

Cyclotrons

Chapter 4 : theory versus reality

- Cyclotron versus **synchrotron or linac**
- superconducting **Kf limitation** (focusing limitation)
- Isochronism and Phase measurement
- Isochronism 2nd approach
- **Resonances and tunes in a cyclotron**

- **Research applications**
- **Medical applications**

Cyclotron vs other RF accelerators

	Radius	Fr _f	Particles
Cyclotrons	not constant	constant :CW (isochronous)	Ions limit $\gamma < 2$
Synchro-cyclotrons	not constant	not constant pulsed, Fr _f (t)	Ions
Synchrotrons	constant	not constant pulsed Fr _f (t)	Ions, electrons no limits for γ , limit €
Linacs	constant ∞	constant	Ions, electrons limit €

LINAC VERSUS CYCLOTRON

Proton Source

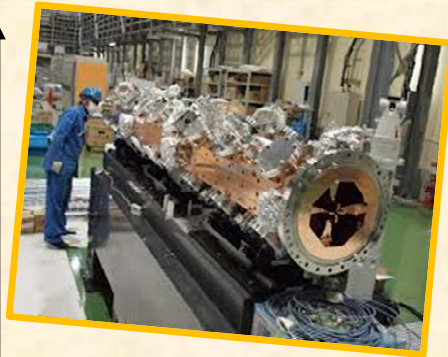
LEBT

RFQ

MEBT

DTL

Drift Tube Linac



~20 m total



~ 10 m

to get ~20 MeV protons



Internal source

H⁻ with stripping extraction

Rextraction ~0.4 m

Total size < 2m

I cw ~ up to 1-5 mA

LINAC Cost ~ up to 10 Meuros
+ building (~ 300m²)

- Not Compact (beam lines+linac)
- specific design € € € € € € €

I cw ~ 0.05 up to 0.3 mA in H⁻
CYCLO Cost ~ 1.5- 3 Meuros

Several industrial manufacturers

Compact

Standard design (300 cyclo in the world)
Operation easy

Max Energy for Superconducting Cyclotrons not limited by (B x Rextraction)

We can demonstrate that **isochronism** imply $n(R) = (1 - \gamma^2) < 0$

Stability : **isochronous field condition compensated by Flutter (B(R,θ))**

$$v_z^2 = [1 - \gamma^2] + \frac{N^2}{N^2 - 1} F (1 + 2 \tan^2 \epsilon) > 0$$

At high energy $[1 - \gamma^2] \ll 0$ compensation not possible

the **max energy** is not given by $Kb \sim 48 (B.Rextraction)^2$

but **K_f** the so-called “*focusing* factor”:



• **Focusing limitation (stronger than B limitation)**

$$\left[\frac{E}{A} \right]_{\max} = K_f \cdot \left\{ \frac{Q}{A} \right\}^2 < K_b \cdot \left\{ \frac{Q}{A} \right\}^2$$

$$K_b \sim 48 (B.Rextraction)^2$$

$$K_f \sim f(\text{FLUTTER})$$

Tutorial : $n(R)=1-\gamma^2$

demonstrate that **isochronism** imply

$$n(R) = (1 - \gamma^2) < 0$$

Answer : we have $\frac{dB_z}{dr} = B_0 \frac{d\gamma}{dr}$

And $\gamma(R) = 1/(1 - (R\omega/c)^2)^{1/2}$

The field index definition

Correspond to $B_z(r) = B_0 r^{-n}$

$$\begin{aligned} \frac{dB_z}{dr} &= \frac{d[B_0 r^{-n}]}{dr} \\ &= -n B_0 r^{-n-1} \\ &= -n B_z / r \end{aligned}$$

$$\frac{dB_z}{dr} = -n B_z / r \quad [1]$$

$$\frac{dB_z}{dr} = B_0 \frac{d\gamma}{dr}$$

$$\begin{aligned} B_0 \frac{d\gamma}{dr} &= B_0 \frac{d(1 - (r\omega/c)^2)^{-1/2}}{dr} \\ &= B_0 \times -2r(\omega/c)^2 \times \frac{1}{2} \times (1 - (r\omega/c)^2)^{-3/2} \\ &= B_0 \times -(\beta^2)/r \times (1 - (r\omega_p/c)^2)^{-3/2} \\ &= B_0 / r \times \beta^2 \times \gamma^3 \\ &= B_0 \gamma / r \times \beta^2 \times \gamma^2 \end{aligned}$$

Remember $\gamma^2 = 1/(1-\beta^2) \Rightarrow \beta^2 \gamma^2 = -1 + \gamma^2$

$$\frac{dB_z}{dr} = -B_z / r \times (1 - \gamma^2) \quad [2]$$

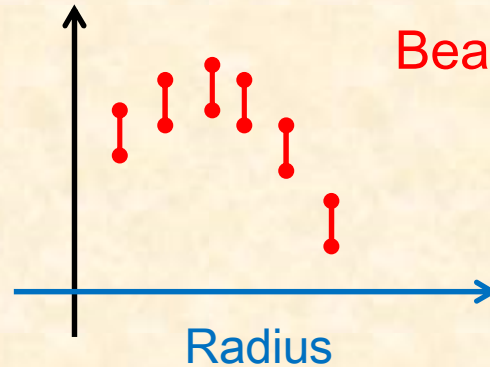
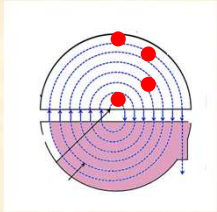
so [1+2]

$$B_z / r \times (1 - \gamma^2) = -n B_z / r$$

So $n(R) = 1 - \gamma^2$

Phase measurement: $n(R)$ check and field correction : ΔB

Relative phase
Beam vs RF

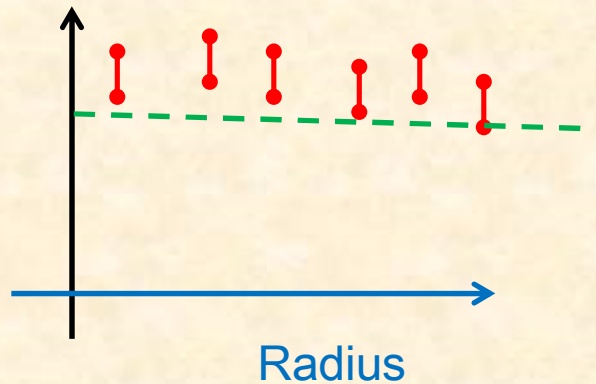
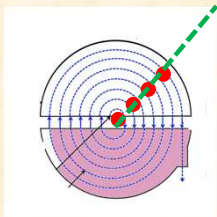


Beam not in phase with RF : **B** field not OK

$$B_z(R) \neq B_{z0} / \sqrt{1 - (R\omega_{rev})^2 / c^2}$$

$B_z(R)$ not correct

Relative phase
Beam vs RF

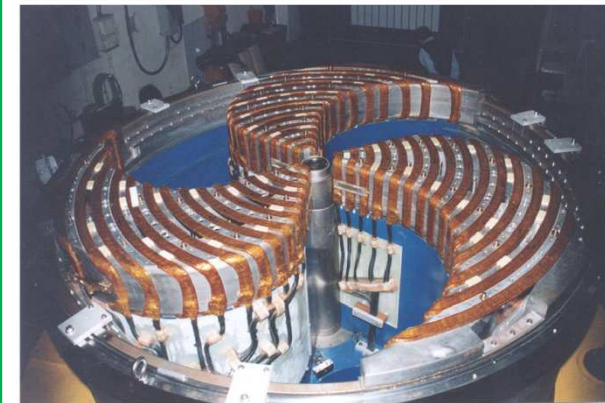


Correction of **B** : $B(\neq \Delta I)$

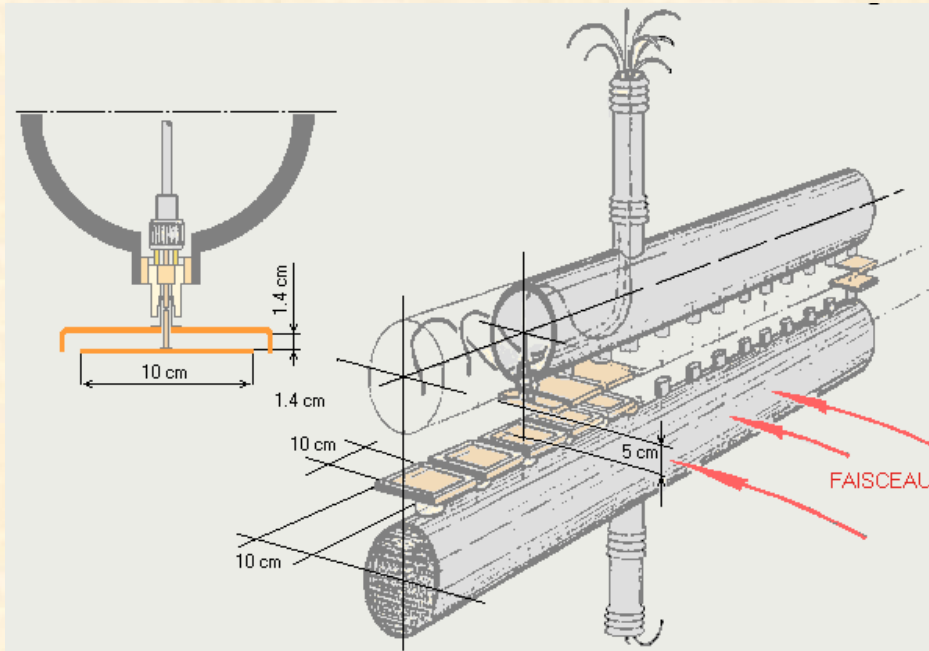
$B_z(R) + \Delta B_z$

$B_z(R)$ correct : **Beam in phase
with RF**

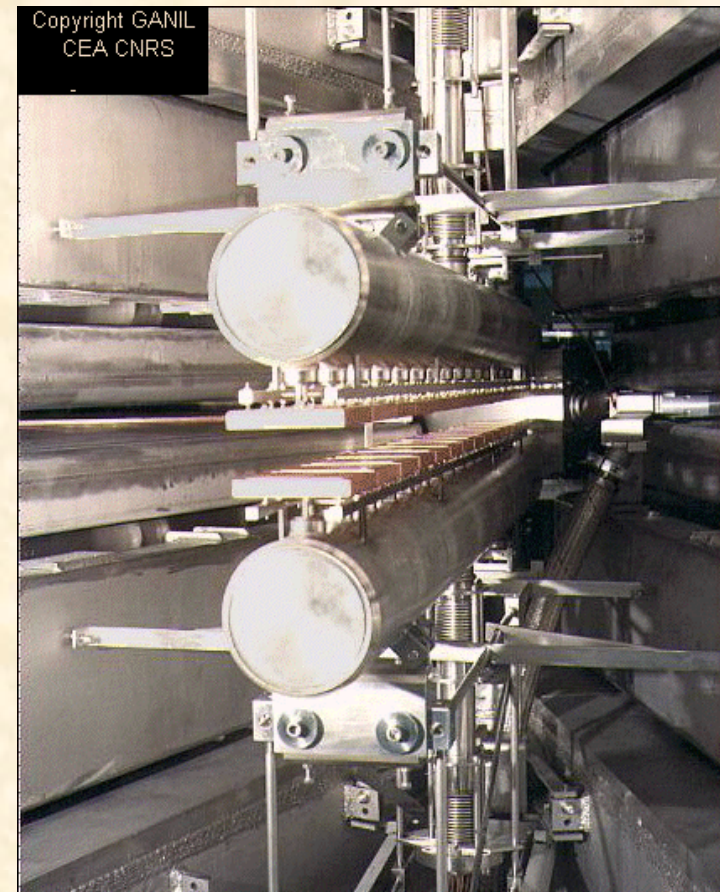
Correction trim coils, AGOR



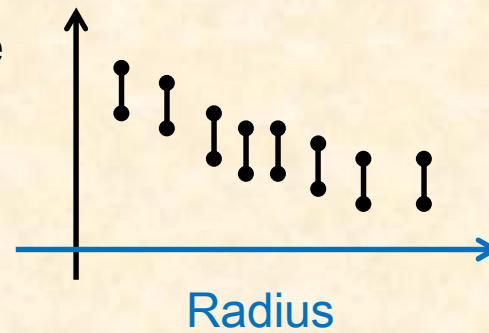
Phase measurement: Isochronism



Ganil, Caen (Fr) : CSS1

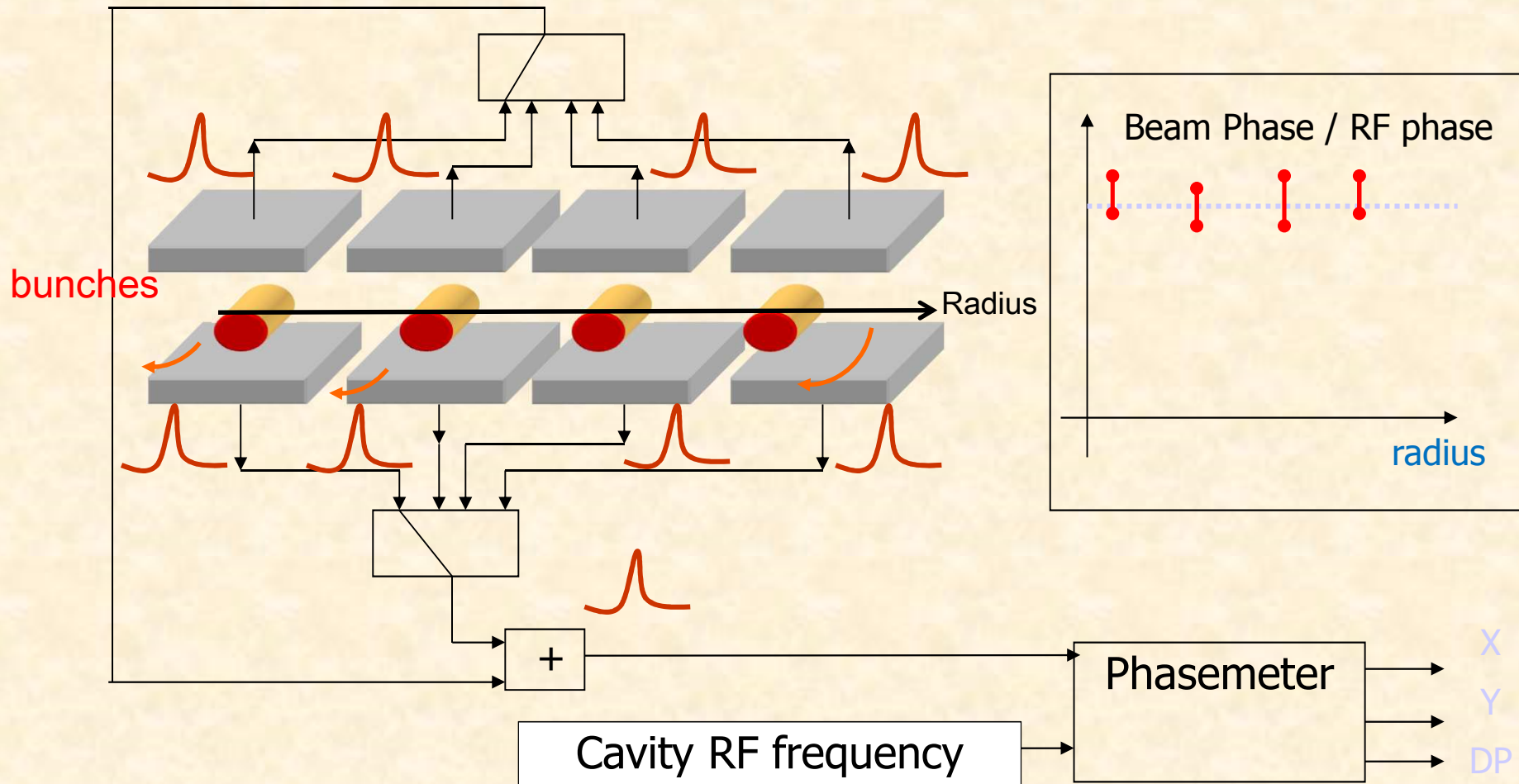


Relative phase
Beam vs RF



Isochronism & Phase measurement

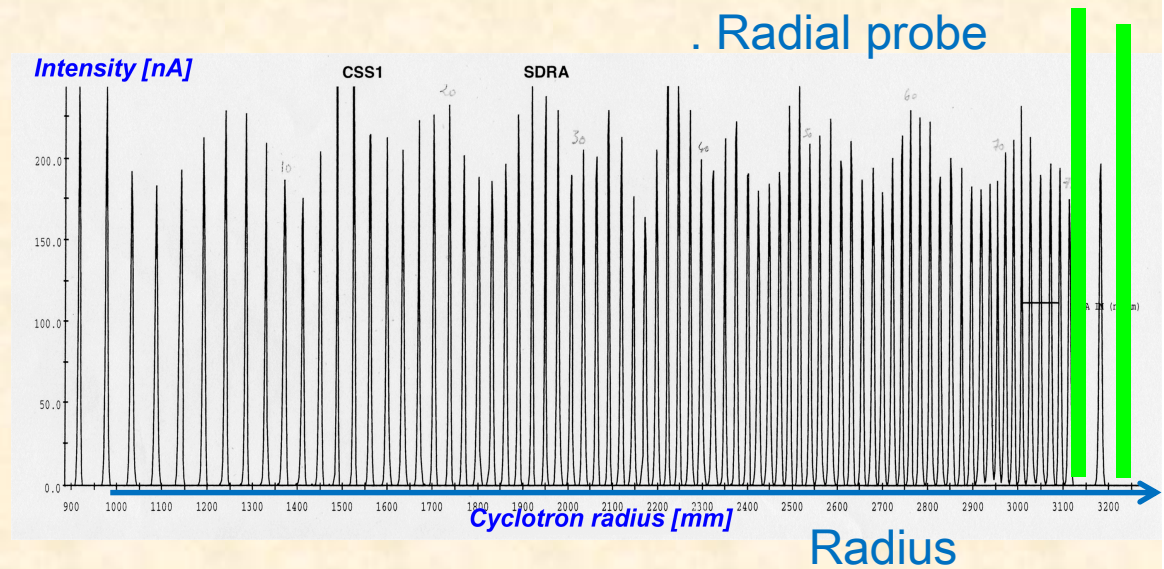
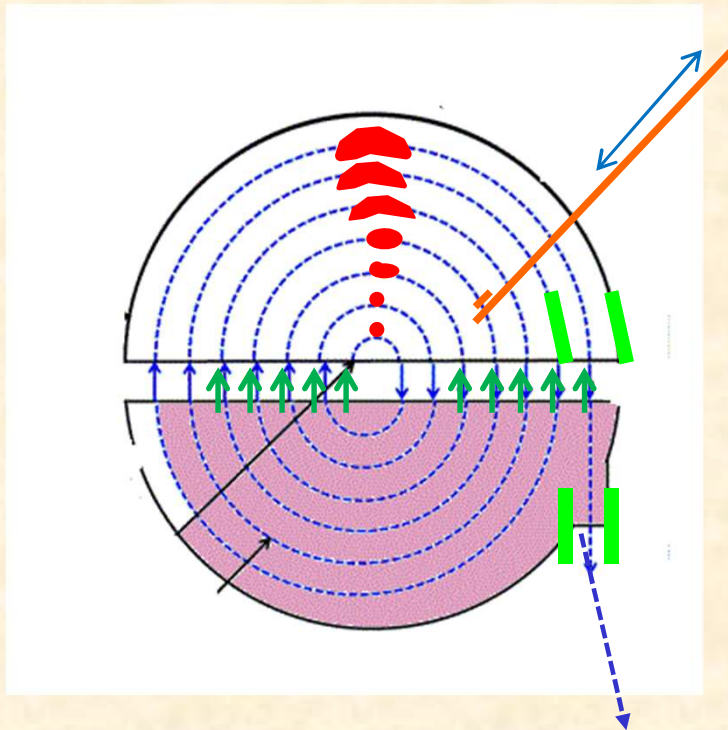
Measuring $B(R)$ or $n(R)$ is difficult, While the $\Phi(R)$ is more sensitive



Radial probes

usefull tool for acceleration, precession study

Monitoring turns with a
Radial Probe



Radial probe : $I = F(\text{Radius})$

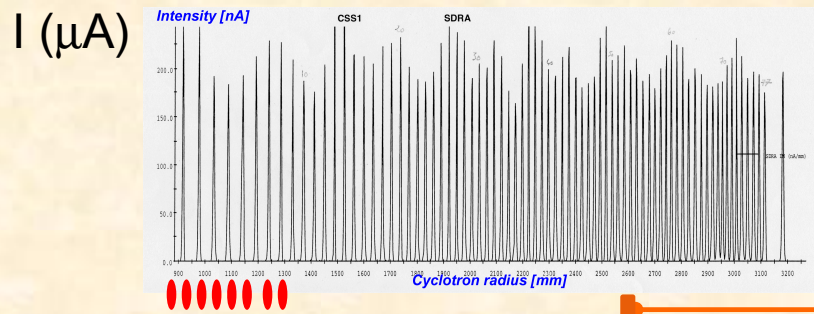
Turn separation : $\delta r = R(\text{turn } N) - R(\text{turn } N-1)$

$\delta r = \text{Acceleration} + \text{Oscillation}$

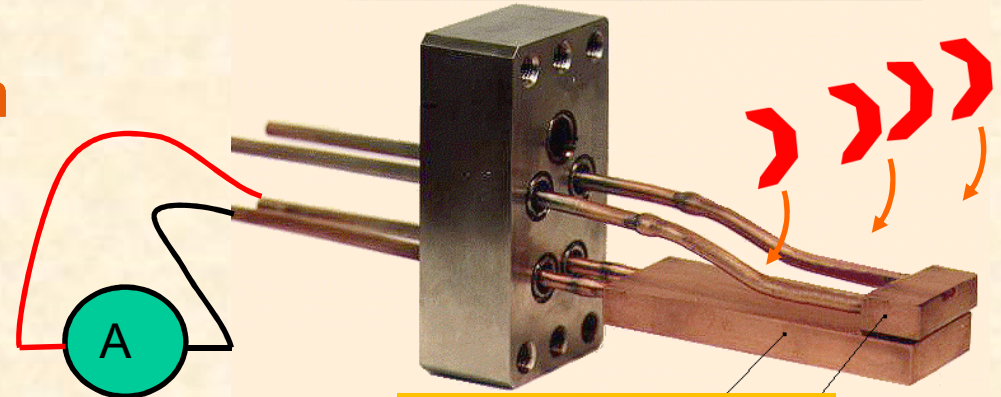
$\delta r \sim \alpha VRF \cos(\phi) + \text{Oscillation}$

Current measurement $I_{\text{beam}}=F(R)$: Radial probe

A Faraday cup on mobile Arm



Ganil Css1 (Fr)
Kb=380 MeV , heavy ions



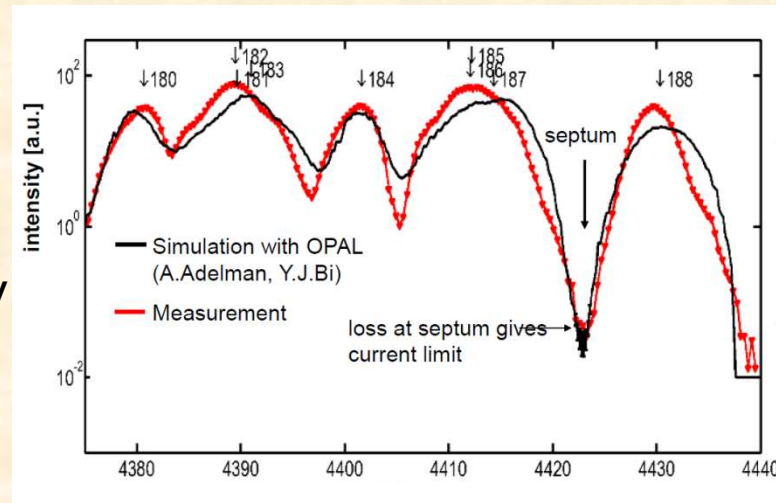
$I (\mu A) = F(\text{radius})$

Radius

Check of the acceleration & extraction

PSI
 Kb=590 MeV

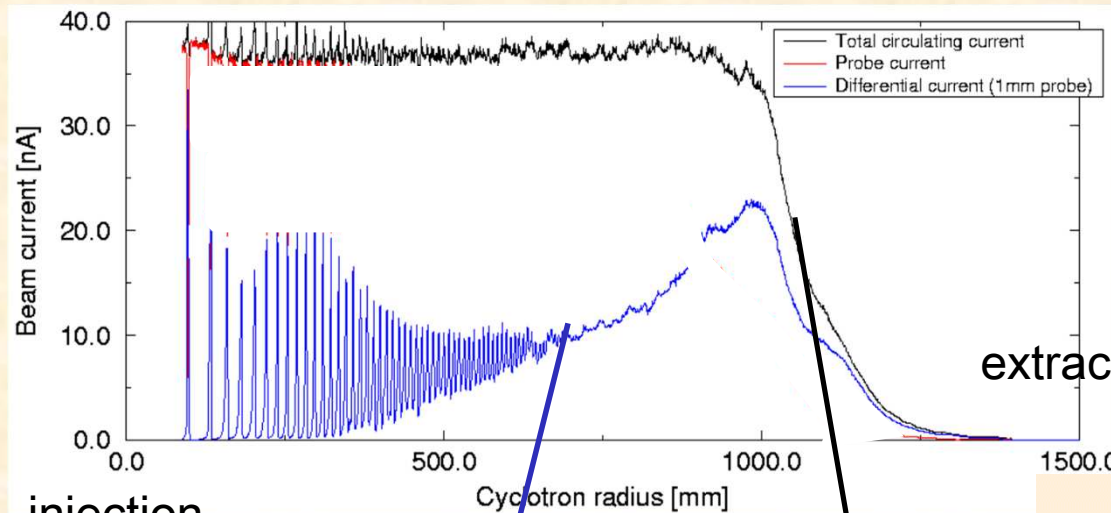
Check of the extraction



Radius

Current measurement with a Radial probe

A full check of the dynamics



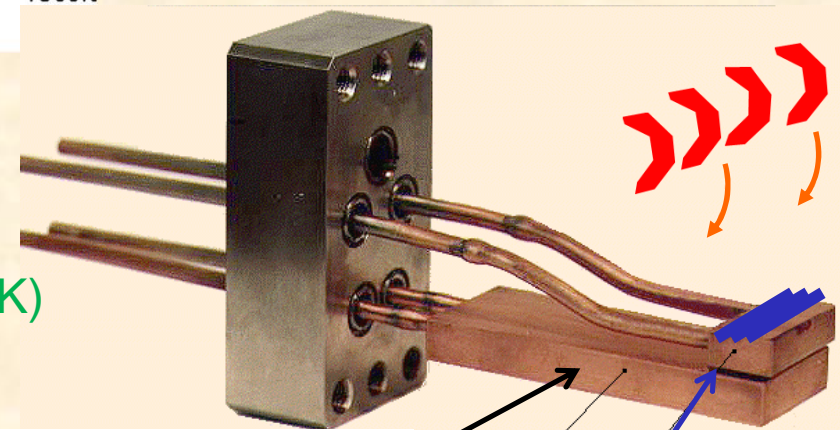
injection

extraction

Radial probes with
2 diagnostics :
-1 small finger
-1 large plate

Beam losses
Before extraction
(B field was not OK)

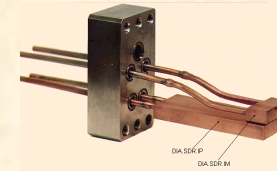
Current on small finger
of the radial probe



Total current
with a large plate

Finger current
Good resolution in r (turn separation)

Tune ν_r measurement with radial probes



Turn separation δr gives ν_r :

$$r(t) = \langle R(t) \rangle + X_0 \cos(\nu_r \omega_0 t)$$

$$\omega_0 t = \text{PHASE} = \theta$$

$$\nu_r = \sqrt{1 - n} + ..$$

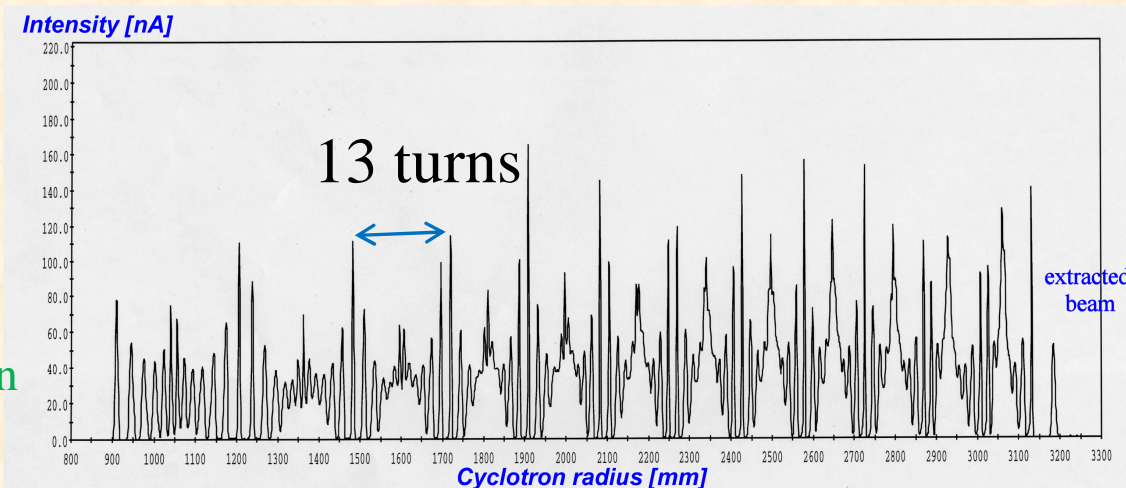
$$\omega_0 \Delta t = 360^\circ = 1 \text{ turn}$$

Precession : $\delta r = \delta r_{\text{acceleration}} + X_0 \cos(\nu_r \omega_0 t)$

Centering error
At injection

Large X_0

Large Oscillation



1 period for

13 turns; so \Rightarrow

$$\nu_r = 1/13 = 0.08$$

Back to dynamics and resonances at high energy

During the acceleration, ν_r and ν_z change because $\nu_{r,z} \propto B(r)$

The plot of ν_r vs ν_z is called the **working point diagram**.

Like any **oscillatory phenomenon**, the amplitude of a betatronic motion can grow uncontrolled whenever an external source excites it with its own frequency.

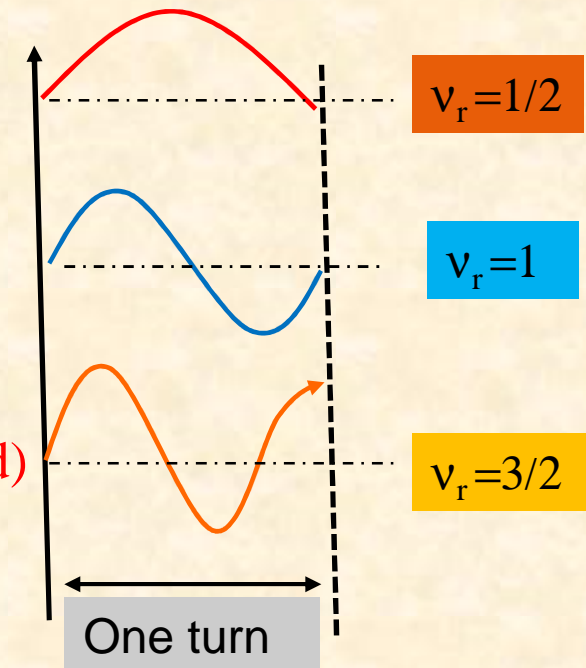
$$\ddot{x} + \nu_r^2 \omega_{rev}^2 x = 0$$

Several kind of radial resonances

$$x(t) = x_0 \cos(\nu_r \omega_0 t) = x_0 \cos(\nu_r 2\pi N \text{turn})$$

can be excited with field defects, injection angle (unwanted)

with field bump, injection angle (desired)



What happens with P field perturbations on 1 turn

$$\ddot{z} + [v_z \omega_0]^2 z = \Delta \cos(P \omega_0 t)$$

Search a particular solution

$$z(t) \propto \cos(P \omega_0 t) ???$$

$$\frac{z(t)}{\Delta} = \frac{\cos(P \omega_0 t)}{[v_z \omega_0]^2 - [P \omega_0]^2}$$

z/Δ diverge at $v_z=P$ (integer)

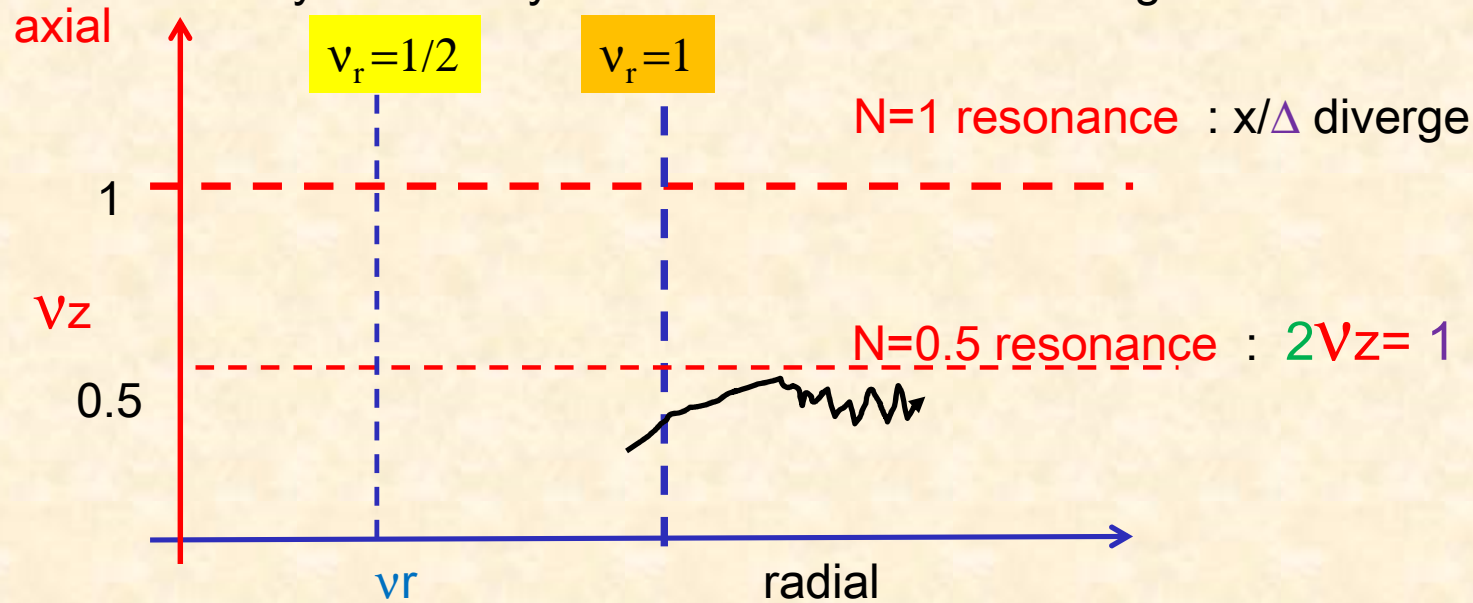
z(t) is very sensitive to any perturbation Δ

(not good : instabilities = beam losses)

With P field perturbations on L turns

z/Δ diverge at $L \cdot v_z = P$ (integer)

The Cyclotrons try to avoid resonance crossing



Resonances

Systematic resonances : This resonance occurs as the betatronic frequency is a multiple of the "geometrical frequency" of the cyclotron. In this case, any kick given to the particle because of its particular position will be experienced again and again.

$$(\mathbf{K} \cdot \mathbf{v}_r = \mathbf{P}) \quad // \quad (\mathbf{L} \cdot \mathbf{v}_z = \mathbf{P})$$

Coupling resonances

Under proper circumstances and frequency ratios, the 2 oscillators can be coupled and the energy stored in one motion, transferred to the other.

These are **coupling resonances** $(\mathbf{K} \cdot \mathbf{v}_r + \mathbf{L} \cdot \mathbf{v}_z = \mathbf{P})$.

$$\mathbf{K} \cdot \mathbf{v}_r + \mathbf{L} \cdot \mathbf{v}_z = \mathbf{P} \quad \mathbf{K}, \mathbf{L} \text{ and } \mathbf{P} \text{ integer}$$

The particle's working point curve should avoid or cross as fast as possible those lines.

Tunes and resonances at Triumf (Canada)

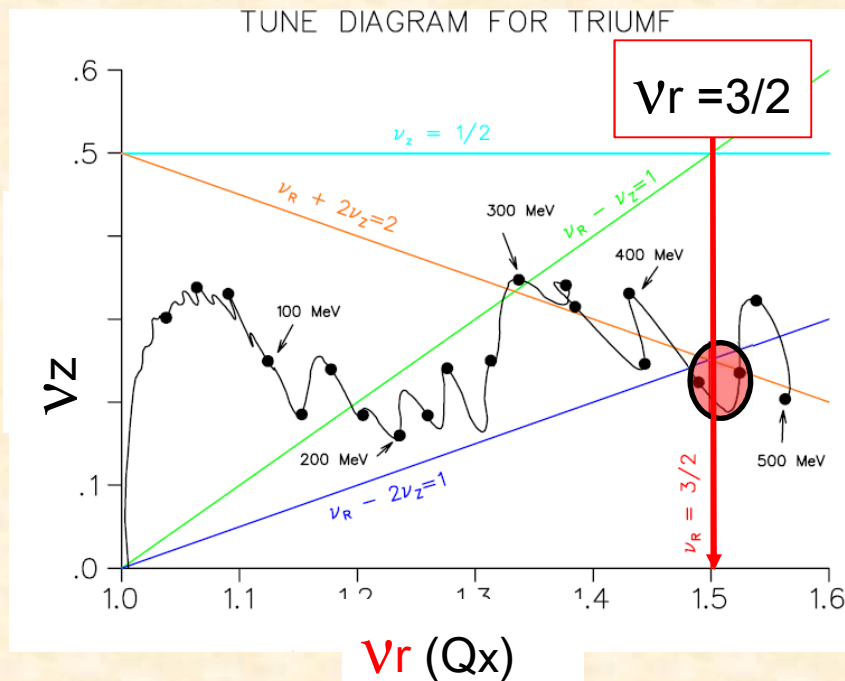
(H⁻ cyclo, $K_b=520$ MeV, 6 sectors)



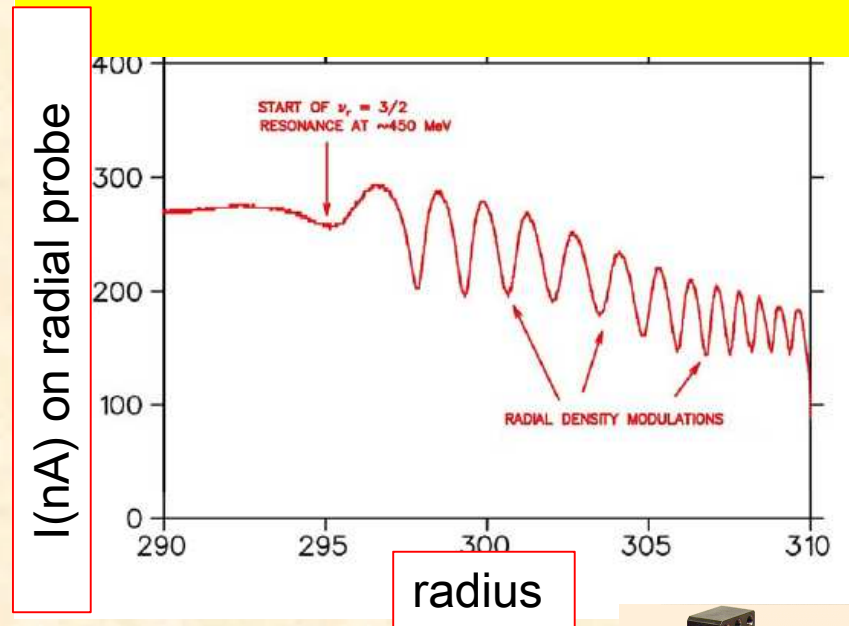
$$\mathbf{K} \cdot \mathbf{v}_r + \mathbf{L} \cdot \mathbf{v}_z = \mathbf{P}$$

$|\mathbf{K}| + |\mathbf{L}|$ is called the resonance order (1, 2, 3 ...)

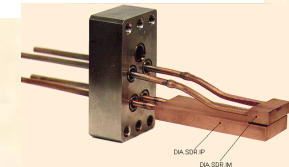
Effect on the crossing of the resonance $2 \cdot \nu_r = 3$ (order 2)



Radial Oscillations : $2 \cdot \nu_r = 3$



$$\nu_r^2 = 1 - n + \dots = 1 - (1 - \gamma^2) + \dots \sim \gamma^2 + \dots$$



Cyclotrons in the world

Some research laboratories with Cyclotron(s)

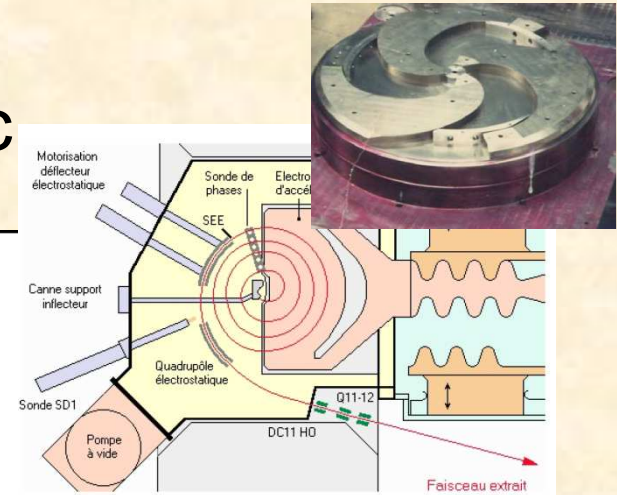
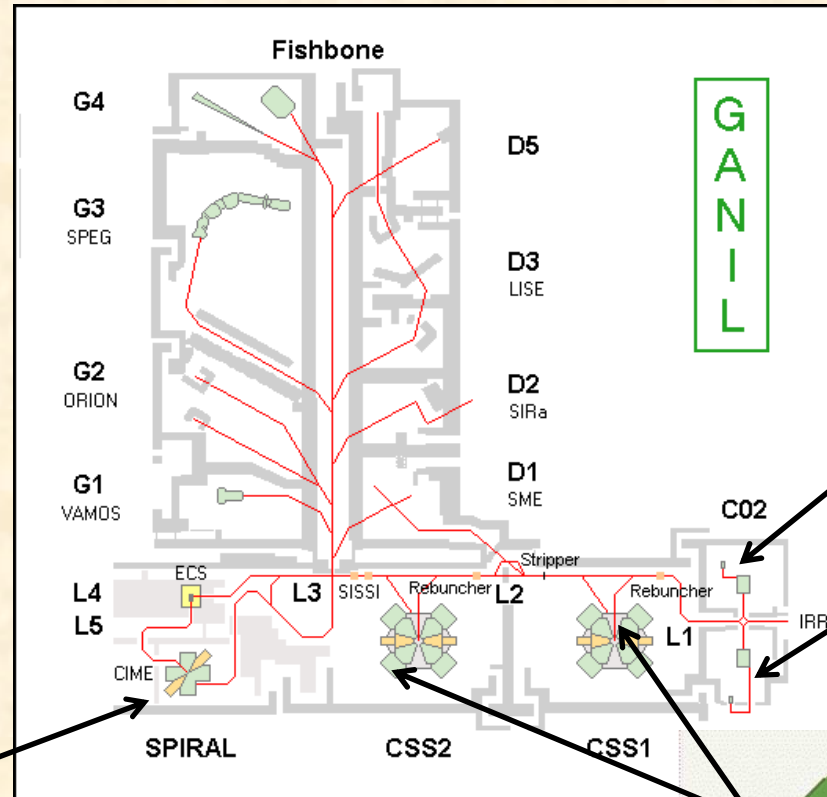


Some of the Research Facility in the world using cyclotrons

GANIL Facility (Caen, Fr)

5 cyclotrons + a new Linac

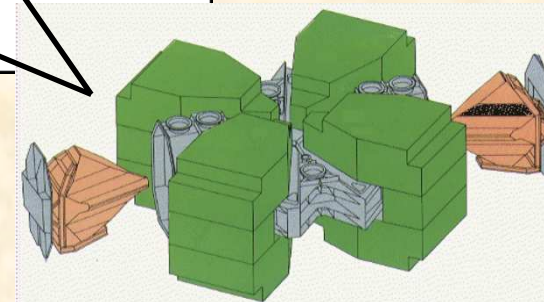
- Nuclear physics
- Atomic physics
- Solid state
- Radiobiology
- Irradiation for industry



2 compact cyclotrons
Kb=28 MeV



1 compact cyclo Kb =265
For radioactive ions

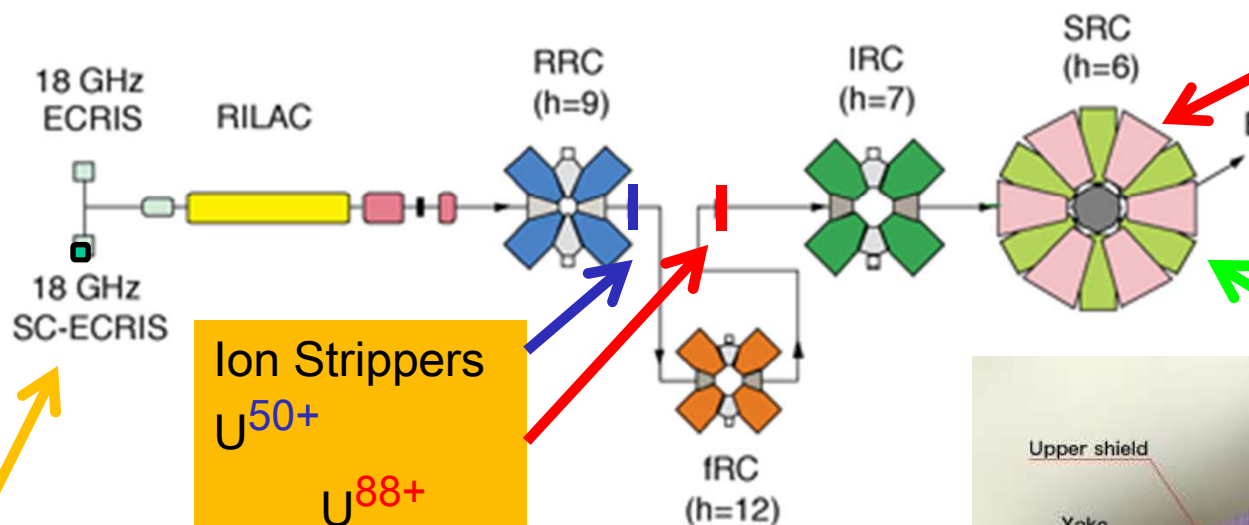


2 Separated Sectors cyclotrons
Kb =380 MeV

RIBF (Tokyo, Japan) :

Uranium beam $^{238}\text{U}^{88+}$ @ 345 MeV/A cw

Mode (1): RILAC + RRC + (stripper2) + fRC + (stripper3) + IRC + SRC



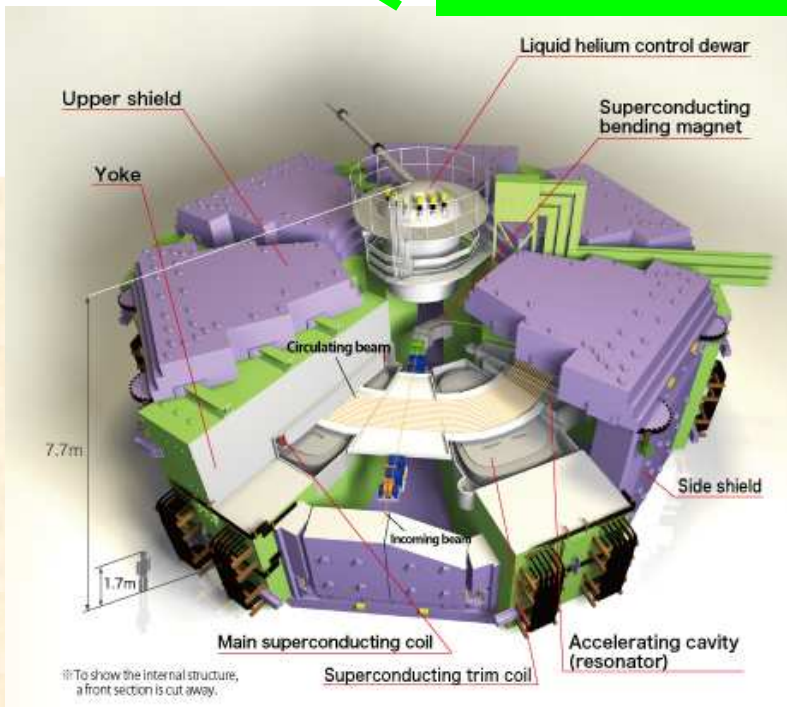
4 RF cavities

Radius=6m
=3.8T

Ion Strippers
 U^{50+}
 U^{88+}

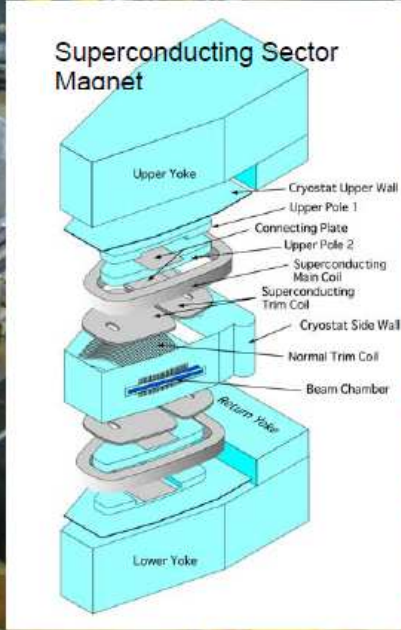
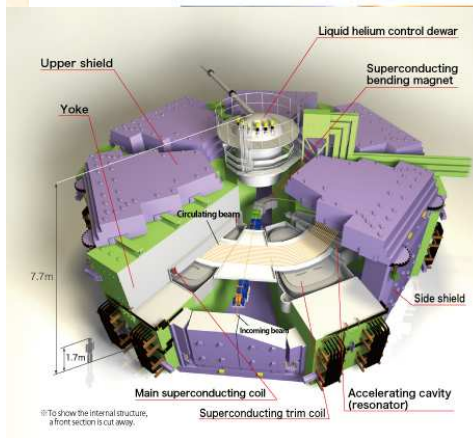
« external »
ECR source
 U^{30+}

5 accelerators
1 LINAC
+ 4 Cyclotrons



To show the internal structure, a front section is cut away.

RIBF (Japan) : SRC (K=2600 MeV) the largest cyclotron in the world Superconducting Ring Cyclo

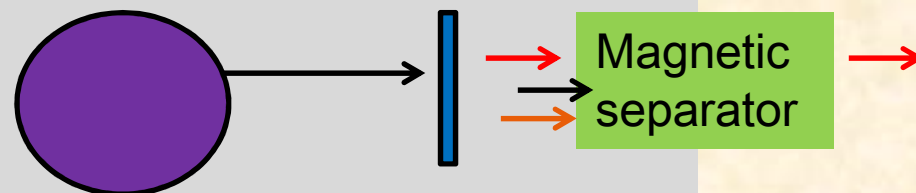


6 sectors :
« Iron dominated magnets »
With superconducting coils

Completed November 2005 - the 140-ton cold mass cooled to 4.5K.

TREND in Nuclear physics : Physics at RIBF (Tokyo)

Create Unknown "exotic" nuclei
by nuclear fragmentation
or by fission



Heavy ions
Cyclotrons

target

Radioactive
nuclei

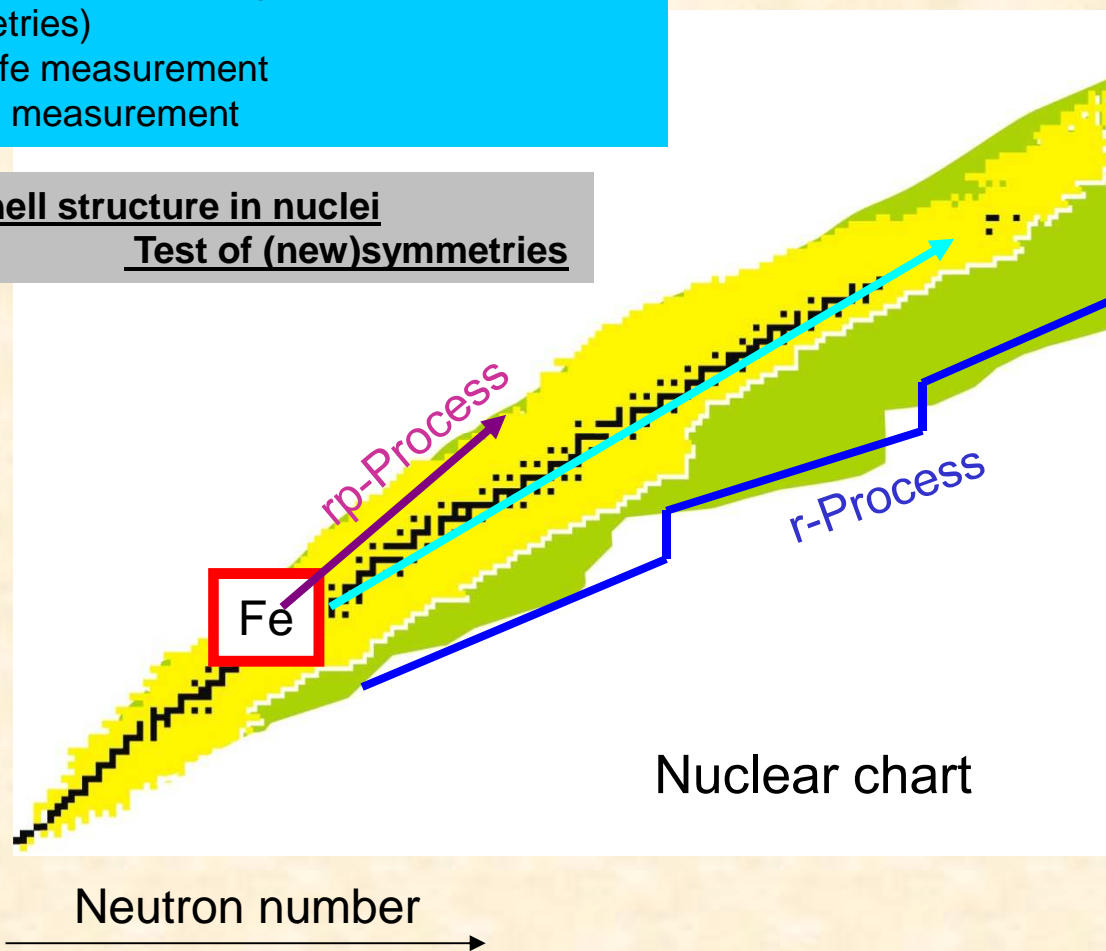
Exotic Nuclei (Very large proton-neutron asymmetries)

- half life measurement
- Mass measurement

Shell structure in nuclei
Test of (new)symmetries

Proton number

Neutron number



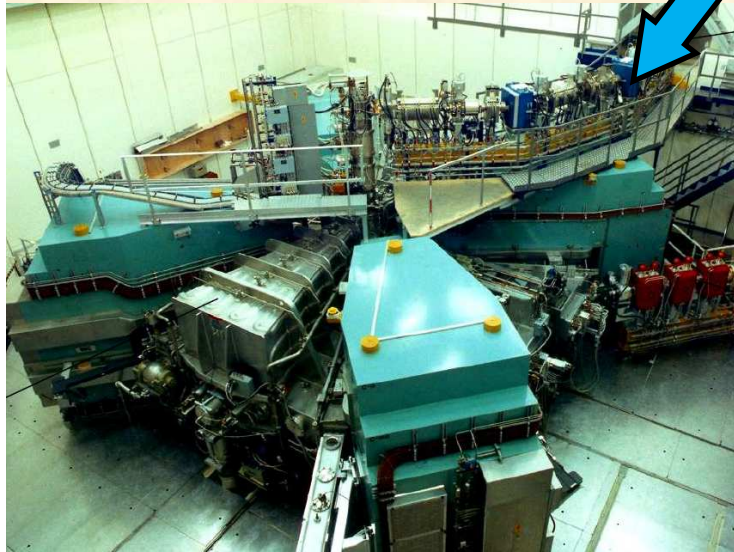
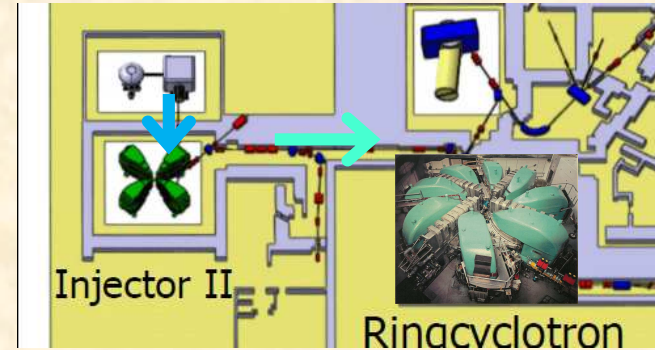
Nuclear chart

	Degrees of Freedom	Energy (MeV)
Physics of Hadrons	Quarks, Gluons 	
	Constituent Quarks 	940 Neutron Mass
	Baryons, Mesons 	140 Pion Mass
Physics of Nuclei	Protons, Neutrons 	8 Proton Separation Energy in Lead
	Nucleonic Densities and Currents 	1.32 Vibrational State in Tin
	Collective Coordinates 	0.043 Rotational State in Uranium

PSI 590Mev proton (Ch)

870 keV

Cockcroft-Walton

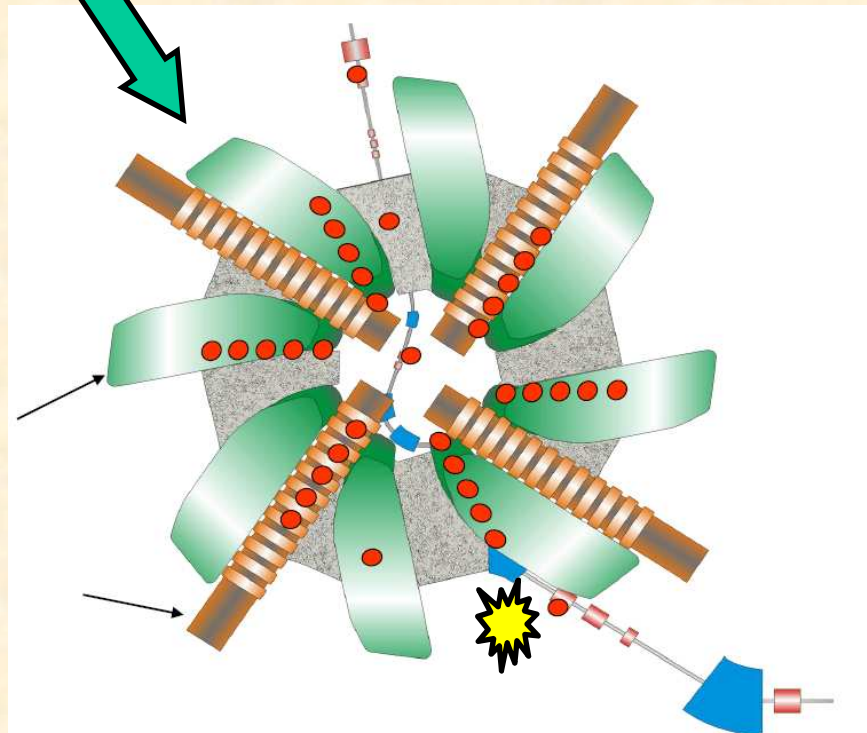


kb=72 MeV (injector 2)

Coupling of 2 cyclotrons :

$$\left[\frac{F_{HF} \cdot R_{extrac}}{h} \right]_{cycloA} = \left[\frac{F_{HF} \cdot R_{injection}}{h} \right]_{cycloB}$$

590 MeV Ring cyclotron

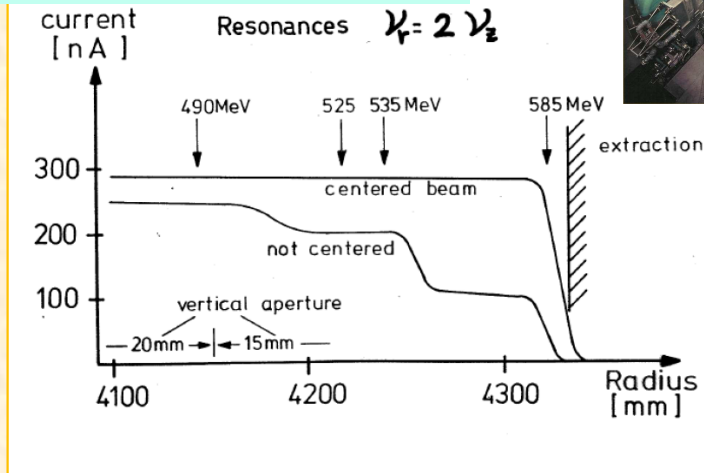


PSI :K= 590 MeV ring cyclotron $P_{\text{beam}}= 1\text{Mwatt}$



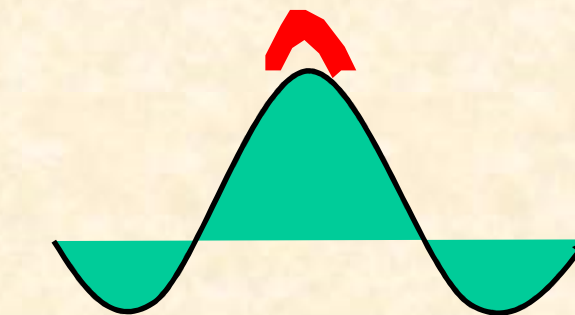
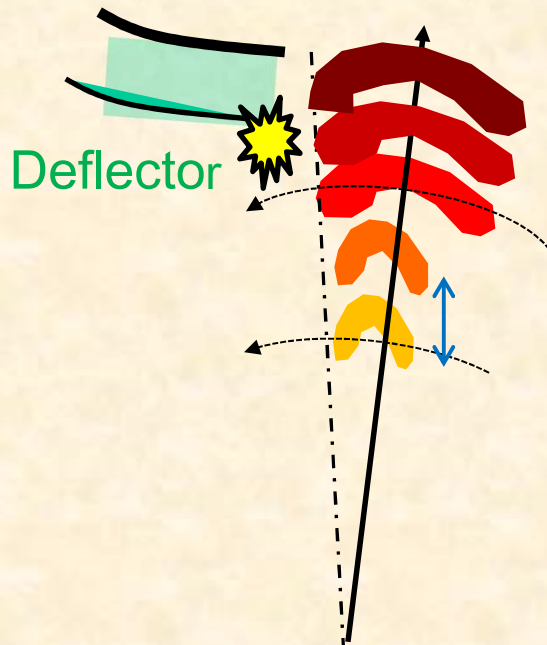
1) Injection centering :

Watch the resonance $2 \cdot \nu_z + \nu_r = 2$



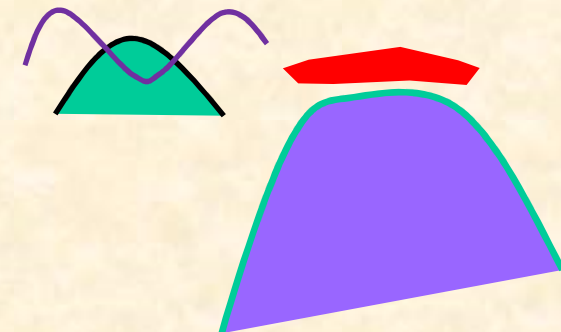
2) Extraction : Watch the beam losses !

A Flat top RF cavity has been added to reduce losses: $V_{\text{RF}} = \text{Cos}(\omega t) - \alpha \cdot \text{cos}(3\omega t)$



Standard cavity :

$$V_{\text{RF}} = \text{cos}(\omega_{\text{RF}} t)$$



Flat top cavity

$$\text{cos}(\omega_{\text{RF}} t) - \text{cos}(3\omega t)$$

Some Commercial Cyclotrons : manufacturers

IBA (Belgium)

			Cost estimate 2008
Cyclone 5/10	proton/D		1 Meuros
Cyclone 9/18	H-/D-		1.5 Meuros
Cyclone 30	H-/D-	
ProteusOne 250 MeV	p	synchro-cyclo superconducting	
C70	p/D or H-/D-		

Sumitomo HI (Japan)

HM-12	p
HM-18	p

>300 commercial cyclotrons in the World
 - (10-20MeV) protons
 - (230MeV) protons :develloping market

EBCO (Canada)

TR 9/ 18	H-/D-
TR 15/ 30	H-/D-

GE-Scanditronix (USA-Sweden)

MINI TRACE	9/18
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Accel // VARIAN

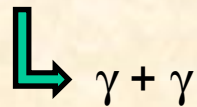
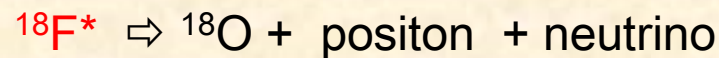
250 MeV	proton	cyclo superconducting
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Commercial Cyclotron

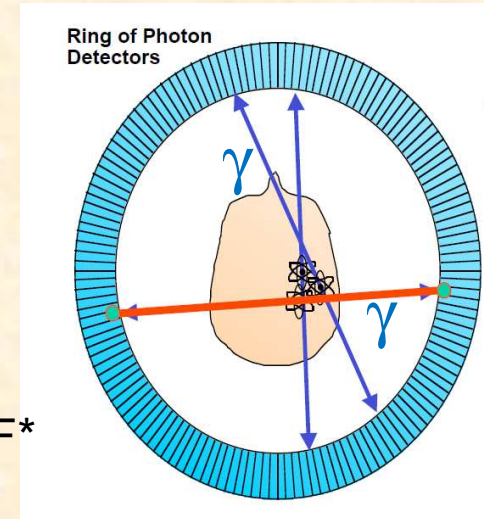
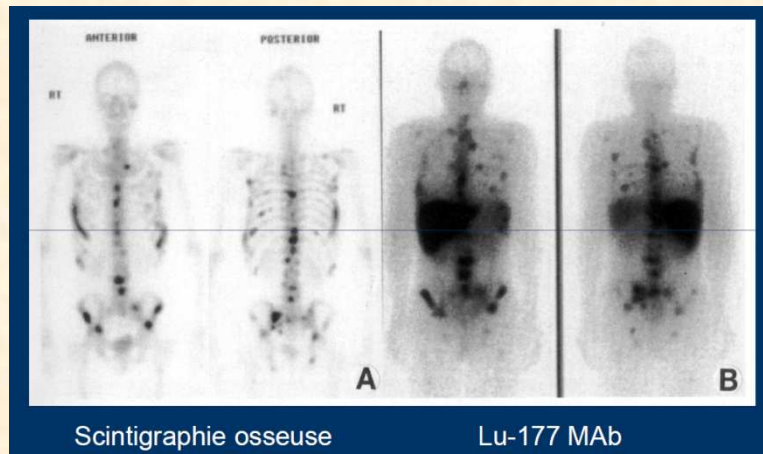
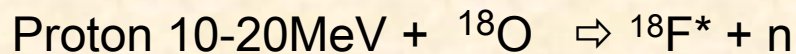
Radioisotope production (5-20 MeV)

Radiotracer $^{18}\text{F}^*$: « Beta+ » emitter
Fluorine 18

$T_{1/2} = 109.7 \text{ min}$

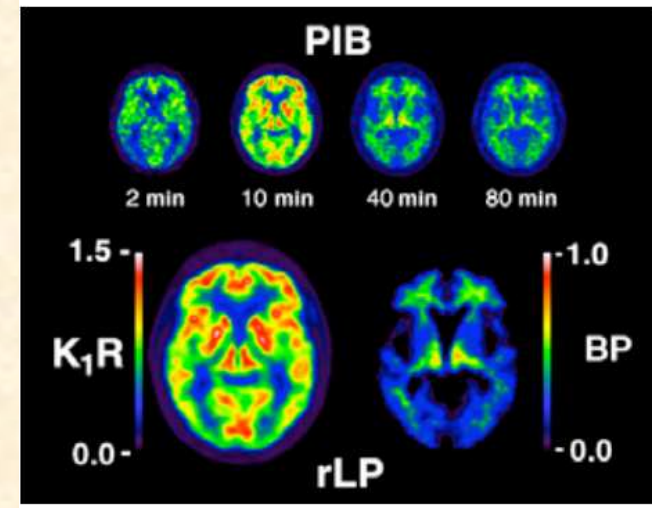


Production With cyclotrons



Injection of $^{18}\text{F}^*$
TEP camera

Reconstruction of the emitter position



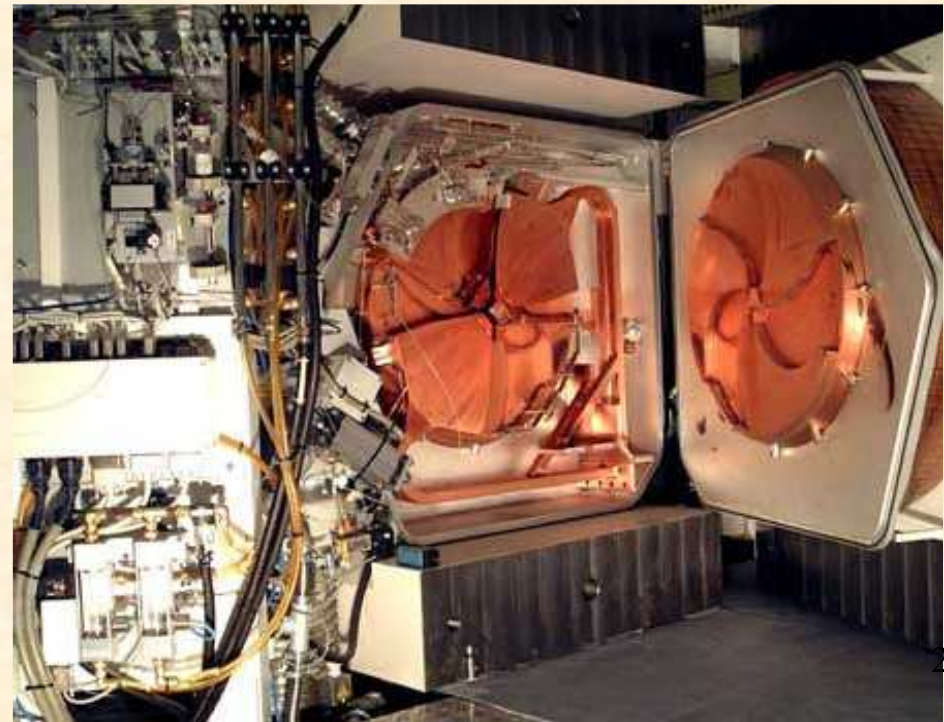


Cyclone 10/5 MeV

= 10 MeV proton ($K_b=10\text{MeV}$)

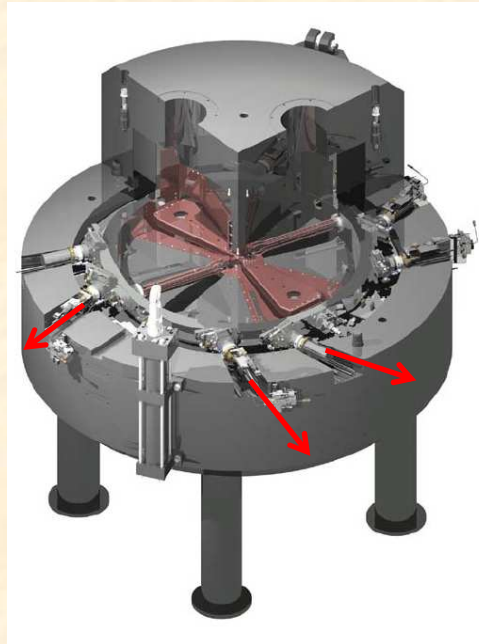
= 5 MeV Deuteron

cyclone 3D ("vertical implantation")



CYCLONE 18/9 (IBA) : H⁻ 18 MeV

Designed for medical applications (radiotracers production)



Kb=18 MeV

Fixed energy ;

4 straight sector 50°

$\langle B \rangle = 1.35$ Tesla

Hill //valley gap 3cm// 67cm

fixed Frf =42Mhz

2 Dee = 30° , 32 kV

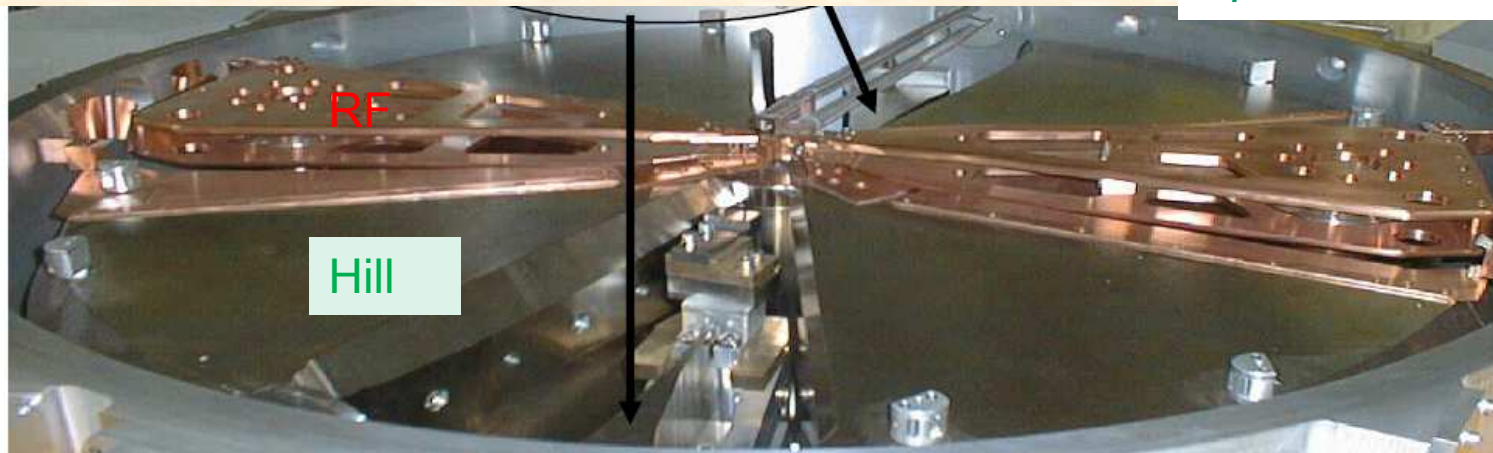
Harmonic h=2(p) ,4 (D)

Internal source

Rextraction=0.46 m

$B_{pmax} = 0.46 \times 1.35 = 0.62$ T.m

Internal PIG source, H⁻ stripping extraction



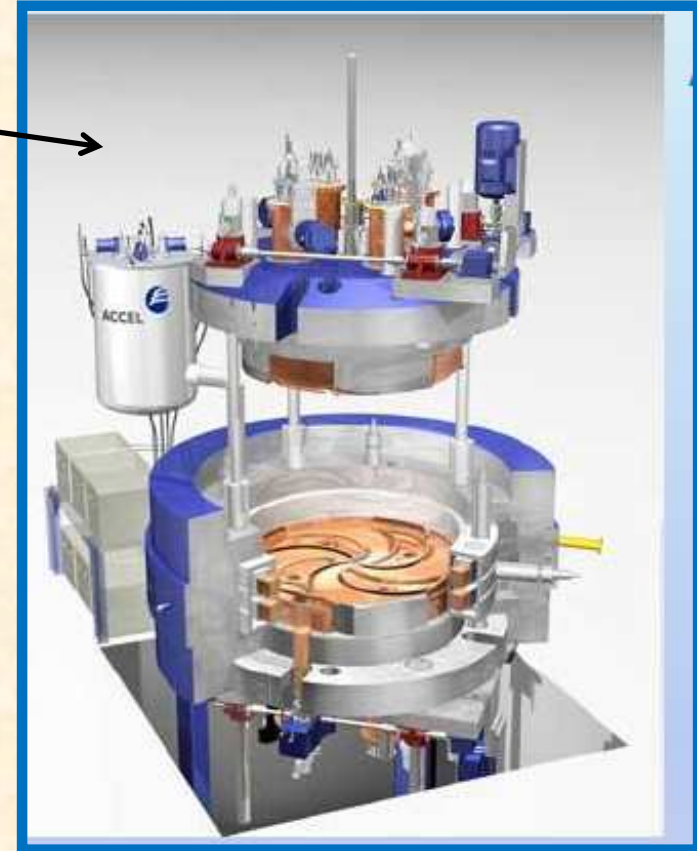
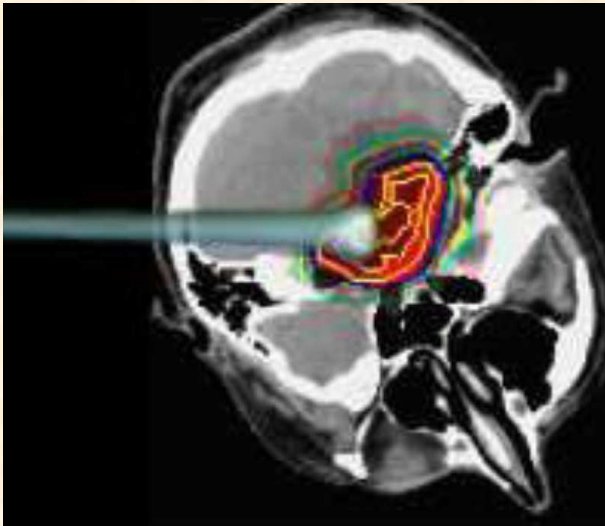
Commercial Cyclotron : proton therapy (230-250 MeV)

Superconducting cyclotron

or

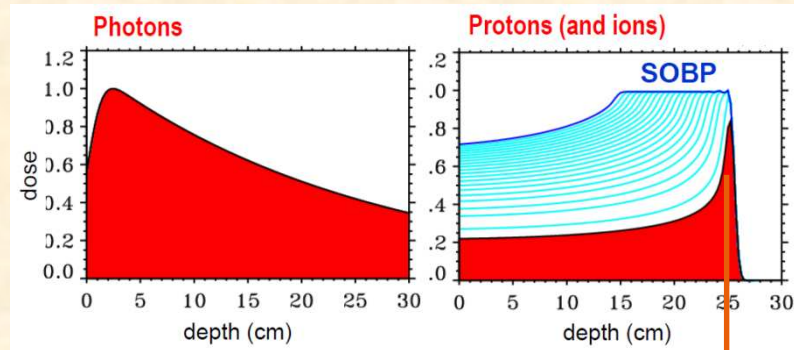
Superconducting Synchro-cyclotron

Brain tumor treatment with protons



Commercial Cyclotron : For proton therapy (230 MeV)

Photon :
(Radiotherapy)
A Dose in the
whole body

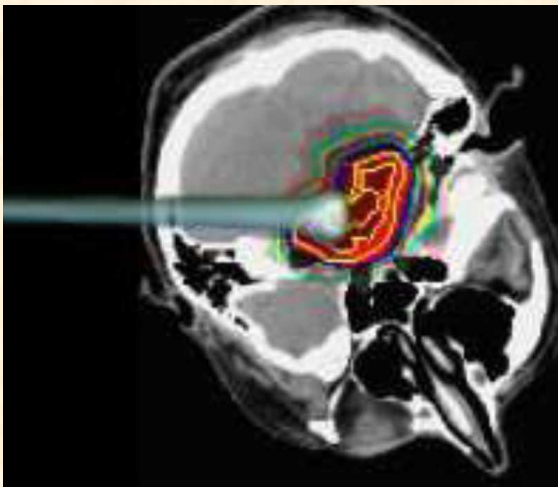


Protons :
Better than
Radiotherapy (photons)

Dose inside the tumor

Bragg Peak

Brain tumor treatment with protons



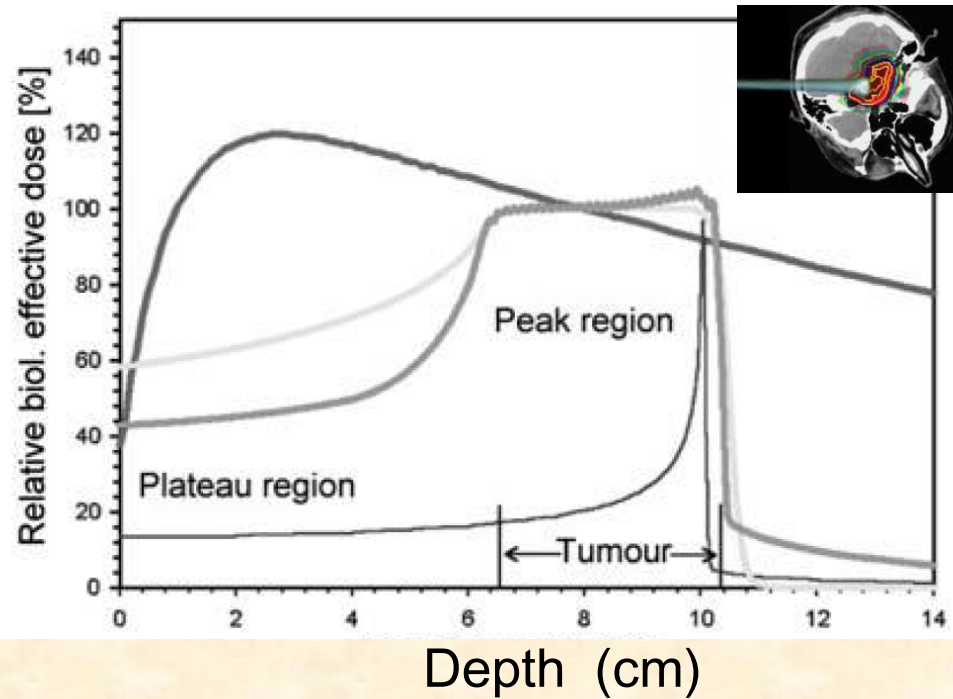
Eyes tumor treatment



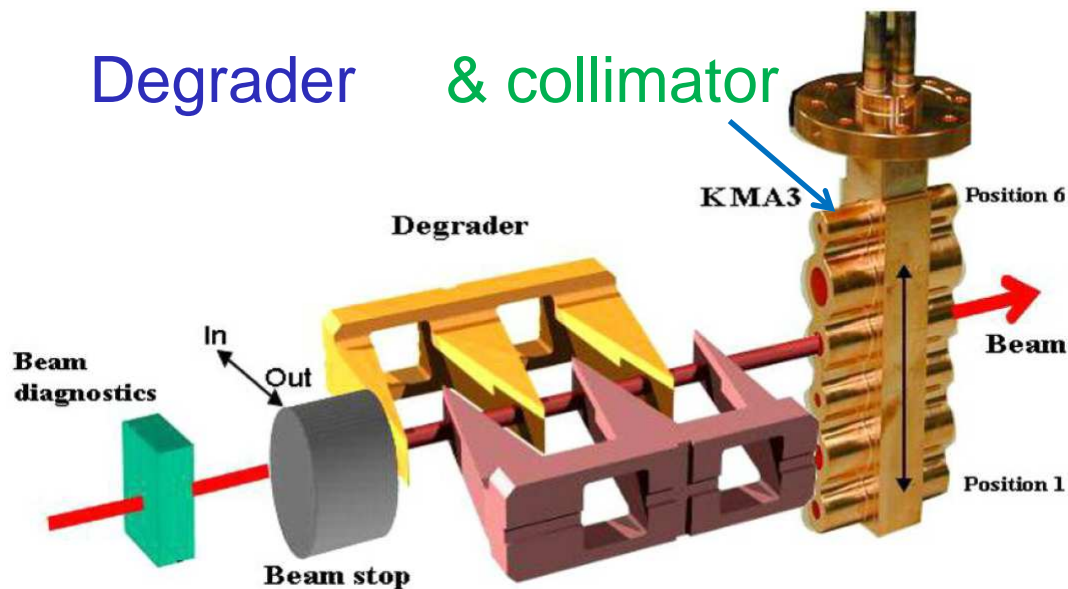
Proton therapy (230 MeV)
-Energy variation with **degrader**

Scanning the tumors :

With energy variations
induce by a degrader



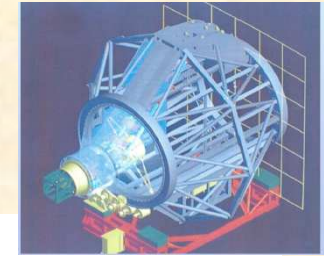
Degrader & collimator



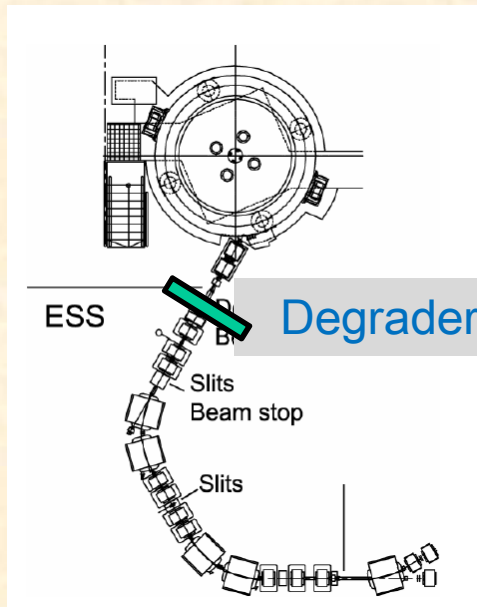
Energy variation
=
Range variation
(tumour scanning)

Proton therapy (230 MeV)

Energy variation with degrader + Rotating gantry

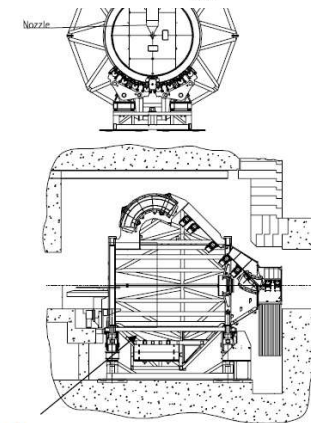
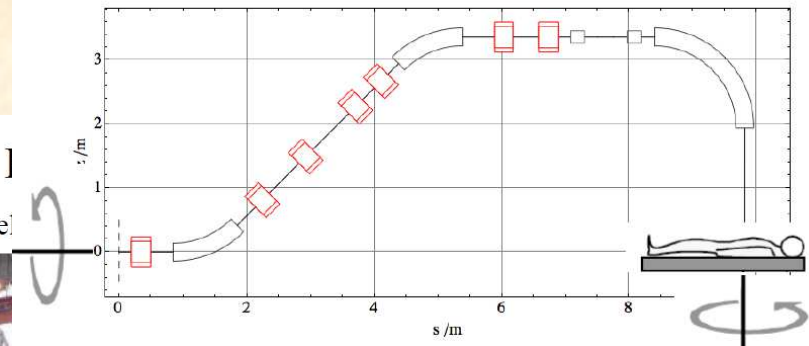


Optimal dose delivery
Scan the tumors :



Depth variation in
patient body:
Tumor scanning

Gantry



Polyethylene
plates

국립암센터
NATIONAL CANCER CENTER

The gantry reduces dose in healthy tissues

END

References & Acknowledgements :

F. Chautard, Juas 2015

M. Craddock lecture on Cyclo //FFAG

W.Joho lecture on PSI facility

S. Brandenburg lecture on beam optics

P. Heikinnen, CAS Jyväskylä 1992, CERN 94-01 (1994) “Cyclotrons“ and “Injection and Extraction“

T. Stammbach, CAS La Hulpe, 1994, CERN 96-02 (1996) “Introduction to Cyclotrons“

... & Many others

Few other slides for questions.....

Isochronous field $B(R)$ = good field index $n(R)$

$$n = - \frac{R}{B_{0z}} \frac{\partial B_z}{\partial R}$$



$$\frac{dB}{B} = - n \frac{dR}{R}$$

$$B \rho = \langle B \rangle \cdot \langle R \rangle = \frac{p}{q}$$

Longitudinal dynamics lecture

$$\frac{dp}{p} = \frac{dB}{B} + \frac{dR}{R} = (1 - n) \frac{dR}{R}$$

$$\frac{dp}{p} = \gamma^2 \frac{d\beta}{\beta} = \gamma^2 \frac{d(\omega_{rev} \cdot R)}{\omega_{rev} \cdot R} = \gamma^2 \frac{dR}{R}$$

$$1 - n = \gamma^2$$

$$n(R) = (1 - \gamma^2)$$

« At high energy » isochronism requires $n \ll 0$

Bz Azimutal modulations are not sufficient
It is a (Focusing) limit for high energy isochronous cyclotron

Max Energy for Superconducting Cyclotrons not limited by (B x Rextraction)

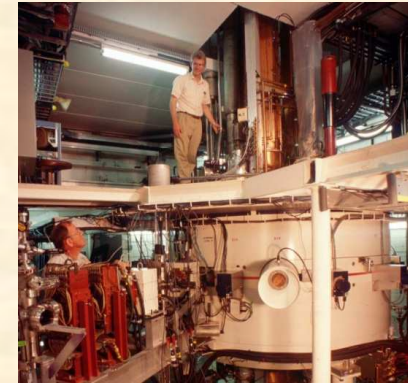
Because of the focusing limitation due to the Flutter dependence on the B field, the max energy is not given by $Kb \sim 48 (B.Rextraction)^2$

but K_f the so-called “focusing factor”:

$$\left[\frac{E}{A} \right]_{\max} \neq Kb \cdot \left\{ \frac{Q}{A} \right\}^2$$

vertical oscillation+ isochronous field condition

$$v_z^2 = \left[1 - \gamma^2 \right] + \frac{N^2}{N^2 - 1} F (1 + 2 \tan^2 \epsilon) > 0$$



•Focusing limitation (stronger than B limitation)

$$\left[\frac{E}{A} \right]_{\max} = Kf \cdot \left\{ \frac{Q}{A} \right\}^2 < Kb \cdot \left\{ \frac{Q}{A} \right\}^2$$

$$Kb \sim 48 (B.Rextract)^2$$

$$Kf \sim f(\text{FLUTTER})$$

The axial oscillations of ion beams in a isochronous cyclotron is described by

Where the vertical tune should respect $\nu_z^2 > 0$,

$$\ddot{z} + [\nu_z \omega_0]^2 z = 0$$

- Why? (1)
- Give a particular solution of the differential equation when $\nu_z^2 < 0$ (1)
- What is the axial physically ν_z

Answer

a. Otherwise the beam is unstable (beam size increase exponentially)

$$z(t) \sim Z_0 \exp(\pm i \nu_z \omega_0 t)$$

b. $z(t) = Z_0 \exp(\pm |\nu_z| \omega_0 t)$

c. ν_z correspond to number of oscillations per turn for beam not injected at the reference orbit.

An exotic cyclotron = the FFAG

RF pulsed (like a Synchrotron) ; B fixed

FFAG = "Fixed-Field Alternating-Gradient"

A kind of synchro-cyclotron

- pulsed beam

$$F_{rf} = f(t)$$

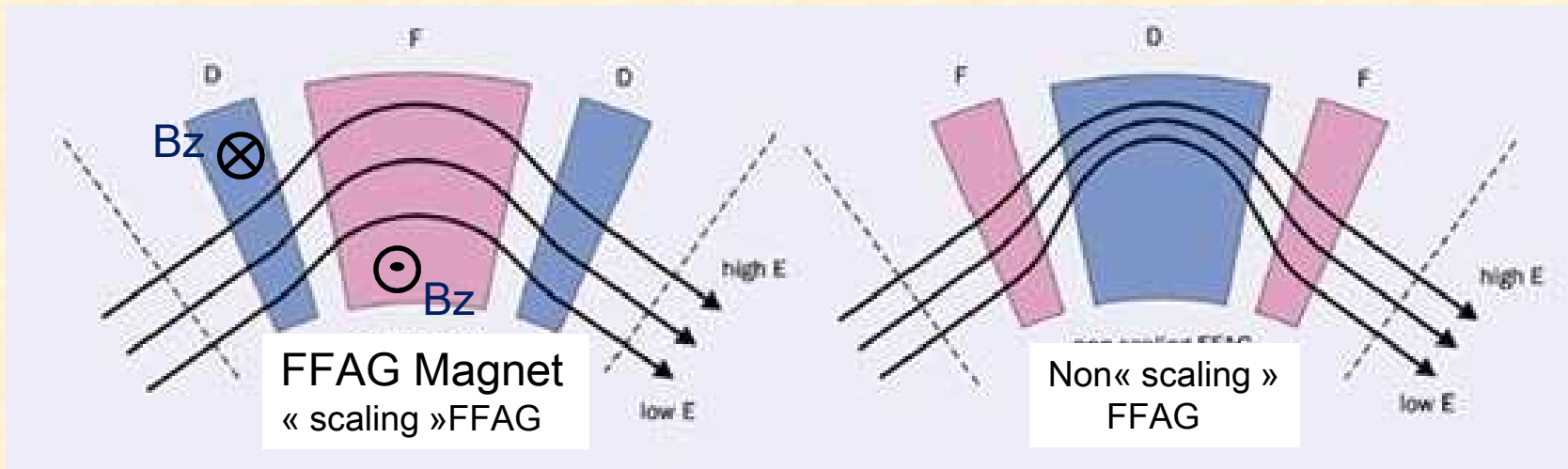
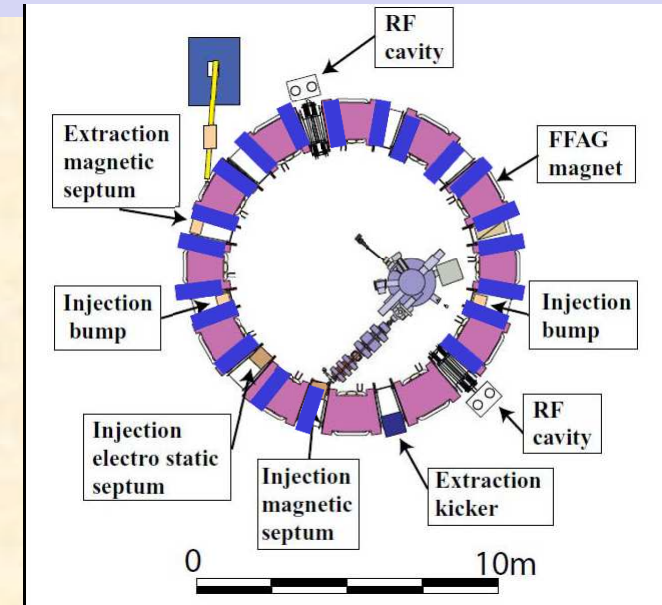
- not Isochronous

- focusing DFD or FDF

with alternating magnet

High repetition Rate (100Hz-1kHz)

Large momentum acceptance



R&D an exotic cyclotron =FFAG

FFAG =Fixed-field alternating-gradient: Higher repetition rate than a synchrotron

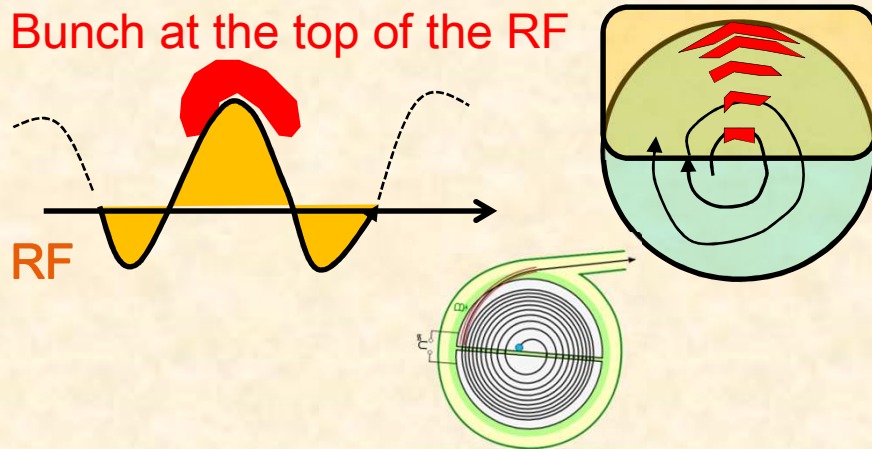


2 coupled FFAG =150 MeV proton in Kyoto

R&D for : - Accelerator Driven System (nuclear reactor)
- muon acceleration

Cyclotron Summary : with pictures

Bunch at the top of the RF



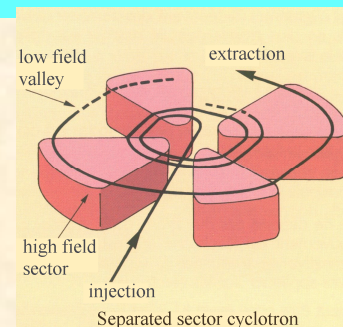
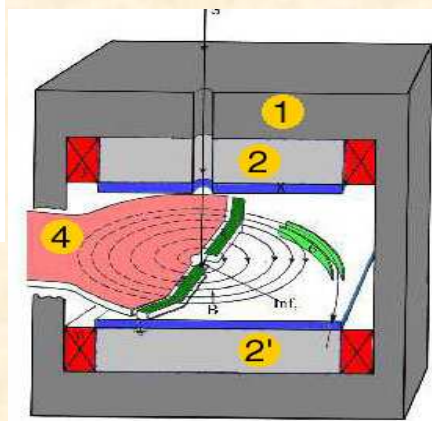
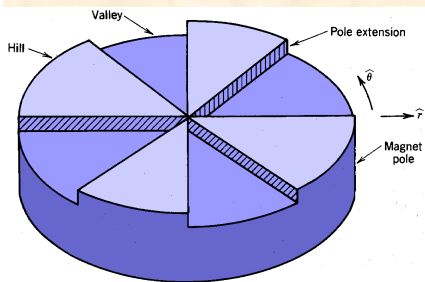
harmonics = h bunches by turn

$$\omega_{\text{rf}} = h \omega_{\text{rev}}$$

$$\omega_{\text{rev}} = \frac{qB_z}{\gamma m}$$

Kb= 30 MeV-200MeV
 Compact cyclotrons (with Hills //Valleys)
 Straight or Spiralled sectors

Kb= 300-600 MeV
 « Ring cyclotrons » : higher energy
 Straight or Spiralled
Separated sectors



30-200 MeV protons

300-600 MeV protons

The Cyclotron Family

isochronous cyclotron

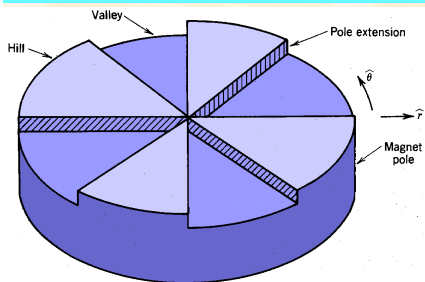
(Azimuthally Varying Field)

$B_z(R, \theta) = \text{NOT uniform}$
 $F_{rev} = \text{Constant}$
 $FRF = \text{constant}$

Isochronous

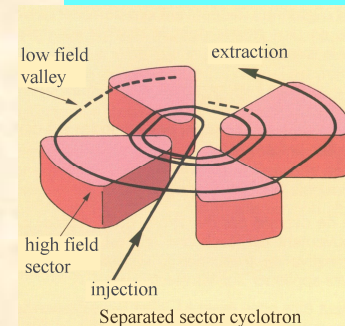
$$\omega_{rev} h = \omega_{RF}$$

Compact cyclotrons (with Hills //Valleys)
 Straight or Spiralled sectors



Ring cyclotrons :
 Straight or Spiralled

Separated sectors



Synchrocyclotrons

$F_{rev} = \text{NOT Constant}$
 $FRF = \text{NOT Constant} = \text{beam pulsed}$

$$\omega_{rev}(R).h = \omega_{RF}(t)$$

Not
 Isochronous