Chapter 2

Cyclotrons : specific techniques

- Acceleration and RF cavities
- Injection (axial or radial)
- Extraction
  (stripping, turn separation, precession…)

Cyclotrons Tutorial 4

- **An cyclotron is supposed to accelerate ions with A nucleons and a charge state Q.**

- **Demonstrate that the maximal kinetic energy *E/A* of a cyclotron is**

$$E/A = K_b \,.\, (Q/A)^2$$

Nota : Give the *Kb* factor in a non relativistic approximation
using the extraction radius R,
the maximal average magnetic field B.

The mass of the ions is $m = Am_0$ & the charge of the ions is $q = Qe_0$

Cyclotrons Tutorial 4

- **An cyclotron accelerate ions with A nucleon and a charge state Q.**

$$E/A = Kb \,.\, (Q/A)^2 \quad ????????$$

Answer : $E=(\gamma-1)mc^2 \sim \frac{1}{2} mV^2 = \frac{1}{2} m (R\omega)^2$

$$E = \frac{1}{2} m (R\, qB/m)^2 = \frac{1}{2} A m_0 (R\, Q\, e_0\, B / A\, m_0)^2$$

$$E/A = \frac{1}{2} (e_0\, R\, B)^2 / m_0 \quad (Q/A)^2$$

$$E/A\ [MeV/A] = Kb \,.\, (Q/A)^2$$

$$Kb \sim (R_{extract} .\, B_{max})^2$$

Cyclotrons Tutorial 5

- A COMPACT CYCLOTRON have a Kb factor of 30 MeV

  $(E/A = K_b \cdot (Q/A)^2)$

What is the maximal energy

we could reach with such a cyclotron magnet (Kb=30 MeV)

a) With a proton beam

b) With a carbon beam (with Q= 6+)

The cyclotron magnet have <B>=1Tesla, what is the revolution frequency ? ($F_{rev} = \omega/2\pi$)

c) of a proton beam

d) of a carbon beam (with Q= 6+)

Can we work with the same RF cavity for the two beams ?

($\omega_{rf} = h\ \omega = h\ qB/m\gamma$)

Acceleration

- The final energy is independent of the accelerating potential $V = V_0 \cos\varphi$.

If V_0 varies, the number of turn varies.   (Bρ*final* =<B>.R*extraction*)

- The energy gain per turn depends on the peak voltage V_0, but is constant, if the cyclotron is isochronous (φ =const):

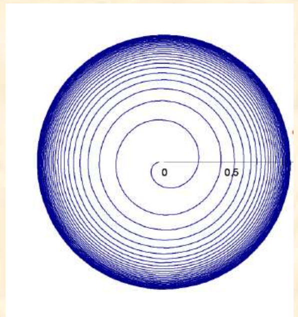
$$\delta E = N_g \; q \; V_0 \cos\varphi$$

N_g : number of gaps per turn

- The radial separation δr between two turns varies as 1/r ($\gamma \sim 1$) :

$$\frac{\delta r}{r} = \frac{\delta B\rho}{B\rho} = \frac{\delta p}{p} = \frac{\gamma}{\gamma+1}\frac{\delta E}{E} \approx \frac{qV_0 \cos\varphi}{2E} \propto \frac{1}{r^2}$$

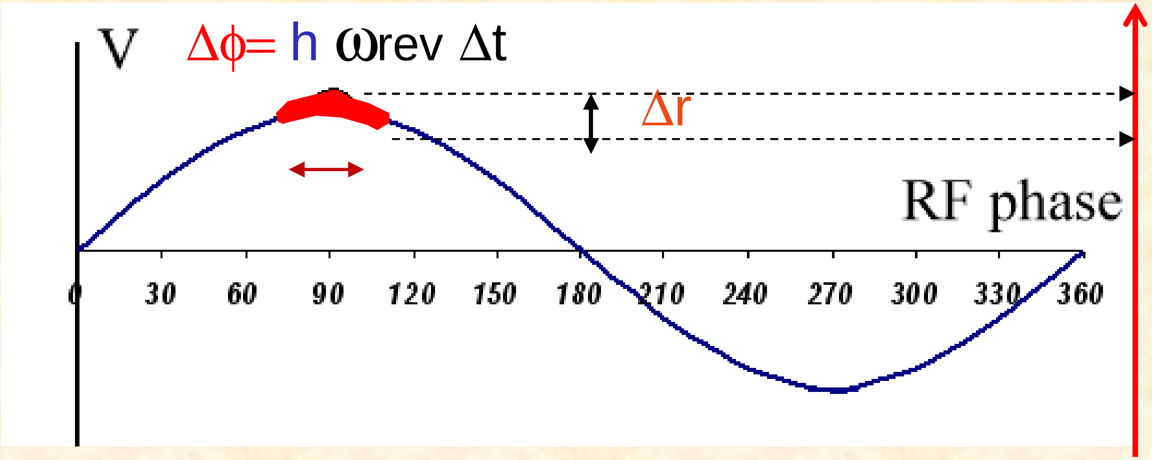
$$\delta r \propto \frac{1}{r}$$



Acceleration & bunch length Δt

•The bunch length **Δt** induces radial dispersion **Δr** & energy dispersion

$$\frac{\Delta r}{r} = \frac{\Delta B\rho}{B\rho} = \frac{\gamma}{\gamma+1}\frac{\Delta E}{E} \approx \frac{1}{2}\frac{\Delta[qV_0\cos(h\omega_{RF}t)]}{E}$$

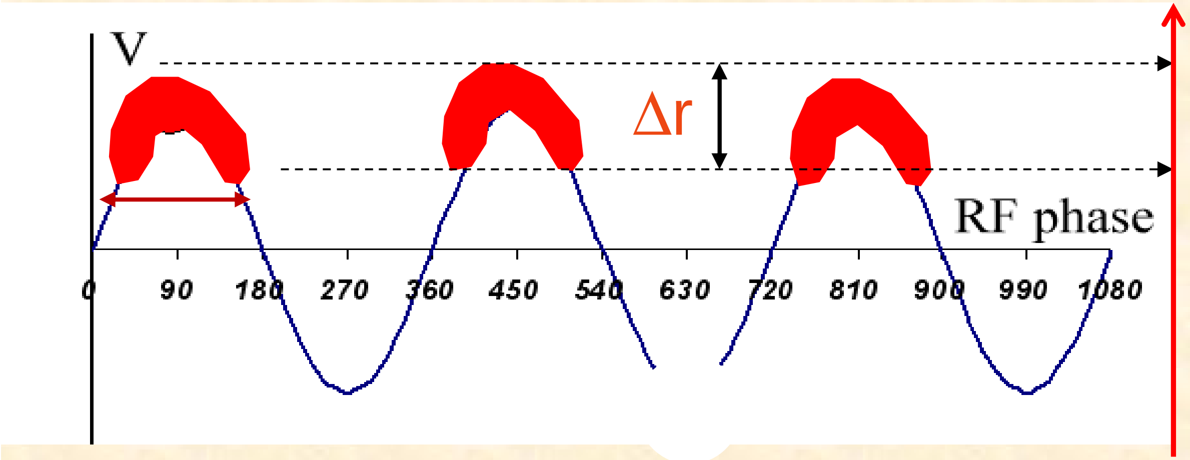


harmonics=1

Δr ~ qV ωrf Δt

harmonics >1

Worst beam quality

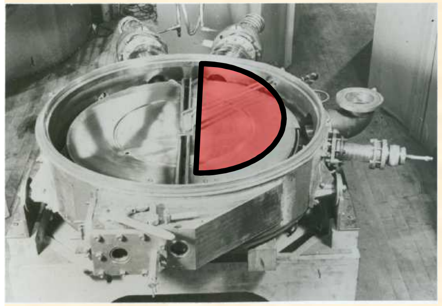


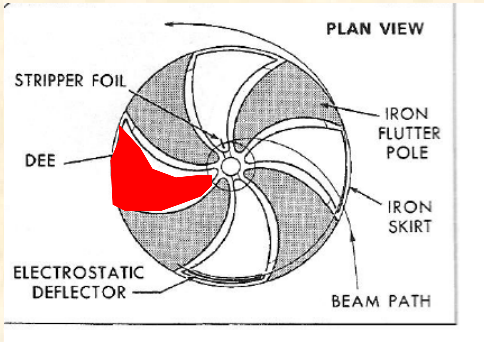
Δr larger ~ qV H ωrf Δt

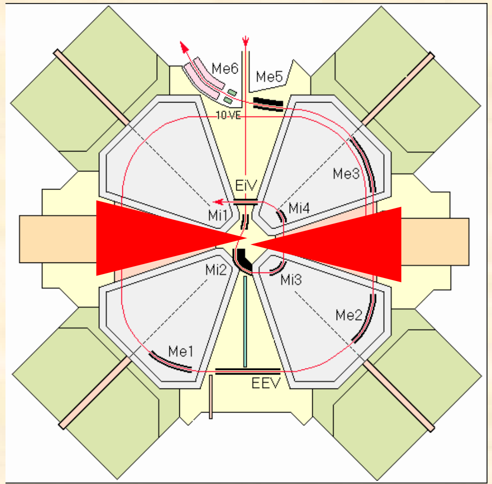
~ Energy dispersion larger

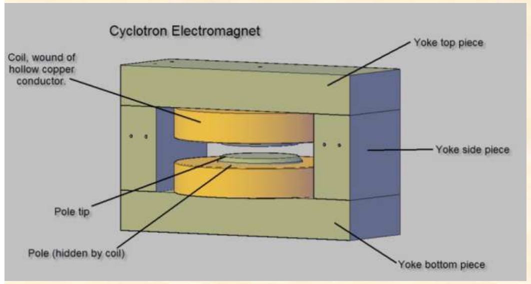
Acceleration RF Technology

Magnetic structure => RF cavity's shape







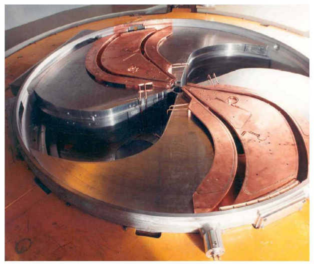


The classical "D" shape

"Curved sector"

For spiral AVF

C235 poles and valleys



"Triangle" shape

For separated sector cyclo

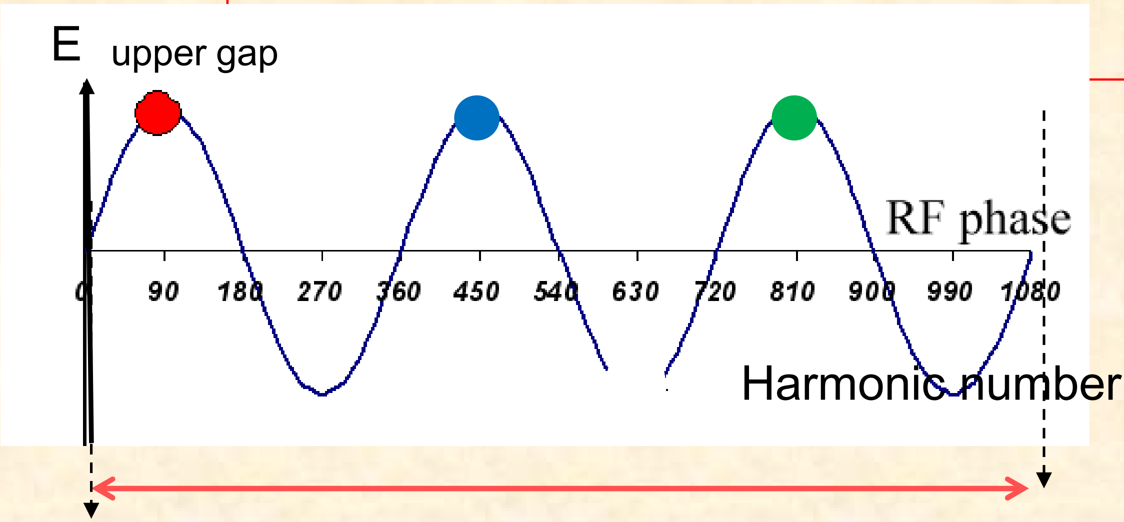
The choice of the pole shape and the number of sectors N have a great impact on the available space for RF systems. Dees have to fit into the gaps and/or valley sections

Harmonic number *h* =FRF/Frev

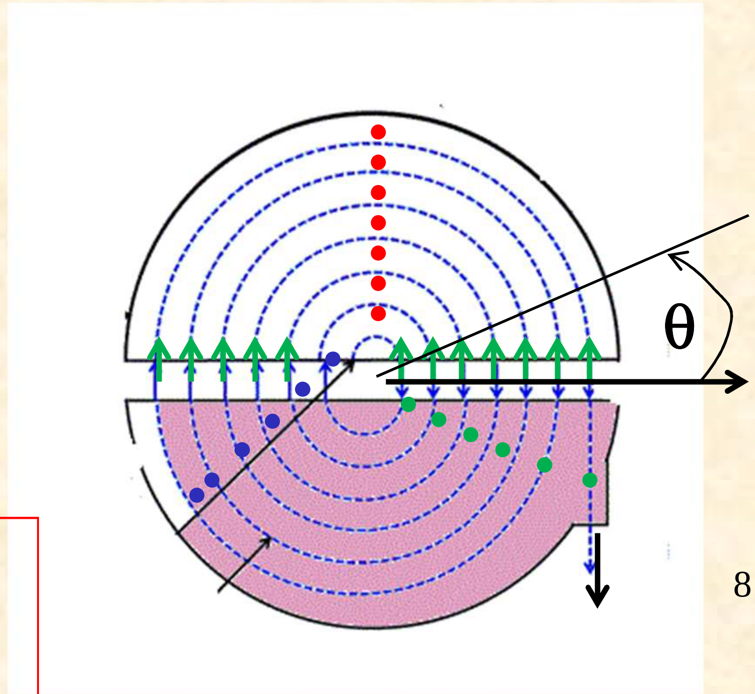
***h* =integer** $\omega_{rf} = \mathbf{h}\ \omega_{rev}$

h Rf oscillations

in a revolution time



1 turn RF= 3 x 360°



h =1,2,3 *Number of bunch per turn*

h bunches by turn $\omega_{rf} = \mathbf{h}\ \omega_{rev}$

RF Cavities
in variable energy cyclotron

Often, in research facilty Cyclotrons must provide
ions at variable energy

How to adjust the final energy to the needs ?

$$\omega_{rev} = \frac{qB}{\gamma\, m} = \frac{\omega_{RF}}{h}$$

$\omega_{rf} = \omega_{rev} \times h$

Adjust Bz which modify ωrev for a given ion (m,q)

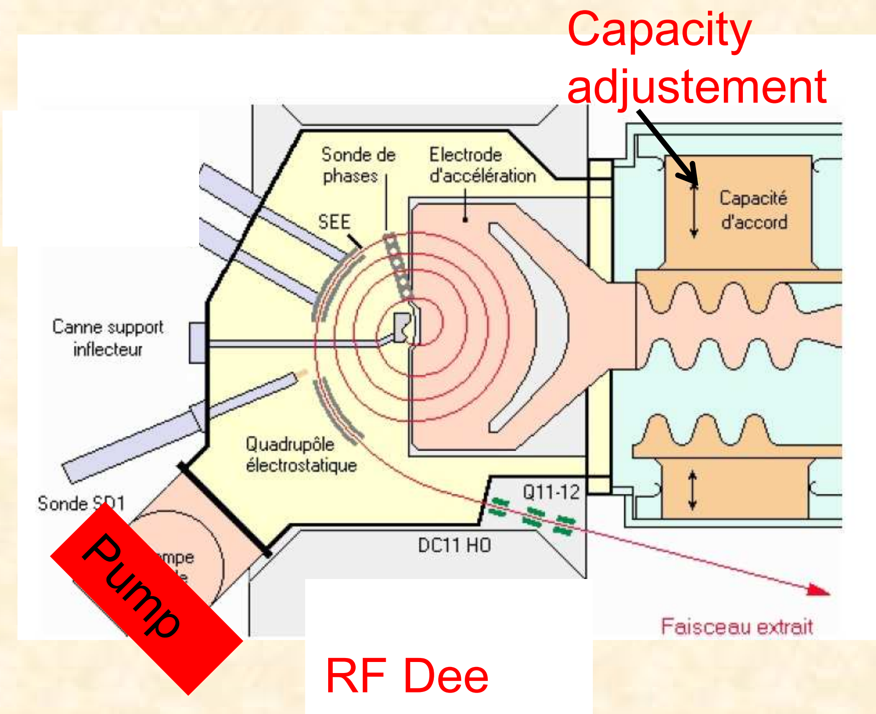
ωRF should be adjusted as well

Variable frequency ACCELERATING CAVITY ARE needed

RF Cavities for "variable ions machine" Adjust the resonance of the cavity

Resonance for a cavity
= minimal impedance (Z) for maximal Voltage

$$\omega_{res}^2 = 1/LC$$



Cyclotron : Variable Energy with B and Frf variable

$$1/Z = 1/R + j\omega(C - 1/L\omega^2)$$

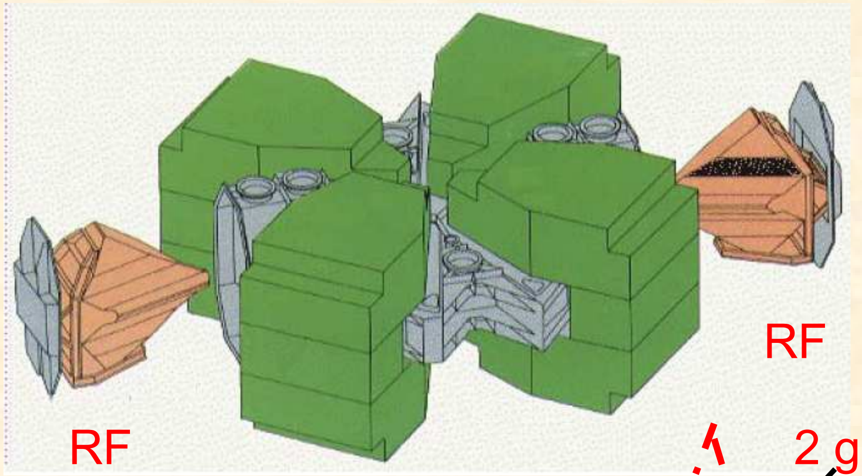
$$\omega_{rev} = \frac{qB}{\gamma m} = h\omega_{RF}$$

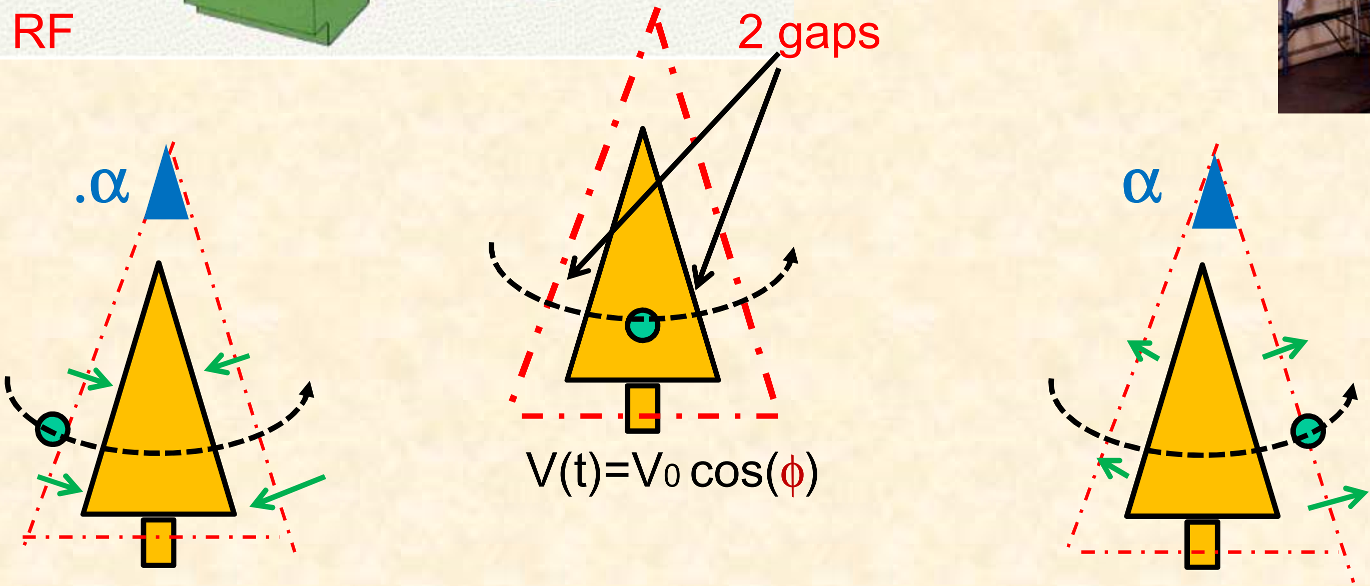
Variation of the Capacity C :

to adjust ωresonance

ωresonance = ωrf= ωrev / h

RF Cavities : example 1
for Separated Sectors Cyclo





$V(t)=V_0 \cos(\phi - h\alpha/2)$

$V(t)=V_0 \cos(\phi)$

$V(t)=V_0 \cos(\phi + h\alpha/2)$

Example 1: RF Cavities (not Dees)

Energy gain in 2 gaps~ $\cos(\phi - h\alpha/2) + \cos(\phi + h\alpha/2)$

$$\delta E = qV_0 \sin\left(\frac{h\alpha}{2}\right)\cos\varphi$$

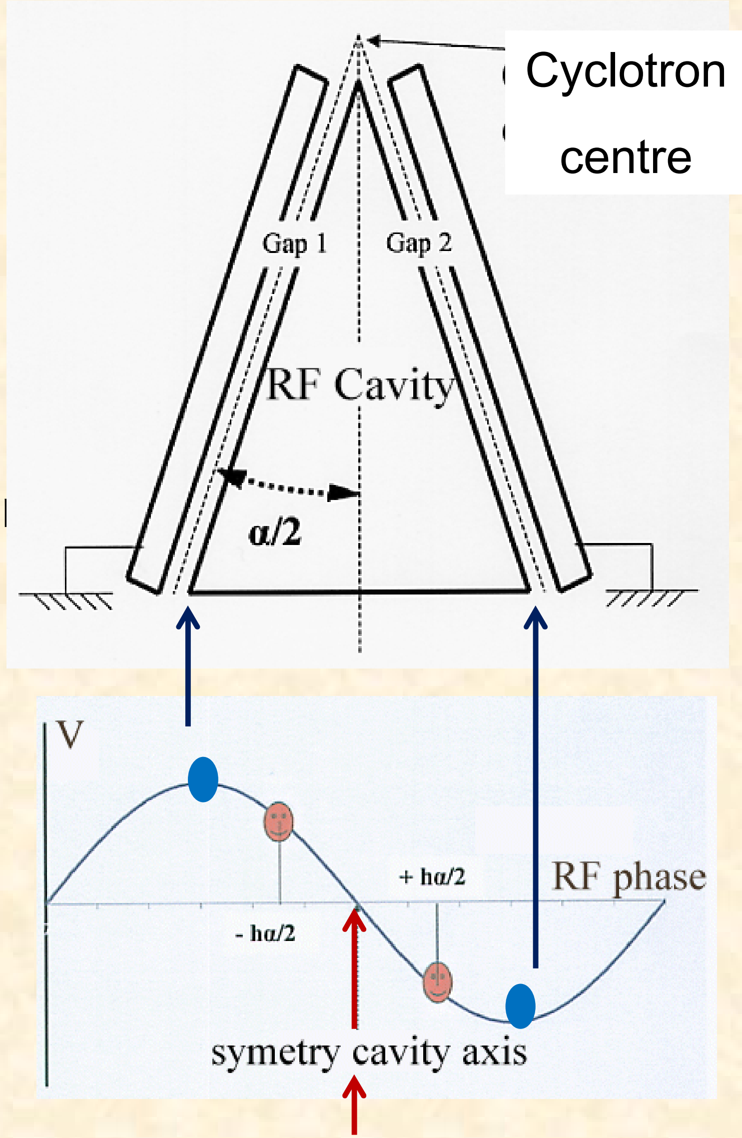
- For a maximum energy gain ($\cos\varphi = 1$: $\phi=0$)

- Energy gain per gap for the various harmonic

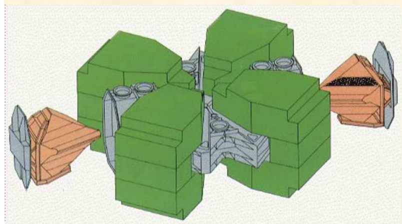
$$\delta E = qV_0 \sin\left(\frac{h\alpha}{2}\right)$$

δE optimum is

for h.α/2 =90 degree

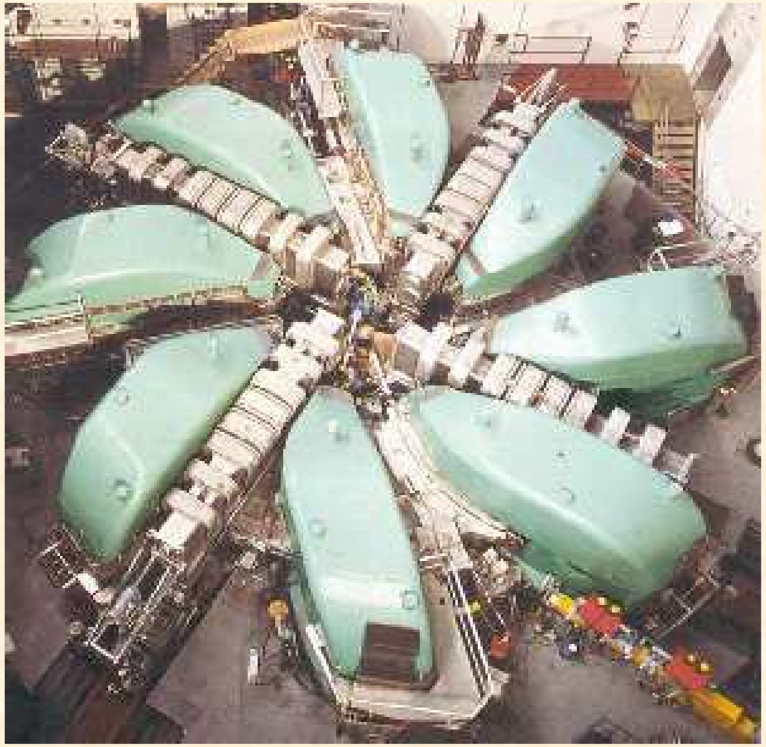


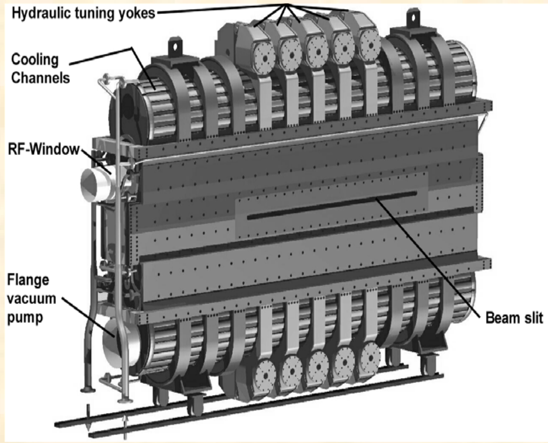
φ=0



example 2 :separated sector cyclotron:

the PSI ring cyclotron (proton kb=590 MeV)





$R_{extraction}$ =4.5 m

Kb= 590 MeV

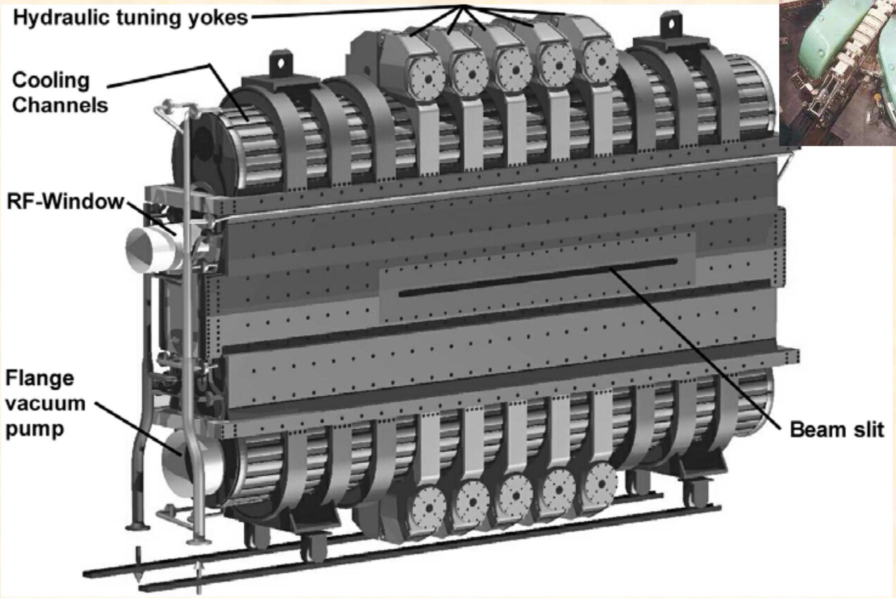
4 RF Cavities

Typical 'Separated Sector Cyclotron' (SSC). the PSI 590 MeV (p) ring cyclotron, with 8 sector magnets and 4 accelerating cavities

PSI Proton Beam ~1 Mwatt

The Challenge :
Single turn extraction

Turn separation δr large
But size Δr small



4 cavities : 50 MHz, CW
Voltage: 0.9-1 MVolt

Harmonics **h**=6

Proton Beam ~1 Mwatt
(I=2 mA)

if δr (~ N*gap* .Vrf) Large
No beam losses
T= 99.99%

Beam injection

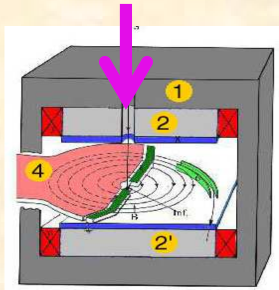
-THE ION SOURCES (internal and external)

Low energy :

AXIAL INJECTION FOR COMPACT CYCLOTRON

- Infector (spiral, hyperboloid;...)

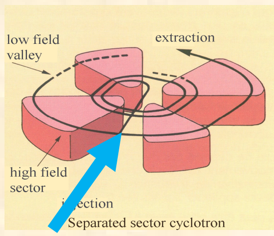
Injection from the top



Higher energy :

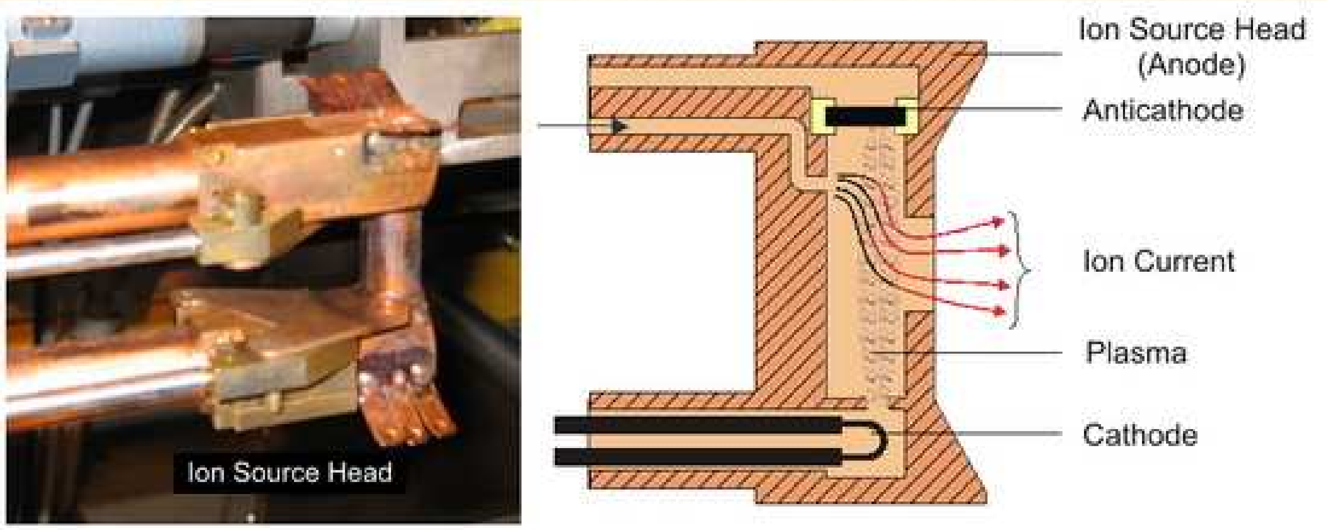
RADIAL INJECTION FOR SEPARATED SECTOR CYCLOTRON

Injection In between sectors



Cold Cathode PIG Ion Source

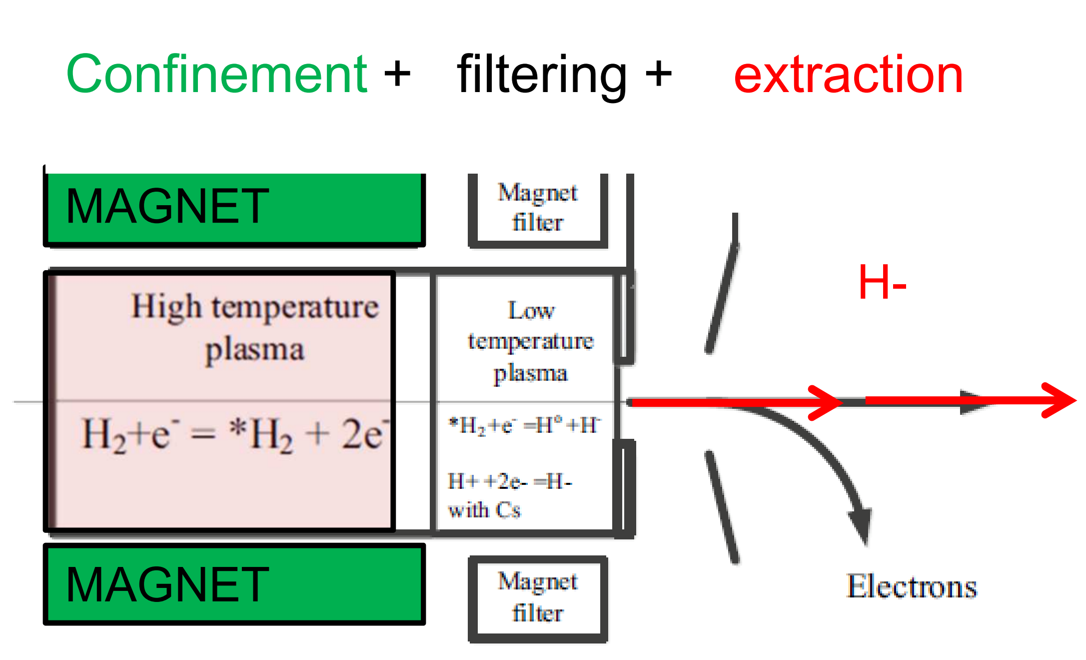
Penning or Philips Ionization Gauge (*PIG*) *ion source*



- Electron emission due to electrical potential on the cathodes
- Electron confinement due to the magnetic field along the anode axis
- Electrons produced by thermionic emission and ionic bombardment
  - Start-up: 3 kV to strike an arc
  - At the operating point : 100 V
- Cathodes heated by the plasma (100 V is enough to pull an outer e- off the gas atoms)

Multi-CUSP source

negativ ions : H-//D-   with high current



e- +H_2 => e- + p +H-

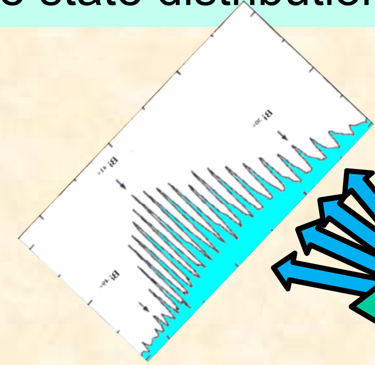
- Larger Than the PIG source (Magnets)
- Better emittance
- Larger current (Magnet confinement+ Filter)

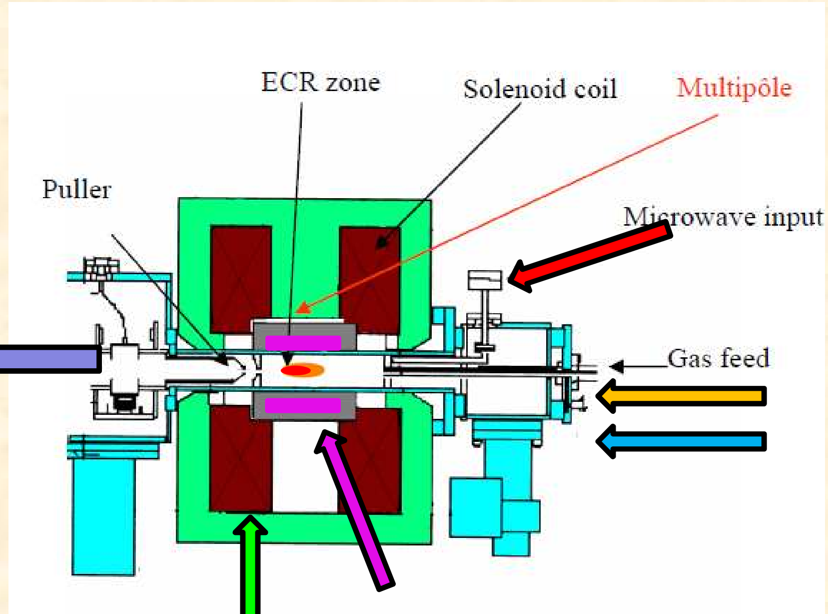
Larger Size ⇨ External Source

ECR ion source :
positiv Heavy ions

Pantechnic
Nanogan®

Charge state distribution





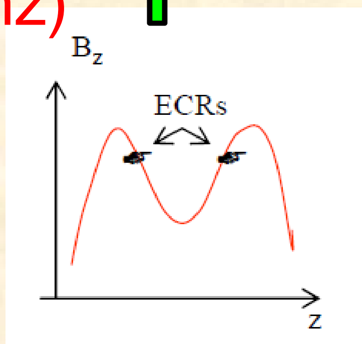
Gas (He,O,..) + RF(microWave 10-18Ghz)

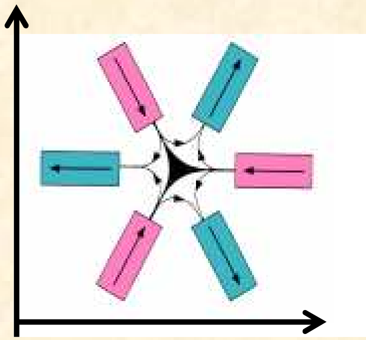
Plasma (ions + electrons) :

+ ATOMS

electrons + ions impacts

Ionize any injected heavy atoms
(He,Li,…….U)



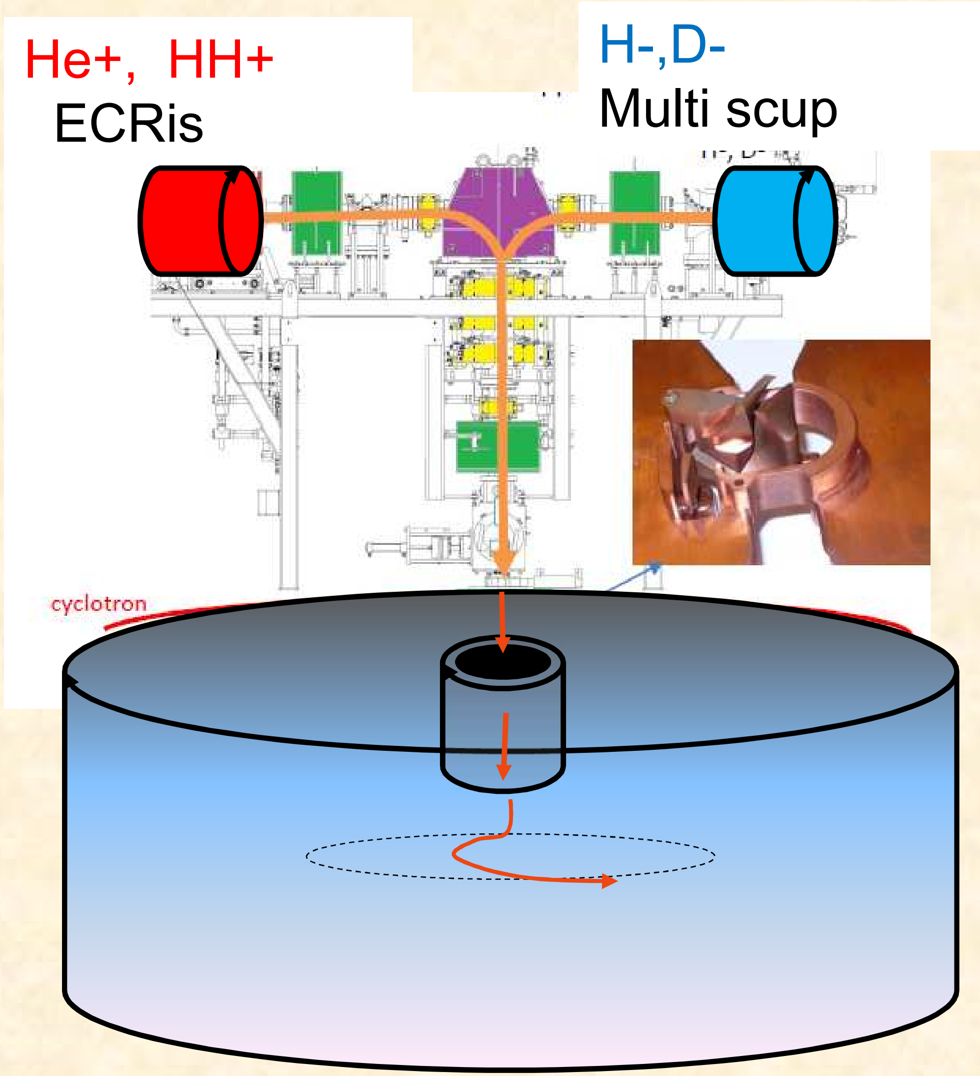


Bz + Br

3D plasma confinement

Exemple ARRONAX (Nantes, France)
2 external sources in a Kb=70 MeV cyclotron

Kb=70 MeV
- RadioIsotopes production
- Radio-Biology studies
- Irradiation

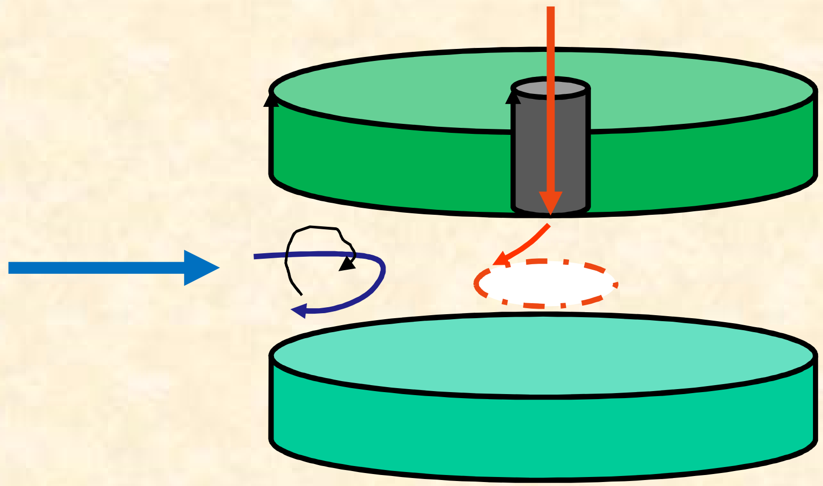


Injection from the Top (AXIAL injection)

injection in compact cylotron

- Goal : Put the beam on the « good orbit »

Can we inject the beam
From the side ?
(horizontal plan)



Not possible !!
Magnetic force too strong !!

Idea : inject Vertically : if Vz // *Bz* F= Vz x *Bz* ~ 0

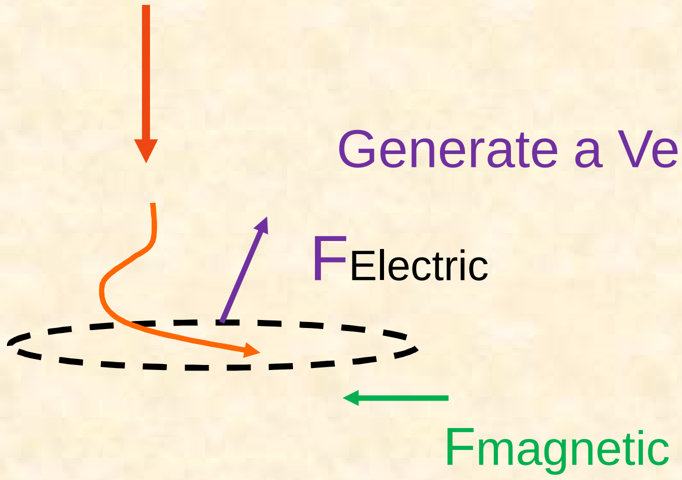
Beam Axial injection from the top of the cyclotron

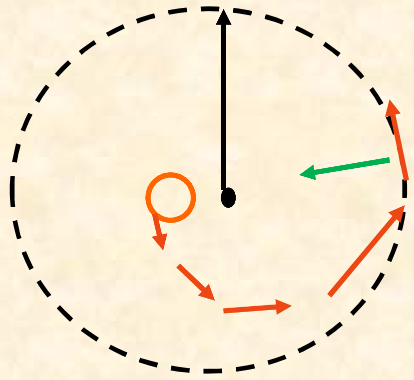
Axial injection with inflector

➢ Goal : Put the beam on the « good orbit » at the good phase

with a very compact geometry

Generate a Vertical force with an electrostatic device





Outside cyclotron

axial motion (vertical)

Inside cyclotron (Magnetic force is radial)

radial motion (horizontal)

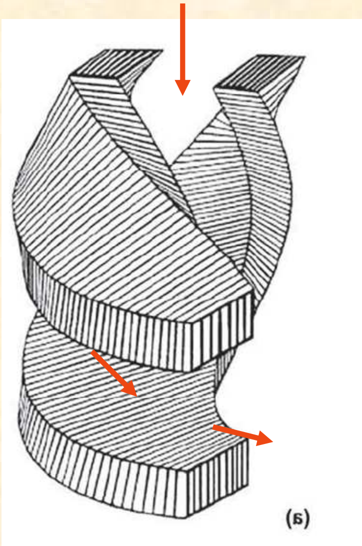
$R_m = B\rho \;/\; B_{center}$

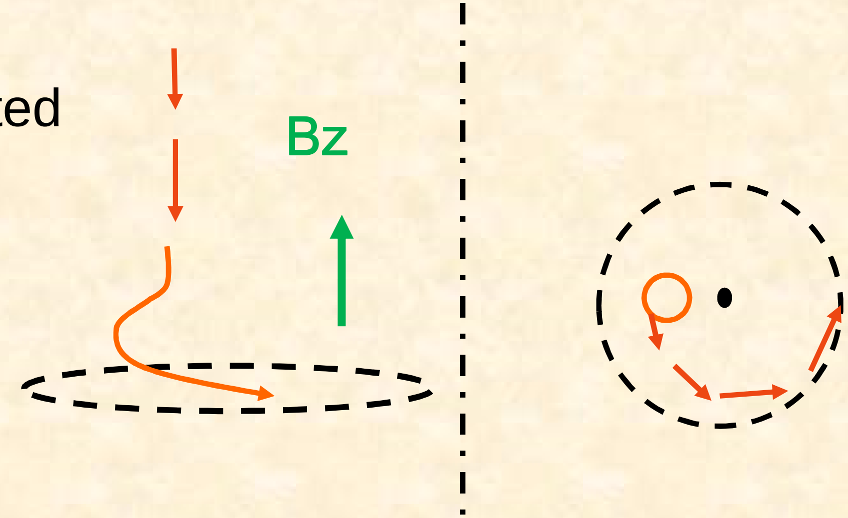
$R_E = mV^2/Q \;/\; E_{inflector}$

Axial injection : Spiral inflector

1. Spiral inflector (or helical channel)

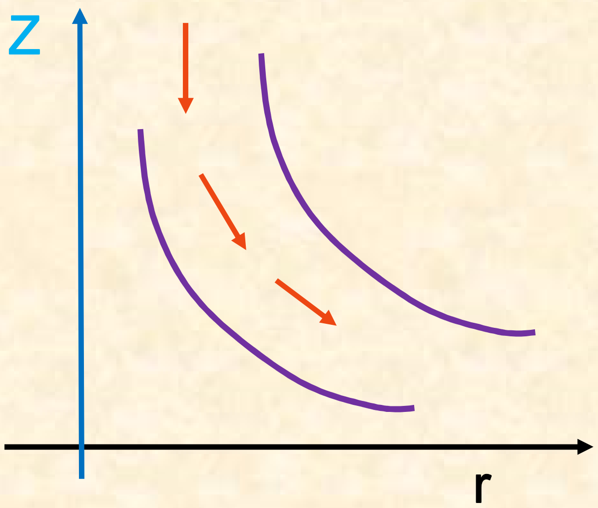
principle: 90° electrostatic deflector twisted



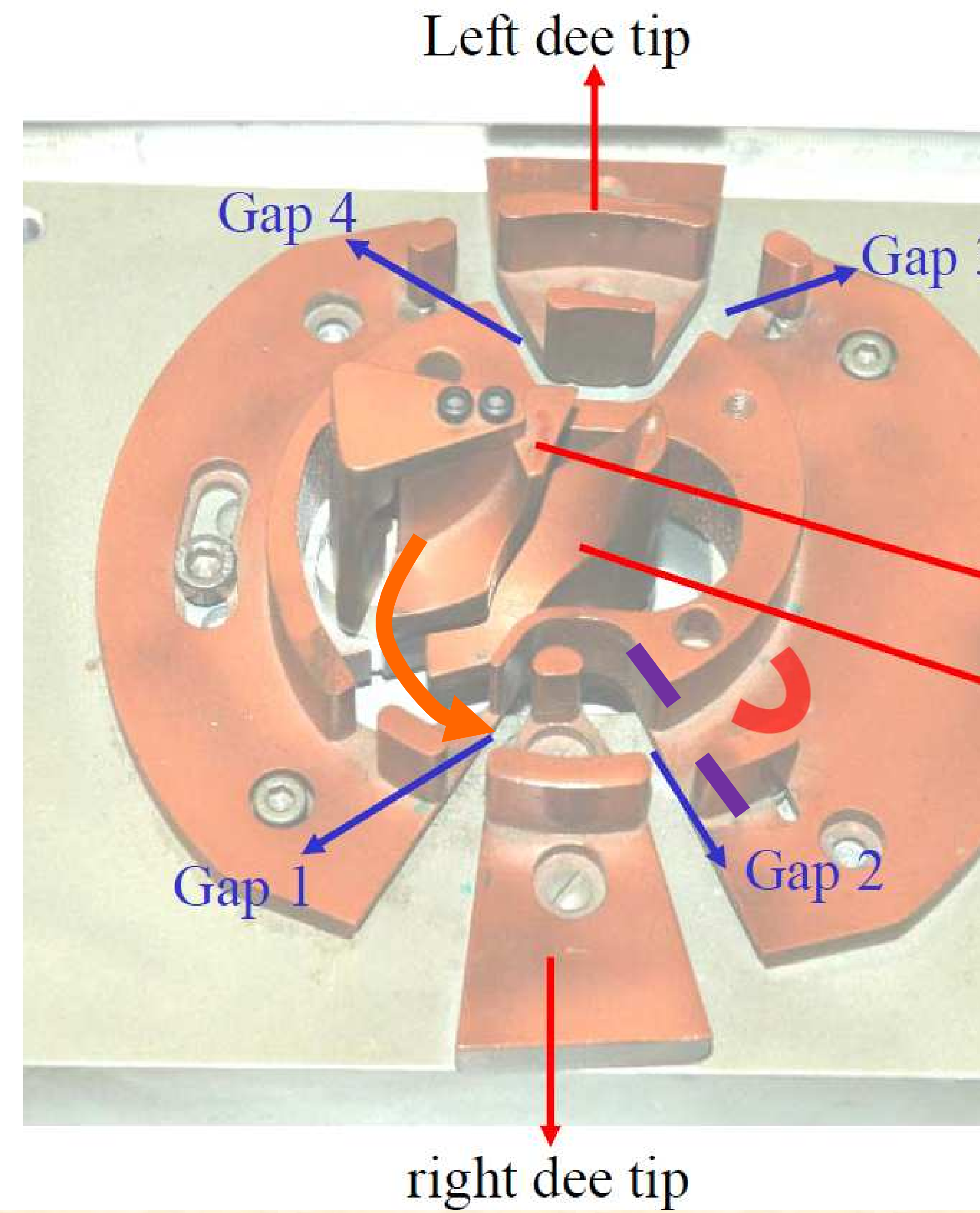


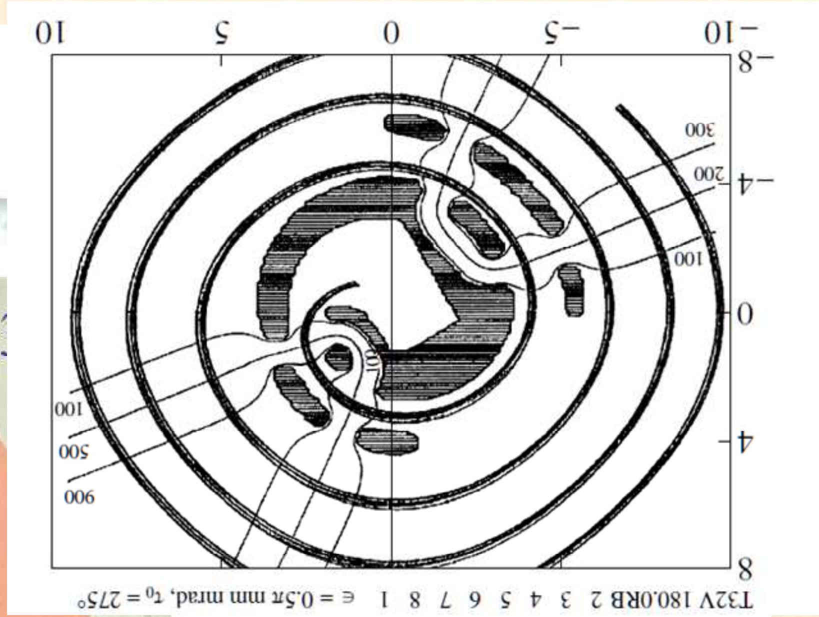
E always perpendicular to v

B=Bz constant (cyclo center)



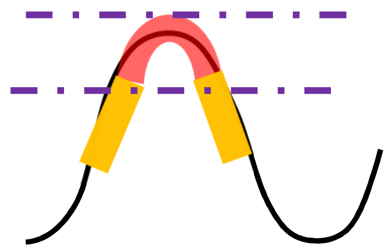
Complex geometry , very compact





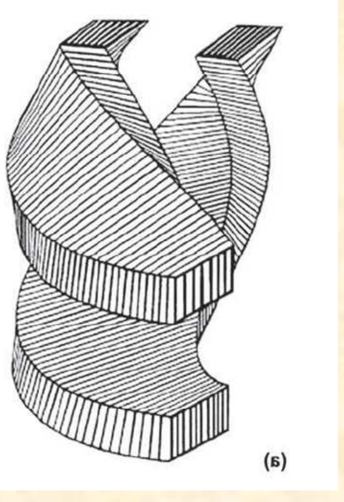
Phase selection

$\Delta\Phi \sim \Delta E/E \;\Rightarrow \Delta R$



Axial injection 1: Spiral inflector

$$m\ddot{x} = qE_x - qv_yB_0,$$
$$m\ddot{y} = qE_y + qv_xB_0,$$
$$m\ddot{z} = qE_z.$$



Trajectory Equations are very funny :

Parametric equation of the trajectory   θ= [0,π/2]

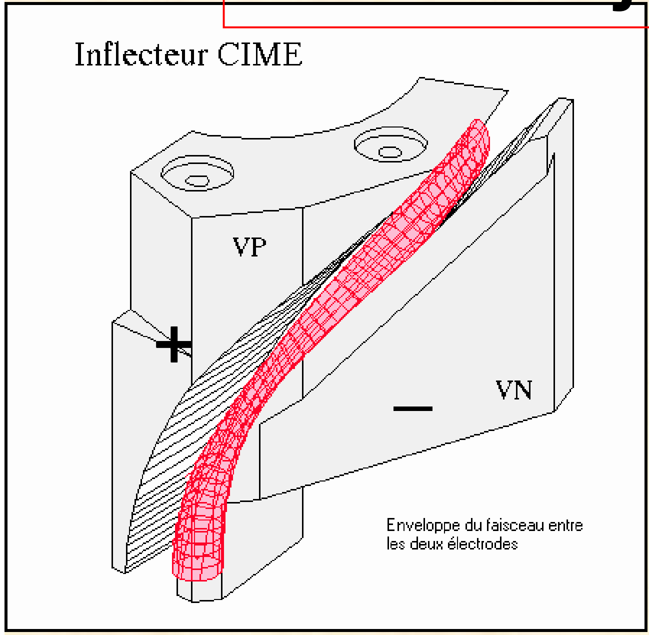
$$x_c = \lambda(1 - \sin k\theta \sin\theta - \cos k\theta \cos\theta)$$
$$y_c = \lambda(\sin k\theta \cos\theta - \cos k\theta \sin\theta) \quad ,$$
$$z_c = A(\sin\theta - 1)$$

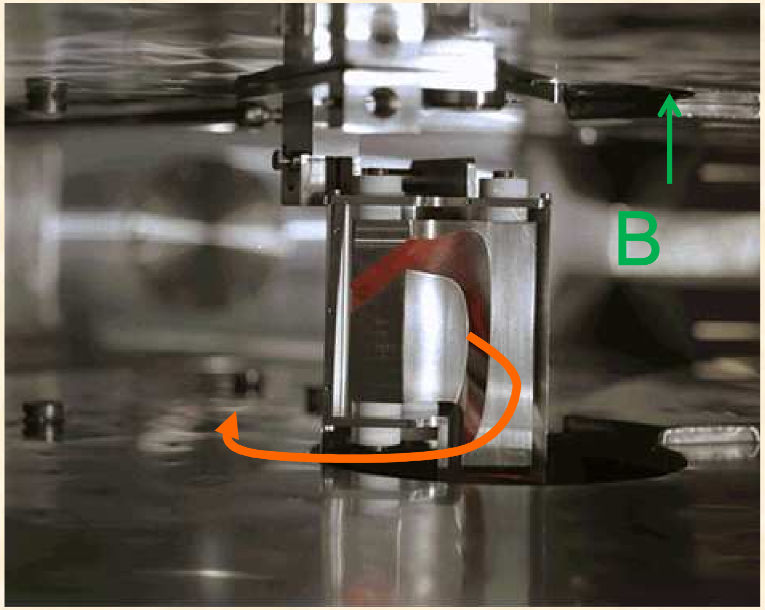
$$k = A/R_m + k'$$
$$\lambda = A/(k^2 - 1)$$

Two parameters :   A the inflector Height
k' the tilt

2 forces bend the beam
Electric radius $A = R_E = mV^2/Q / E_o$
Magnetic radius $R_m = B\rho / B_o$

Axial injection 1: Spiral inflector





•Consists of 2 cylindrical capacitors which have been twisted to take into account the spiraling of the ion trajectory from magnet field.

• $\vec{v}_{beam} \perp \vec{E}$: central trajectory lies on an equipotential surface. Allows lower voltage than with mirrors.

• 2 free parameters (spiral size in z and xy) giving flexibility for central region design

• 100 % transmission

Axial injection 2: hyperboloid inflector

Spiral electrodes are complex :

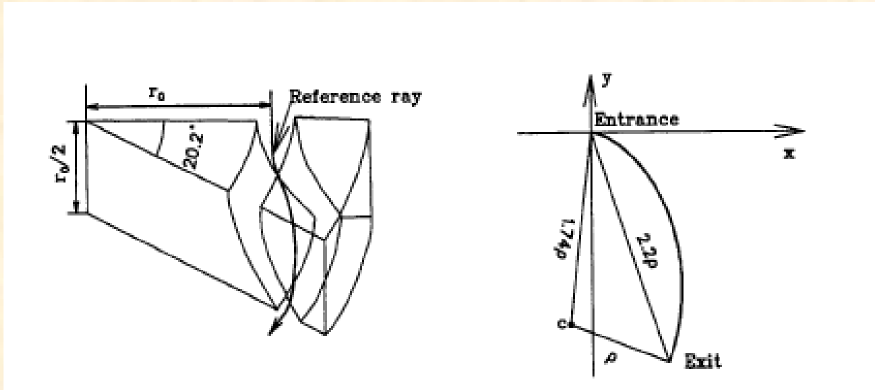
hyperboloid inflector have simpler electrode

two electrodes equation : $r^2 - 2z^2 = r_1 \quad r^2 - 2z^2 = r_2$

$V = -Kz^2/2 + Kr^2/4 + c$

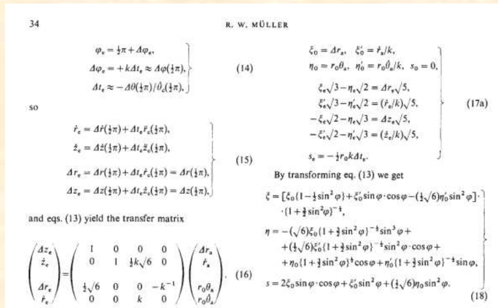
Vertical field $E_z = -Kz$

$$x = \frac{r_0}{2}\{-b\cos(akt) + a\cos(bkt)\},$$
$$y = \frac{r_0}{2}\{-b\sin(akt) + a\sin(bkt)\},$$
$$z = \frac{r_0}{2}\sin(kt),$$
$$k^2 = \frac{qK}{m},$$
$$r_0 = (2\sqrt{6})\rho.$$



$k^2 = -qv^2/2$

$r0 = 2.\,6^{1/2}\,Rm$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By transforming eq. (13) we get

and eqs. (13) yield the transfer matrix

Simpler geometry than spiral inflector

But No free parameter (Rinjection=Rm it fixes all parameters)

Radial injection

Radial Injection for pre-accelerated beam :

- Compact inflector not possible (axial inj. not possible) :

-Higher rigidity (electrostatic field have “low efficiency”)

need space to bend the beam with large magnet !!

1. Injection into separated sector cyclotron (most common)
   - More room for injection pieces and excellent transmission

2. Other Specific examples (not described here)

- Injection with Charge exchange (internal stripper foil)

in a compact superconducting cyclotron NSCL

- 3 ....

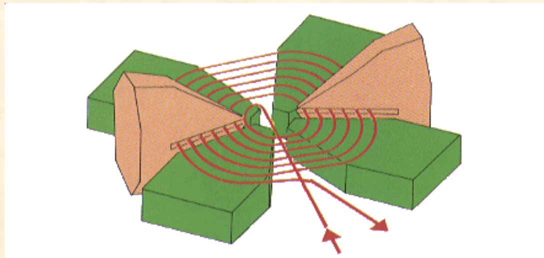
Example: Radial injection in a ring cyclotron

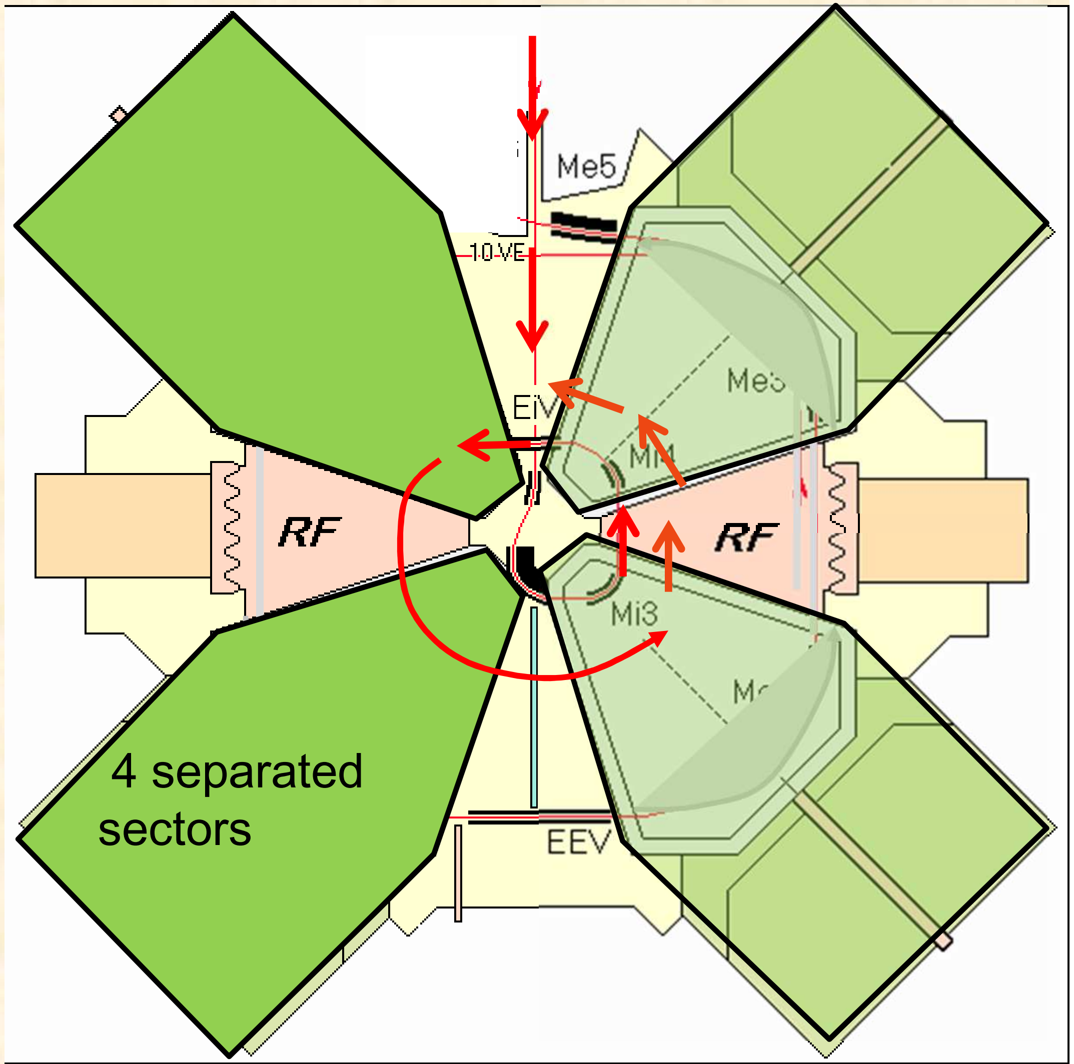
- More room to insert bending elements.

Beam injected between sector magnets

- The beam coming from the pre-injector enters the SSC horizontally.

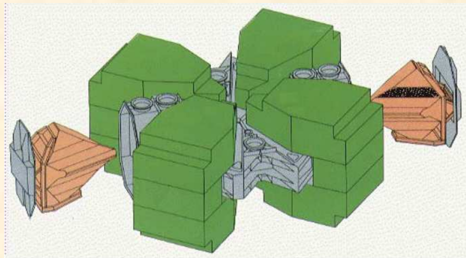
It is guided by 4 magnetic dipoles to the "good trajectory", then an electrostatic inflector deflect the beam behind the dipole yokes.

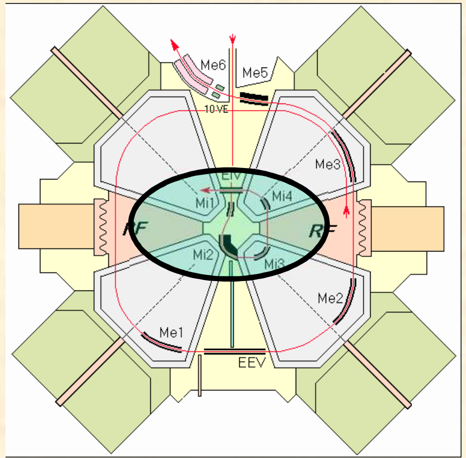


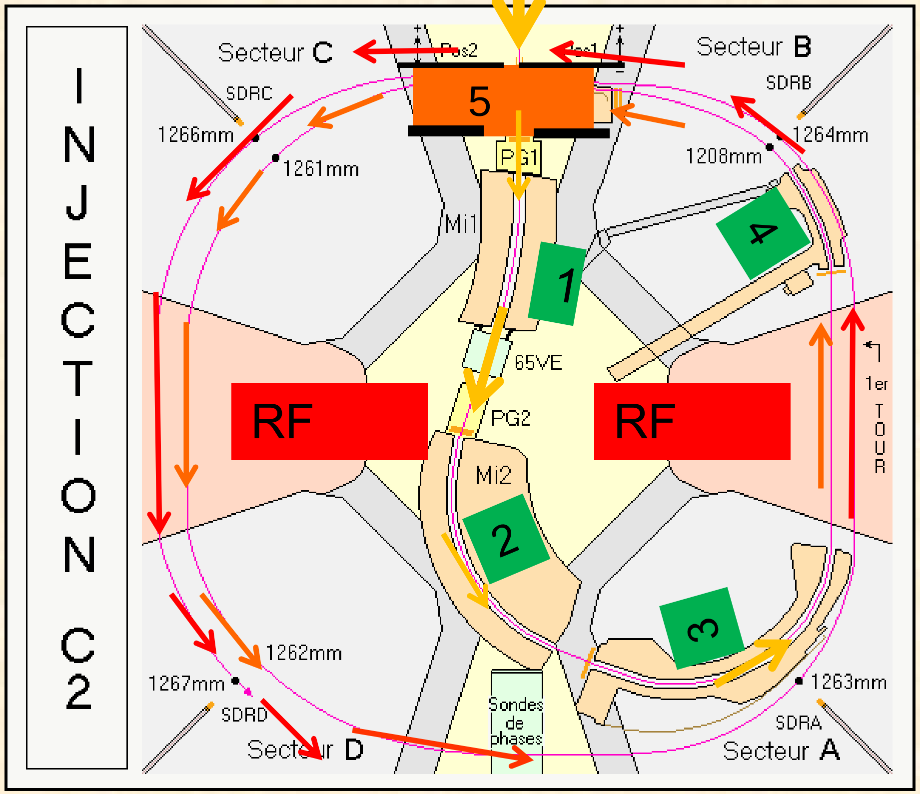


Example: Radial injection

Beam coming from
an other accelerator







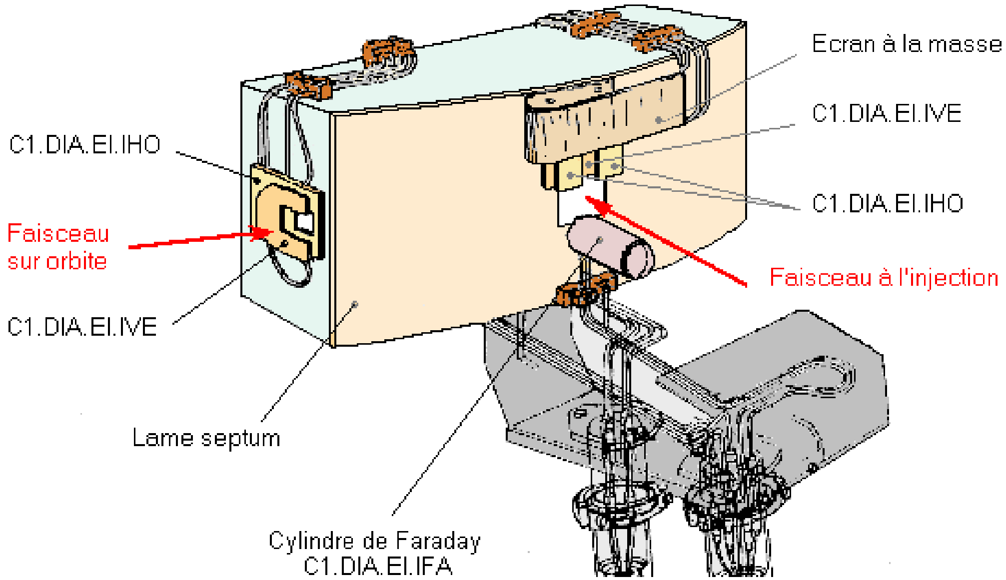
1-4 :magnetic dipoles

5 : electrostatic inflector

-**Very carefull centering**

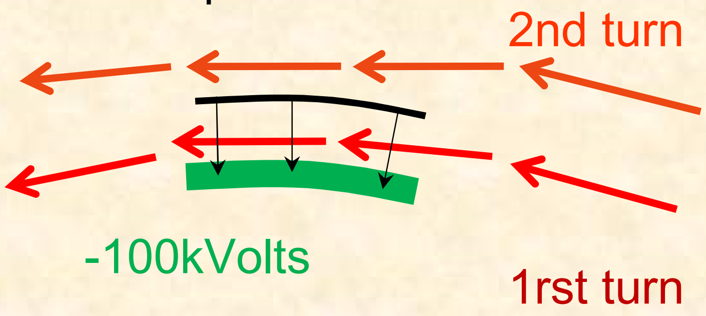
-RF Phase adjustement with bunches

ELETROSTATIC INFLECTOR
SSC1 (GANIL)

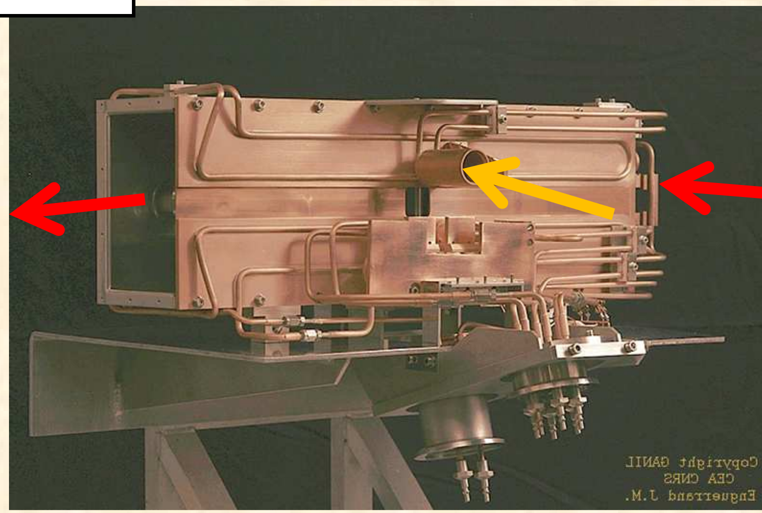


Bending of the first turn

Septum: 0 Volt



Electrostatic inflector
100 kVolt gap~ 1cm



Injection of accelerated beam
(rigidity =difficult to bend)
Magnet requires more space
than electrostatic devices

Cyclotron Extraction

1. Extraction by stripping negative ions

simpler and low cost , but restricted to Hydrogene isotopes

100% efficiency

2. Extraction using the radial separation

between turn n°N & n°N+1

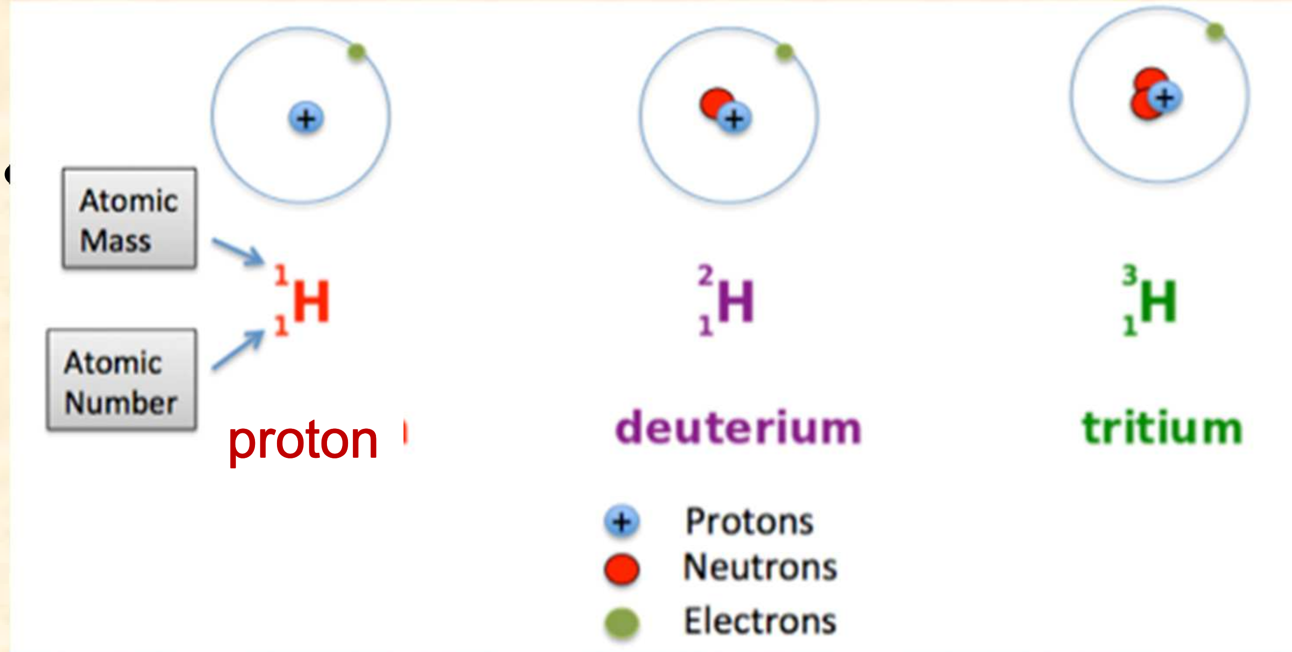
method 2.a : natural acceleration

method 2.b : precession

method 2.c : resonance

Negative Hydrogene isotopes for proton & deuteron beam

very convenient for stripping extraction



Hydrogene isotopes

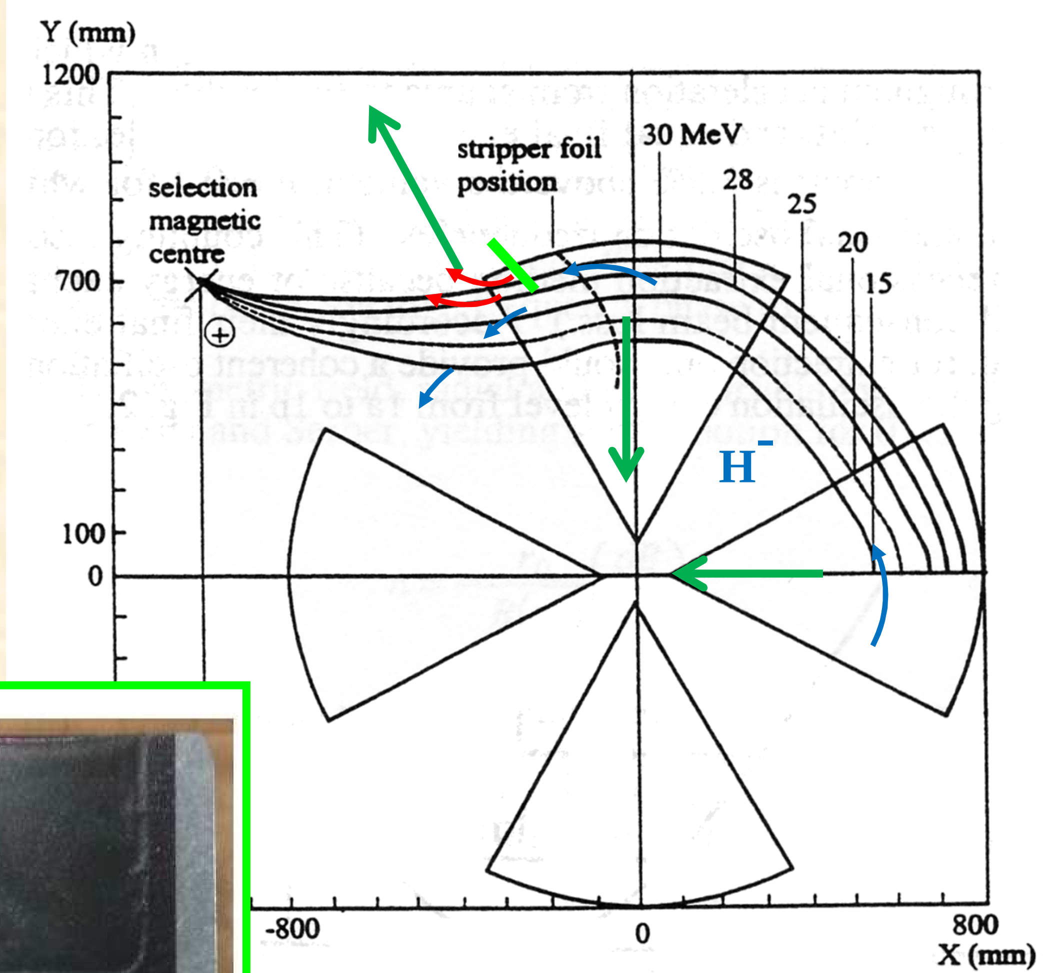
PIG sources or multiscup sources for negative ions of H,D

H- (proton+ 2 e-)

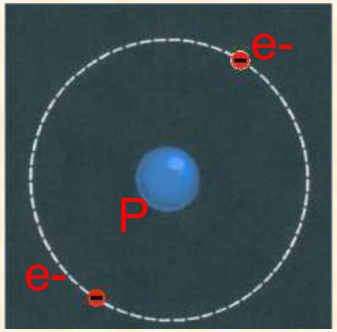
D- (deuteron+2 e-) D= good for several nuclear reactions (radio isotopes production)

1) Extraction by stripping negativ ions

easy and efficient with H^- (1 Proton+2 orbiting electrons)



Extraction orbits in the IBA Cyclone 30

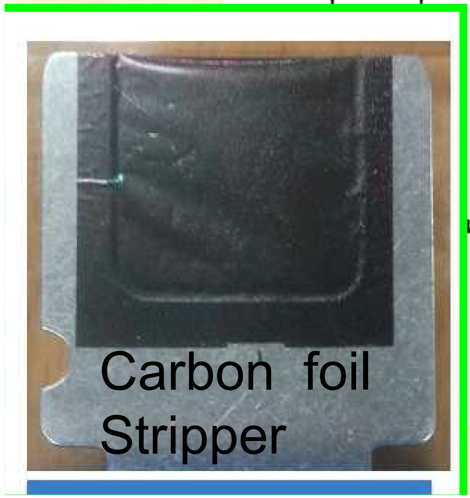


$H^- \Rightarrow H^+ + 2e^-$

The magnetic force is inverted

$F_r \sim - v.B_z \Rightarrow + v.B_z$

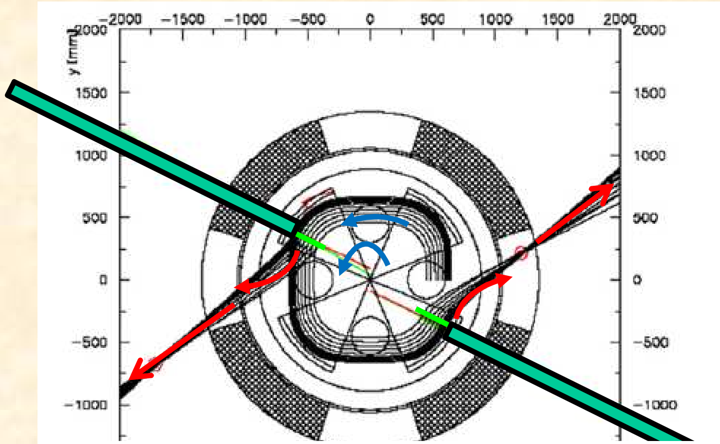
Q= -1 ⇨ Q= +1



1) H^- & D^- commercial cyclotrons with two extracted beams

Low cost extraction beam line(s) :

less complex than electrostatic deflectors



- 2 strippers foils
- = 2 beams at the same time

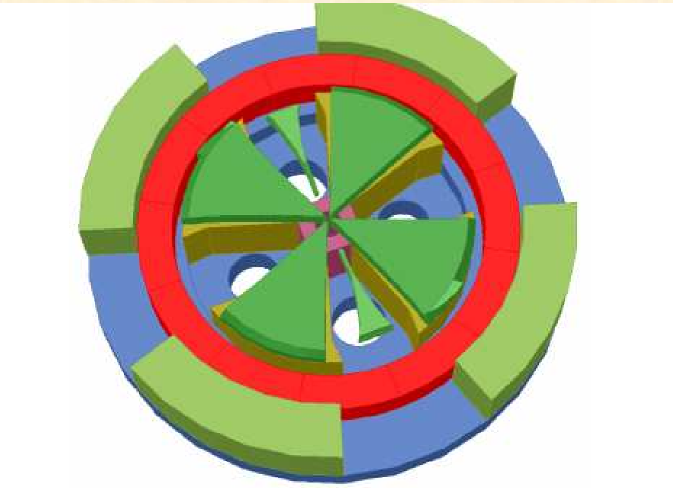


Figure 8: Final Cyclone® 30XP model.

H^- production or D^- production with an internal source (PIG)

2 strippers at extraction radius :

H^- => H^+

good beam quality, easy maintenance

2 Extraction by turn separation

Method 2.a :Extraction by acceleration (and fringe field)

The orbit radial δr separation between 2 turns is :

$$\boxed{\delta r} = r \times \frac{\delta E}{E} \times \frac{\gamma}{\gamma + 1} \times \frac{1}{\nu_r^2}$$

- δE : Energy gain per turn as high as possible (RF)
- ν_r : Accelerate the beam to fringing field (Bz decrease,n>0, $\nu_r \searrow$)

$$\frac{\delta B}{B} = -n \frac{\delta r}{r}$$

Demonstration :

$$\boxed{\frac{\delta r}{r}} = \frac{\delta B\rho}{B\rho} = \frac{\delta < B > R}{< B > R} = \frac{\delta R}{R}\Big|_{acc} - \frac{\delta B}{B}$$

$$= \frac{\delta P_{acc}}{P} + n\frac{\delta r}{r} = \frac{\delta P_{acc}}{P}\frac{1}{(1-n)} \approx \frac{\delta P}{P}\frac{1}{\nu_r^2} \approx \frac{1}{2}\frac{\delta E_{acc}}{E}\frac{1}{\nu_r^2}$$

$$\boxed{\nu_r = \sqrt{1-n}}$$

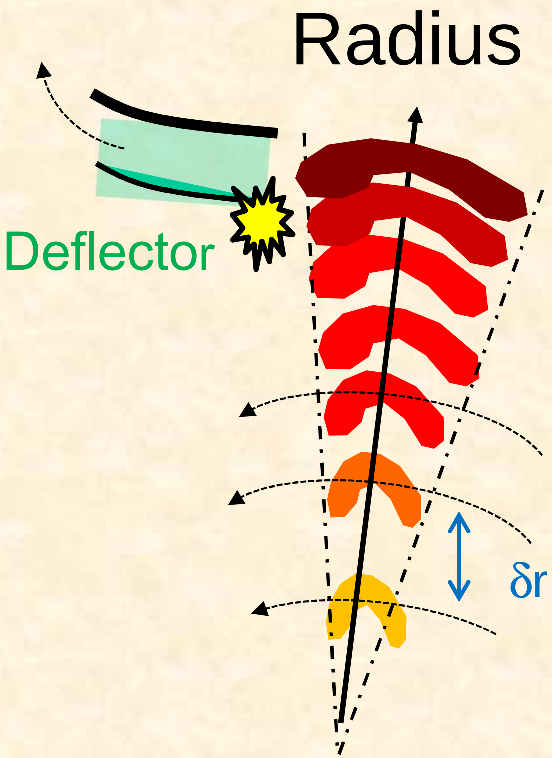
2a: Extraction using turn separation δr

if $\Delta r < \delta r$ Each turn are separated

Extraction with an electrostatic deflector

100% efficiency

SINGLE TURN EXTRACTION



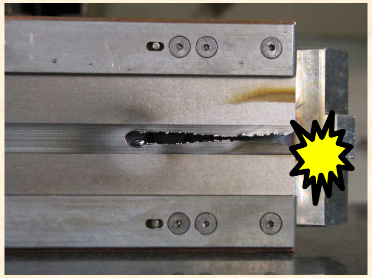
if $\Delta r > \delta r$ (size> turn separation)

last turns are not separated

Beam losses in the extraction channel

Multi TURN EXTRACTION

Deflector sparking or damaged



$$\delta r \propto \frac{1}{Radius}$$

Extraction : 3 mechanisms possible

Goal : High extraction efficiency with well separated orbit

δr = Acceleration + Precession + increase oscillation
by a field bump
(resonance extraction)

a. Extraction by acceleration (and fringe field + deflector)

- Energy gain per turn as high as possible...

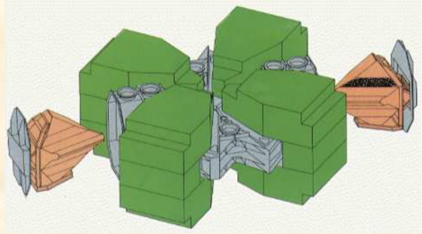
b.Precession extraction : radial oscillations help to separate orbits

$$r(N) = r_0(N) + x_0 \sin(\nu_r . \omega_0 t)$$

c. Resonant extraction : increase the precession by a field bump

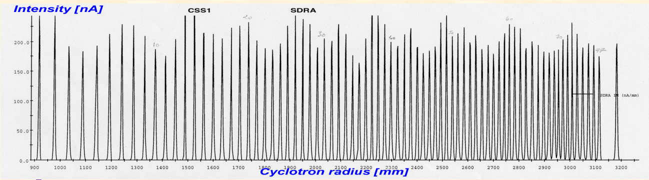
If turn separation not enough then magnetic perturbations are used. Particles are forced to oscillate around their equilibrium orbit with a magnetic bump

Example: Ejection SSC

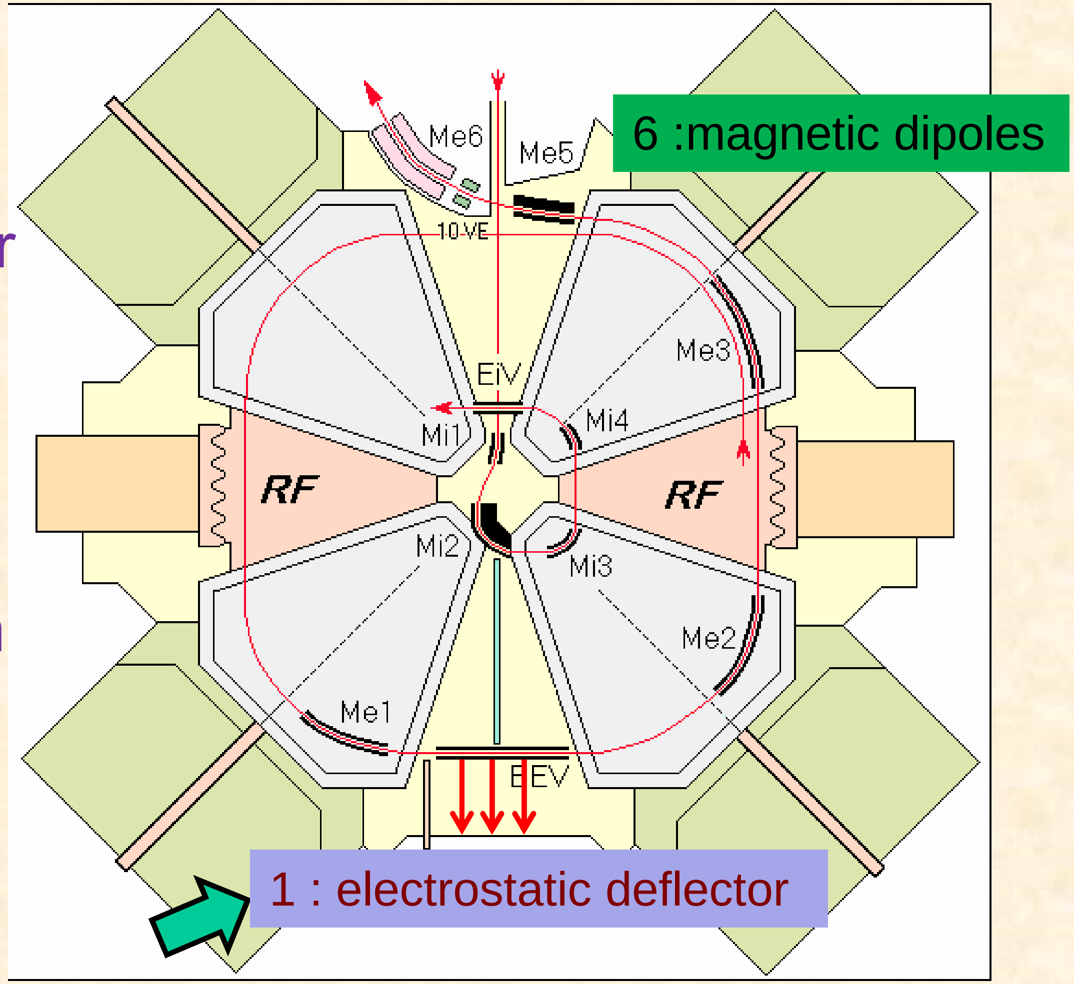


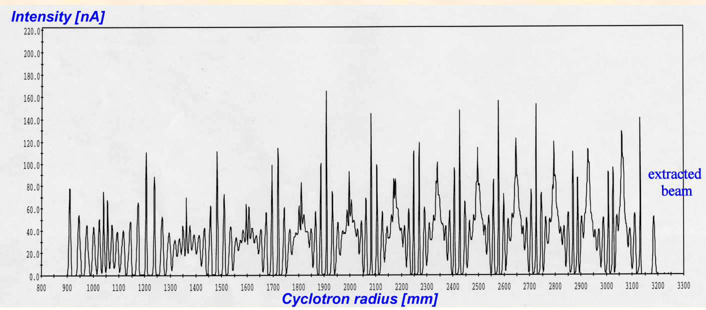
- 1) Acceleration (2 RF cavities)

  \+ Radial kick on the extracted turn With an electrostatic deflector



Precession : excited from injection





Extraction with precession

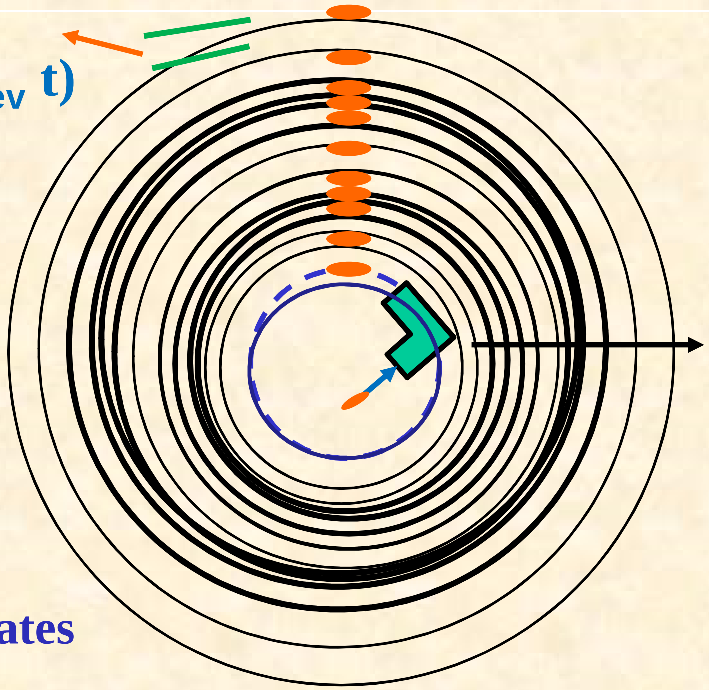
$r(t) = R_0(t) + x_0 \cos(\nu_r \, \omega_{rev} \, t)$

X_0 given by injection tuning

$X_0 = 0$ No precession

$X_0 \neq 0$ precession

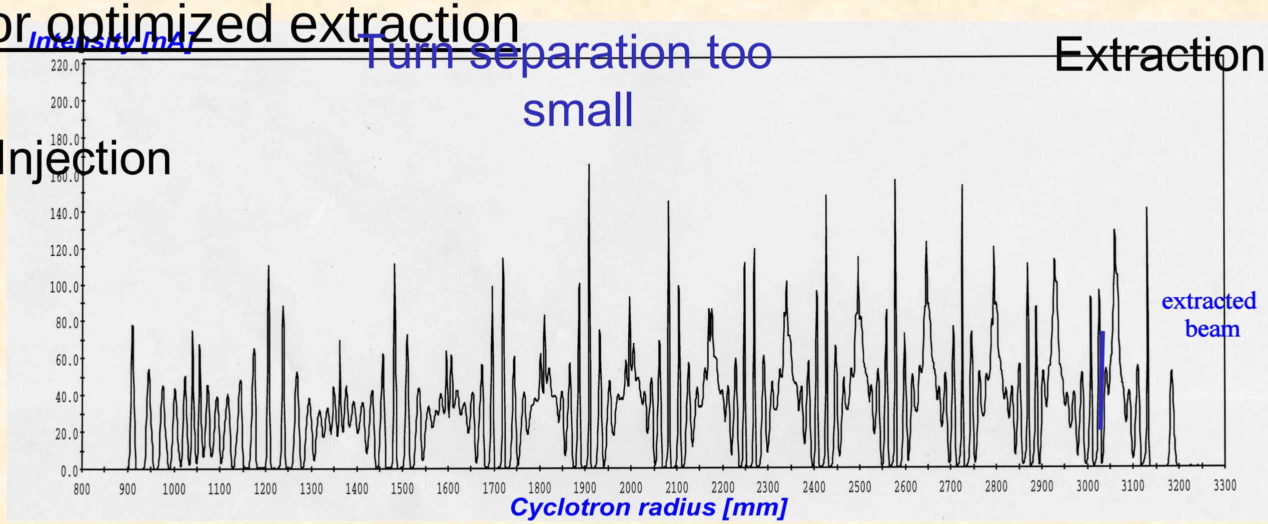
Radial Distance between bunches oscillates



At certain Radius Bunches are close

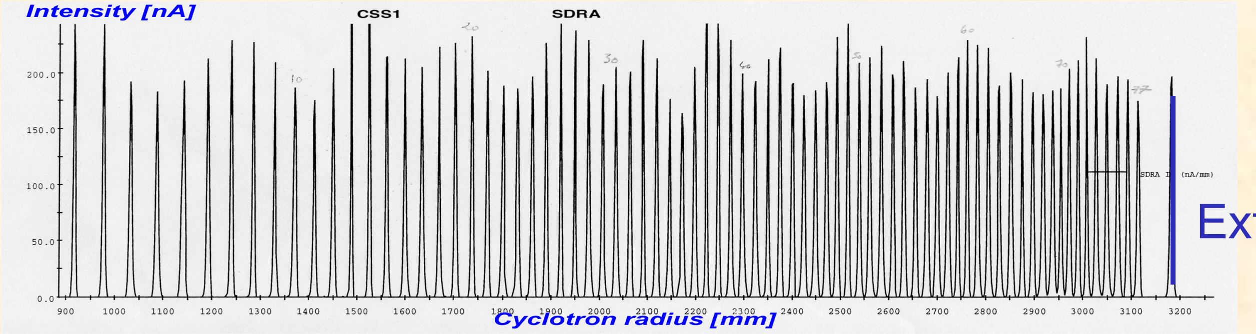
At certain Radius Bunches are well separated Good for extraction

Precession for optimized extraction



Extraction with precession

Well centered beam orbits **Separated Sector Cyclo N°1** GANIL



Turn separation sufficient

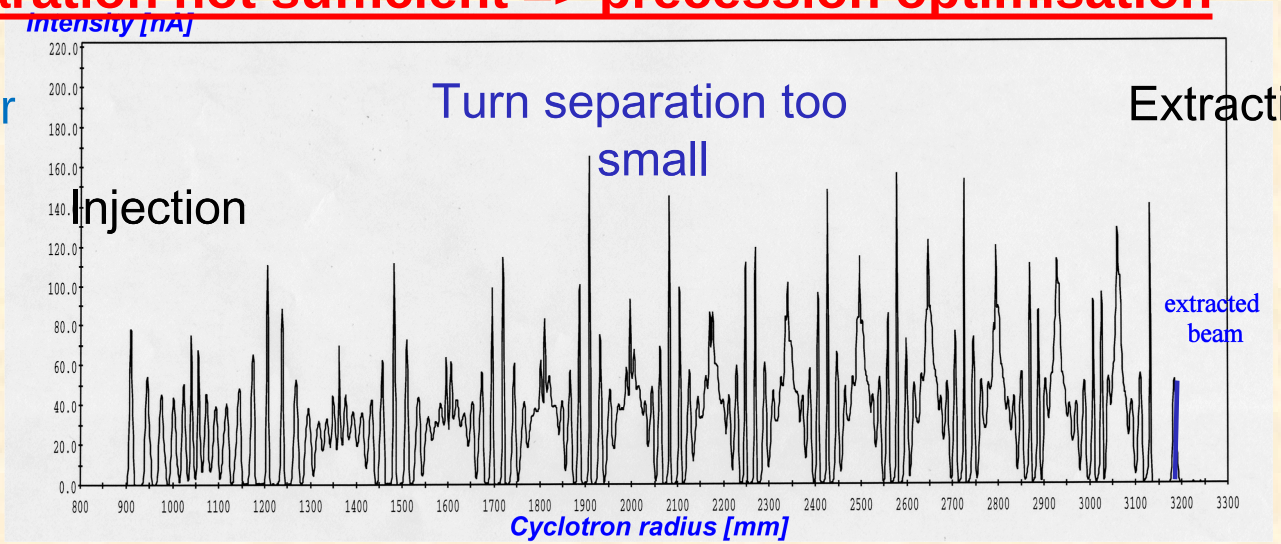
Extracted turn

$\Delta r < \delta r$

Precession for optimized extraction **Separeted Sector Cyclo N°2** GANIL

Turn separation not sufficient => precession optimisation

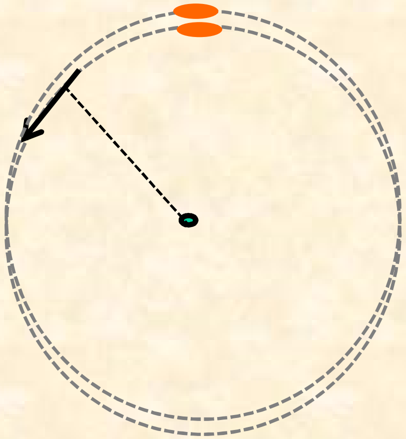
$\Delta r > \delta r$



transmission 95%

Resonant extraction with a magnetic bump : with $Q_x = \nu_r \sim 1$

Step 1 : circular motion
+ small oscillations

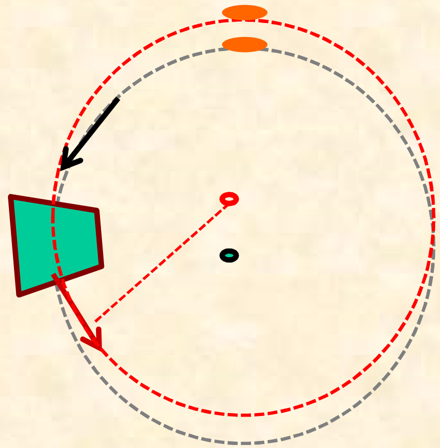


$$\ddot{x} + \nu_r^2 \omega_0^2 \cdot x = 0$$

$\nu_r \sim 1$: 1oscillation per turn

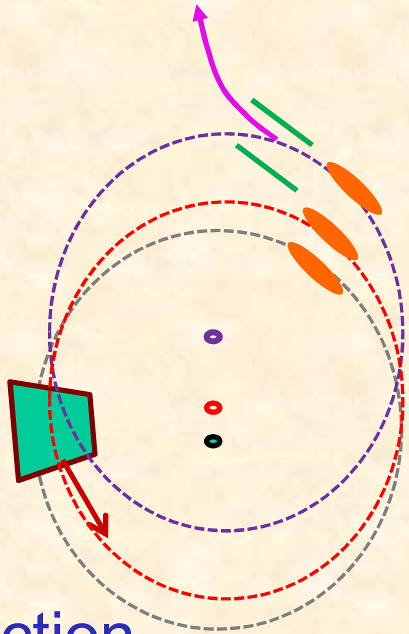
$$\delta r \approx \left[\frac{1}{2} \frac{\delta E_{RF}}{E} \right] + \Delta x_0 \sin(\nu_r . \omega t)$$

Step 2 : A magnetic bump
shift of the orbit center
larger deviation



Step 3 : Several turns
produce
Large amplitude oscillation

Larger &Larger &Larger



Large δr =easy extraction

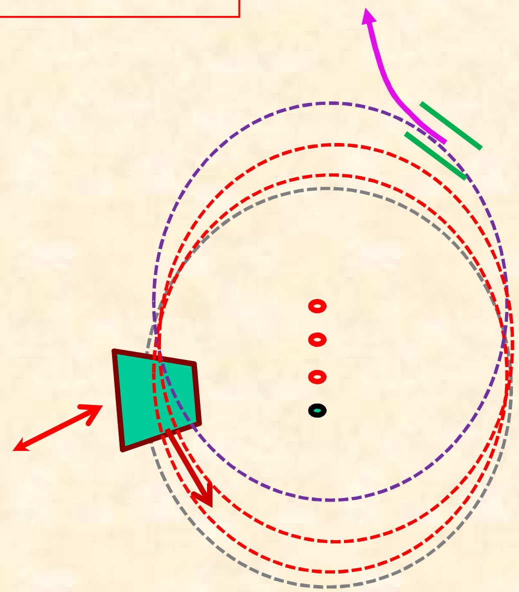
Resonant extraction shown by equations

Radial Equation without magnetic Perturbation

$$\ddot{x} + \nu_r^2 \omega_0^2 \cdot x = 0$$

Equation with Perturbation δBz ~ $b_M(r)$ cos(M θ)

$$\ddot{x} + [\nu_r \omega_0]^2 x = \omega_0^2 \frac{r}{B} \frac{db_M}{dr} \cos(M \omega_0 t)$$



Driven oscillator excited at the « frequency » M

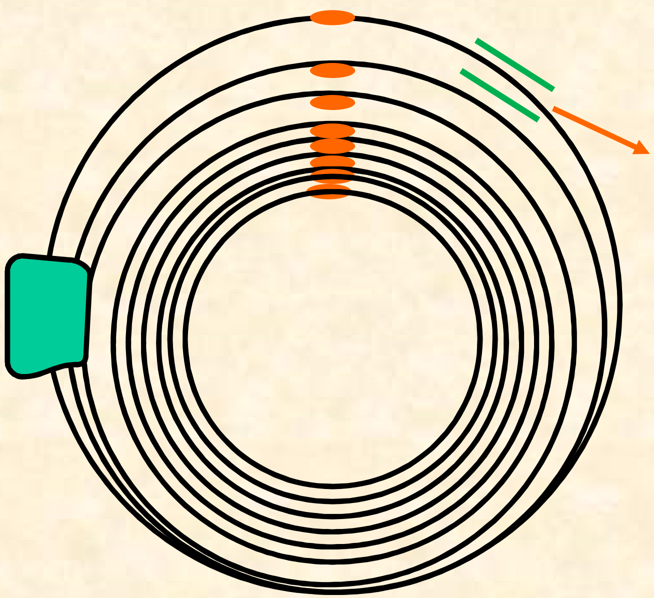
if the excitation is at the resonance frequency M= νr

you get Large amplitude oscillations δr (easy extraction)

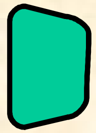
One field Bump correspond to harmonic M=1

coil for fieldbump

Extraction with resonance excitation



Close to extraction radius,
a field bump
Increase the bunch separation δr

Field bump 

The excitation correspond to ν_r

Very small excitation is sufficient
if resonance …..

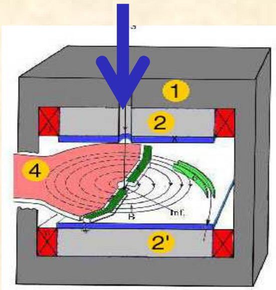
if $\nu_r \sim 1$ One field bump
if $\nu_r \sim 2$ two field bump

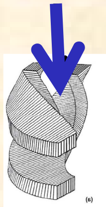
Few other slides for questions

Summary N° 4 :

1) $(E/A)_{max} = Kb \, . \, (Q/A)^2$ $\qquad Kb \sim (Radius.Bmax)^2$

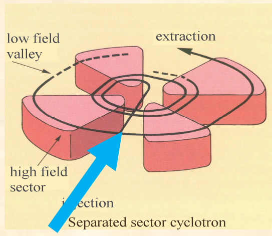
2) "Compact cyclotron" have an axial compact inflector



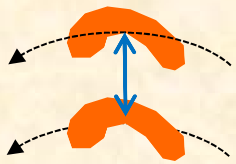


3) "Separated sectors cyclo" have radial injection

injection in between sector magnet



3) Large Turn separation for extraction δr ($> \Delta r$)



δr = distance between bunches induced Acceleration (RF)

+ Eventually Precession (injection angle)

+ Eventally resonance excitation (magnetic bump)

Accelerating gap & Transit Time

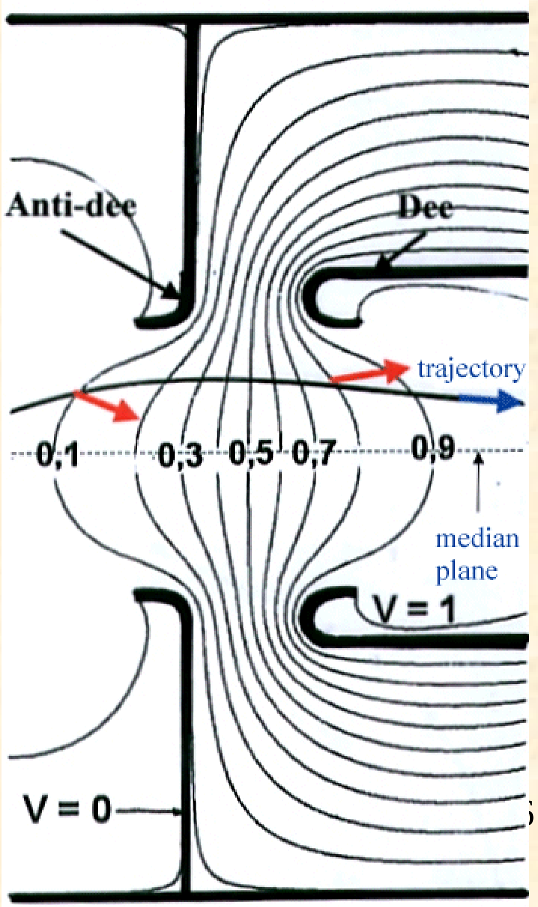
The formula $\delta E = QV_0 \cos\varphi$ corresponds to small accelerating gaps

Because of the gap geometry, the efficiency of the acceleration through the gap (g) **is modulated** by the transit time factor τ:

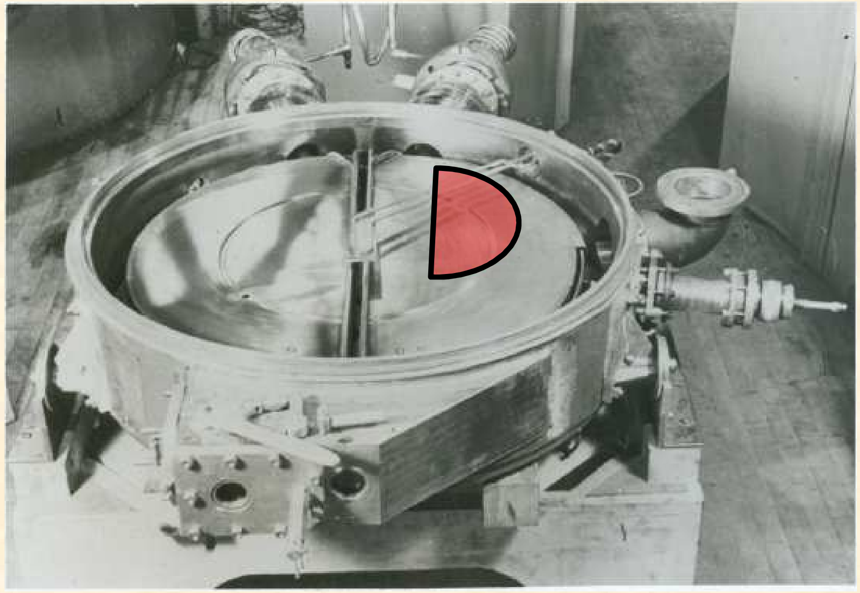
$$\delta E = QV_0 \tau \cos\varphi$$

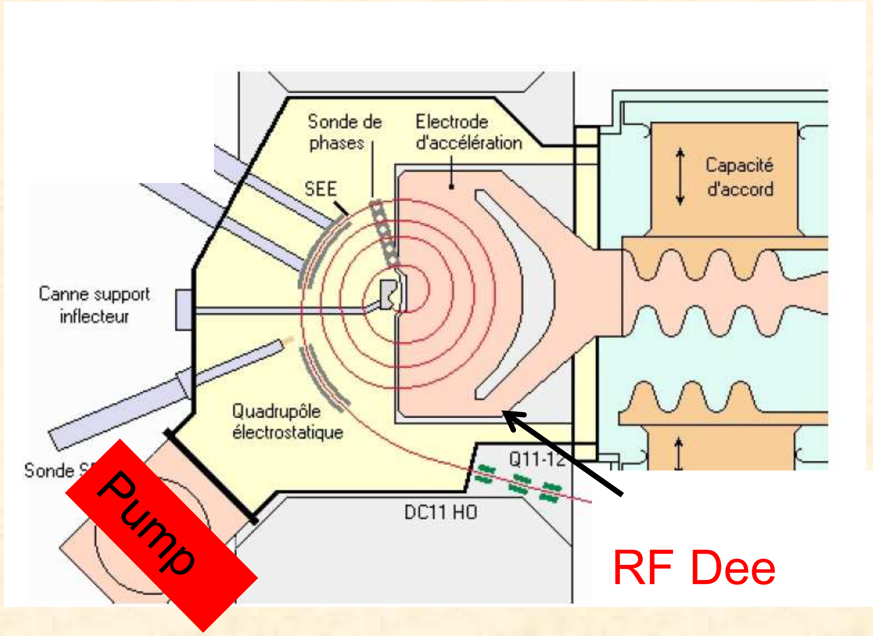
$$\tau = \frac{\sin\left\{\frac{hg}{2r}\right\}}{\frac{hg}{2r}} < 1$$

Finite size of gap decreases the efficiency of accelerating cavity



RF Cavities : with the 180° Dees





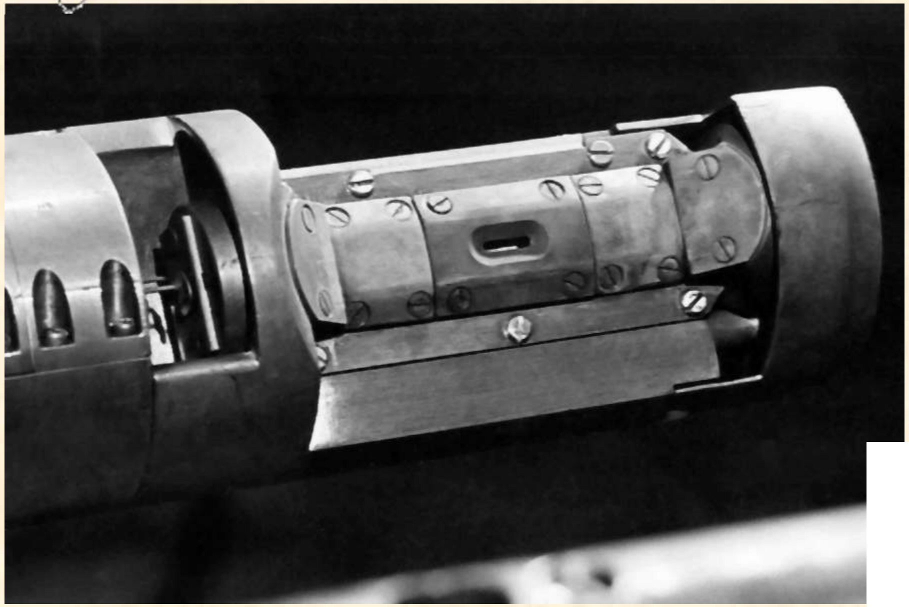
With the specific "180° Dees " :

h=1,3,5 only odd number allowed

h=2,4 even number forbidden

Dee should change its voltage every half turn for a bunch

Example of PIG source



FLNR, PIG test-bed, 1992.
The head of MC400 cyclotron vertical ion source

Small size

Inserted in the cyclotron gap