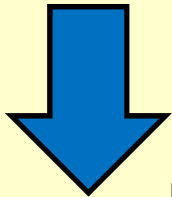


# Cyclotrons $B_z = f(R, \theta)$

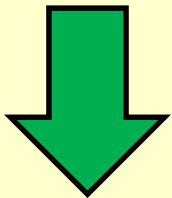
Isochronism condition  
(longitudinal)

$$B_z = B(r)$$

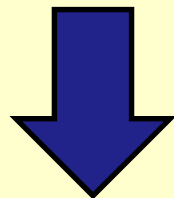
$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)m}$$



$B_z$  should increase with radius ( $B \sim r^{-n}$  with  $n < 0$ )



Unstable Vertical oscillations ( $B_r$  defocus in  $z$  plane)



Additive Vertical focusing is needed

$$\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix}$$

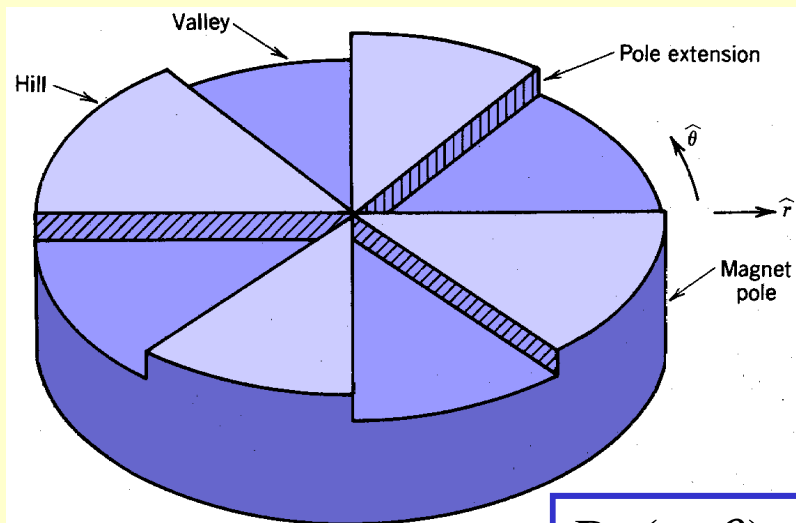
$B_\theta$  component needed ( $F_z = -q v_r B_\theta$ ) : « AVF » Cyclo

# Azimuthally varying Field (AVF) $B_z = f(R, \theta)$

an additive focusing vertical force  $v_r \cdot B_\theta$

$B_\theta$  created by:

- Succession of high field & low field regions :  $B_z = f(R, \theta)$ 
  - **Valley** : large gap, weak field
  - **Hill** : small gap, strong field



$N=4$  sectors

FLUTTER function (definition)

$$F_l = \frac{\langle (B - \langle B \rangle)^2 \rangle}{\langle B \rangle^2} \quad 1 \text{ turn}$$

$$F_l = \frac{\sigma_B^2}{\langle B \rangle^2}$$

$$B_z(r, \theta) = B_0 \cdot [1 + f \cos(N\theta)] \quad \Rightarrow \quad F_l = f^2 / 2$$

# Azimuthally varying Field (AVF) cyclo

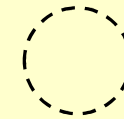
Vertical focusing  $\langle F_z \rangle_\infty \propto v_r \cdot B_\theta$



$V_r = dr/dt \vec{e}_r$  created by :

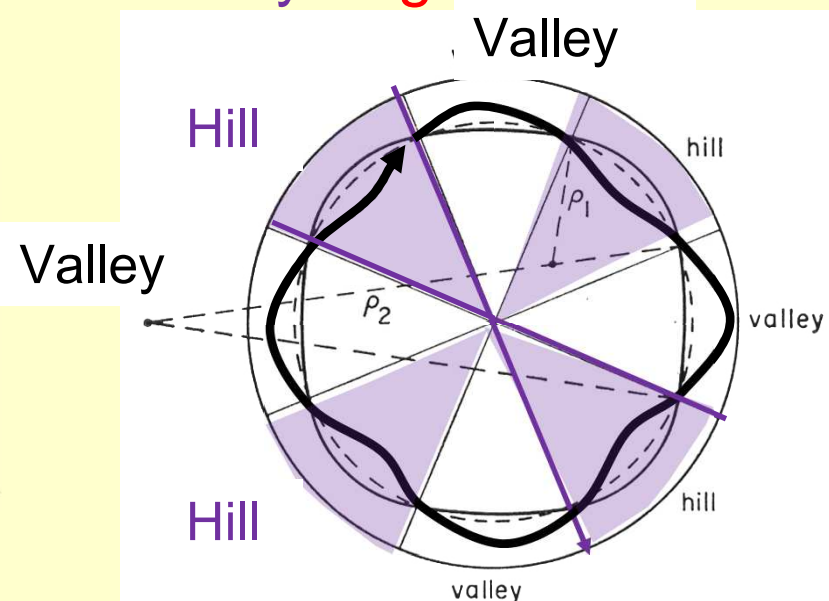
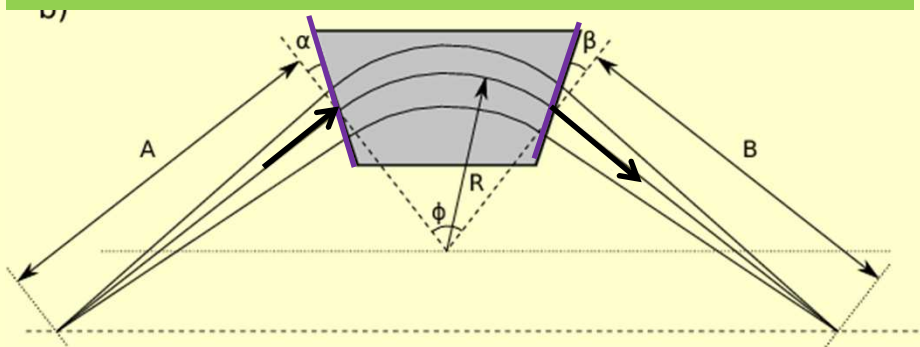
- Valley: weak field, large trajectory curvature
- Hill : strong field, small trajectory curvature

Trajectory is not a circle

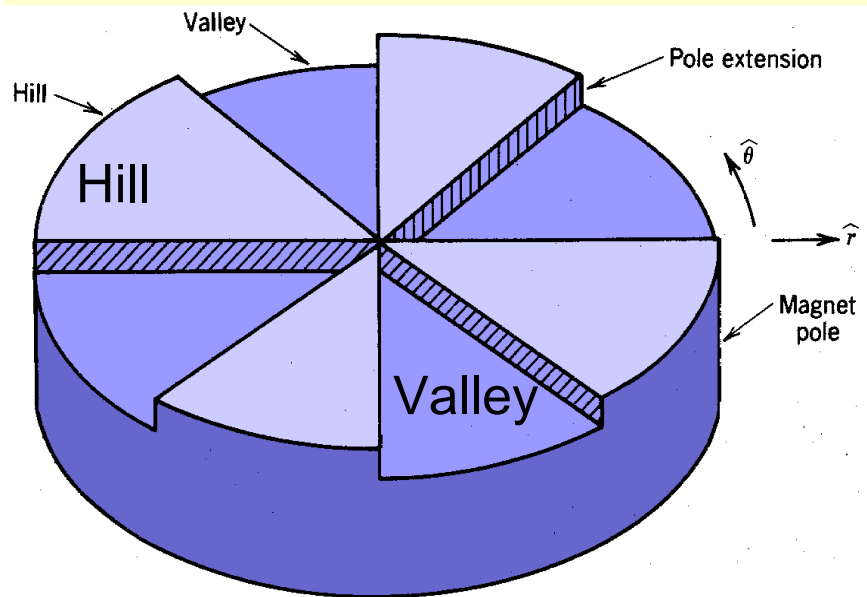


- Orbit not perpendicular to hill-valley edge

Like edge focusing in Dipole Magnet



# Vertical focusing with sectors



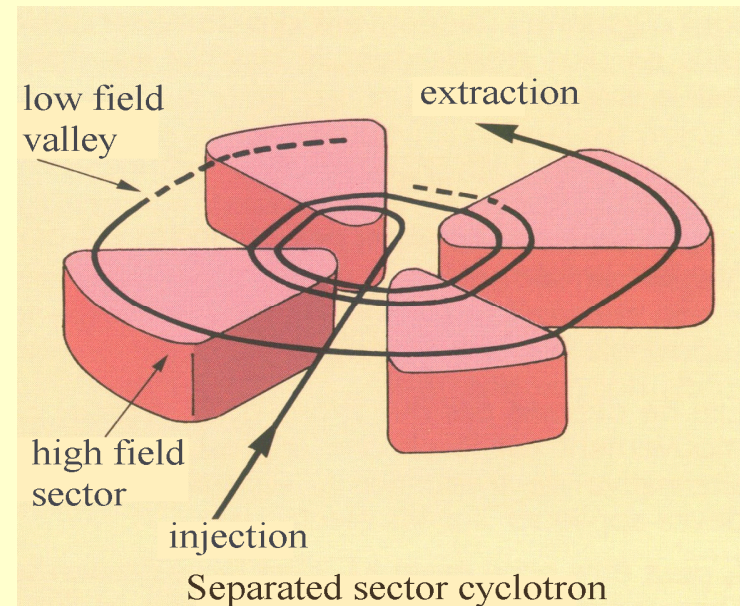
Compact cyclo : pole oscillation in  $\theta$

$$B_z = \langle B_0 \rangle [ 1 + f \cdot \cos (N \theta ) ]$$

$$f < 1 \quad f = 0.5 \cdot (B_{\text{hill}} - B_{\text{valley}}) / \langle B_0 \rangle$$

FLUTTER function (definition)

$$F_l = \frac{\langle (B - \langle B \rangle)^2 \rangle}{\langle B \rangle^2} < 1 \text{turn} >$$



Separated magnets  
generate field oscillation in  $\theta$

$$B_z = \langle B_0 \rangle [ 1 + \cos (N \theta ) ]$$

Separated sector cyclotron

The FLUTTER is larger

Larger vertical focusing

# Tutorial 1 :

Give the **Lorentz force** in a **cyclotron**

and explain the **focusing** and **defocusing** effect in Vertical plane  
of the **B<sub>r</sub>** (**radial**) and **B<sub>θ</sub>** (**azimuthal**) components

$$m\gamma \frac{d^2 \mathbf{r}}{dt^2} = m\gamma \frac{d^2 (r\mathbf{e}_r + z\mathbf{e}_z)}{dt^2} = q(\mathbf{v} \times \mathbf{B}) = ?$$

$$B_z = B_{0z} R^{-n}$$

$$m\gamma \frac{d^2 z}{dt^2} = F_z = ?$$

$$B_r = ?$$

« Curl  $\mathbf{B}=0$  »

# Tutorial 1 : Give the Lorentz force in a cyclotron and explain the focusing and defocusing effect of the $B_r$ (radial) and $B_\theta$ (azimuthal) components

$$m\gamma \frac{d^2 z}{dt^2} = F_z = q(\mathbf{v} \times \mathbf{B})_z = -q(v_r B_\theta - v_\theta B_r)$$

$$\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ v_r & v_z & v_\theta \\ B_r & B_z & B_\theta \end{vmatrix}$$

$$\frac{d^2 z}{dt^2} + \frac{q}{m\gamma} (v_r B_\theta + R\omega_{rev} n \cdot \frac{B_z}{R} z) = 0$$

$$v_r = \frac{dr}{dt} \quad v_\theta = R \frac{d\theta}{dt} = R\omega_{rev}$$

Isochronous cyclotron  $n < 0$   
 $B_z(R)$  increases

$B_z = f(R, \theta)$  and « Curl  $B = 0$  »

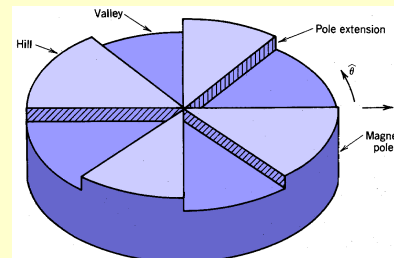
$B_r = h(R, \theta) = -z \cdot n B_z / R \cdot \mathbf{e}_r$

$n < 0$   $B_r$  : Defocusing in vertical plan

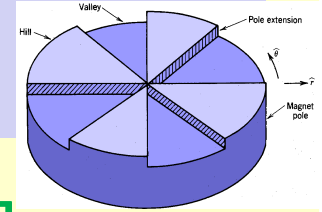
$B_\theta = g(R, \theta) \cdot \mathbf{e}_\theta$

Induced by Sectors (AVF)  
Focusing in vertical plan

it Compensates  $n < 0$



## Tutorial 2 : Flutter $F$ in AVF cyclo



The field of a cyclotron is  $B_z(r,\theta) = B_0(r) [1 + f \cdot \cos(4\theta)]$

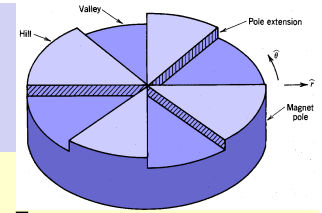
A: How many hills and valleys have such a cyclotron

B: Compute  $B_\theta$  With  $\text{Curl } B = 0$

$$\nabla \times \mathbf{B} = \left[ \mathbf{e}_r \frac{\partial}{r \partial r} + \mathbf{e}_z \frac{\partial}{\partial z} + \mathbf{e}_\theta \frac{\partial}{r \partial \theta} \right] \times [B_r \mathbf{e}_r + B_z \mathbf{e}_z + B_\theta \mathbf{e}_\theta]$$

C: Compute the flutter function «  $F$  »

## Tutorial 2 : Flutter $F$ in AVF cyclo



if the field of a cyclotron is  $B_z(r, \theta) = B_0(r) [1 + f \cdot \cos(4\theta)]$

4 valleys, 4 hills : 4 sectors

$$B_{\text{hill}} = B_0(r) [1 + f]$$

$$B_{\text{valley}} = B_0(r) [1 - f]$$

$$\langle B \rangle = B_0$$

$$\nabla \times \mathbf{B} = \left[ \mathbf{e}_r \frac{\partial}{\partial r} + \mathbf{e}_z \frac{\partial}{\partial z} + \mathbf{e}_\theta \frac{\partial}{r \partial \theta} \right] \times [B_r \mathbf{e}_r + B_z \mathbf{e}_z + B_\theta \mathbf{e}_\theta]$$

$$\frac{\partial B_\theta}{\partial z} - \frac{\partial B_z}{r \partial \theta} = 0$$

$$B_\theta(z, r, \theta) = z \cdot B_0(r) [-4f \cdot \sin(4\theta)] / r + K$$

C:

$$\langle (B - \langle B \rangle)^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} B_0^2 [f \cos(4\theta)]^2 d\theta$$

$$F_l = \frac{\langle (B - \langle B \rangle)^2 \rangle}{\langle B \rangle^2}$$

Flutter function  $F_l = B_0^2 (f.)^2 / 2B_0^2 = f^2 / 2$



# Separated Sectors(ring) Cyclotron

Focusing condition limit: ( $n < 0$ )

$$v_z^2 = n + \frac{N^2}{N^2 - 1} F_l + \dots > 0$$

Increase the flutter  $F_l$ , using separated sectors where  $B_{\text{valley}} = 0$

$$F_l = \frac{\langle (B - \langle B \rangle)^2 \rangle}{\langle B \rangle^2}$$



PSI= 590 MeV proton  
 $\gamma=1.63$

➔ Separated sectors cyclotron  
needed at “High energies” ( $n(R) = 1 - \gamma^2 \ll 0$ )

# Vertical focusing and isochronism

## 2 conditions to fulfill

- Increase the vertical focusing force strength:

$$v_z^2 = n + \frac{N^2}{N^2 - 1} F_l + \dots > 0$$

- Keep the isochronism condition true:  $n < 0$

$$n = -\frac{R}{B_{0z}} \frac{\partial B_z}{\partial R} = 1 - \gamma^2 < 0$$

So we should have:

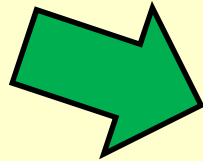
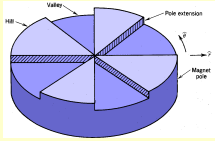
$$\frac{N^2}{N^2 - 1} F_l > \gamma^2 - 1$$

**For High Energy cyclotron : 2 solutions for vertical stability**

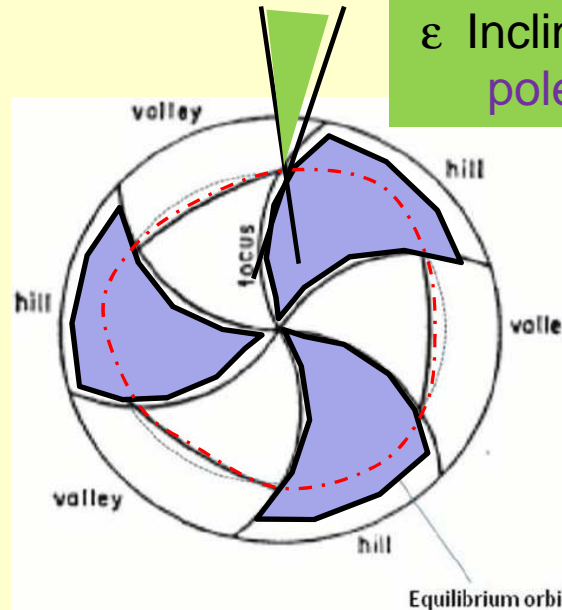
- 1) Larger Flutter (separated sectors)  $F_l$
- 2) Other idea ??? Yes (spiralled sectors)

Increase  $N_{\text{sectors}}$  doesn't help **Z stability** but it reduces resonances

# Better vertical focusing : Spiralled sectors



AVF with straight sectors



$\epsilon$  Inclination angle  
pole edge AND Trajectory

AVF cyclo with spiralled sectors

Larger vertical focusing

$$B_z = \langle B_0 \rangle ( 1 + f(r).g (r, \theta) )$$

$$\text{Spiral eq. } r = A.(\theta + 2\pi j / 2N)$$

$$\tan \epsilon = 2r/A$$

Additive vertical focusing : + FLUTTER  $\cdot (1 + 2 \tan^2 \epsilon)$

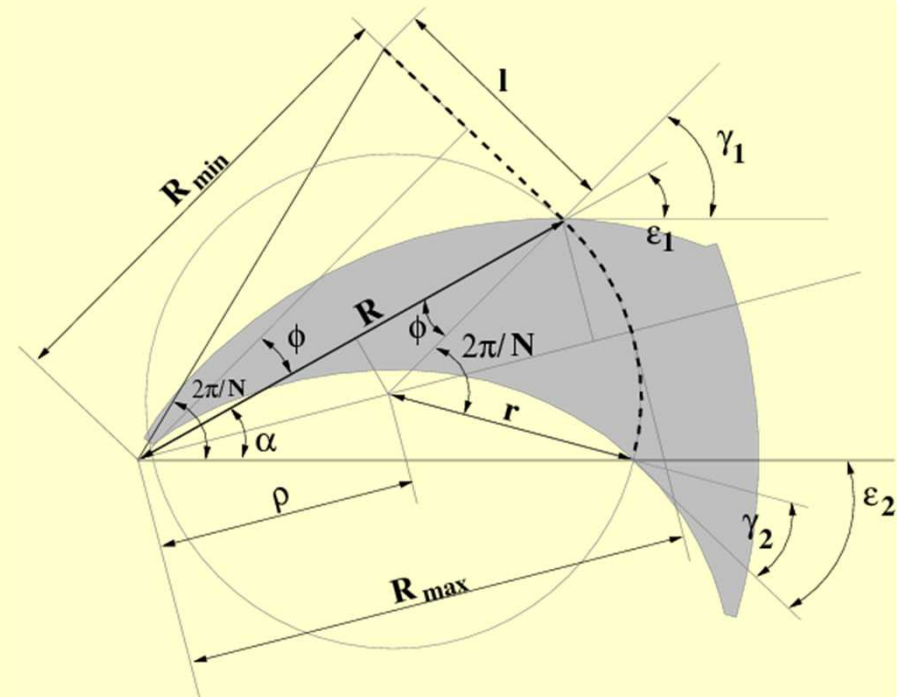
$$v_z^2 = n + \frac{N^2}{N^2 - 1} F_l (1 + 2 \tan^2 \epsilon)$$

# Spiralled sectors

By tilting the edges ( $\varepsilon$  angle) :

- The valley-hill transition became more focusing
- The hill-valley transition became less focusing

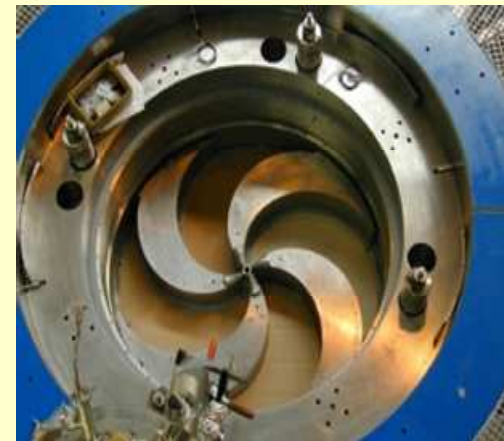
But by the strong focusing principle (larger betatron amplitude in focusing, smaller in defocusing), the net effect is focusing (cf F+D quadrupole).



$$v_z^2 = n + \frac{N^2}{N^2 - 1} F_l (1 + 2 \tan^2 \varepsilon)$$

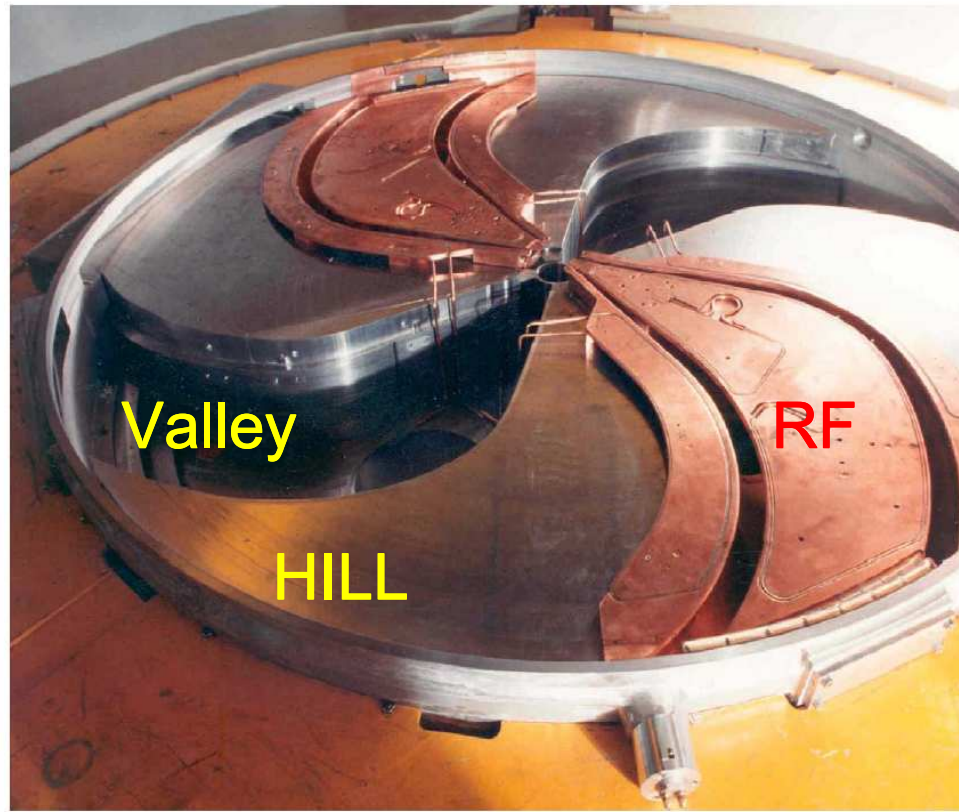
$n < 0$

$F_l > 0$



# Exemple : 235 MeV compact proton cyclo 4 spiralled sectors

C235 poles and valleys

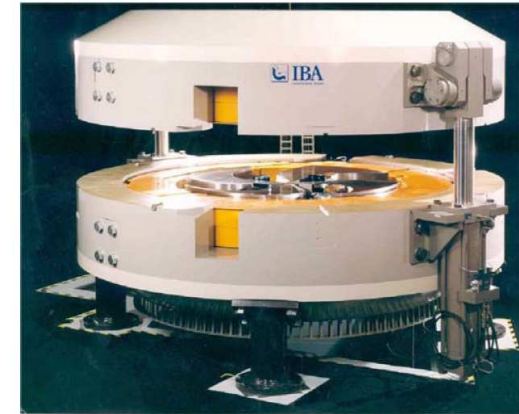


-2 RF cavities (Dees)  
Inserted in the valleys

4 Spiralled sectors:

Higher energy=  
Higher axial focusing  
required

# Exemple : 235 MeV compact proton cyclo



## C235 lower part



Without RF

4 Spiralled sectors:

Valley gap = 60cm

Hill gap = 10cm-1cm

B<sub>Hill</sub> ~ 2-3 Teslas

B<sub>valley</sub> ~ 1 Tesla

$\langle B \rangle \sim 1.7$  T in center

$\langle B \rangle \sim 2.1$  T extraction

# Beam dynamics in the ISOCHRONOUS cyclotrons

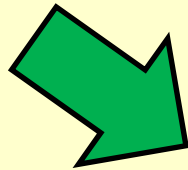
$B = \text{Constant}$  ≠ Isochronism condition

A STRONG LIMITATION in energy  $\gamma = 1$   
to get the ions synchronise With RF

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R) m}$$

$$B_z = B_0 \cdot g(R)$$

$B_z$  increase with  $R$  (field index  $n < 0$ )

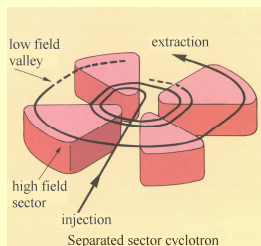
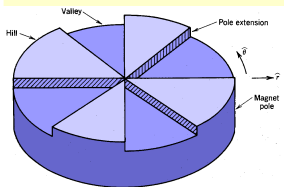


Unstable **Vertical oscillations**  
strong limitation in transmission



Additive **Vertical focusing** is needed : -N sectors (**Hills//valleys**)

- **separated straight sectors**
- **spiralled sectors**
- **separated spiralled sectors**



$$B_z = B_0 \cdot g(R, \theta)$$

4 techniques

## One other possibility

### SYNCHRO CYCLOTRON (NOT ISOCHRONOUS)

Acceleration condition with  $B_z$  uniform or decreasing ( $n \geq 0$ )

$\omega_{\text{rev}} = \text{not constant}$

**Not isochronous !!**

But no vertical instabilities

$v_z^2 = n \geq 0$  **stable oscillation**  
Revolution frequency evolves  $F_{\text{rev}}(t) = F_{\text{rev}}(\text{Radius})$

beam has to be synchronous With RF :

$\omega_{\text{rev}} / h = \omega_{\text{RF}}$  synchronous

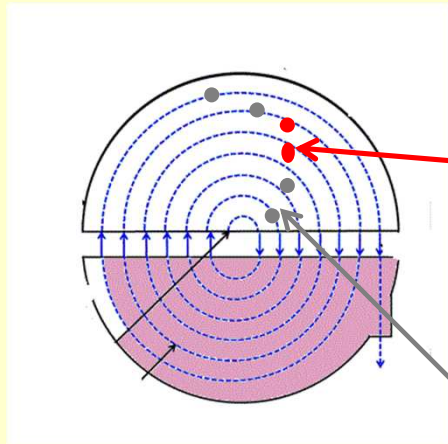
**But**  $\omega_{\text{rev}} / h = \omega_{\text{RF}} = f(\text{time}) = f(\text{Radius})$  NOT constant

Revolution frequency is evolving during acceleration  $F_{\text{RF}}(t)$   
(like a synchrotron)

Pulsed Machine  $\omega_{\text{RF}}(t)$  : SYNCHRO CYCLOTRON



# Synchro-cyclotrons : RF cycled, but stable in z

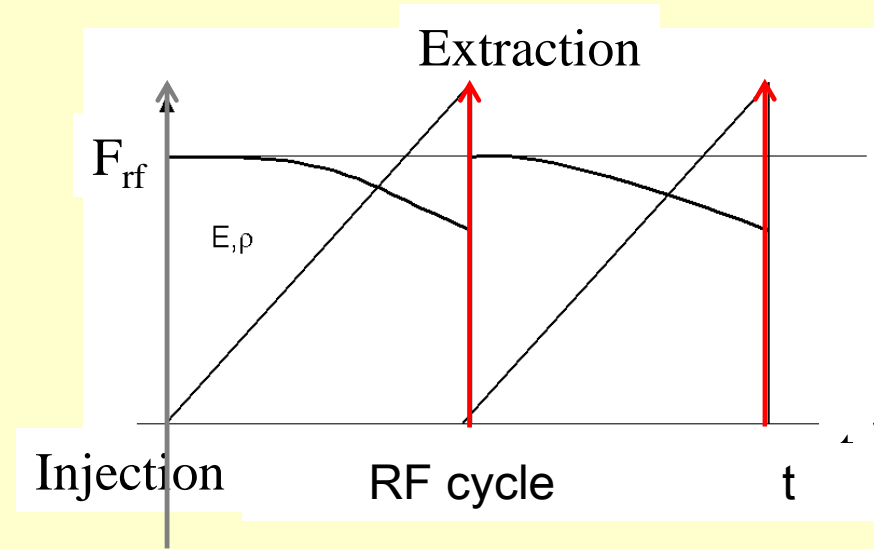


Pulsed beam

Accelerated bunches

$$F_{RF} = h F_{rev}$$

Not accelerated



Less intensity (pulsed) available (not cw)

Exemple : medical aplication  
Superconducting synchrocyclo.



ProteusOne® (IBA) : 250 MeV proton

$B_z = 5.7 - 5.0$  Tesla (very compact)

Reextraction = 0.6 m / harmonics = 1

$F_{RF} = 93$  MHz - 63 MHz (Reextraction)

Beam pulse : Every 1 ms

*Few other slides  
for questions*

# Summary N° 2 : without equations

*Isochronous Cyclotron* :  $B_z = f(R, \theta)$  complex magnet  
 $F_{rf} = \text{constant}$

**Isochronism** :  $\omega_{RF} = H \omega_{rev} = \text{constant}$   
(particle are synchronous with RF :  $F_{RF} = \text{const}$ )

with  $\langle B_z \rangle = f(\text{radius}) \sim \gamma(\text{radius})$

**Defocusing Vertical force** :  $B_z \sim \gamma(r) \Rightarrow F_z = v_\theta B_r e_z$

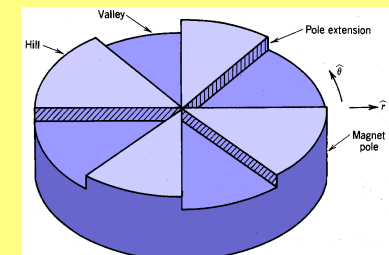
unstable oscillation

Compensation with focusing force with  $\theta$  field oscillations

Vertical stability can be obtained with azimuthal field variations

$$B_z = f(r, \theta)$$

cyclotron pole  
valley and hill



# Summary N° 3 : without equations

*Synchro-Cyclotron* :  $B_z \neq \text{constant}$  (uniform) = simpler magnet  
 $F_{rf} = \text{Not constant} = \text{Not CW}$

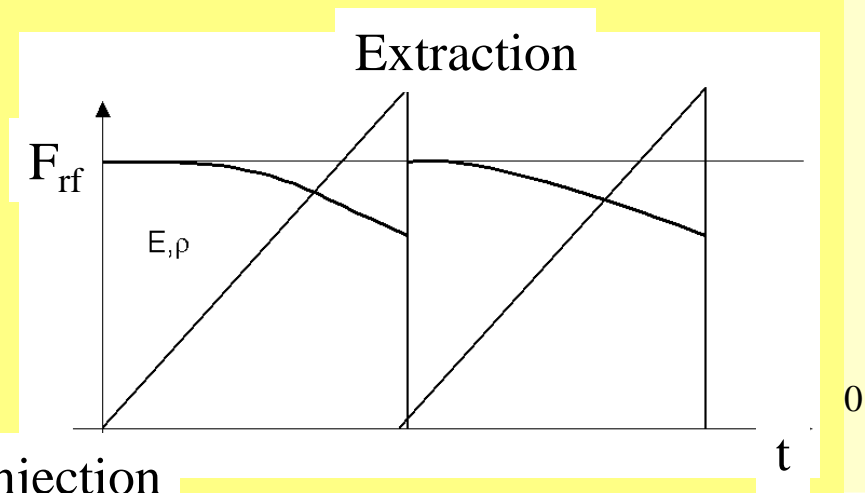
-Not Isochron :  $\omega_{rev} = \text{NOT constant}$

$$\omega_{rev} = q B_z / m \gamma (R)$$

NO Defocusing Vertical force  $B_r = 0$

- Synchronicity with RF:  $\omega_{rev} = q B_z / m \gamma = f(\text{Radius}) = \omega_{RF}(t)$

*For longitudinal stability !  
(RF to be varied)*



# CYCLOTRONS

## The Family

$$\omega_{rev} = \frac{qB_z}{\gamma m}$$

1

Conventional cyclotrons

$B_z = \text{uniform}$   
 $F_{rev} = \text{evolves with } \gamma \text{ !!!}$   
 $FRF = \text{constant}$

NOT USED anymore  
 Limited in energy :  $EK < 1\text{MeV}$

2: isochronous

Compact cyclotrons

1 magnet with modulation

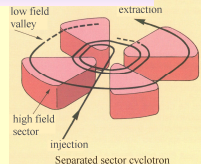


$B_z = \text{NOT uniform} = f(\text{radius})$   
 $F_{revolution} = \text{Constant}$   
 $FRF = \text{constant}$

Isochronous

$$\omega_{rev} / h = \omega_{RF}$$

Separated sectors



Vertical focusing with  
 $B_z = f(r, \theta)$

3 : non isochronous

Synchro-cyclotrons

$F_{rev} = \text{NOT Constant}$   
 $FRF = \text{NOT Constant} = \text{beam pulsed}$

Not Isochronous

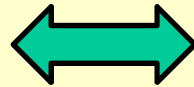
Less intensity (pulsed)  $\neq$  cw

$$\omega_{rev}(R) / h = \omega_{RF}(t)$$

## Tutorial 3 : What is the field index $n(R)$ ?

$n(R)$  : it gives the radial evolution of  $B_z$

$$n = - \frac{R}{B_0} \frac{\partial B_z}{\partial R}$$



Equivalent definition

$$B_z \sim B_0 (r / R_0)^{-n}$$

The field index is not constant in a cyclotron  $n = n(\text{radius})$

Isochronous cyclotron  $n(r) < 0$  :  $\langle B_z(r, \theta) \rangle_{\text{turn}}$  increases with  $R$

$$\begin{aligned} B_z(R) &= \gamma(r) B_0 \\ &= k r^{-n} \end{aligned}$$

$$\langle B_z(R) \rangle = \frac{B_0}{\sqrt{1 - [R \omega / c]^2}} = B_0 \cdot R^{-n(R)}$$

$$\gamma = \frac{1}{\sqrt{1 - [v/c]^2}} = \frac{1}{\sqrt{1 - [R\omega/c]^2}}$$

# Dynamics in cyclotron

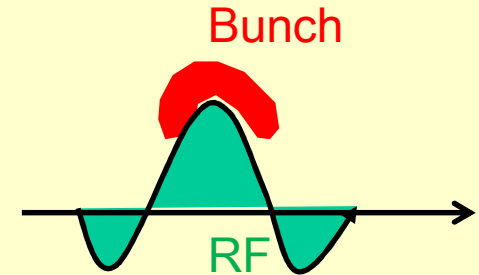
## summary

$$Qe_0 \hat{V} \cos \phi \cdot N_{gap}$$

**Energy gain per turn**

$$\phi_0 \approx 0^\circ$$

**Central RF phase ,  
Ion bunches are centered at  $0^\circ$**



$$\omega_{RF} = h\omega_{rev} = const$$

**RF synchronism = Isochronism  
(h - harmonic number)**

$$R = R(t) = R(N^\circ turn)$$

**Orbit evolving**

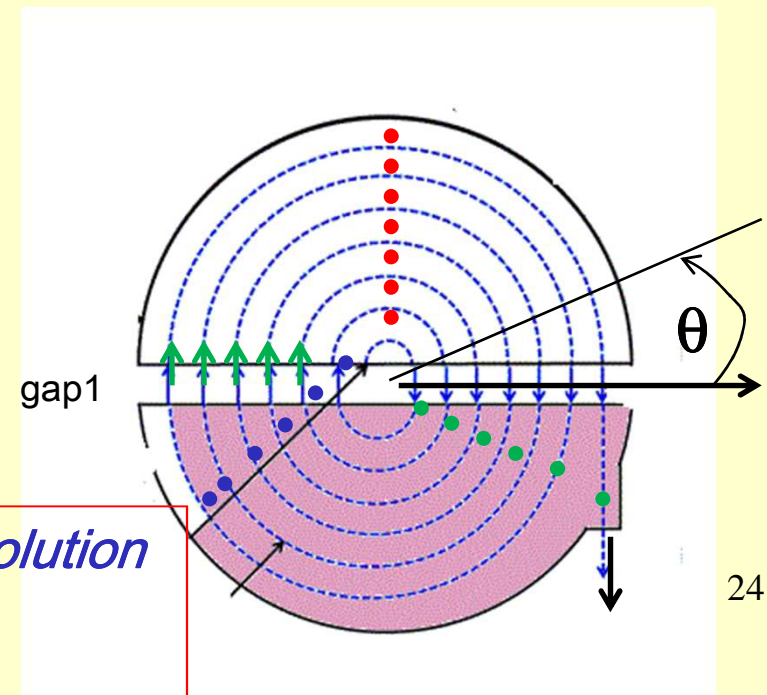
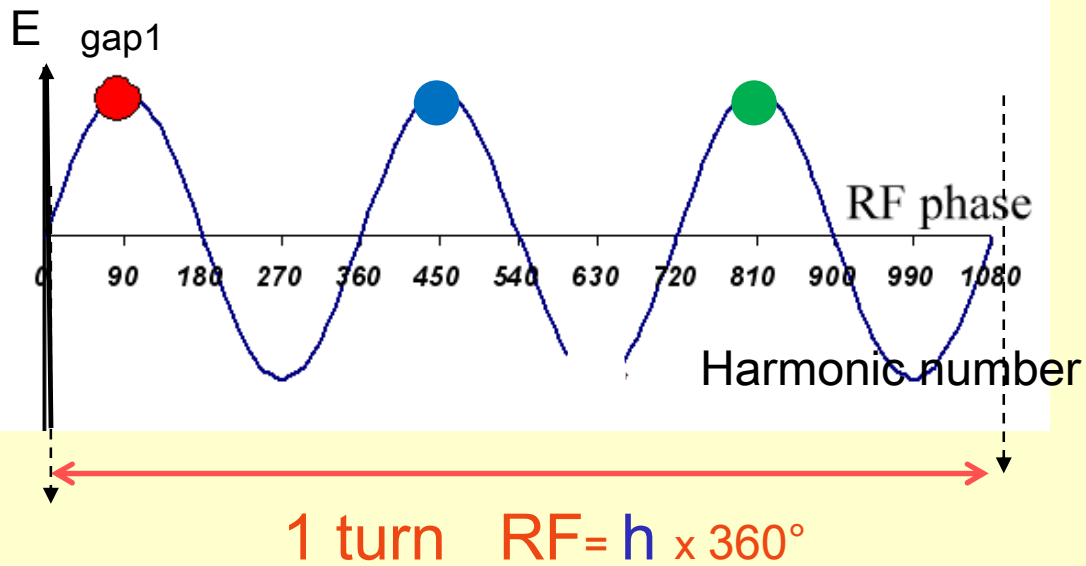
$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R) m} = const$$

$$B\rho(t) = \frac{P}{q} \Rightarrow \langle B \rangle = B\rho / R$$

**Average Magnetic field**

# Harmonic number $h = F_{RF} / F_{rev}$

$h = \text{integer}$        $\omega_{RF} = h \omega_{rev}$

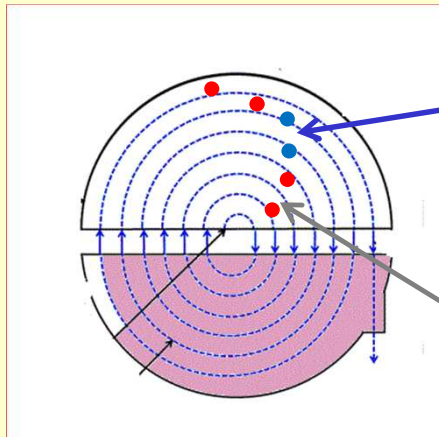


$h = 1, 2, 3$     Number RF oscillations per revolution

→  $h$  bunches by turn     $\omega_{rf} = h \omega_{rev}$



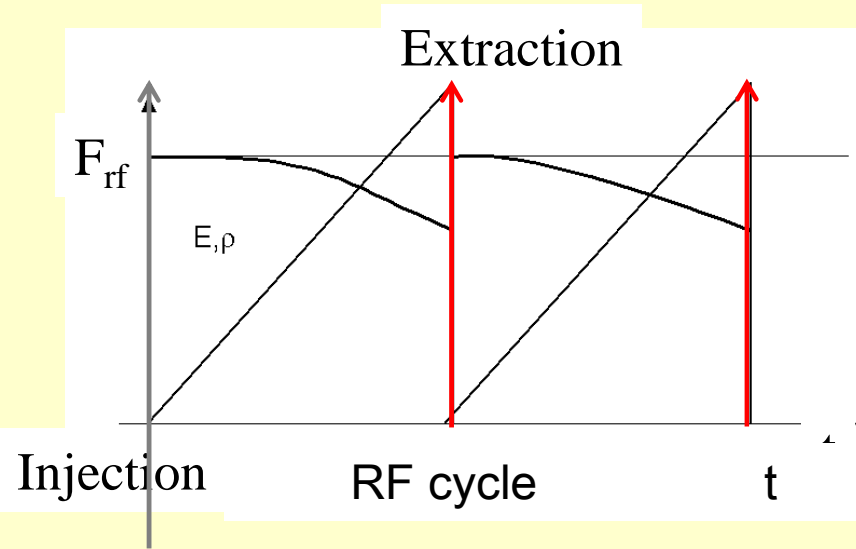
# Synchro-cyclotrons : pulsed accelerator



Accelerated bunches

$$F_{RF} = h F_{rev}$$

Not accelerated (beam losses)



Less intensity (pulsed) available (since not *cw*)

$$\omega_{rev} = \frac{\omega_{RF}}{h} = \frac{qB_z}{\gamma(R)m}$$

$$\omega_{RF}(injection) = h \frac{qB}{\gamma m} \approx h \frac{qB}{m}$$

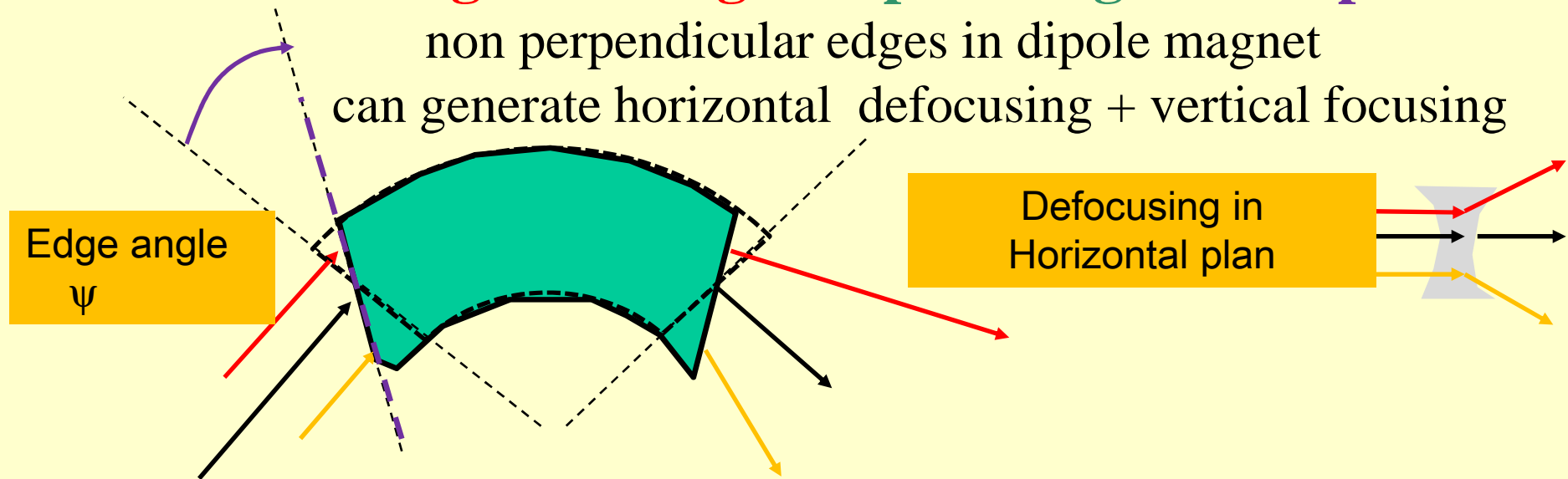
$$\omega_{RF}(extraction) = h \frac{qB_0}{\gamma(R).m}$$

2-3 bunches are synchronized with RF

Other bunches are lost (not accelerated properly)

## Edge focusing in dipole magnet recap

non perpendicular edges in dipole magnet  
can generate horizontal defocusing + vertical focusing

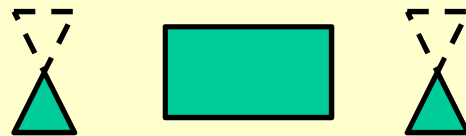


Non perpendicular edge in dipole magnet can provide

- 1) additive focusing in vertical + 2) defocusing in horizontal plane

The optical Transfer Matrix  $M$  is

$$M_{\text{dipole}} = M_{\text{edge1}} \cdot M_{\text{body}} \cdot M_{\text{edge2}}$$



$$M_{z_{\text{edge}}} \begin{bmatrix} z \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{\tan \psi}{R} & 1 \end{bmatrix} \begin{bmatrix} z_0 \\ z_0' \end{bmatrix}$$

*Equivalent effect in AVF Cyclo*