Cyclotrons

Chapter 3 Cyclotron Design

- Isochronism

- Maximal energy (Bmax, R, stability)

- **Simulation // tracking**

   **(numerical integration, realistic cyclo simulation)**

Design Strategy for K=10 MeV cyclo

Design Strategy for K=250 MeV cyclo

Design Strategy for a research facility (E/A vs I)

Cyclotron Summary : without formulas

Isochronous cyclotron   =   constant revolution frequency

Bz

Bz=Bz(R)

Radius

radial component
Br defocusing
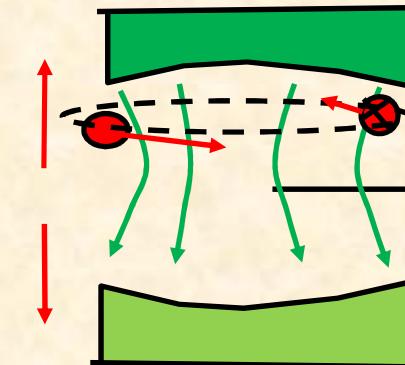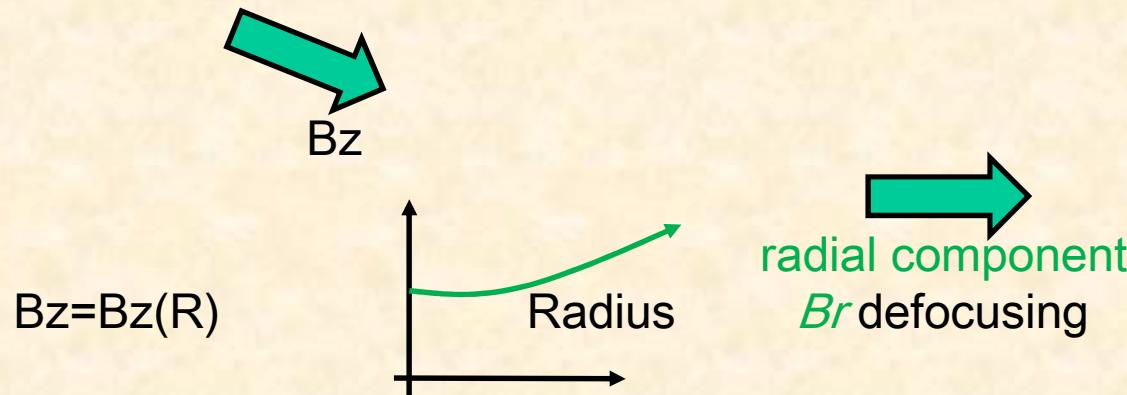
Transverse dynamics : vertical defocusing forces have to be compensated

Azimutal(θ) Field modulation

straight sectors in one magnet
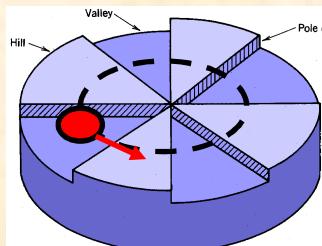
spiralled sector

Separated sectors

low field valley

extraction

high field sector

injection

Separated sector cyclotron

Separated spiralled sectors
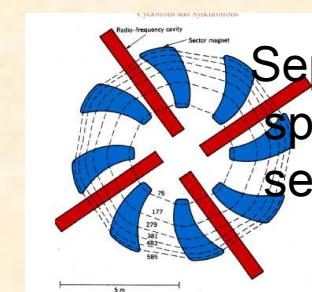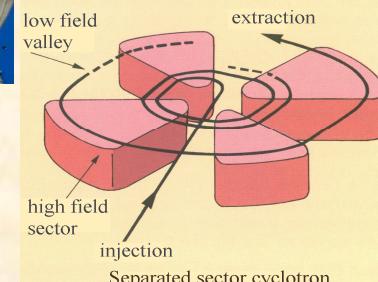
Cyclotron Summary : with formulas

Isochronous cyclotron = constant revolution frequency

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\,m} = const$$

field index n <0

$$B_z \sim B_0\,(r/R_0)^{-n}$$

$$<R> = \frac{B\rho}{<B_z>} = \frac{\gamma\,m v}{q<B_z>}$$

$$\omega_{rev}\,h = \omega_{RF}$$

$$E/A = K_b \cdot (Q/A)^2$$

Vertical stability in isochronous cyclotron $B_z = F(R,\theta)$

requires Azimuthal Field Modulation (N sectors)

$$\ddot{z} + \left[\nu_z\,\omega_{revolution}\right]^2 z = 0$$

$$\nu_z^2 = n + ... < 0$$

$$z(t) \sim z_0 \exp(-i\,\nu z\,\omega\,t) \quad : \text{vertical tune } \nu z; \text{ real for stability}$$

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1}F_l\,(1 + 2\,\tan^2 \varepsilon) > 0$$

Magnet design

How to adjust B(R) : $\langle B \rangle \sim R^{-n}$

- Pole Gap evolution $\langle B (R) \rangle$: $n(R) = 1 - \gamma^2$
- Correction coils (trim coils)

FLUTTER $= \dfrac{< (B - <B>)^2 >}{\langle B \rangle^2}$

$FLUTTER \approx \dfrac{\left(B_{hill} - B_{val} \right)^2}{8 \langle B \rangle^2}$

Sufficient FLUTTER F for axial stability($\nu_z^2 > 0$)

- Valley // hill B field
- sector angle
- spiral angle ε

$$\nu_z^2 = n + \dfrac{N^2}{N^2 - 1} F \ (1 + 2 \tan^2 \varepsilon) > 0$$

Space for injection beam line and RF

- Azimutally Varying Field or   Separated Sectors : $B = B(R, \theta)$
-        large Number of sectors N (4, 6, 8)


4
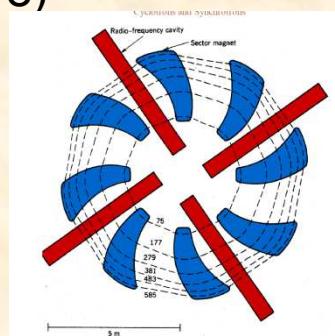
Isochronism : Field B=f(r)

Bz(R) adjusted to get   h ωrev = ωrf

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\,m}$$

$$\gamma(R) = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - (R\omega_{rev})^2/c^2}}$$

$$<B_z(R)>=<B_{z0}>/\sqrt{1 - (R\omega_{rev})^2/c^2}$$

$\Delta\Phi = \Delta T/ \omega rf$

$\Delta\Phi=90°$
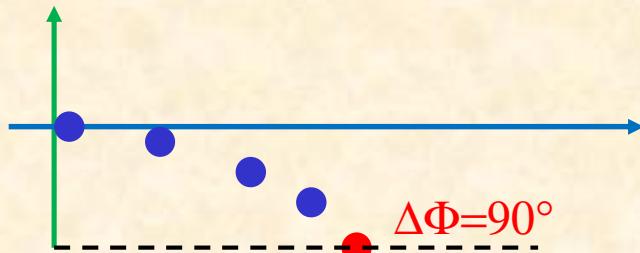
I=f(R)
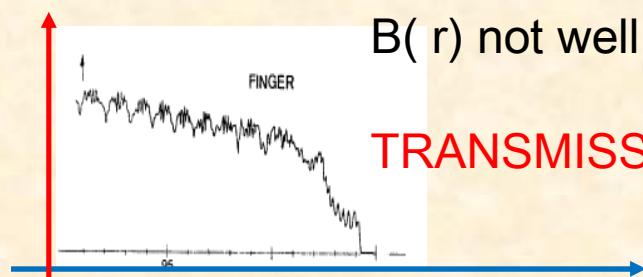
B(r) not well tuned

TRANSMISSION=0%

B(Radius) obtained   -with correction coils
                   or -with pole shapes

FINGER
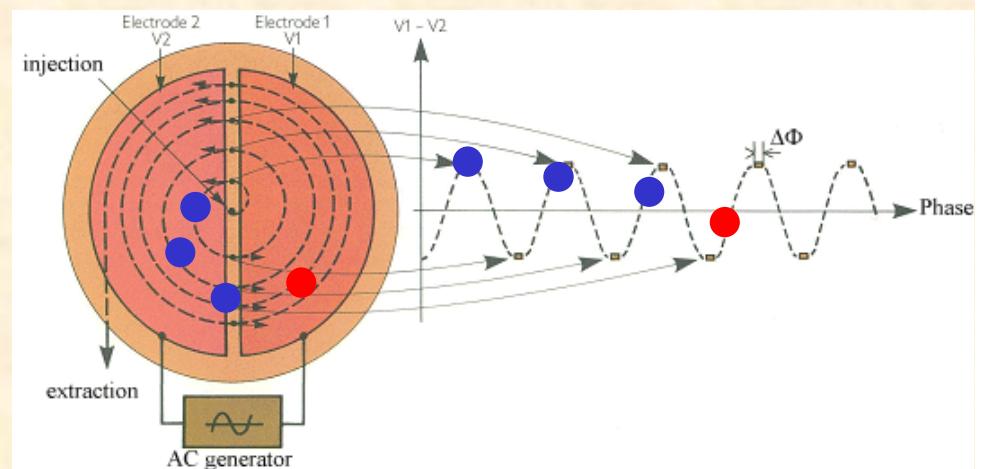
Max Energy for Cyclotrons

Heavy Ion
A= nucleon number
Q= charge number

$$\omega_{rev} = \frac{qB}{\gamma m} \approx \frac{qB}{m}$$

Max Kinetic Energy

$$(\gamma-1)mc^2 \approx \tfrac{1}{2} \, m \, v^2$$

$$= \tfrac{1}{2} \, m \, (R_{extraction}. \, \omega rev)^2$$

For ions: $m = A \, m0 = A. \, [1.6 \, 10^{-27} \, kg]$ $q = Q \, e0$

ex : $^{12}C^{4+}$ A=12 Q=4

$$-[E \, / \, A]_{max} \, (MeV \, / \, nucleon \,) = K_b \left\{ \frac{Q}{A} \right\}^2$$

with $K_b \approx 48.2 \left(\langle B \rangle . R_{ext} \right)^2$

<B> limitation and size limitation (: Rextraction) for *Emax*

Cyclotron Design

1) Particle kind : proton, heavy ion ?   Q/A

2) Max Kinetic Energy of reference ions Emax

3) Magnet   (Bmax, size ,sectors, hill/valley gap)   : Compute a field Map

4) Number of cavities (Ngap) : energy gain per turn

Then, let's start the SIMULATIONs

Multi_particlelcode in « realistic » magnetic field
 - compute reference orbit
 - simulate injection
 - simulate extraction

IS IT OK ?, IF NOT restart at 1)

Analytical B Field + RF Kick
Computed field map B + RF Kick
Computed field map B + E

Cyclotron simulation :
Particle Tracking with a computer code

SIMULATION : tracking ions (M,Q,v0)

Multi-particle-code
        in « realistic » magnetic field

In cylindrical coodinates

TRANSPORT (1rst order)
        MAD   (3rd order)
                CPO

Cyclotron Magnets are too complex

$$\mathbf{r} = r \cdot \mathbf{e}_r + z \cdot \mathbf{e}_z$$

Comoving Frame :   er = f(t)

$$d\mathbf{e}_r = \mathbf{e}_\theta \cdot d\theta \qquad d\mathbf{e}_z = 0 \qquad d\mathbf{e}_\theta = -\mathbf{e}_r \cdot d\theta$$

Velocity : $\dot{\mathbf{r}} = \dfrac{d\mathbf{r}}{dt}$?

$$\dot{\mathbf{r}} = \dot{r} \cdot \mathbf{e}_r + \dot{z} \cdot \mathbf{e}_z + r \cdot \dot{\mathbf{e}}_r + z \cdot \dot{\mathbf{e}}_z$$

$$\frac{d}{dt}\left[m\gamma\dot{\mathbf{r}} \right] = q \cdot (\mathbf{E} + \dot{\mathbf{r}} \times \mathbf{B})$$
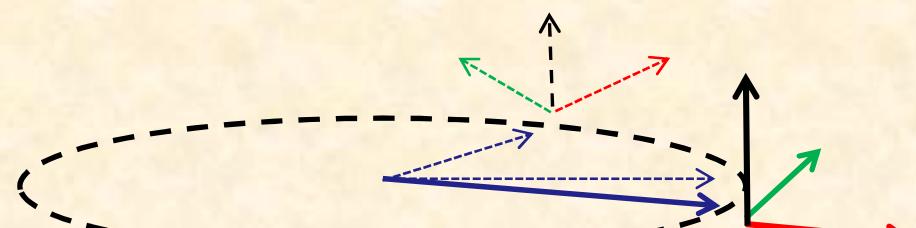
Unit vectors are evolving in time !!!
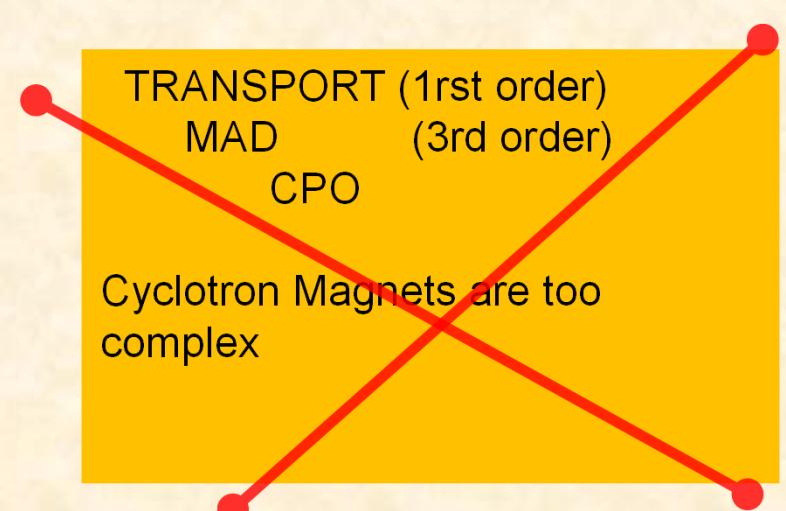
Cyclotron simulation :
Particle Tracking with a computer code

SIMULATION : tracking ions (M,Q,v0)   In cylindrical coodinates

Let's track one particle   Start $\theta=\theta_0$ (At t=0)

r = r0   pr = pr_0
z =z0   pz =pz_0
                 pθ =pθ_0

What is the particle position at $\theta=\theta_0+\Delta\theta$ (At t=0+ $\Delta\theta$ [dt/dθ])

$r(\theta_0 + \Delta\theta)$ = r0+ $\Delta\theta$ [dr/dθ]   (first order extrapolation= euler algorithm)

z =z0 + $\Delta\theta$ [dz/dθ]

pr = pr_0 + $\Delta\theta$ [d pr /dθ]

pz =pz_0 + $\Delta\theta$ [d pz /dθ]

pθ =pθ_0 + $\Delta\theta$ [d pθ /dθ]

$\theta_0 + \Delta\theta$

θ_0

[d r /dθ]   =

……

[d pr /dθ] = cylindrical equation of motion   = f [B(r, θ z)]
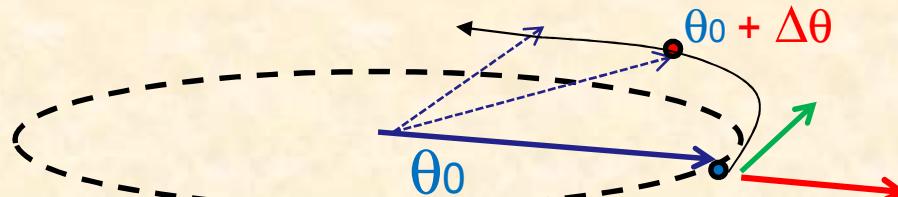
Cyclotrons simulation: cylindrical equation

$$\frac{d\mathbf{p}}{dt} = q \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix} =$$

$$= (\dot{z} \cdot B_\theta - r\dot{\theta} \cdot B_z) \cdot \mathbf{e}_r + (r\dot{\theta} \cdot B_x - \dot{r} \cdot B_\theta) \cdot \mathbf{e}_z + (\dot{r} \cdot B_z - \dot{z} B_r) \cdot \mathbf{e}_\theta$$

Evolution in time t is not convenient, evolution in θ is better !!!

$$\frac{d}{dt} = \frac{d\theta}{dt}\frac{d}{d\theta} = \dot{\theta}\frac{d}{d\theta} \qquad\qquad \frac{d\mathbf{p}}{dt} = \dot{\theta}\frac{d\mathbf{p}}{d\theta} = q \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\frac{d}{d\theta}\left[m\gamma\dot{r}\right] = \frac{d}{d\theta}[p_r] = m\gamma\dot{r}\theta + \frac{q}{\dot{\theta}}(\dot{z}\cdot B_\theta - r\dot{\theta}\cdot B_z)$$

$$\frac{d}{d\theta}\left[m\gamma\dot{z}\right] = \frac{d}{d\theta}[pz] = \frac{q}{\dot{\theta}}(r\dot{\theta}\cdot B_x - \dot{r}\cdot B_\theta)$$

$$\frac{d}{d\theta}\left[m\gamma r\dot{\theta}\right] = \frac{d}{d\theta}[p_\theta] = \frac{q}{\dot{\theta}}....$$

$$\frac{dr}{rd\theta} = \frac{\dot{r}}{r\dot{\theta}} = \frac{p_r}{p_\theta} \qquad\qquad \frac{dz}{rd\theta} = \frac{\dot{z}}{r\dot{\theta}} = \frac{p_z}{p_\theta}$$

Cyclotrons simulation: trajectory in (r,z)=f(θ)

The integration of particle's equation can be obtained with numerical methods

The equationq to be solved is a set of Ordinary Differential Equations (ODE).

START at θ=0 (r_0,z_0,Pr,Pz,Pθ) :   what are (r,z)   at θ=0+Δθ ?

At first order , we can compute (r,z) and (pr,pz)

$$\frac{d}{d\theta}[pr] = m\gamma \dot{r}\dot{\theta} + \frac{q}{\dot{\theta}}(\dot{z}\cdot B_\theta - r\dot{\theta}\cdot B_z)$$

$$p_r(\theta_0 + d\theta) = p_r(\theta_0) + \frac{dp_r(\theta_0)}{d\theta}d\theta + 0(d\theta^2) + ...$$

$$r(\theta = \theta_0 + d\theta) = r_0 + \frac{dr}{d\theta}d\theta + ...$$

$$\frac{dr}{r\,d\theta} = \frac{dr/dt}{r\,d\theta/dt} = \frac{v_r}{v_\theta} = \frac{p_r}{p_\theta}$$

$$\frac{dr}{d\theta} = r \cdot \frac{p_r}{p_\theta}$$

$$t = t_0 + \frac{dt}{d\theta}d\theta + ...$$

$$\frac{dt}{d\theta} = \frac{1}{d\theta/dt} = \frac{m\gamma\,r}{P_\theta}$$

This the EULER method ! = 1rst order expansion

Cyclotrons simulation : the algorithm

Loop   j=1,Nparticles

INITIAL position and momentum   : $\theta = 0$ r, z pr, pz, pθ

Loop i=1,Nstep   **// step in $\Delta\theta$**

$$B_r = BR(r,z,\theta) \qquad B_z = BZ(r,z,\theta) \qquad B_\theta = B\theta(r,z,\theta)$$

$\theta = \theta_0 + \Delta\theta$

FIELD MAP

$r(\theta = \theta + d\theta) = r + \dfrac{dr}{d\theta}\Delta\theta \qquad p_r(\theta) = pr_0 + \dfrac{dp_r}{d\theta}\Delta\theta$

$\dfrac{d}{d\theta}[pr] = m\gamma r\dot{\theta} + \dfrac{q}{\dot{\theta}}(\dot{z}\cdot B_\theta - r\,\dot{\theta}\cdot B_z)$

$z(\theta = \theta + d\theta) = z + \dfrac{dz}{d\theta}\Delta\theta \qquad p_z(\theta) = ...$

Endloop I   **// end θ loop**
Endloop J   **// Nparticle loop**
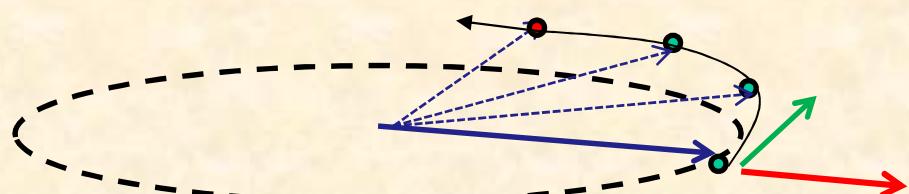
Euler algorithm (second order accurate in **dθ**)

Numerical integration of the equations of motion

Euler algorithm (second order accurate in $\Delta\theta$) is not the best !!

RK4 (runge kutta order 4) is better (4th order accurate in $\Delta\theta$)
 See a Numerical analysis Lecture

SPECIAL ATTENTION to FIELD INTERPOLATION
between the points of the field map

How to simulate a cyclotron in 4 steps

- **1) Define the basic parameters of the cyclotron (B,R,F) :** $B\rho = <B>\ R_{extraction}$

START The simulation in the "middle" of cycloton
With a defined magnetic structure

- 2) Find the closed orbit (1 particle) without acceleration at R=Rref

- 3) Find a matched beam in the cyclotron (multiparticles)
     backward tracking toward injection

- 4) Forward tracking (multiparticles) toward extraction
- Extraction (multiparticles) : (deflector, precession, resonance)

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Iterative process

Basic parameters R,<B>,Sectors,Flutter (1/4)
Ex: cyclotron design 20 MeV/A for carbon ion 4+

What is the Max energy (MeV/A) : 20 MeV/A for carbon 4+

What are the ions (Q/A) = 4+/12 ⇨ Bρextraction= 2 T.m

ION (M,Q)
FINAL ENERGY

Bρextraction= 2 T.m   γ= 1.02   **Bρ =< B> Rextraction**

Reasonnable Fied
<B> =1.5 T

Rextraction= Bρextrac /<B> =1.4 m

ION (M,Q)
Source Voltage
(30kV -100kVolts ?)

Bρinjection= 0.04 T.m

Rinjection= Bρinjection /<B>=0.04m

Vertical stability
        B(r,θ) Nsector , Hill//Valley gap

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l (1 + 2 \tan^2 \varepsilon) > 0$$

let's take 4sectors & gap hill =12 cm //gap valley =30 cm //…ε=0

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Dynamic of 1 particle in the middle of the cyclotron
- Choose reference particle (1 particle) $(M, Q, B\rho_0)$
- **Choose a field Level B(r,θ) =k. FIELD MAP**

Find the reference radius Rref ?

r'

r
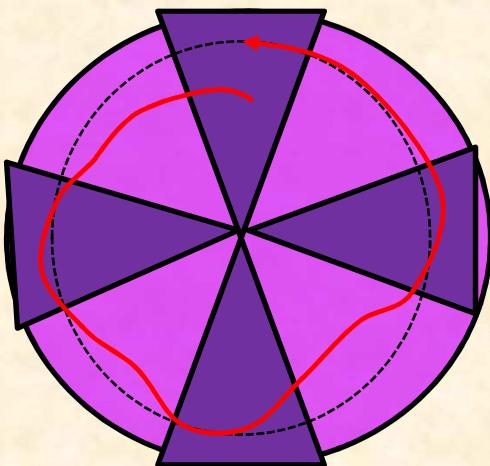
Trajectory is not a closed orbit
Not a good starting point

Trajectory is a closed orbit :OK

$$<Rref> = \ <B(r,\theta)> \ / \ B\rho$$
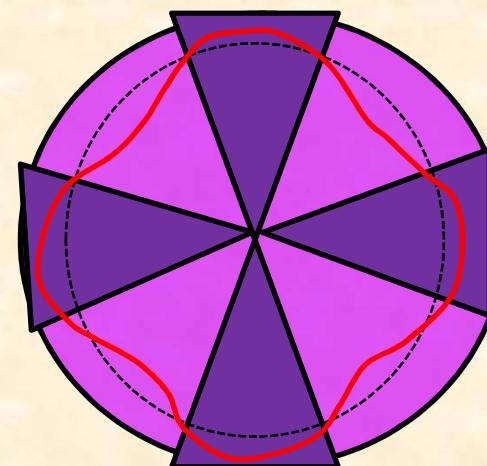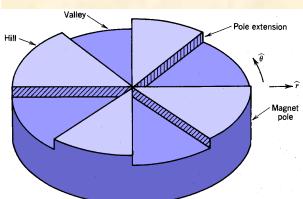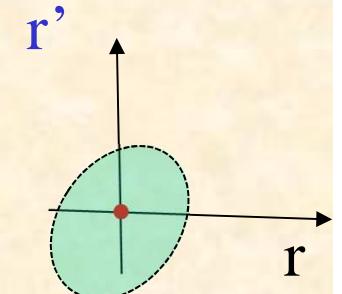
Find a matched beam in the cyclotron (3/4)

**Around the reference trajectory, send a particle for many turns**

Hill-Valley is a periodic lattice

Turn 3

r'_{max}

Turn 2

r_{max}

Turn 1
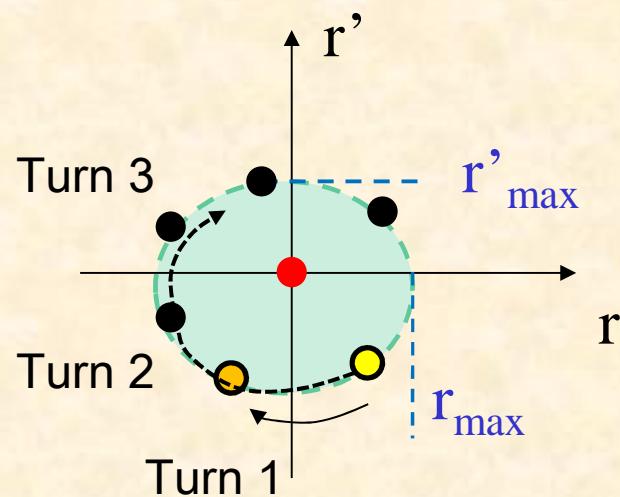
A particle trajectory follows an ellipse

$n.$
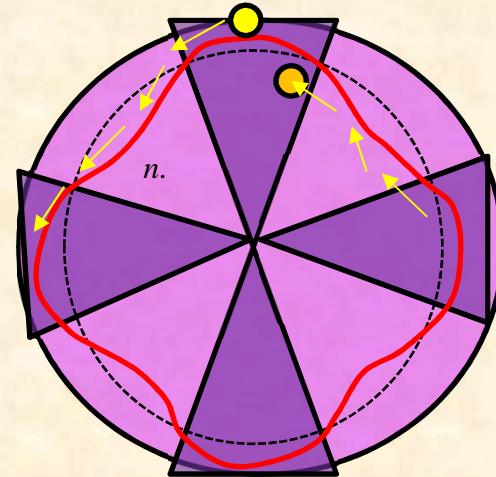
Beam matching =
Choose a beam ellipse with
$$\Delta r'/ \Delta r = r'_{max}/ r_{max}$$

$$r(t) = r_0 + r_{max} \cos(\nu_r \omega_0 t)$$

$$r'(t) = r'_{max} \sin(\nu_r \omega_0 t)$$
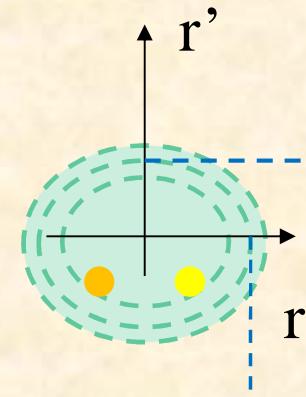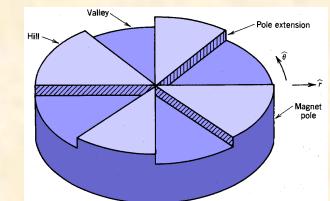
This ellipse occupy the minimal size in the cyclotron

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Mismatched beam recall

Because of each individual trajectory

$$r(t) = r_0 + r_{max} \cos(\nu_r \omega_0 t)$$
$$r'(t) = r'_{max} \sin(\nu_r \omega_0 t)$$

it exist an optimal ellipse

for a given beam   Emittance : $\varepsilon = \pi \Delta r_{max} . \Delta r'_{max}$

Betatron oscillation with mismatched beam   MisMatched beam

r' r' r' 4

r'_{max} 3

r r 5 r

r_{max} 2

$\nu_r = 1/5$

1
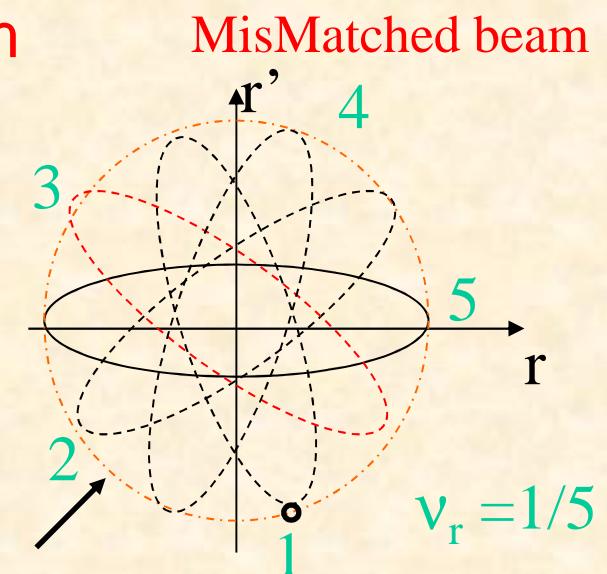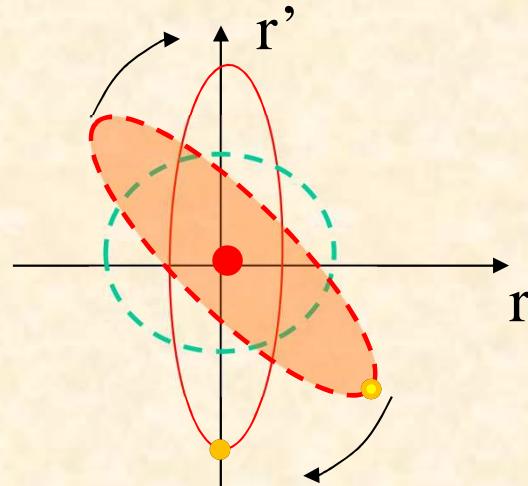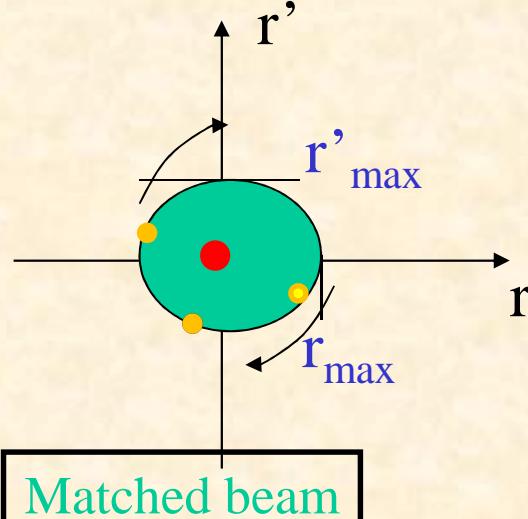
Matched beam

Larger acceptance required for the cyclo : not good

Matched beam recall

$$r(t) = r_0 + r_{max} \cos(\nu_r \omega_0 t)$$

$$r'(t) = dr/ds = dr / R\omega_0 dt = -(r_{max} \nu_r / R) \sin(\nu_r \omega_0 t)$$

the Matched ellipse $\qquad |r'_{max}| = |r_{max} \nu_r / R|$

⇨ Initial beam conditions depend of the tune (ν_r) of the cyclotron at the matching point.

 qquad ⇨ Betatron oscillation disappears

qquad ⇨ Matched beam

qquad ⇨ Minimum of acceptance


r'

$r'_{max} = r_{max}\nu_r/R$

r

r_{max}
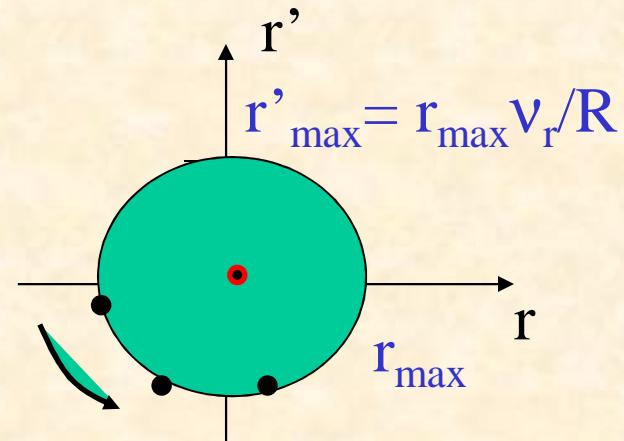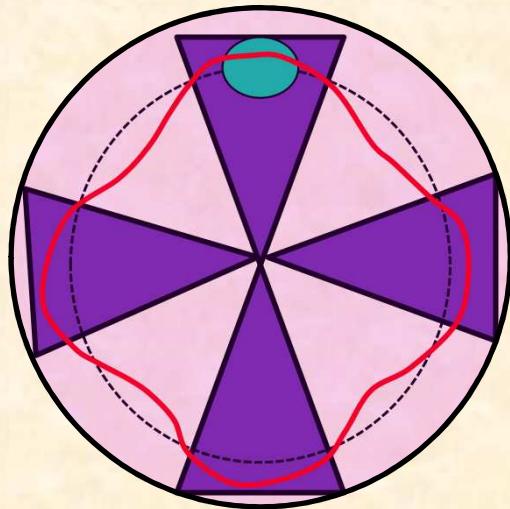
Central Trajectory is a closed orbit :OK

Beam matched : OK
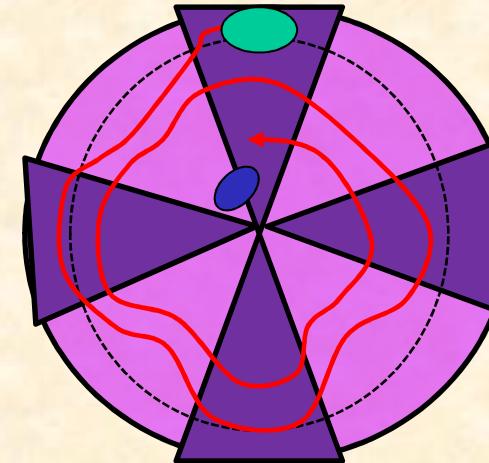
turn on RF : backward toward injection
Adjust Vrf, central field.....

r'_{max}

r

r_{max}

r'

r
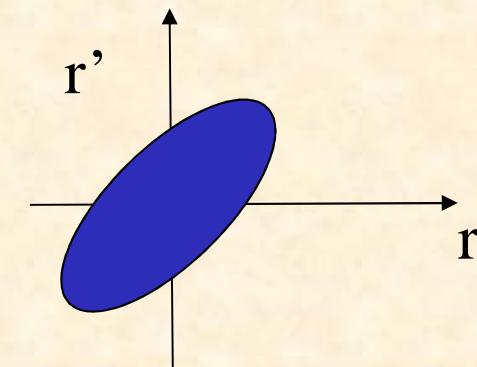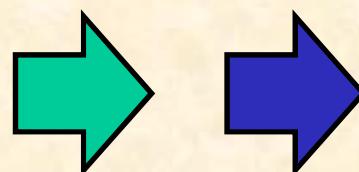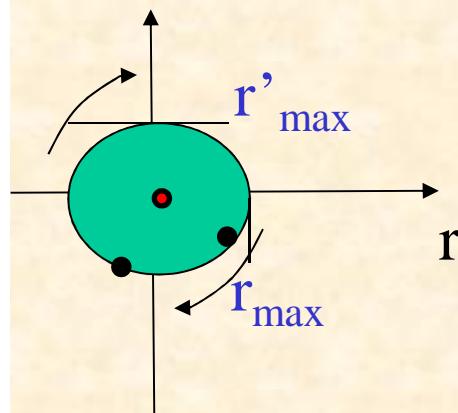
Corresponding optimal beam at injection radius

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Drive the beam backward

Start with matched beam in the cyclotron (multiparticles) at large radius
Then Adjust Vrf, central field to reach injection Radius

Find the optimal beam at injection radius

**CIME injection**
Injection radius 34 mm
Harmonic 2, 3 and 4

RF cavity

Inflector

45.8mm

19,3mm

RF cavity

Pillars

Harmonique 3

r'

r'$_{max}$

r$_{max}$
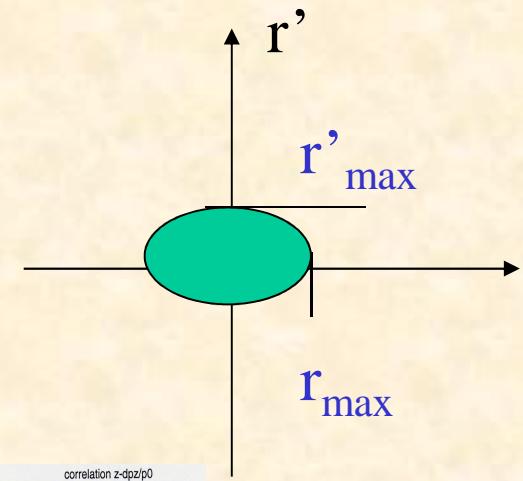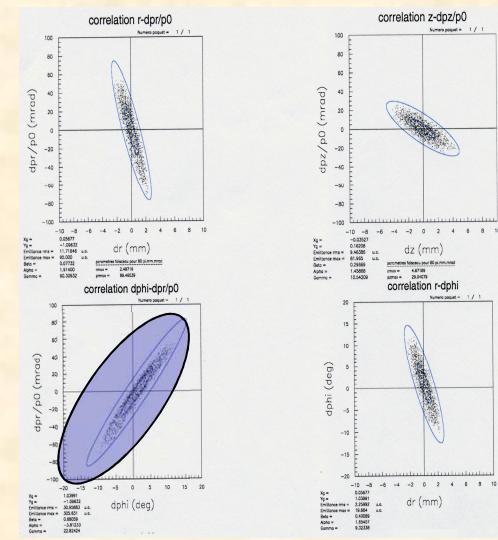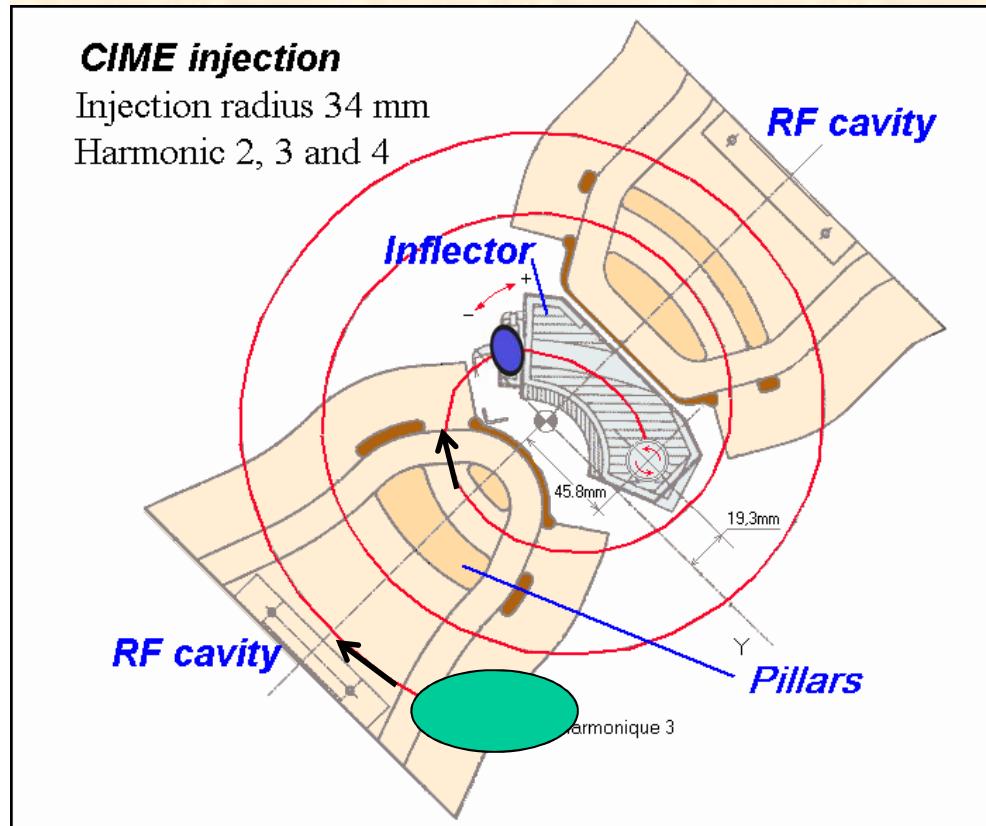
Beam
Obtained
With
backward
tracking

(r, r', z, z', E, ϕ)

Simulate the injection beam line

to get the perfect beam at injection

Matching at this point

$$x'_{max} = x_{max} \nu_r / R$$

Buncher (Energy,time)
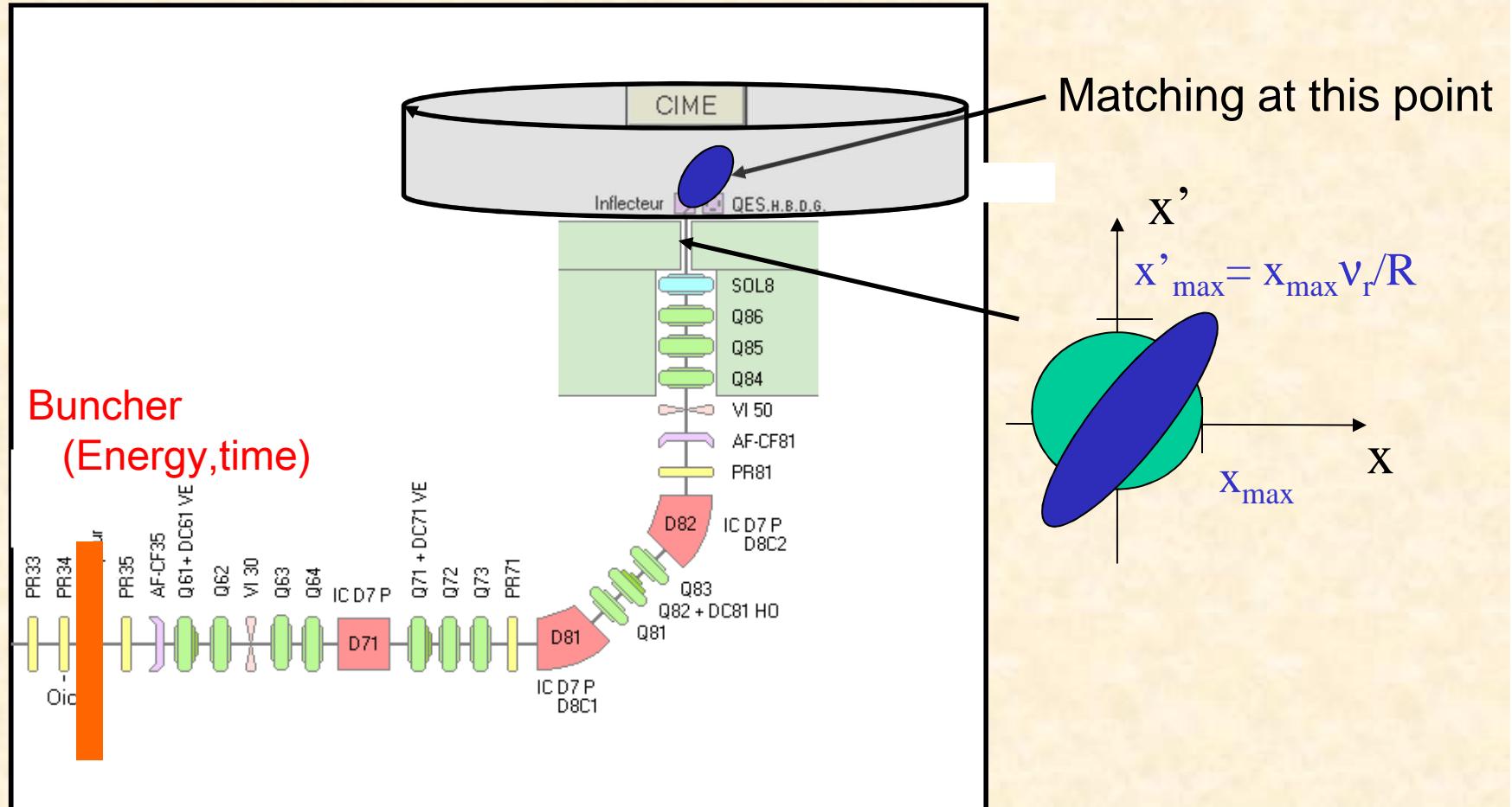
Classical transport line problems :

Adjust quad to get desired beam at injection (r,r') (z,z') (t,E)
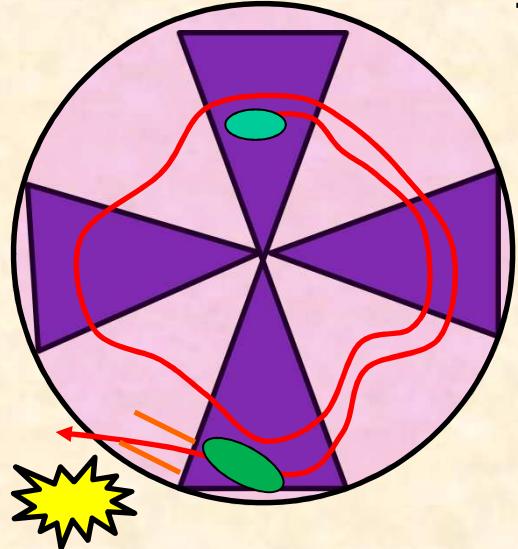
turn on RF : Forward toward extraction

tune the isochronism $\langle B(r)\rangle = \langle B\rangle\,\gamma(r)$

Extraction
- design the extraction (deflector+..)
- turn separation (RF +precession? + magnetic bump?)
- beam losses ?

Beam matched

r'_{max}

r_{max}

r
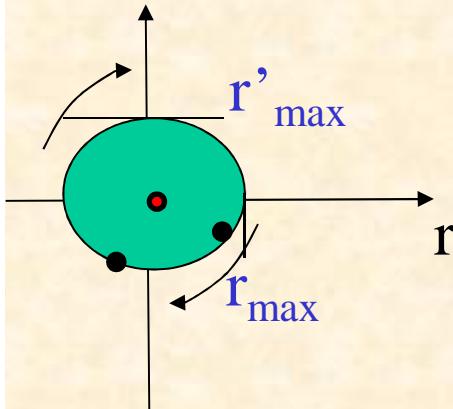
beam at extraction radius : Watch beam losses

r'

r

Beam losses
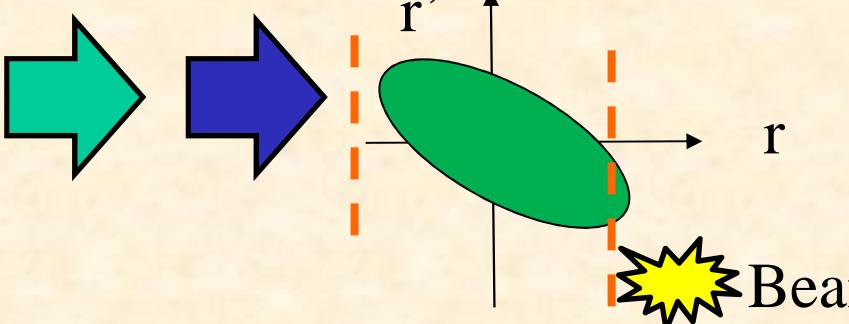
THE CYCLOTRON IS COMPUTED, Let's construct it !

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Cyclotron Design strategies

Radio-Isotopes production
cost & reliability

**Medical applications : Cancer treatment**
cost & reliability

Nuclear physics& Research facility
performance , intensity

Mode (1): RILAC + RRC + (stripper2) + fRC + (stripper3) + IRC + SRC

24
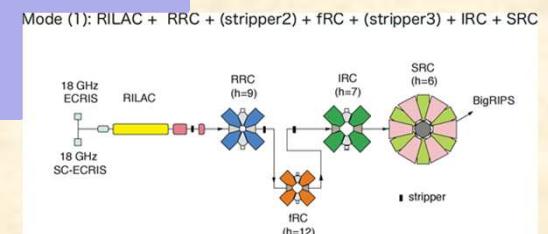
Strategy for Radio-Isotopes production medical applications

10 MeV Protons / 5 MeV Deutons : @ low cost

Bρmax = 0.458 T.m = $\langle B \rangle$ Rextraction

Rextract = 0.34 m

$\langle B \rangle$ = 1.35 Tesla   [hill = 1.8 T // valley = 0.5 T]

AVF with 4 straight sectors (sufficient z-focusing)

I_{beam} ~ 0.1 -0.05 mA

Rf Dees : 2   (so 4 gaps)

2 possibilities for extraction

Extraction By stripping :
external target (18F, radiotracer)

No Extraction :
internal target (in yoke)

A « low energy » industrial Cyclotron Cyclone 10/5 : 2particles : ^{1}H & D

Kb=10 MeV

Fixed energy ;
  4 straight sectors 50°
  fixed **Frf =42Mhz**
      <B> =1.35 Tesla
Harmonic   h=2(p) ,4 (D)
Internal source
Rextraction=0.33m
Bρmax=0.33x 1.35=0.45 T.m
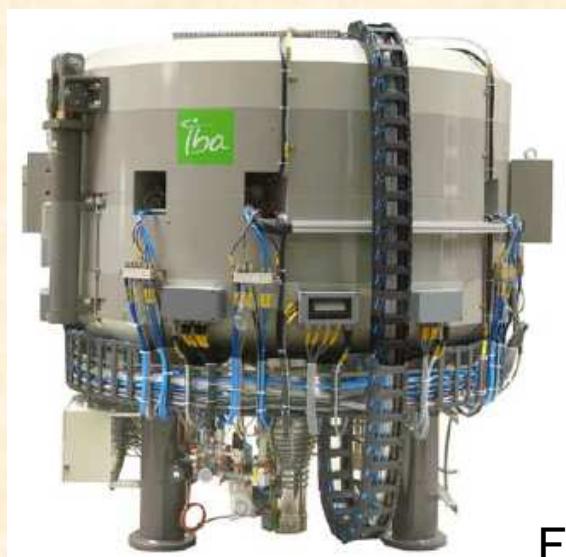
$$\left[\frac{E}{A}\right]_{max} (MeV\ /\ nucleon\ ) = K_b \left\{\frac{Q}{A}\right\}^2$$

Eprotons=10 MeV   protons= ^{1}H^{1+} A=1 Q=1
  (E/A=Kb*1^2 =10MeV/A)
    RF Harmonic =2   Frev=42 Mhz/ h= 21 Mhz

EDeutons=5 MeV   Deutons= ^{2}H^{1+} A=2 Q=1
        (E/A=Kb*0.5^2= 2.5 MeV/A)
  RF Harmonic =4

$$F_{rev} \propto \frac{Q.B_{cyclo}}{A}$$

Q/A=2   Q/A=1
Deutons   protons

<B>

1.35T

Frev= Frf / H

Frev~ <B>.Q/A

10.5 Mhz   21 Mhz
H=4   H=2
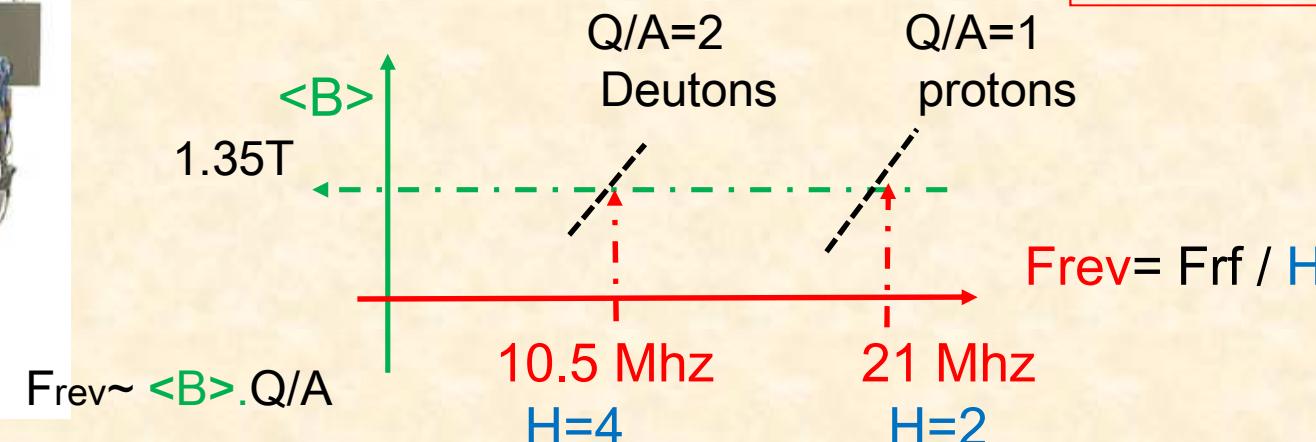
Strategy for cancer treatment
proton therapy (>80 facilities in the world)

-250 MeV   Protons

Accel VARIAN Isochronous cyclo

Superconducting   <B> = 2.2 Tesla

Rextrac~1.2m
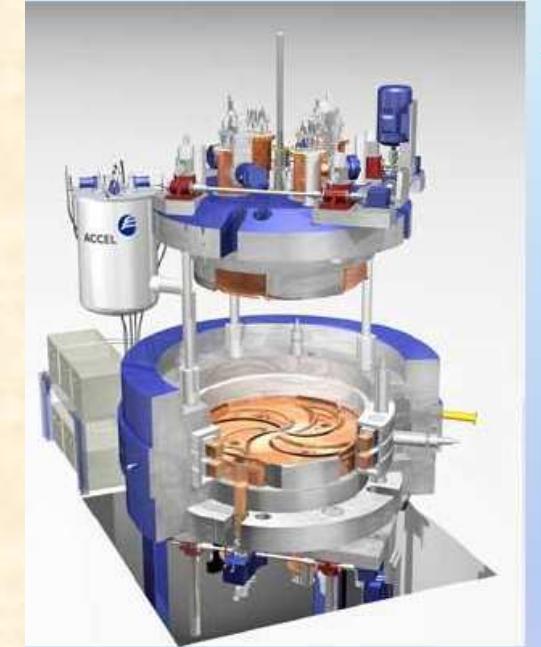
-230 MeV   Protons

IBA   Synchro cyclotron

Superconducting   <B> = 5. Tesla

Rextract ~0.6 m

Very compact

   Hill/valley not needed

Strategy for a Cyclotron in a research facility

High energy

$$E/Amax = Kb \cdot (Q/A)^2$$

High $Kb \sim (R.B)^2$

R Large magnet (Radius)

High Bz (superconducting)

Vertical stability
Increase « Flutter »
Separated sectors

High ion charge Q

« external » ECR source

Ion stripping at high energy
« 2 Stages accelerators »

Strategy for a Cyclotron in a research facility

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Increase « Flutter »
Separated sectors

High ion charge Q

« external » ECR source

Ion stripping at high energy
« 2 Stages accelerators »

**High Intensity**

Good ion source

No losses at extraction

High RF voltage

Many gaps

RIBF (Japan) : SRC (K=2600 MeV) –the bigest cyclo
Uranium beam $^{238}U^{88+}$ @345 MeV/A cw

Mode (1): RILAC + RRC + (stripper2) + fRC + (stripper3) + IRC + SRC

4 RF cavities

Radius=6m
<B>=3.8T

Ion Strippers
U^{50+}

U^{88+}

ECR source
U^{30+}

5 Stages accelerators

1 LINAC
+ 4 Cyclotrons

30
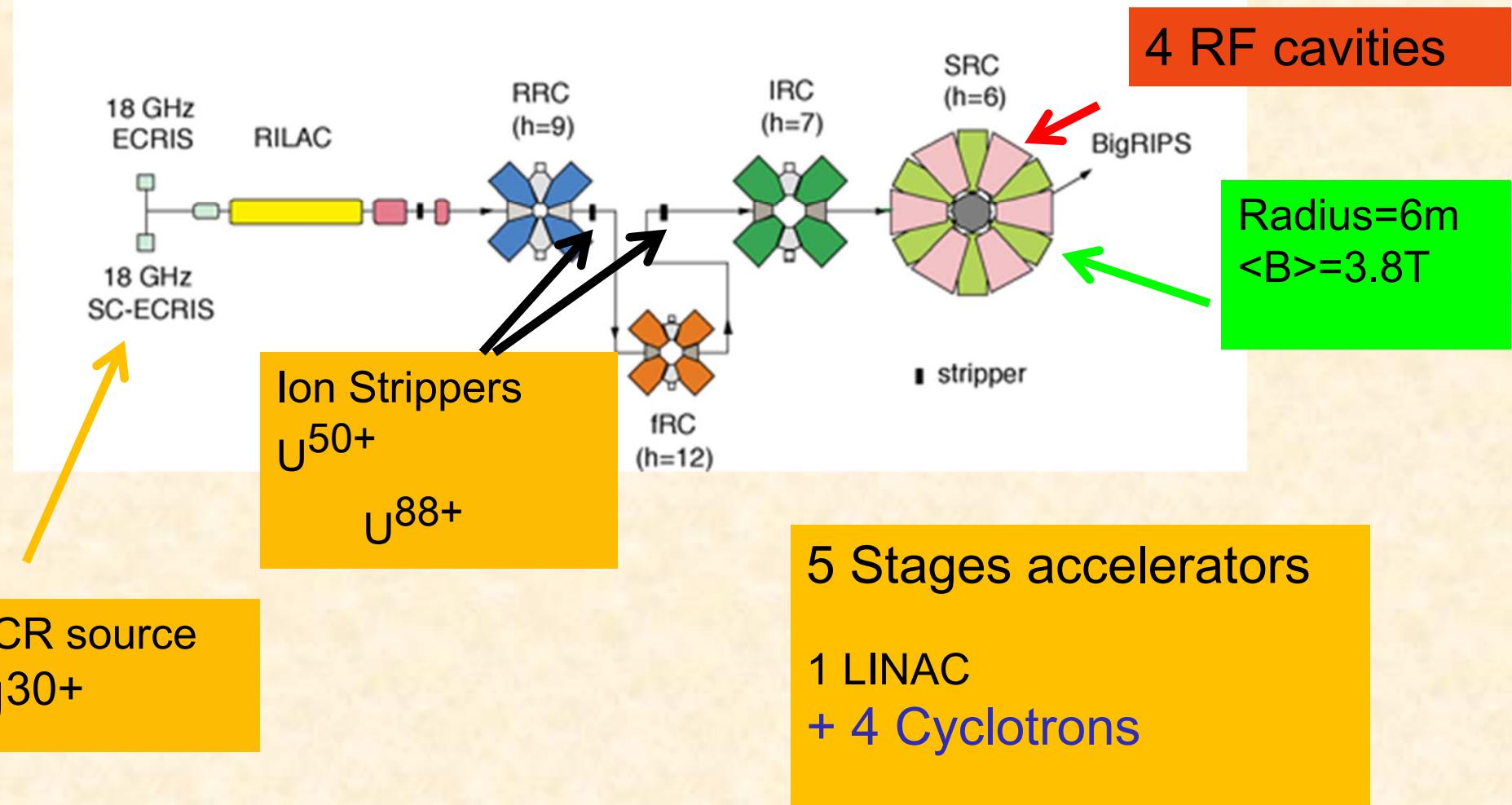
Ion Stripping at high energy

Heavy ions are not fully stripped by ion sources :

Incoming Ions

$A^{Q_{1+}}$ →

Magnetic spectrometer

Stripping some of residual electrons

final Ions $A^{Q_{2+}}$

$Q_2 > Q_1$

$B\rho_2 < B_{\rho 1}$

Carbon foil Stripper

Number of events/channel

Kr^{-23+} (444 MeV)
Carbon Thickness : 50 µg/cm2
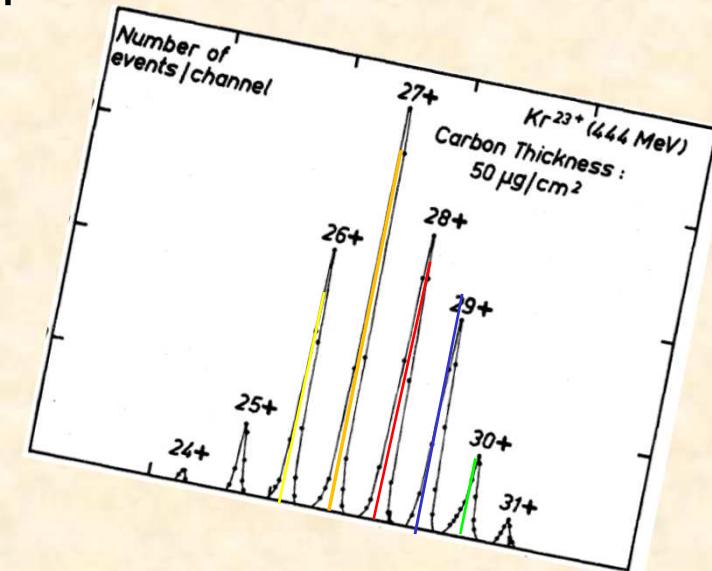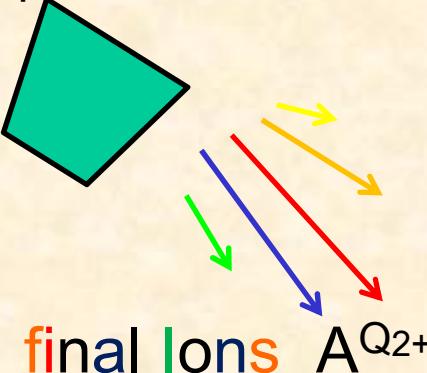
27+
26+
28+
25+
29+
24+
30+
31+

$$B\rho = \frac{P}{q} = \frac{\gamma \, m.v}{q}$$

Ion Stripping help to increase the maximal energy of a given cyclotron....

$$[E/A]\max = Kb\left[\frac{Q}{A}\right]^2$$

- **End Chapter 3**

important facts for cyclotron :

1) Simulations are done with realistic magnetic field (not transport matrices)

2) Magnetic structure should provide the vertical stability (field index n compensated by sectors)

3) The Beam matching at injection for better transverse acceptance

Additive slides …..

Diagram for
The variable energy cyclotrons

$$F_{rev} \propto \frac{Q.B_{cyclo}}{A} \propto h \cdot F_{RF}$$

A= nucleon number
Q= charge number

$$\omega_{rev} = \frac{qB}{\gamma\, m}$$

Q/A=3

$^{12}C^{4+}$ Q/A=2

Q/A=1
proton

<B>

2) Compute
<B>
For a given
(Q,A)

Frev= Frf / h

h=3   h=2   h=1
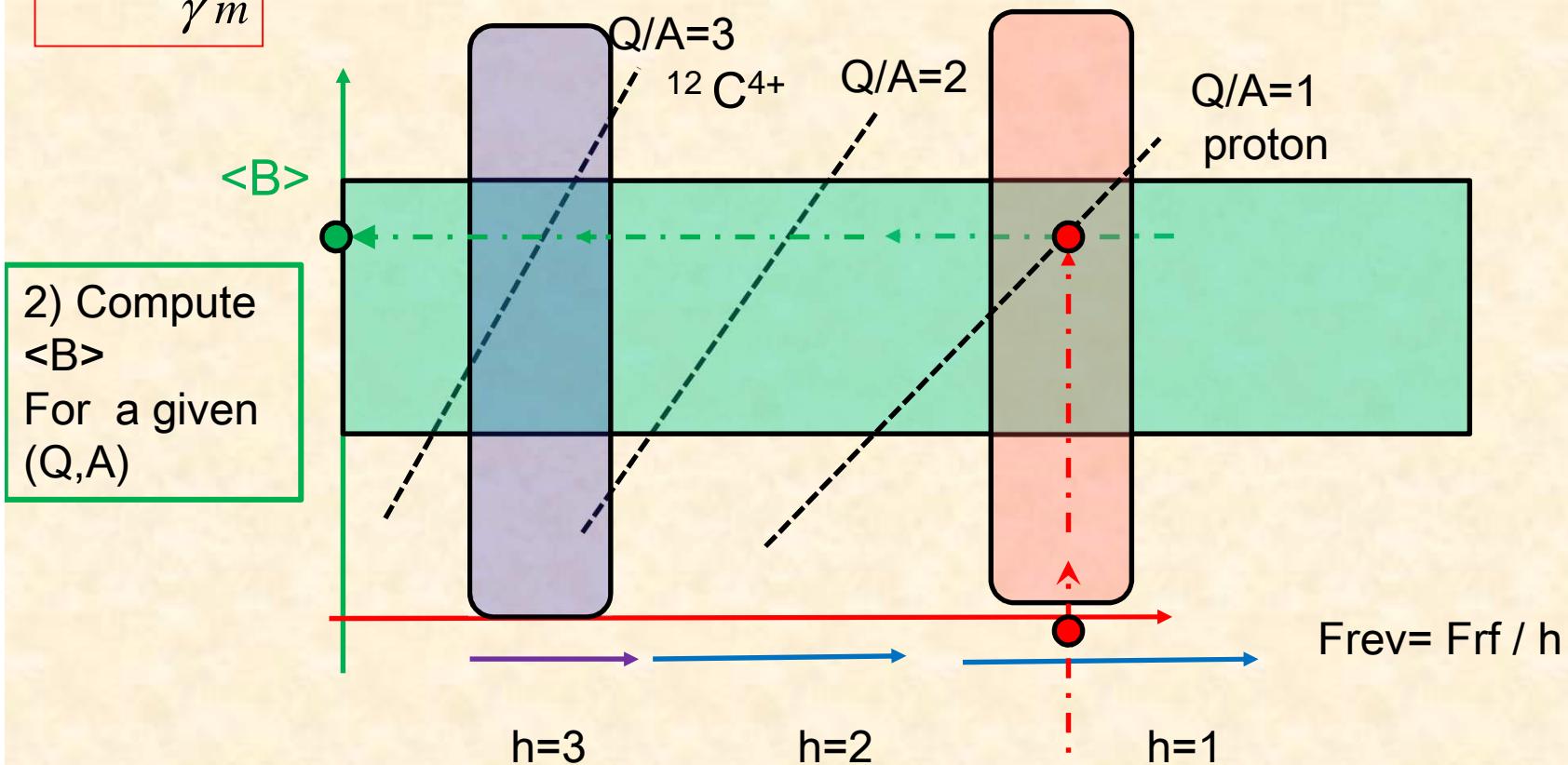
1) Select the energy   (Frev)
Select the ions (Q,A)
Adjust <B>

Bρ # <B> Rextract

E/A (MeV/A) # K (Q/A) ^{2}

33

Coupling of 2 Cyclotrons : velocity matching

Two cyclotrons can be used to reach higher energy :

- Harmonic&Radius of the 2 cyclotrons have to be matched

$$\frac{v}{2\pi} = \left[\frac{F_{HF}.R_{ejec}}{h}\right]_{cycloA} = \left[\frac{F_{HF}.R_{inj}}{h}\right]_{cycloB}$$

The velocity of extraction CycloA

= velocity of injection CycloB

- Ion stripping can be used, to increase Q before injection into the second cyclo

large Q ⇨ large *Emax*

$$\left[\frac{E}{A}\right]_{max} = K_b \left\{\frac{Q}{A}\right\}^2$$