PAUL SCHERRER INSTITUT PSI

juas

Synchrotron Radiation — Exercises 1

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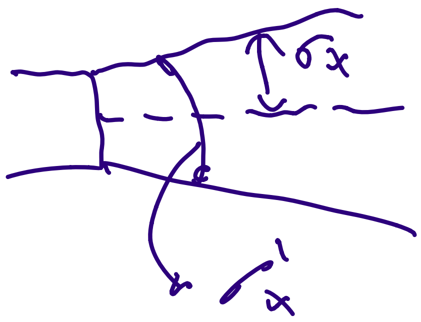
1 Brilliance

Estimate the brilliance of my flashlight. How does it change when I add a lens?

$$B = \frac{\dot{N}_\gamma}{4\pi^2 \sigma_x \sigma_y \sigma_x' \sigma_y' (0{,}1\% \, BW)}$$



$P = 1\,W$

$E_{ph} \approx \hbar\omega = h f \approx 2{,}75\,eV$

$N_{ph} \approx 2{,}767 \cdot 10^{18}\,ph/s$

$\sigma_x = 10\,mm \approx \sigma_y$

$\sigma_x' = \cancel{1\,mrad} \longrightarrow 500\,mrad$

$$BW = \frac{\Delta W}{W_0} \approx \frac{340}{600} \approx 0{,}566$$

$$B = 495{,}32 \frac{ph}{s\,(mm)^2\,(mrad)^2}$$

Remember: $1\,eV = 1.6 \cdot 10^{-19}\,J$

2 Large Hadron Collider

A proton circulates in LHC. Assume a circumference of 26.7 km, a particle energy of 6.5 TeV, and a magnetic field of 7.7 T. Calculate

- The Lorentz factor γ
- The radius of curvature that the protons make in the dipoles
- The critical energy of the synchrotron radiation
- The energy emitted through synchrotron radiation in the dipoles by one proton in one turn
- Which fraction of the circumference is occupied by dipole magnets?

- $\gamma = \frac{E}{E_0} = \frac{6500\,GeV}{0{,}938\,GeV} = 6929{,}63$

- $B\rho = \frac{1}{0{,}3} p \qquad \rho = \frac{1}{0{,}3}\frac{p}{B} = \frac{6500}{0{,}3 \cdot B} = 2{,}813\,km$

- $\varepsilon_c = \frac{3\hbar c}{2\rho} \cdot \gamma^3 = \frac{3}{2}\frac{1}{\rho} \cdot (6500)^3 \cdot 0{,}197 \cdot 10^{-6} =$
  $= 35\,eV$

- $U_0 = \frac{e^2 \gamma^4}{3\varepsilon_0 \rho} = \frac{\gamma^4}{3\rho} \cancel{e^2} \frac{2\alpha\hbar c}{\cancel{e^2}} = 4940\,eV$

  $\varepsilon_0 = \frac{e^2}{2\alpha\hbar c}$

$$N \cdot \theta = 2\pi$$

$$\theta = \frac{L}{\rho}$$

$$\Rightarrow \frac{N \cdot L}{\rho} = 2\pi$$

$$N \cdot L = 2\pi\rho$$

$$\frac{2\pi\rho}{Circ} = 66\%$$

3 Synchrotron

Consider an electron storage ring at an energy of 800 MeV, a circulating current of 1 A, and a bending radius of ρ = 1.784 m. Calculate the energy loss of each electron per turn, and the total synchrotron radiation power from all bending magnets.
What would the radiation power be if the particles were 800 MeV muons?

$$e \left\{ \begin{array}{l} U_0 = \dfrac{e^2 \gamma^4}{3 \varepsilon_0 \rho} = 3,25\ 10^{-15}\ \mathrm{J} = 20\ \mathrm{keV} \\[2ex] I = N_{bc} \times f \rightarrow N_{bc} = N_b \cdot e = I \cdot \dfrac{2\pi\rho}{v} \\[2ex] \qquad \hookrightarrow N_b = \dfrac{I \cdot 2\pi\rho}{v e} \\[2ex] P_{tot} = N_b \cdot \dfrac{e^2 c}{6\pi\varepsilon_0} \cdot \dfrac{\gamma^4}{\rho^2} = N_b \cdot U_0 \cdot \dfrac{c}{2\pi\rho} = 20\ \mathrm{kW} \end{array} \right.$$

$$(m_0 c^2)_{muon} \simeq 100\ \mathrm{MeV}$$

$$U_0 = 11\ \mu\mathrm{eV}$$

$$P_{tot} = 11\ \mu\mathrm{W}$$

4 Swiss Light Source 2.0

Calculate how much energy is stored in the electron beam in the SLS-2.0 storage ring, with a circumference of 290.4 m and an average current of 400 mA. The particle energy is 2.4 GeV. Assume the RF trips off. Knowing that the momentum acceptance is ±5%, compute how long the beams survives in the ring before hitting the wall.

$$E_{tot\,beam} = N_b \cdot E_{tot\,1part} = \frac{I \cdot 2\pi R}{\beta c e} \cdot E_{tot\,1part} = 929\ J$$

$$\Delta E = \Delta p \cdot c = (1 - 0{,}95) \cdot 2{,}4\ GeV = 1{,}2\ 10^8\ eV$$

$$U_0 = \frac{e^2 \gamma^4}{3 \varepsilon_0 \rho} = 6{,}3\ 10^4\ eV$$

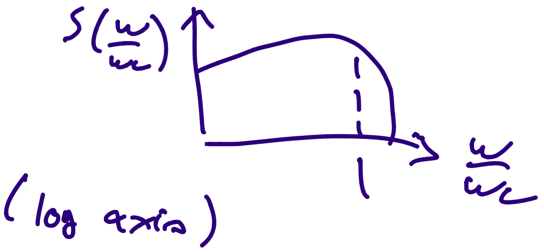
$$N_{turn} = \frac{\Delta E}{U_0} = 1{,}905\ 10^3\ turns$$

$$\Delta t = \frac{N_{turns}}{f} = \frac{N_{turns} \cdot 2\pi R}{c} = 1{,}84\ ms$$

5 Critical Energy

For the electron beam of the previous exercise, calculate the critical photon energy ε_c that is emitted by the superbends with $B = 6$ T, and draw a sketch of the radiation spectrum. What is the useful photon energy range for experiments, assuming that the spectral intensity should be within 1% of the maximum value?

$$\varepsilon_c = 0,665 \times E^2 \times B = 23\,\mathrm{keV}$$



$$\omega_{max} = 0,286\,\omega_c$$

$$\varepsilon_{max} = \hbar\omega_{max} = 6,578\,\mathrm{keV}$$

$$\omega_{max} = 4\omega_c \Rightarrow \varepsilon = 4\hbar\omega_c \approx 100\,\mathrm{keV}$$

6 Diffraction Limited Storage Ring

Assume an undulator of 18 mm period and 5.4 m length. The pole tip field is $B_t = 1.5$ T, and the gap can be varied between 10 and 20 mm.
This undulator is placed in a storage ring, with an electron beam energy of $E = 4$ GeV, and a beam current of 400 mA. The beam is focused to a waist of $\sigma_x = \sigma_y = 20$ µm inside the undulator.

a)
- What range can be reached with the fundamental photon energy?
- What brilliance can be reached at the fundamental photon energy?
- Is there a significant flux higher harmonics?

a) $\varepsilon_c = 0{,}665 \cdot E^2 \cdot B = \langle 960\,eV; 5{,}48\,eV \rangle$

$$B_0 = \frac{B_t}{\cosh\left(\frac{\pi \cdot g}{\lambda_u}\right)} \quad g: \langle 10; 20 \rangle\ mm$$

B_0: $\langle 0{,}091; 0{,}151 \rangle$ T

b) $$K = \frac{e \cdot B_0 \lambda_u}{2\pi m_e c} \cdot \frac{c}{c} = \langle 0{,}153; 0{,}857 \rangle$$

$$B_r = \frac{\dot{N}_{ph}}{4\pi^2 \cdot \sigma_x \sigma_y \cdot \sigma_{x'} \sigma_{y'} \cdot (0{,}1\% BW)} \quad ; \quad \sigma_{x'} = \sqrt{\frac{\lambda}{L}}$$

$$\lambda[\mu m] = \frac{1{,}2398}{E[eV]}$$

$\sigma_{x,y'} = \langle 6{,}5 \cdot 10^{-6}; 1{,}5 \cdot 10^{-5} \rangle$

K_{LOW}

$$B_{R\,LOW} = \frac{1{,}43 \cdot 10^{14} \cdot 0{,}4 \cdot 0{,}01 \cdot 5{,}4}{4\pi^2 \cdot (20)^2 \cdot 10^{-12} \cdot (1{,}5 \cdot 10^{-5})^2 \cdot 0{,}018} = 4{,}8 \cdot 10^{31}\ u.w.$$

$$B_{R\,HIGH} = \frac{1{,}43 \cdot 10^{14} \cdot 0{,}4 \cdot 0{,}5 \cdot 5{,}4}{4\pi^2 (20 \cdot 10^{-6})^2 \cdot (6{,}5 \cdot 10^{-6})^2 \cdot 0{,}018} = 1{,}29 \cdot 10^{32}\ u.w.$$

Note on calculating Brilliance:
in the most general case, you will need to calculate the effective beam size and divergence.

$$\sigma_{x'(eff)} = \sqrt{\sigma_{r'}^2 + \sigma_{x'(e\,beam)}^2}$$

from radiation

from Twiss parameters

In the most general case, need to consider both terms!

Electron beam:

$$\sigma_{x'} = \sqrt{\varepsilon \cdot \gamma}$$

Twiss γ, not Lorentz γ

In the case, that the electron beam is focused to a waist

$$\sigma_{x'} \cdot \sigma_x = \varepsilon_x$$

Note: here we define emittance without the π.

[The area of the phase space ellipse is then $\pi\,\varepsilon$]

→ use this to calculate σ_x , $\sigma_{x'}$

put this into :

$$\sigma_{x(\text{eff})} = \sqrt{\sigma_r^2 + \sigma_x^2}$$

$$\sigma_{x'(\text{eff})} = \sqrt{\sigma_{r'}^2 + \sigma_{x'}^2}$$

where

$$\sigma_r = \frac{1}{4\pi}\sqrt{\lambda L} \qquad \sigma_{r'} = \sqrt{\frac{\lambda}{L}}$$

7 Neutron Star

A proton with energy $E_p = 10$ TeV moves through the magnetic field of a neutron star with strength $B = 10^8$ T.

- Calculate the diameter of the proton trajectory and the revolution frequency
- How large is the power emitted by synchrotron radiation?
- How much energy does the proton lose per revolution?

$$B\rho = \frac{p}{0,3} \Rightarrow \rho = 3,3 \cdot 10^{-4} \text{ m}$$

$$f_{rev} = \frac{\beta c}{2\pi\rho} \qquad \gamma = \frac{E}{E_0} = 1 \cdot 10^4 \Rightarrow \beta = 1$$

$$= 0,143 \text{ THz}$$

$$P = \frac{e^2 c}{6\pi \varepsilon_0} \frac{\gamma^4}{\rho^2} \qquad \varepsilon_0 = 8,85 \cdot 10^{-12} \text{ F} \cdot \text{m}^{-2}$$

$$P = 5,35 \text{ kW}$$

$$\Delta E = \frac{P}{f_{rev}} = 0,234 \text{ TeV per turn}$$

8 Undulators

Assume an undulator with a given undulator period λ_u. How does the brilliance of the emitted radiation at resonance energy depend on the undulator length?

$\mathcal{B} \sim L^2$

9 In-Vacuum Undulators

What is the advantage of using in-vacuum permanent magnet undulators?

▷ smaller gap

▷ higher field on axis

▷ for permanent magnets: no current feed-throughs, no cooling necessary

10 Fundamental Limits

The SLS 2.0, a diffraction limited storage ring, aims for an electron energy of 2.4 GeV and an emittance of 126 pm. How far is this away from the de Broglie emittance, i.e. the minimum emittance given by the uncertainty principle?

$$\varepsilon_{dB} = \frac{\lambda_{dB}}{4\pi}$$

$$\hbar c = 197 \, \mathrm{MeV} \cdot \mathrm{fm}$$

$$\lambda_{dB} = \frac{h}{p} = \frac{2\pi \hbar c}{pc}$$

$$= 0{,}516 \, \mathrm{fm}$$

$$\varepsilon_{dB} = 4{,}1 \cdot 10^{-2} \, \mathrm{fm}$$