t)$$

compute the velocity and the acceleration.



Answer

$$V_x = -\omega R\sin(\omega t)$$
$$V_y = +\omega R\cos(\omega t)$$

$$v_\theta = |\mathbf{v}| = (V_x^2 + V_y^2)^{1/2} = \omega R$$

$$|\mathbf{a}| = (a_x^2 + a_y^2)^{1/2} = \omega^2 R = v^2 / R$$

v perpendicular to **a** (since $\mathbf{v}.\mathbf{a} = 0$)

$$\mathbf{a} = -v^2/R$$

Radial vector e_r and longitudinal vector e_θ

$$e_r = \begin{vmatrix} \cos(\omega t) \\ \sin(\omega t) \end{vmatrix}$$

$$de_r/dt = \omega\, e_\theta$$
$$d^2 e_r/dt^2 = -\omega^2 e_r = v^2/R^2\, e_r$$

Transverse dynamics in cyclotrons

Steenbeck 1935, Kerst and Serber 1941

We will use cylindrical coordinates (**er**, **eθ**, **ez**)

We will show that

In Radial plane (horizontal) :

$$\mathbf{radius(t) = R(t) + x_0 \cos(\nu_r\ \omega_{rev}\ t)}$$

Radial tune ν_r

In the Vertical (axial) plane :

$$\mathbf{z(t) = z_0 \cos(\nu_z\, \omega_{rev}\ t)}$$

axial tune ν_z

3 slides to compute ν_z

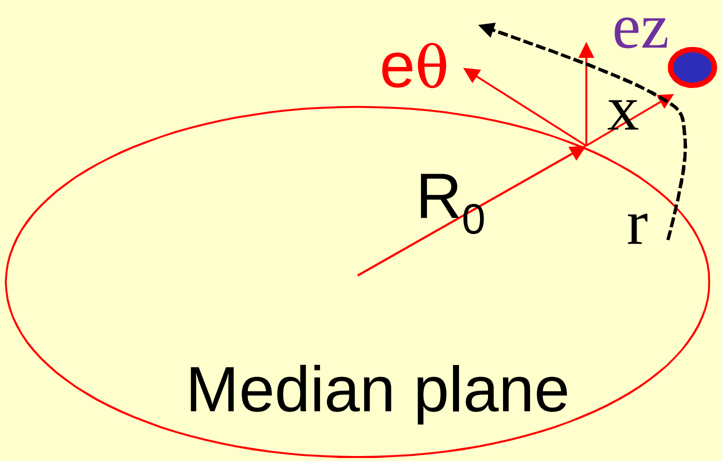
slide 1) Transverse dynamics with *Bz*(R)

cylindrical coordinates (**er**, **e**θ, **ez**)

and

define **x & z** a small orbit deviation with **B***z*=Bz(r) (not constant)

$$\vec{\mathbf{r}} = [R_0 + x(t)]\,.\overrightarrow{er} + z(t)\,.\overrightarrow{ez}$$



Circular Reference orbit

$$Bz(r) = \gamma(r)\quad B_0 \quad \text{Isochron field}$$

$$= r^{-n}\, B_0$$

Uniform Circular motion *x*=0

Motion Equat. With x≠ 0 & z ≠ 0???

$$\boxed{m\frac{d(\vec{\mathbf{v}})}{dt} = m\frac{d^2(\mathbf{r})}{dt^2} = ?}$$

Slide 2) Transverse dynamics with Bz(R) *(No RF)*

- Taylor expansion of the field B_z around the median plane:

definition of n(r) $Bz \sim B_0 \, (r/R_0)^{-n}$ n =field index (Bz is never uniform)

$$\text{with} \quad n = -\frac{R_0}{B_0}\frac{\partial B_z}{\partial r} = -\frac{R_0}{B_0}\frac{\partial B_z}{\partial x}$$

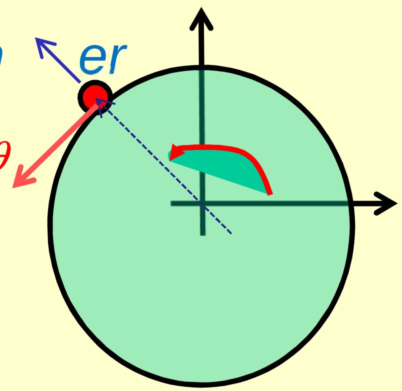
so $Bz\,(R_0+x) = B_0(R_0) + (dB/dx)\,x + \ldots = B_0\,(1 - nx/R_0)$

How evolves an ion, in this non uniform Bz :

$$\mathbf{r}(t) = (R + x(t))\,\mathbf{e_r} + z(t)\,\mathbf{e_z}$$

$m\,\gamma\, d^2\,(x\,\mathbf{er})/dt^2 = \mathbf{Fr}\ \mathbf{er}$ *Horizontal motion*

$m\,\gamma\, d^2\,(z\,\mathbf{e_z})/dt^2 = \mathbf{Fz}\ \mathbf{ez}$ *Vertical motion* e_θ



Fz vertical plan : ***ez**=constant*

$$m\gamma\frac{d^2 z}{dt^2} = F_z = q\,(\mathrm{v}\times B)_z = -q(\dot{r}\,B_\theta - r\,\dot{\theta}\,B_r)$$

slide 3) Vertical dynamics with B (r)

$$m\gamma\frac{d^2z}{dt^2} = F_z = q(\mathrm{v}\times B)_z = -q(\dot{r}\cancel{B_\theta} - r\dot{\theta}B_r)$$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & 0 \end{vmatrix}$$

$Br=?$ $\quad \nabla\times B=0 \quad \frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} = 0$ $\quad + \quad Bz = B_0\, r^{-n}$

$$\Rightarrow \quad B_r = -n\frac{B_{oz}}{r}z$$

Motion equation $\boxed{\ddot{z} + [\nu_z\omega_{rev}]^2 z = 0}$

Harmonic oscillator with the frequency $\boxed{\nu_z = \sqrt{n}}$

$$z(t) = z_0\cos(\nu_z\,\omega_{rev}\,t)$$

For isochronism n < 0 (so νz imaginary)

$$\boxed{\nu_z = \sqrt{n}} \quad \Rightarrow \quad \boxed{\nu_z = i\sqrt{|n|}}$$

Watch the vertical oscillations !!

Isochronism condition : **n < 0** **: Bz(r)~ r^{-n} :increase with Radius**

$$\ddot{z} + \left[\nu_z \omega_{rev}\right]^2 z = 0$$

Vertical tune $\nu_z = i\sqrt{|n|}$

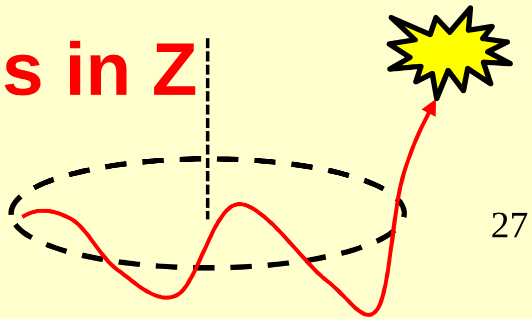
Isochronism condition will induce Unstable oscillations

$$z(t) = z_0 \cos(\nu_z \omega_{rev} t)$$

$$z(t) \sim z_0 \exp(\pm |n|^{1/2} \omega_{rev} t)$$

Unstable oscillations in Z

= exponential growth =beam losses



ion trajectory in cyclotrons

We have shown that

In the Vertical (axial) plane :

$$\nu_z = i\sqrt{|n|}$$

$B_z(r) \sim r^{-n}$

$$z(t) = z_0 \cos(\nu_z\, \omega_{rev}\, t) \sim z_0 \exp(+|n|^{1/2}\, \omega_{rev}\, t)$$

vertical defocusing force : unstable motion

any particle with $z_0 \neq 0$ with be lost ($z(t) \Rightarrow \infty$) : beam losses

In Radial plane (horizontal) : as well, we can show

$$\nu_r = \sqrt{1-n}$$

$$\text{radius}(t) = R(t) + x_0 \cos(\nu_r\, \omega_{rev}\, t)$$

focusing horizontal force : stable oscillations

Radial (horizontal) dynamics with $B_z(R)$ *(No RF)*

- Taylor expansion of the field B_z around the median plane:

- *definition of n(R)* $\quad Bz \sim B_0\,(r/R_0)^{-n}$ $\quad$ n=field index

so $Bz\,(R_0+x) = B_0(R_0) + (dB/dx)\,x + \ldots = B_0\,(1-nx/R_0)$

- How evolves an ion, in this non uniform Bz : $\quad r(t) = R + x(t)$

$$m\gamma \frac{d^2\vec{r}}{dt^2} = -q\,\mathbf{v}\times\mathbf{B}$$

$$d^2 \mathbf{er}/dt^2 = -\,\omega^2\,\mathbf{er} = v^2/r^2\,\mathbf{er}$$

$$d\mathbf{er}/dt = \omega\,\mathbf{e}\theta$$

$$\frac{d^2(r\cdot\vec{er})}{dt^2} = (\ddot{x} - \frac{v_\theta^2}{r})\vec{er} + 2\dot{x}\dot{e}_r$$

$$r = R(1 + x/R)$$

$$= \left[\ddot{x} - \frac{v_\theta^2}{R}(1-\frac{x}{R})\right]\vec{\mathbf{er}} + 2\dot{x}.\vec{\mathbf{e}_\theta}$$

*radial motion (projection on **er**)*

$$m\gamma\left(\ddot{x} - \frac{v_\theta^2}{R}(1-\frac{x}{R})\right) = -q\,\mathbf{v}_\theta B_{0z}(1-n\frac{x}{R})$$

$$\frac{1}{r} = \frac{1}{R\,(1+\frac{x}{R})} \approx \frac{1}{R}(1-\frac{x}{R})$$

Radial dynamics with $B_z(R)$

$$m\gamma\left(\ddot{x}-\frac{v_\theta^2}{R}(1-\frac{x}{R})\right)=-q\,B_{0z}\,(1-n\frac{x}{R}).v_\theta$$

After simplification :

and $$\omega_{rev}=\frac{qB_{0z}}{\gamma m}=\omega_0\approx\frac{v_\theta}{R}$$

$$\ddot{x}+\omega_0^2\cdot(1-n)x=0\;\Rightarrow$$

$$\ddot{x}+[\nu_r\omega_0]^2\,x=0 \qquad \nu_r{}^2=(1-n)$$

Harmonic oscillator with the frequency $$\boxed{\omega_r=\sqrt{1-n}\;\omega_0}$$

Horizontal stability condition (ν_r real) : **n < 1**

n <1 : Bz could decrease//or increase with the radius R

Horizontal stability is generally easy to obtain

Radial dynamics (ν_r real) : Stable oscillations

Harmonic oscillator with the frequency

$$\ddot{x} + [\nu_r \omega_{rev}]^2 x = 0$$

$$\boxed{\nu_r = \sqrt{1-n}}$$

ν_r Radial tune

$$x(t) = x_0 \cos(\nu_r \, \omega_{rev} \, t)$$

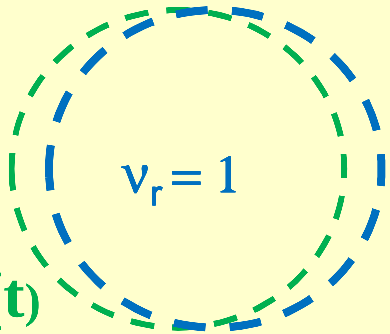
Horizontal stability if n <1

$\nu_r^{\,2} = 1 - n \quad > 0$

n<0 : isochronism condition Bz **should increase**

stability condition ($\nu_r^{\,2} > 0$)

$$r(t) = R_0(t) + x_0 \cos(\nu_r \, \omega_{rev} \, t)$$

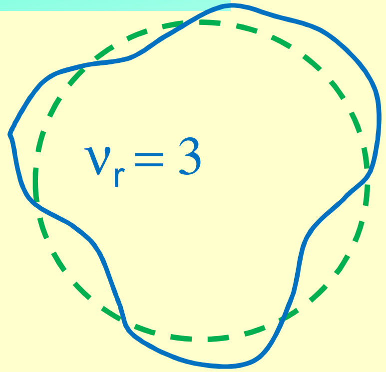


$r(t) = R_0(t)$

Tunes : ν_r & ν_z

oscillations around reference trajectory

$$r(t) = R_0(t) + x_0 \cos(\nu_r\, \omega_{rev}\, t)$$



$r = R_0(t) + x_0 \cos(3\, \omega_0\, t)$

ν_r :Number of radial oscillations per cyclotron turn
in horizontal (radial) plan

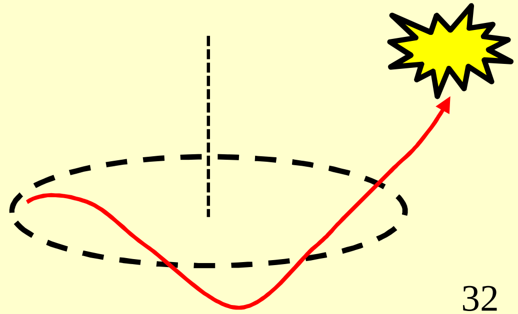
$\nu_r^2 = 1 - n$ stable oscillations

$$z(t) = z_0 \cos(\nu_z\, \omega_{rev}\, t) = z_0 \cos(\nu_z\, \theta)$$

$\nu_z^2 = n < 0$ unstable oscillations

$(\nu_z = i\, |\nu_z|)$

$$z(t) \sim z_0 \exp(\pm |\nu_z|\ \theta)$$



Summary N° 1 : without equations

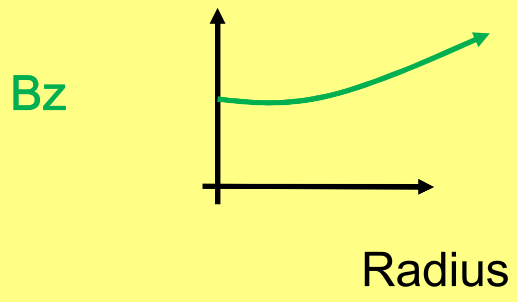
Isochronous Cyclotron :

Isochronism : ωRF = H ωrev = constant

(particles are synchronous with RF : F_{RF} =const)

- RF frequency constant
- Ion Revolution frequency constant

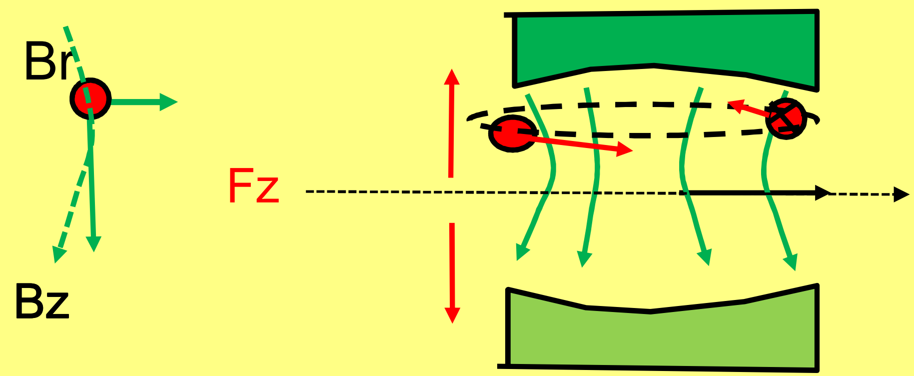
with Bz= f(R) ~ γ (R)



ωRF = H ωrev H = harmonic (integer)

Defocusing Vertical force appears with Bz= f(R)

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ 0 & \dot{z} & r\,\dot{\theta} \\ B_r & B_z & 0 \end{vmatrix}$$



Isochronism condition will induce Unstable Vertical oscillations

Vertical stability $\neq$ Isochronism

Isochronism condition
(longitudinal)

$\boxed{B=B_z(R)}$

$$\boxed{\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m}}$$

Bz should increase with radius ($B_z \sim B_0\, r^{-n}$ $n<0$)

Unstable Vertical oscillations (B_r defocus in z plane)

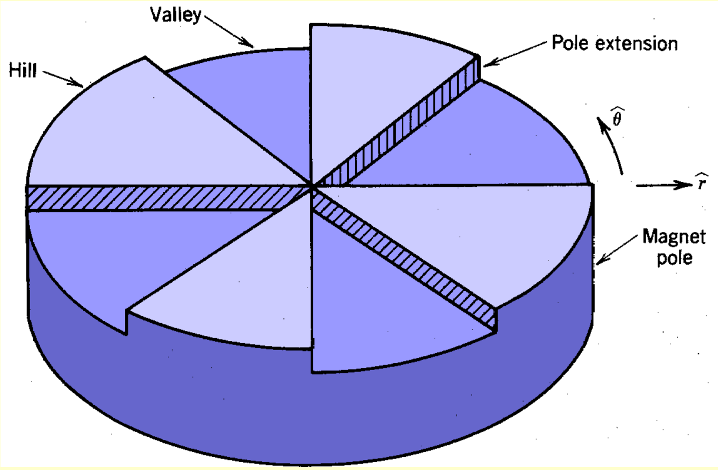
Additive Vertical focusing is needed

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix}$$

$B\theta$ component needed ($F_z = -q\, v_r B_\theta$) :
« Azimutally Varying Field » Cyclotron $B= B(r,\theta)$

Azimuthally Varying Field ("AVF")
Vertical weak focusing : $B_z = f(R,\theta)$

- $F_z \sim \langle q\, v_r\, B_\theta \rangle$: Vertical focusing



$B_z = f(R,\theta)$ $\quad \nabla \times B = 0$

$B_\theta = g(R,\theta)$

Like edge focusing in dipole magnet :
Bz variation
can produce vertical forces

Isochronism n<0 : $B_z(R)$ increase with R

Vertical stability : $B_z(R)$ Defocus + $B\theta$ *Focus*

Bz should oscillate with θ to compensate the instability

- Vertical force Fz , with component **B**θ

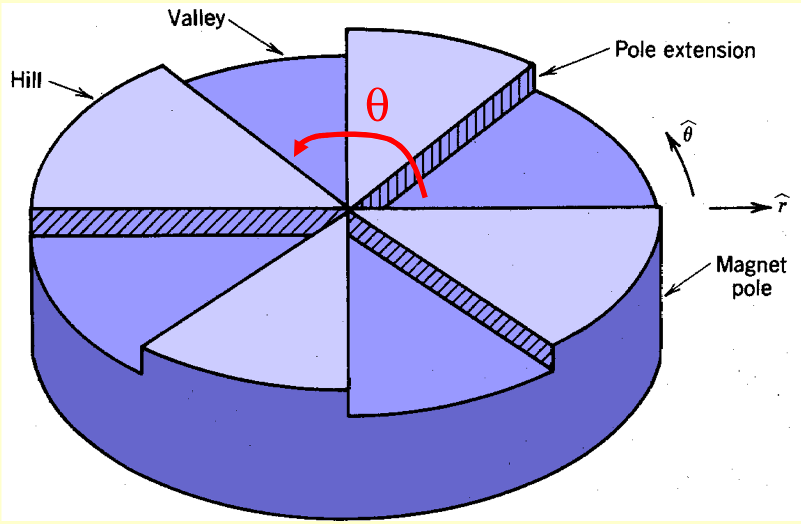
Azimuthally varying Field (AVF)

an additive focusing vertical force $\langle F_z \rangle = q \langle v_r \cdot B_\theta \rangle$

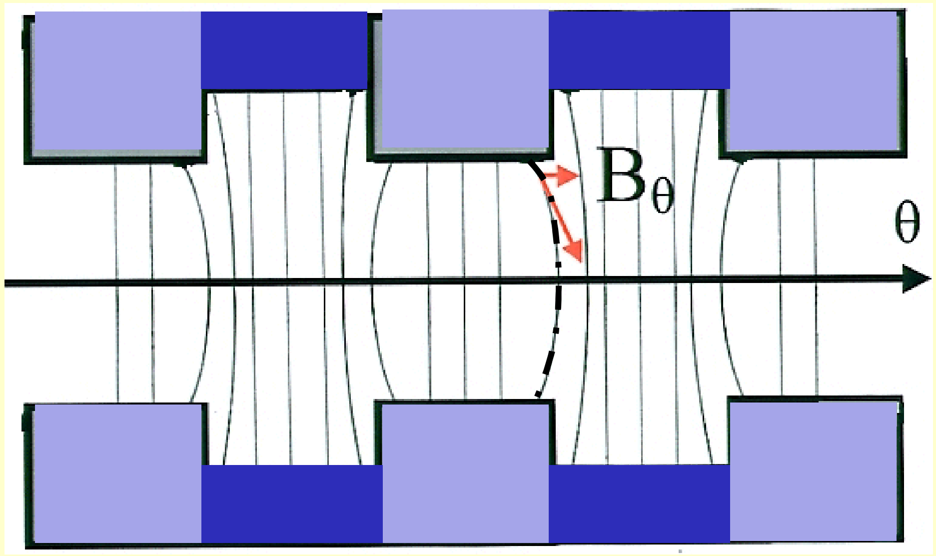
B_θ created by gap modulation

Succession of high field and low field regions : $B_z = f(R, \theta)$

- B_θ appears around the median plane
  - valley : large gap, weak field
  - Hill : small gap, strong field



N=4 sectors

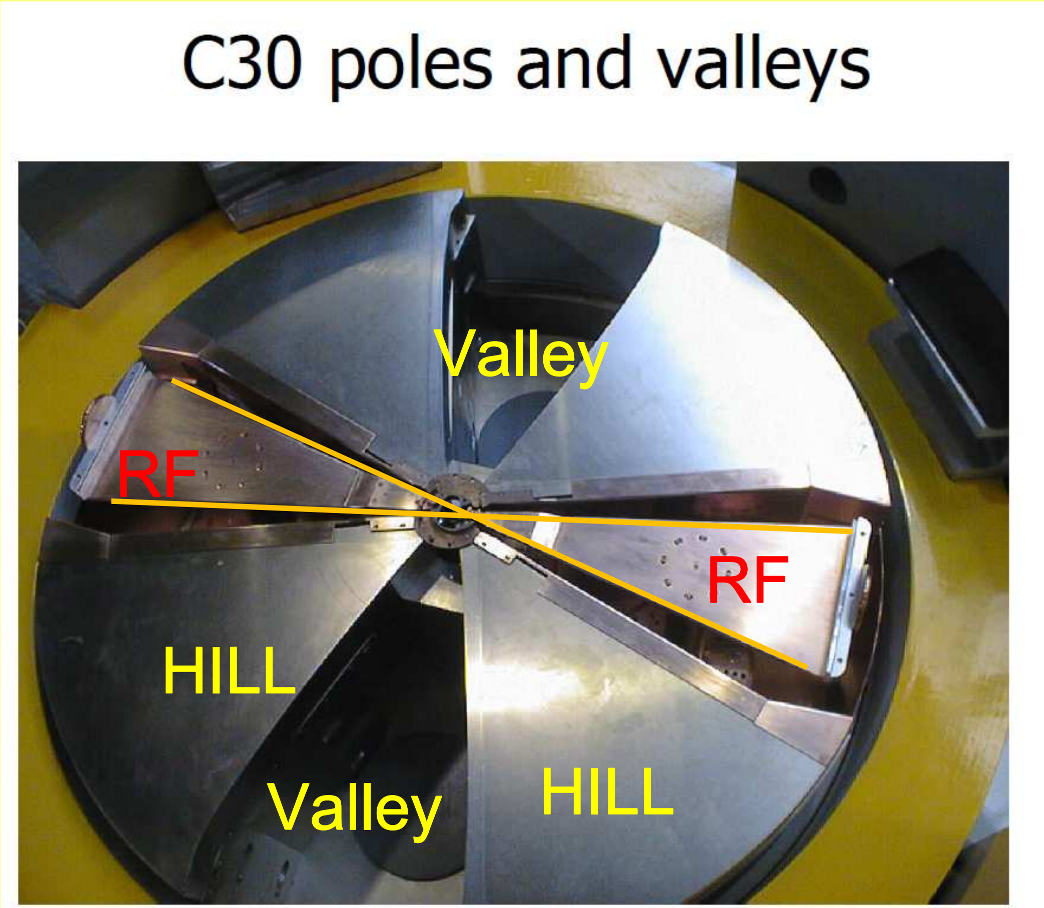


Hill valley Hill valley

Azimuthally varying Field (AVF)

Exemple : 30 MeV compact proton cyclo.

4 straight sectors



-2 RF cavities
Inserted in the valleys

= 4 accelerating gaps

4 Hills + 4 Valleys
$B = B(r,\theta)$

B Field varies
with azimuth θ