Measurement of transverse Emittance

The emittance characterizes the whole beam quality, assuming linear behavior as described by second order differential equation.

It is defined within the phase space as: $\varepsilon_x = \frac{1}{\pi}\int_A dx dx'$

The measurement is based on determination of:

either profile width $\boldsymbol{\sigma_x}$ and angular width $\boldsymbol{\sigma_x'}$ at one location
or $\boldsymbol{\sigma_x}$ at different locations and linear transformations.

Different devices are used at transfer lines:

- Lower energies $\boldsymbol{E_{kin}}$ < 100 MeV/u: slit-grid device, pepper-pot (suited in case of non-linear forces).
- All beams: Quadrupole variation & 'three grid' method using linear transformations (**not** well suited in the presence of non-linear forces)

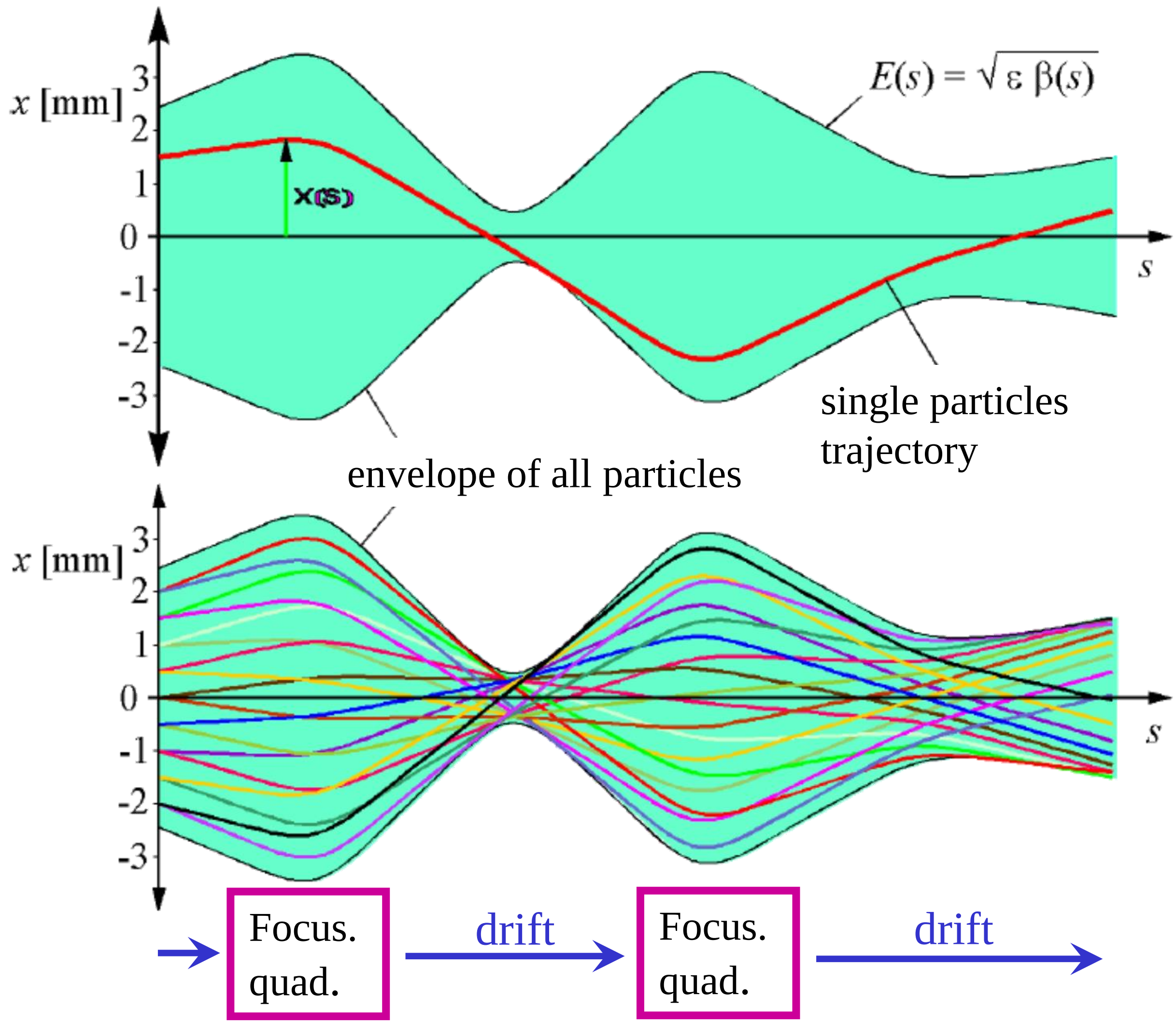
Synchrotron: lattice functions results in stability criterion

$\Rightarrow$ beam width delivers emittance: $\varepsilon_x = \frac{1}{\beta_x(s)}\left[\sigma_x^2 - \left(D(s)\frac{\Delta p}{p}\right)\right]$ and $\varepsilon_y = \frac{\sigma_y^2}{\beta_y(s)}$

Outline:

- Definition and some properties of transverse emittance
- Slit-Grid device: scanning method
- Quadrupole strength variation and position measurement
- Summary

Excurse: Particle Trajectory and Characterization of many Particles



- Single particle trajectories are forming a beam
- They have a distribution of start positions and angles

⇒ Characteristic quantity is the **beam envelope**

- **Goal:**

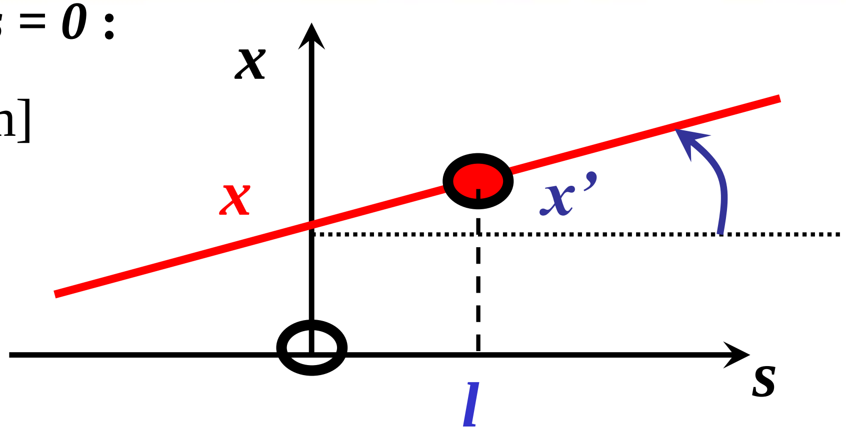
Behavior of whole ensemble

Plot: Wille

Horizontal and vertical coordinates at $s = 0$:

- x **:** Offset from reference orbit in [mm]
- x'**:** Angle of trajectory in unit [mrad]

  $x' = dx / ds$



Assumption: par-axial beams:

- x is small compared to ρ_0
- Small angle with $p_x / p_s << 1$

Longitudinal coordinate:

- Longitudinal orbit difference: $l = - v_0 \cdot (t - t_0)$ in unit [mm]
- Momentum deviation: $\delta = (p - p_0) / p_0$ sometimes in unit [mrad] = [‰]

For **continuous** beam: l has no meaning $\Rightarrow$ set $l \equiv 0$ **!**

Reference particle: no horizontal and vertical offset $x \equiv y \equiv 0$ and $l \equiv 0$ for all s

Excurse: Definition of Coordinates and basic Equations

The basic vector is 6 dimensional:

$$\vec{x}(s) = \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix} = \begin{pmatrix} \text{hori. spatial deviation} \\ \text{horizontal divergence} \\ \text{vert. spatial deviation} \\ \text{vertical divergence} \\ \text{longitudinal deviation} \\ \text{momentum deviation} \end{pmatrix} = \begin{pmatrix} [\text{mm}] \\ [\text{mrad}] \\ [\text{mm}] \\ [\text{mrad}] \\ [\text{mm}] \\ [‰] \end{pmatrix}$$

The transformation of a single particle from a location s_0 to s_1 is given by the Transfer Matrix R:

$$\vec{x}(s_1) = \mathrm{R}(s) \cdot \vec{x}(s_0)$$

The transformation of a the envelope from a location s_0 to s_1 is given by the Beam Matrix σ:

$$\sigma(s_1) = \mathrm{R}(s) \cdot \sigma(s_0) \cdot \mathrm{R}^T(s)$$

6-dim Beam Matrix with *decoupled* hor. & vert. plane:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 & 0 & \sigma_{15} & \sigma_{16} \\ \sigma_{12} & \sigma_{22} & 0 & 0 & \sigma_{25} & \sigma_{26} \\ 0 & 0 & \sigma_{33} & \sigma_{34} & 0 & 0 \\ 0 & 0 & \sigma_{34} & \sigma_{44} & 0 & 0 \\ \sigma_{15} & \sigma_{25} & 0 & 0 & \sigma_{55} & \sigma_{56} \\ \sigma_{16} & \sigma_{26} & 0 & 0 & \sigma_{56} & \sigma_{66} \end{pmatrix}$$

horizontal
vertical
longitudinal
hor.-long. coupling
→ 13 values

Beam width for the three coordinates:

$$x_{rms} = \sqrt{\sigma_{11}}$$
$$y_{rms} = \sqrt{\sigma_{33}}$$
$$l_{rms} = \sqrt{\sigma_{55}}$$

Horizontal beam matrix:

$$\sigma_{11} = \langle x^2 \rangle$$
$$\sigma_{12} = \langle x\, x' \rangle$$
$$\sigma_{22} = \langle x'^2 \rangle$$

Excurse: Some Examples for linear Transformations

The 2-dim sub-space (x,x') can be used in case there is coupling like dispersion $R_{16}=(x\,|\,\delta)=0$

Important examples are:

- Drift with length L: $\mathbf{R}_{\text{drift}} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$

- Horizontal **focusing** with quadrupole constant k end effective length l:

$$\mathbf{R}_{\text{focus}} = \begin{pmatrix} \cos\sqrt{k}\,l & \frac{1}{\sqrt{k}}\sin\sqrt{k}\,l \\ -\sqrt{k}\cdot\sin\sqrt{k}\,l & \cos\sqrt{k}\,l \end{pmatrix} \Rightarrow \mathrm{R}^{\text{thin lens}}_{\text{focus}} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

- Horizontal **de-focusing** with quadrupole constant k end effective length l:

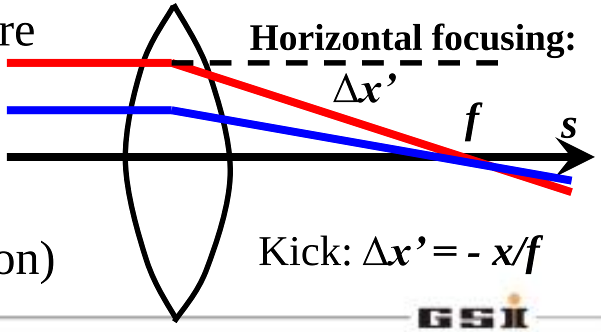
$$\mathbf{R}_{\text{de-focus}} = \begin{pmatrix} \cosh\sqrt{k}\,l & \frac{1}{\sqrt{k}}\sinh\sqrt{k}\,l \\ \sqrt{k}\cdot\sinh\sqrt{k}\,l & \cosh\sqrt{k}\,l \end{pmatrix} \Rightarrow \mathrm{R}^{\text{thin lens}}_{\text{de-focus}} = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}$$

Ideal quad.: field gradient $g = B_{pole}/a$, B_{pole} field at poles, a aperture

$\rightarrow$ quadrupole constant $k = |\,g\,| / (B\rho)_0$

Thin lens approximation: $l \rightarrow 0 \;\Rightarrow\; kl \rightarrow \text{const} \;\Rightarrow kl \equiv 1/f$

$\Rightarrow$ simple transfer matrix (math. proof by 1st order Taylor expansion)



The emittance characterizes the whole beam quality: $\varepsilon_x = \frac{1}{\pi} \int_A dx\,dx'$

Ansatz:

Beam matrix at one location: $\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} = \varepsilon \cdot \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$ with $\vec{x} = \begin{pmatrix} x \\ x' \end{pmatrix}$

It describes a 2-dim probability distr.

The value of emittance is:

$$\varepsilon_x = \sqrt{\det \boldsymbol{\sigma}} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

For the profile and angular measurement:

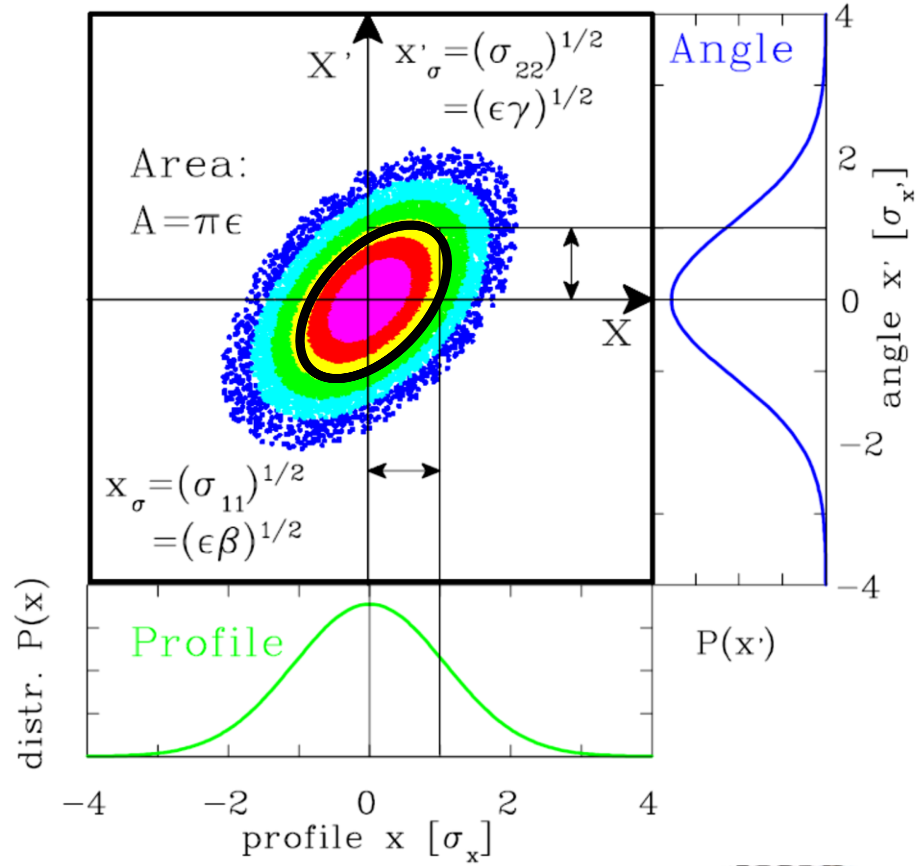
$$x_\sigma = \sqrt{\sigma_{11}} = \sqrt{\varepsilon\beta} \quad \text{and}$$

$$x'_\sigma = \sqrt{\sigma_{22}} = \sqrt{\varepsilon\gamma}$$

Geometrical interpretation:

All points $\boldsymbol{x}$ fulfilling $\boldsymbol{x}^t \cdot \boldsymbol{\sigma}^{-1} \cdot \boldsymbol{x} = \mathbf{1}$ are located on a **ellipse**

$$\boldsymbol{\sigma_{22} x^2 - 2\sigma_{12} x x' + \sigma_{11} x'^2} = \det \boldsymbol{\sigma} = \varepsilon_x^2$$



The density function for a 2-dim Gaussian distribution is:

$$\rho(x,x') = \frac{1}{2\pi\epsilon}\exp\left[-\frac{1}{2}\,\vec{x}^T\,\sigma^{-1}\,\vec{x}\,\right]$$

$$= \frac{1}{2\pi\epsilon}\exp\left[\frac{-1}{2\det\sigma}\left(\sigma_{22}x^2 - 2\sigma_{12}xx' + \sigma_{11}x'^2\right)\right]$$

It describes an ellipse with the characteristics

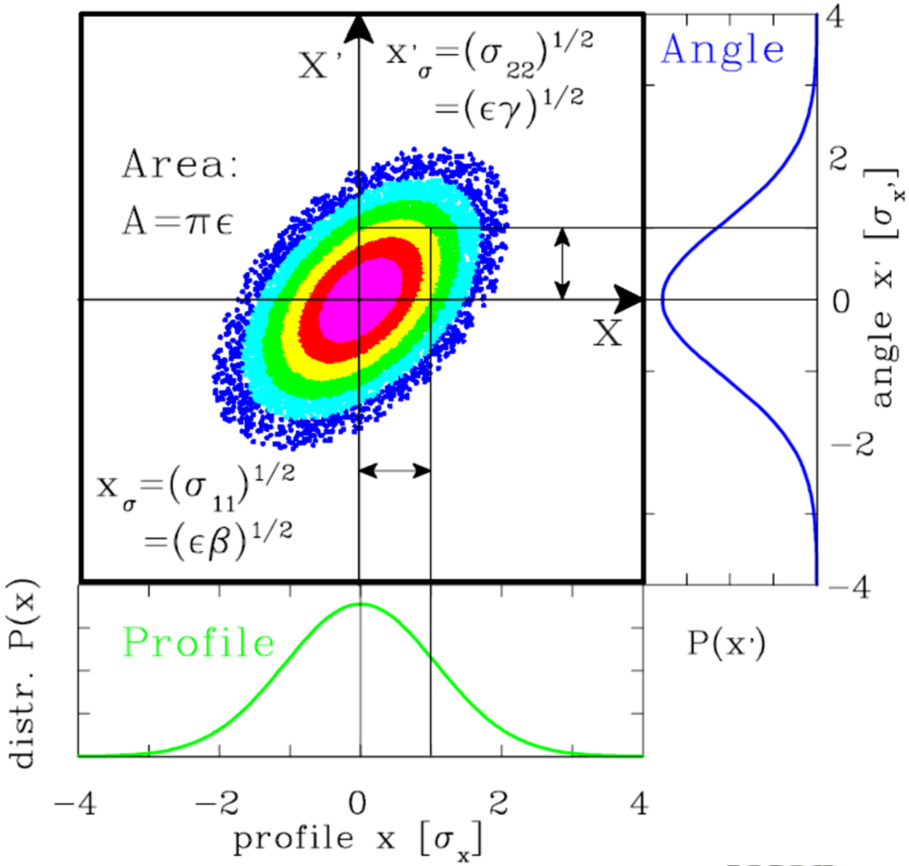
profile and angle Gaussian distribution of width

$$x_\sigma \equiv \sqrt{\langle x^2\rangle} = \sqrt{\sigma_{11}} \quad \text{and}$$

$$x'_\sigma \equiv \sqrt{\langle x'^2\rangle} = \sqrt{\sigma_{22}}$$

and the correlation or covariance

$$\text{cov} \equiv \sqrt{\langle xx'\rangle} = \sqrt{\sigma_{12}}$$



For $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ it is $\mathbf{A}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

assuming $\det(\mathbf{A}) = ad - bc \neq 0 \Leftrightarrow$ matrix invertible

The Emittance for Gaussian and non-Gaussian Beams

The beam distribution can be non-Gaussian, e.g. at:

- beams behind ion source
- space charged dominated beams at LINAC & synchrotron
- cooled beams in storage rings

General description of emittance using terms of 2-dim distribution:

Variances — Covariance i.e. correlation

$$\varepsilon_{rms} = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

It describes the value for 1 standard derivation

$$\langle x\rangle \equiv \mu = \frac{\iint x\cdot\rho(x,x')\,dxdx'}{\iint \rho(x,x')\,dxdx'} \qquad \langle x'\rangle \equiv \mu' = \frac{\iint x'\cdot\rho(x,x')\,dxdx'}{\iint \rho(x,x')\,dxdx'}$$

$$\langle x^n\rangle = \frac{\iint (x-\mu)^n\cdot\rho(x,x')\,dxdx'}{\iint \rho(x,x')\,dxdx'} \qquad \langle x'^n\rangle = \frac{\iint (x'-\mu')^n\cdot\rho(x,x')\,dxdx'}{\iint \rho(x,x')\,dxdx'}$$

$$\text{covariance}:\ \langle xx'\rangle = \frac{\iint (x-\mu)(x'-\mu')\cdot\rho(x,x')\,dxdx'}{\iint \rho(x,x')\,dxdx'}$$

For discrete distribution:

$$\langle x\rangle = \frac{\sum_{i,j}\rho(i,j)\cdot x_i x'_j}{\sum_{i,j}\rho(i,j)}$$

and correspondingly for all other moments

The Emittance for Gaussian and non-Gaussian Beams

The beam distribution can be non-Gaussian, e.g. at:

- beams behind ion source
- space charged dominated beams at LINAC & synchrotron
- cooled beams in storage rings

General description of emittance

using terms of 2-dim distribution:

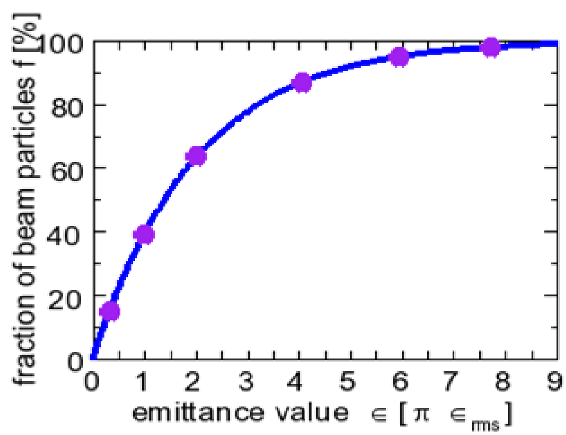
It describes the value for 1 stand. derivation

Variances **Covariance i.e. correlation**

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

For <u>Gaussian</u> beams only: ε_{rms} ↔ interpreted as area containing a fraction ***f*** of ions:

$$\varepsilon(f) = -2\pi\varepsilon_{rms} \cdot \ln(1-f)$$



Emittance *ε(f)*	Fraction *f*
$1 \cdot \varepsilon_{rms}$	15 %
$\pi \cdot \varepsilon_{rms}$	39 %
$2\pi \cdot \varepsilon_{rms}$	63 %
$4\pi \cdot \varepsilon_{rms}$	86 %
$8\pi \cdot \varepsilon_{rms}$	98 %

Care:
No common definition of emittance concerning the fraction ***f***

Outline:

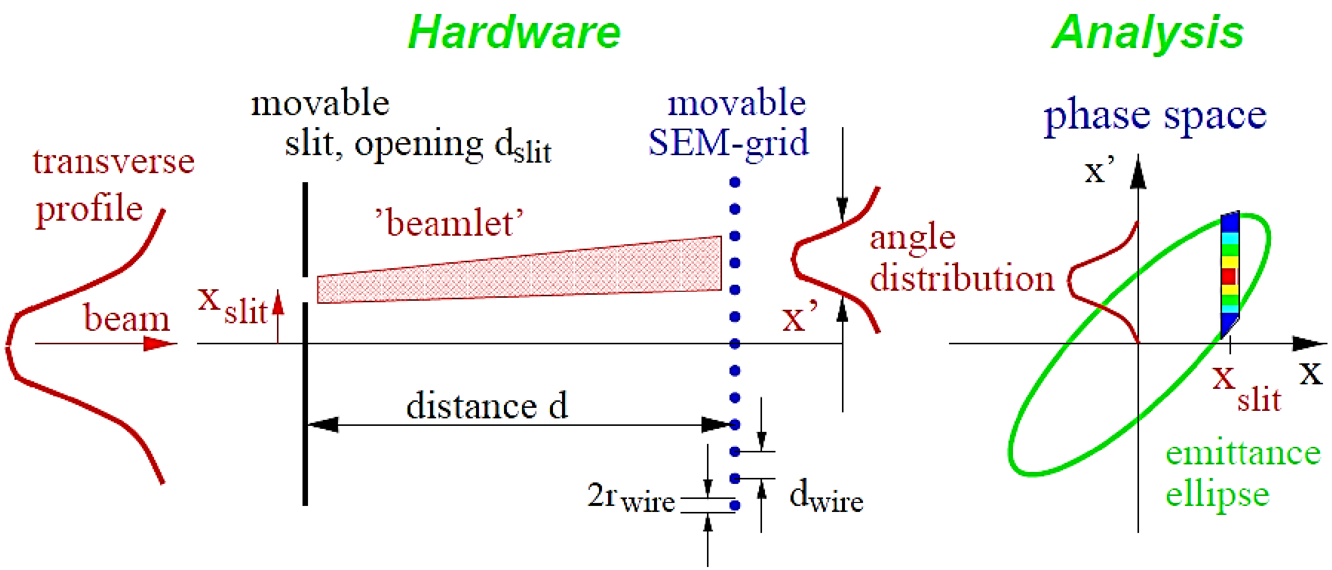
- Definition and some properties of transverse **emittance**
- Slit-Grid device: scanning method

  scanning slit → beam position & grid → angular distribution
- Quadrupole strength variation and position measurement
- Summary

The Slit-Grid Measurement Device

Slit-Grid: Direct determination of position and angle distribution.

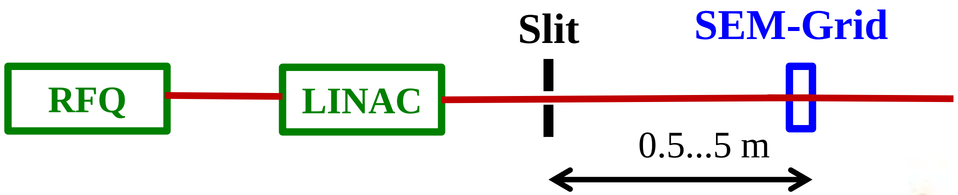
Used for protons with E_{kin} < 100 MeV/u ⇒ range R < 1 cm.



Slit: position ***P(x)*** with typical width: 0.1 to 0.5 mm

Distance: typ. 0.5 to 5 m (depending on beam energy 0. 1 ... 100 MeV)

SEM-Grid: angle distribution ***P(x')***

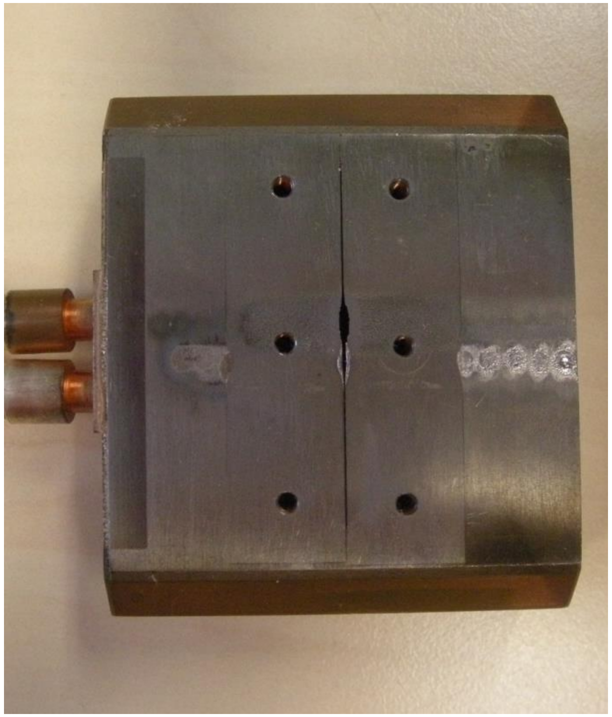


Slit with e.g. 0.1 mm thickness

→ Transmission only from ***Δx***.

Example: Slit of width 0.1 mm (defect)

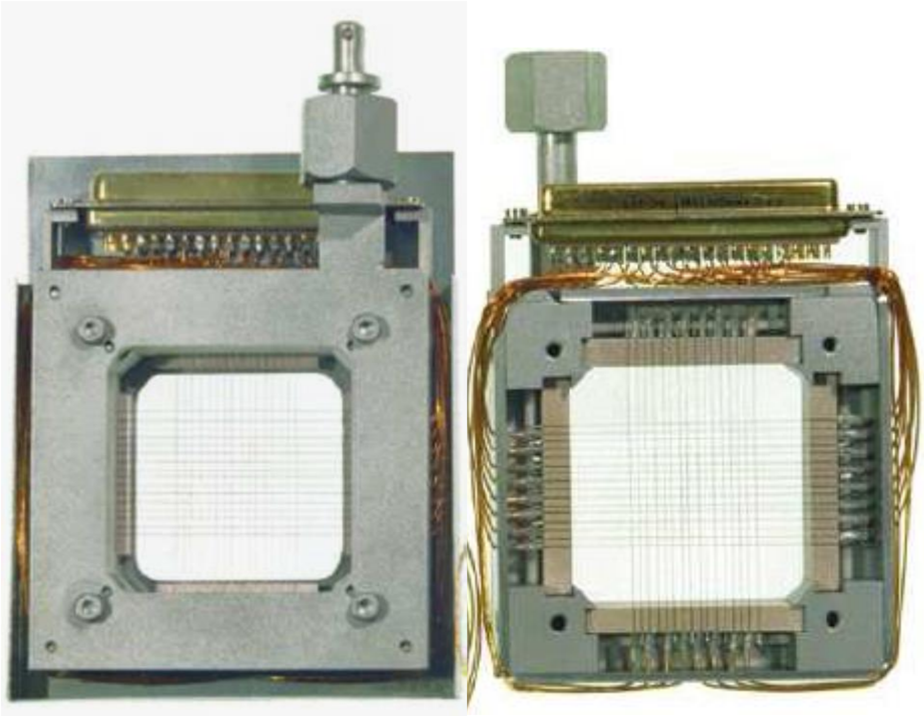
Moved by stepping motor, 0.1 mm resolution



Beam surface interaction: e⁻ emission

→ measurement of current.

Example: 15 wire spaced by 1.5 mm:



Each wire is equipped with one I/U converter

different ranges settings by R_i

→ very large dynamic range up to 10^6.

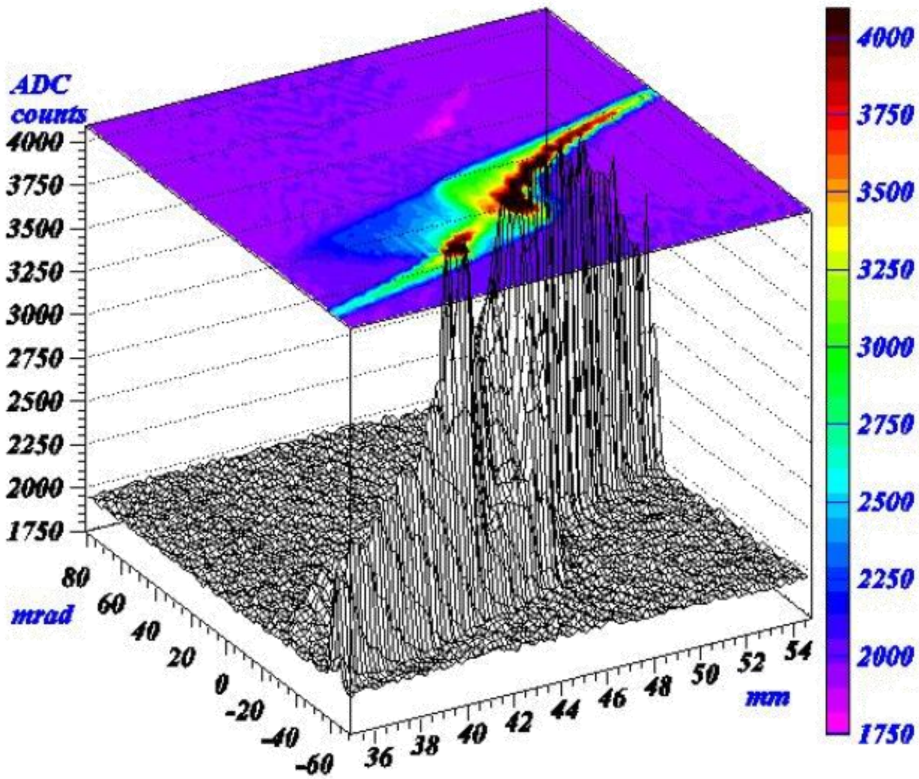
Display of Measurement Results

The distribution of the ions is depicted as a function of

- Position [mm]
- Angle [mrad]

The distribution can be visualized by

- Mountain plot
- Contour plot



Calc. of 2nd moments $\langle x^2 \rangle$, $\langle x'^2 \rangle$ & $\langle xx' \rangle$

Emittance value ε_{rms} from

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

Problems:

- Finite **binning** results in limited resolution
- **Background** → large influence on $\langle x^2 \rangle$, $\langle x'^2 \rangle$ and $\langle xx' \rangle$

Or fit of distribution i.e. ellipse to data

⇒ **Effective emittance only**

Beam: Ar^{4+}, 60 KeV, 15 µA

at Spiral2 Phoenix ECR source.

Plot from P. Ausset, DIPAC 2009

The width of the slit d_{slit} gives the resolution in space $\Delta x = d_{slit}$.

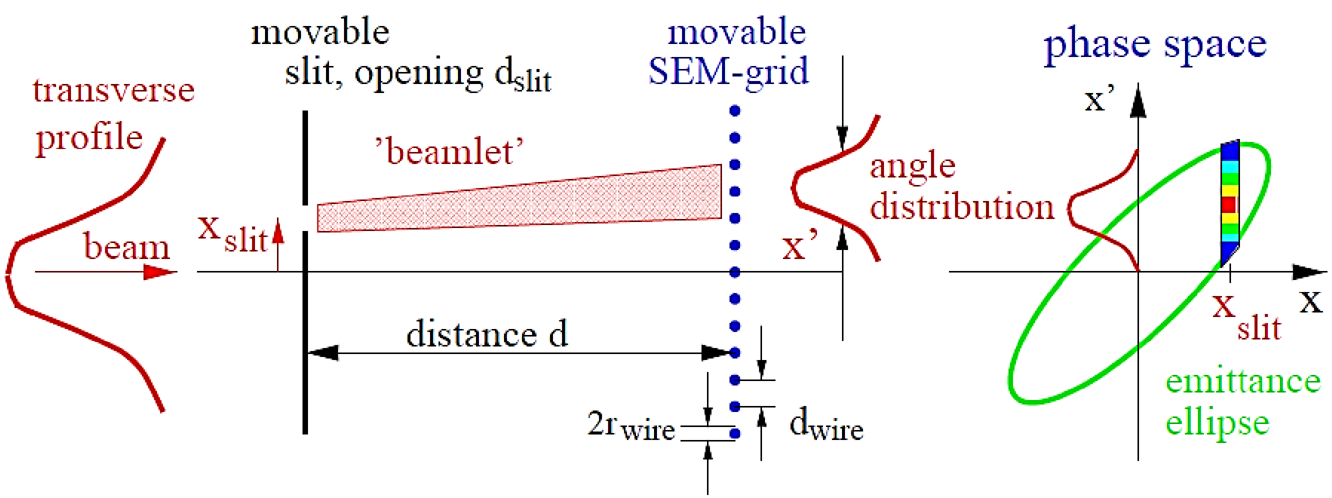
The angle resolution is $\Delta x' = (d_{wire} + 2r_{wire})/d$

$\Rightarrow$ discretization element $\Delta x \cdot \Delta x'$.

By scanning the SEM-grid the angle resolution can be improved.

Problems for small beam sizes or parallel beams.

Hardware **Analysis**



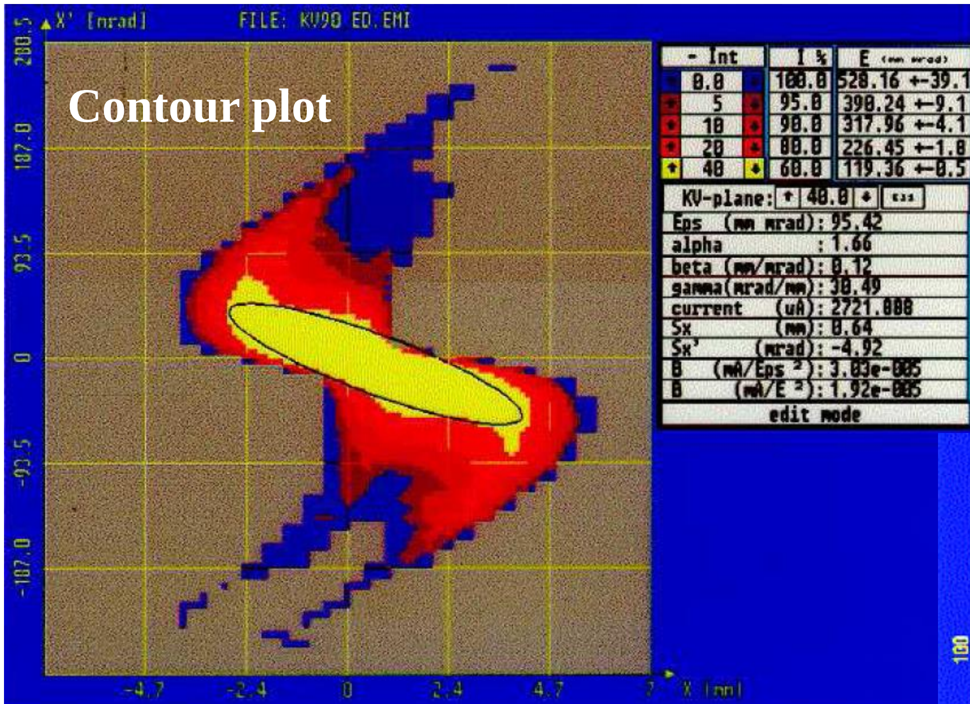
For pulsed LINACs: Only one measurement each pulse → long measuring time required.

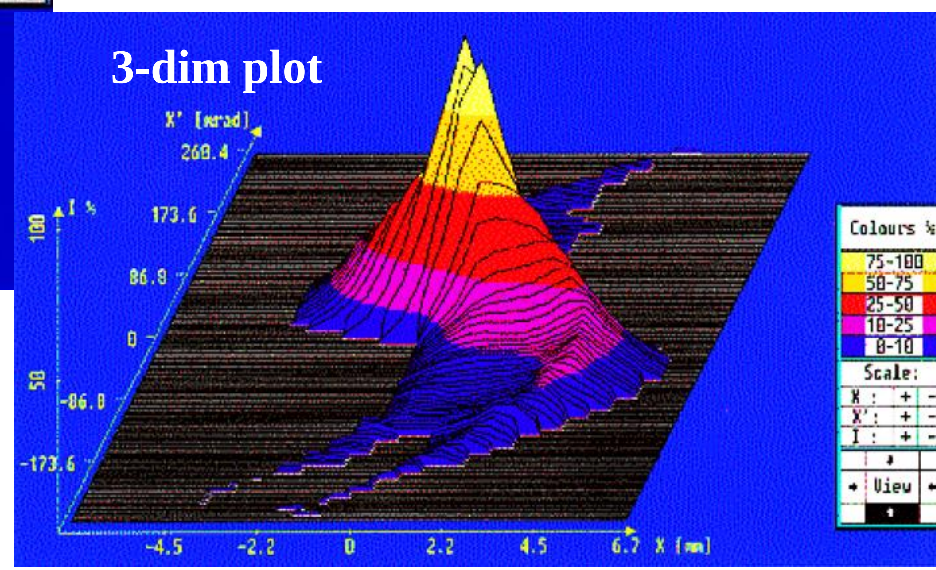
Result of an Slit-Grid Emittance Measurement

Result for a beam behind ion source: ➢ here aberration in quadrupoles due to large beam size

➢ different evaluation and plots possible

➢ can monitor any distribution





Low energy ion beam:
→ well suited for emittance showing space-charge effects or aberrations.

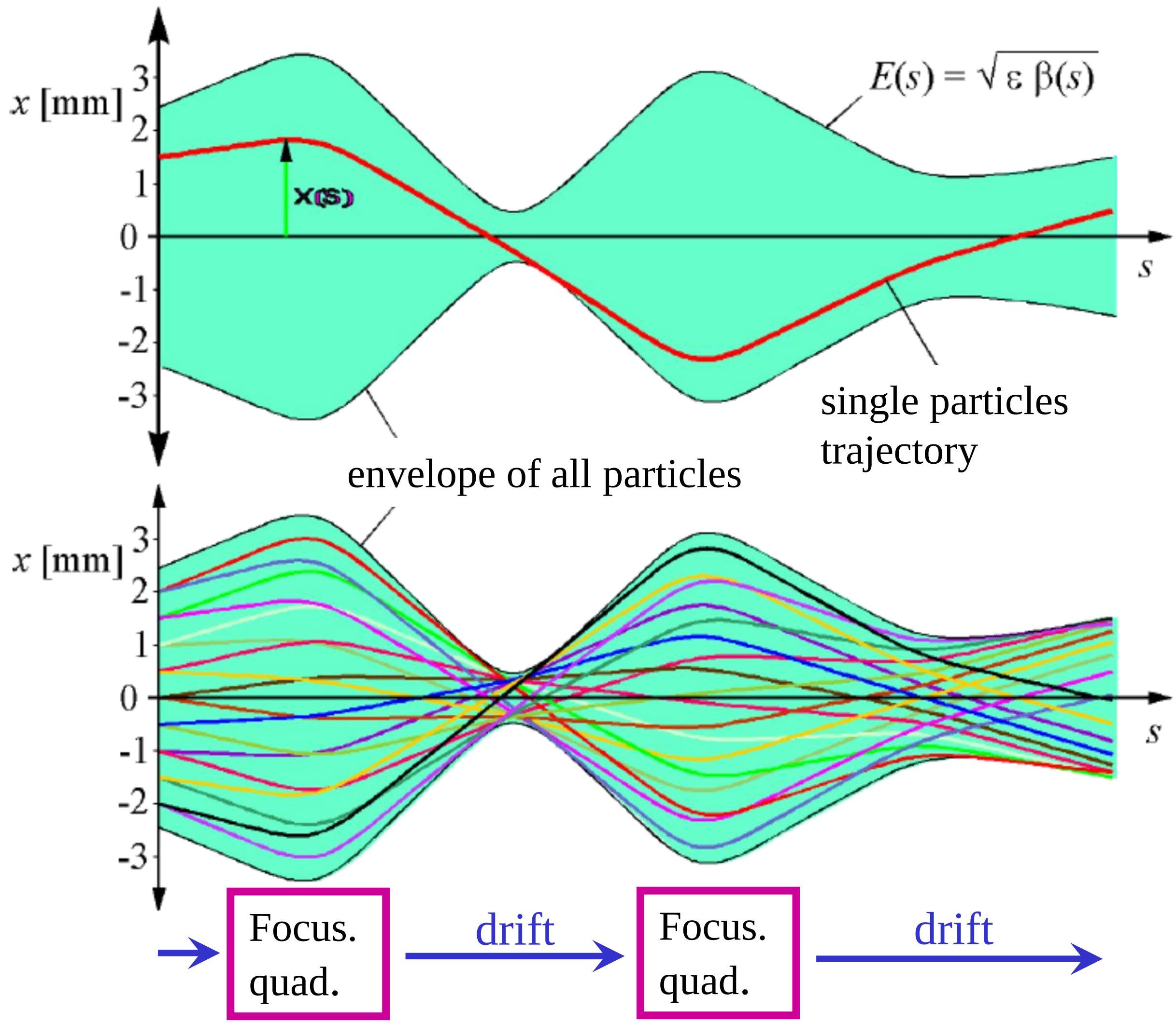
Outline:

- ➢ Definition and some properties of transverse emittance
- ➢ Slit-Grid device: scanning method

  scanning slit → beam position & grid → angular distribution
- ➢ **Quadrupole strength variation and position measurement**

  emittance from several profile measurement and beam optical calculation
- ➢ Summary

Excurse: Particle Trajectory and Characterization of many Particles



- Single particle trajectories are forming a beam
- They have a distribution of start positions and angles

⇒ Characteristic quantity is the **beam envelope**

Transformation of envelope

- **Goal:**

Behavior of whole ensemble

Plot: Wille

Liouville's Theorem:

The phase space density can not changes with conservative e.g. linear forces.

The beam distribution at one location s_0 is described by the beam matrix $\boldsymbol{\sigma}(s_0)$

This beam matrix is transported from location s_0 to s_1 via the transfer matrix

$$\boldsymbol{\sigma}(s_1) = \mathbf{R} \cdot \boldsymbol{\sigma}(s_0) \cdot \mathbf{R}^T$$

6-dim beam matrix with *decoupled* horizontal, vertical and longitudinal plane:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 & 0 & 0 & 0 \\ \sigma_{12} & \sigma_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{33} & \sigma_{34} & 0 & 0 \\ 0 & 0 & \sigma_{34} & \sigma_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{55} & \sigma_{56} \\ 0 & 0 & 0 & 0 & \sigma_{56} & \sigma_{66} \end{pmatrix}$$

Horizontal beam matrix:

$$\sigma_{11} = \langle x^2 \rangle$$

$$\sigma_{12} = \langle x\,x' \rangle$$

$$\sigma_{22} = \langle x'^2 \rangle$$

Beam width for the three coordinates:

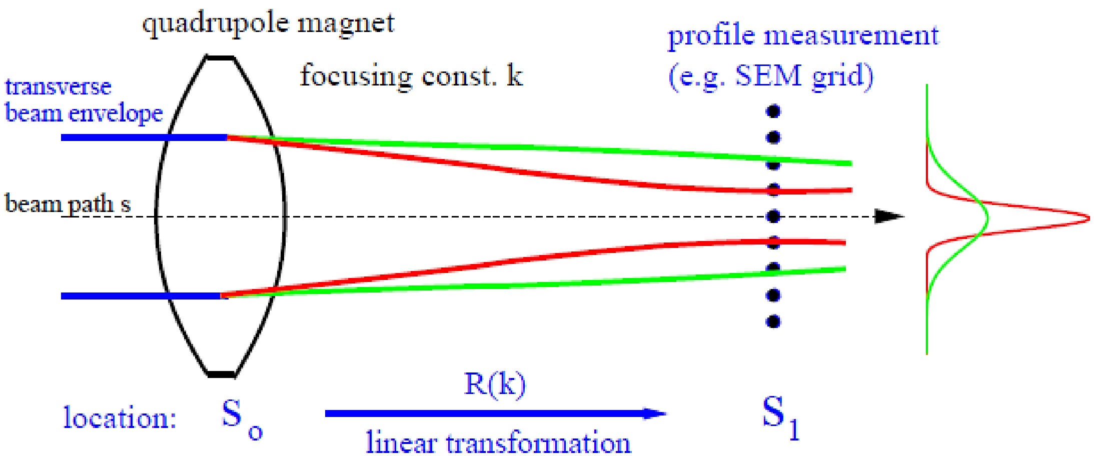
$$x_{rms} = \sqrt{\sigma_{11}} = \sqrt{\langle x^2 \rangle}$$

$$y_{rms} = \sqrt{\sigma_{33}} = \sqrt{\langle y^2 \rangle}$$

$$l_{rms} = \sqrt{\sigma_{55}} = \sqrt{\langle l^2 \rangle}$$

Emittance Measurement by Quadrupole Variation

From a profile determination, the emittance can be calculated via linear transformation, if a well known and constant distribution (e.g. Gaussian) is assumed.



- Measurement of beam width

$$x^2_{max} = \sigma_{11}(1, k)$$

matrix **R(*k*)** describes the focusing.

- With the drift matrix the transfer is

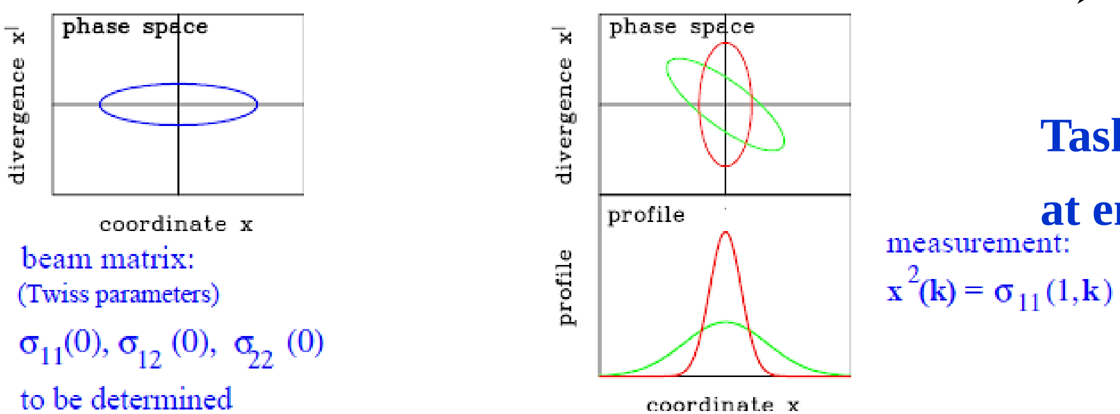
$$\mathbf{R}(k_i) = \mathbf{R}_{drift} \cdot \mathbf{R}_{focus}(k_i)$$

- Transformation of the beam matrix

$$\sigma(1,k_i) = \mathbf{R}(k_i) \cdot \sigma(0) \cdot \mathbf{R}^T(k_i)$$

Task: Calculation of $\sigma(0)$

at entrance s_0 i.e. all three elements



Measurement of transverse Emittance

- The beam width $x_{max}(s_1)$ at s_1 is measured ⇔ matrix element $\sigma_{11}(1, k_i) = x^2_{max}(k_i)$
- Different focusing of quadrupoles $k_1, k_2 \dots k_n$ are used ⇒ $\mathbf{R}_{focus}(k_i)$
- After the drift the transfer matrix is $\mathbf{R}(k_i) = \mathbf{R}_{drift} \cdot \mathbf{R}_{focus}(k_i)$
- **Task: Calculation of beam matrix $\sigma(0)$ at entrance s_0 (matrix elements give orientation)**
- **The transformation of the beam matrix is:** $\sigma(1,k_i) = \mathbf{R}(k_i) \cdot \sigma(0) \cdot \mathbf{R}^T(k_i)$

⇒ **Result: Redundant system of linear equations for matrix elements $\sigma_{ij}(0)$**

$$\sigma_{11}(1,k_1) = R^2_{11}(k_1) \cdot \sigma_{11}(0) + 2\,R_{11}(k_1)\,R_{12}(k_1) \cdot \sigma_{12}(0) + R^2_{12}(k_1) \cdot \sigma_{22}(0) \quad \text{focusing } k_1$$

$$\dots$$

$$\sigma_{11}(1,k_n) = R^2_{11}(k_n) \cdot \sigma_{11}(0) + 2\,R_{11}(k_n)\,R_{12}(k_n) \cdot \sigma_{12}(0) + R^2_{12}(k_n) \cdot \sigma_{22}(0) \quad \text{focusing } k_n$$

- To have an error estimation at least three measurements must be done

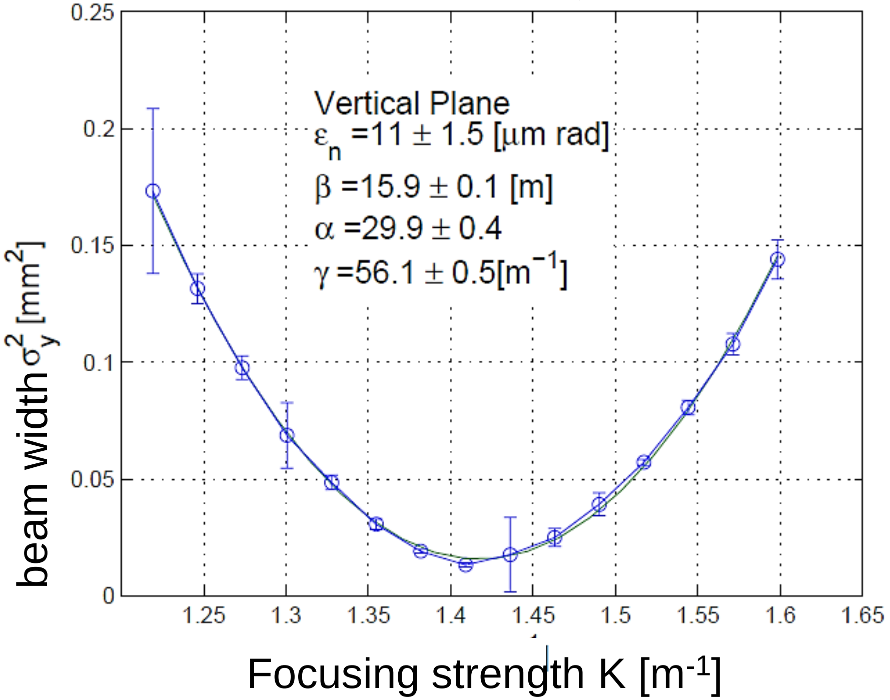
Assumptions:
- Constant emittance, in particular no space-charge broadening
- Only elliptical shaped beam distribution is considered
- No misalignment, i.e. beam center equals center of the quadrupoles
- If **not** valid: A self-consistent algorithm can be used .

Using the 'thin lens approximation' i.e. the quadrupole has a focal length of f:

$$\mathrm{R}_{focus}(K) = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix} \Rightarrow \mathrm{R}(L,K) = \mathrm{R}_{drift}(L) \cdot \mathrm{R}_{focus}(K) = \begin{pmatrix} 1+LK & L \\ K & 1 \end{pmatrix}$$

Measurement of the matrix-element $\sigma_{11}(1,K)$ from $\sigma(1,K) = \mathrm{R}(K) \cdot \sigma(0) \cdot \mathrm{R}^T(K)$

Example: Square of the beam width at ELETTRA 100 MeV e⁻ Linac, YAG:Ce:



G. Penco (ELETTRA) et al., EPAC'08

For completeness: The relevant formulas

$$\sigma_{11}(1,K) = L^2\sigma_{11}(0)\cdot K^2 + 2\cdot(\ L\sigma_{11}(0) + L^2\sigma_{12}(0)\ )\cdot K + L^2\sigma_{22}(0) + \sigma_{11}(0)$$

$$\equiv a\cdot K^2 - 2ab\cdot K + ab^2 + c$$

The three matrix elements at the quadrupole:

$$\sigma_{11}(0) = \frac{a}{L^2}$$

$$\sigma_{12}(0) = -\frac{a}{L^2}\left(\frac{1}{L} + b\right)$$

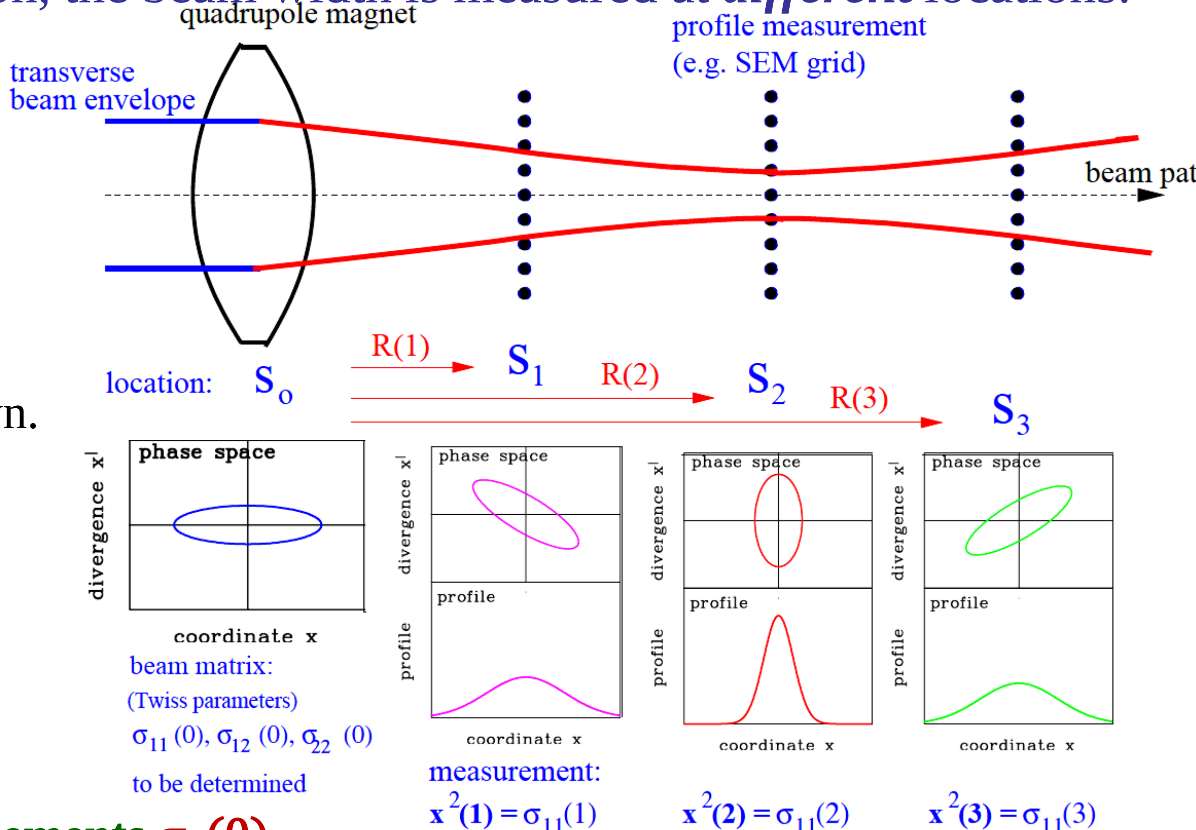
$$\sigma_{22}(0) = \frac{1}{L^2}\left(ab^2 + c + \frac{2ab}{L} + \frac{a}{L^2}\right)$$

$$\varepsilon_{rms} \equiv \sqrt{\det \sigma(0)} = \sqrt{\sigma_{11}(0)\cdot\sigma_{22}(0) - \sigma_{12}^2(0)} = \sqrt{ac}\,/\,L^2$$

Instead of quadrupole variation, the beam width is measured at ***different*** locations:

The procedure is:

- Beam width $x(i)$ measured at the locations s_i
  $\Rightarrow$ beam matrix element $x^2(i) = \sigma_{11}(i)$.
- The transfer matrix **R(*i*)** is known. (without dipole a 3 × 3 matrix.)
- The transformations are:
  $\sigma(i) = R(i) \cdot \sigma(0) \cdot R^T(i)$
  $\Rightarrow$ redundant equations:



$\Rightarrow$ **Result: at least equations for elements $\sigma_{ij}(0)$**

$$\sigma_{11}(1) = R_{11}^2(1) \cdot \sigma_{11}(0) + 2\, R_{11}(1)\, R_{12}(1) \cdot \sigma_{12}(0) + R_{12}^2(1) \cdot \sigma_{22}(0) \quad \text{for } \mathbf{R}(1)\text{: } s_0 \rightarrow s_1$$

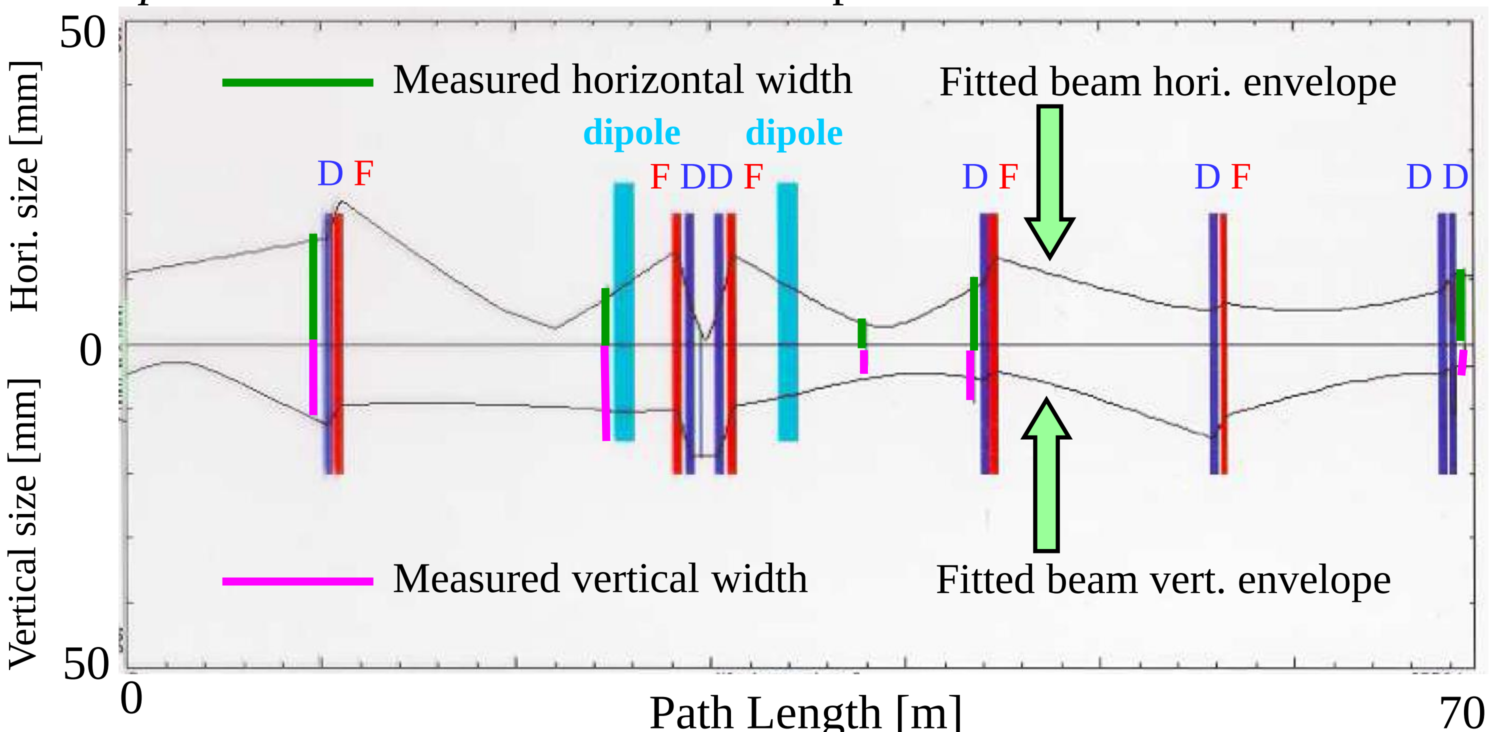
...

$$\sigma_{11}(n) = R_{11}^2(n) \cdot \sigma_{11}(0) + 2\, R_{11}(n)\, R_{12}(n) \cdot \sigma_{12}(0) + R_{12}^2(n) \cdot \sigma_{22}(0) \quad \text{for } \mathbf{R}(n)\text{: } s_0 \rightarrow s_n$$

Results of a 'Three Grid Method' Measurement

Solution: Solving the linear equations like for quadrupole variation or fitting the profiles with linear optics code (e.g. MADX, TRANSPORT, WinAgile).

Example: The hor. and vert. beam envelope and the beam width at a transfer line:



Assumptions: ➢ constant emittance, in particular no space-charge broadening

➢100 % transmission i.e. no loss due to vacuum pipe scraping

➢ no misalignment, i.e. beam center equals center of the quadrupoles.

Emittance measurements are very important for comparison to theory.

It includes size (value of ε) and orientation in phase space (σ_{ij} or α, β and γ)

i.e three independent values $\varepsilon_{rms} = \sqrt{\sigma_{11} \cdot \sigma_{22} - \sigma_{12}} = \sqrt{< x^2 > \cdot < x'^2 > - < xx' >^2}$

Low energy beams → direct measurement of x- and x'-distribution

- ***Slit-grid:*** movable slit → ***x***-profile, grid → ***x'***-profile
- Variances exists: slit-slit, slit-kick, pepperpot .... method

All beams → profile measurement + linear transformation:

- ***Quadrupole variation:*** one location, different setting of a quadrupole
- ***'Three grid method':*** different locations
- ***Assumptions:***
  - well aligned beam, no steering
  - no emittance blow-up due to space charge.

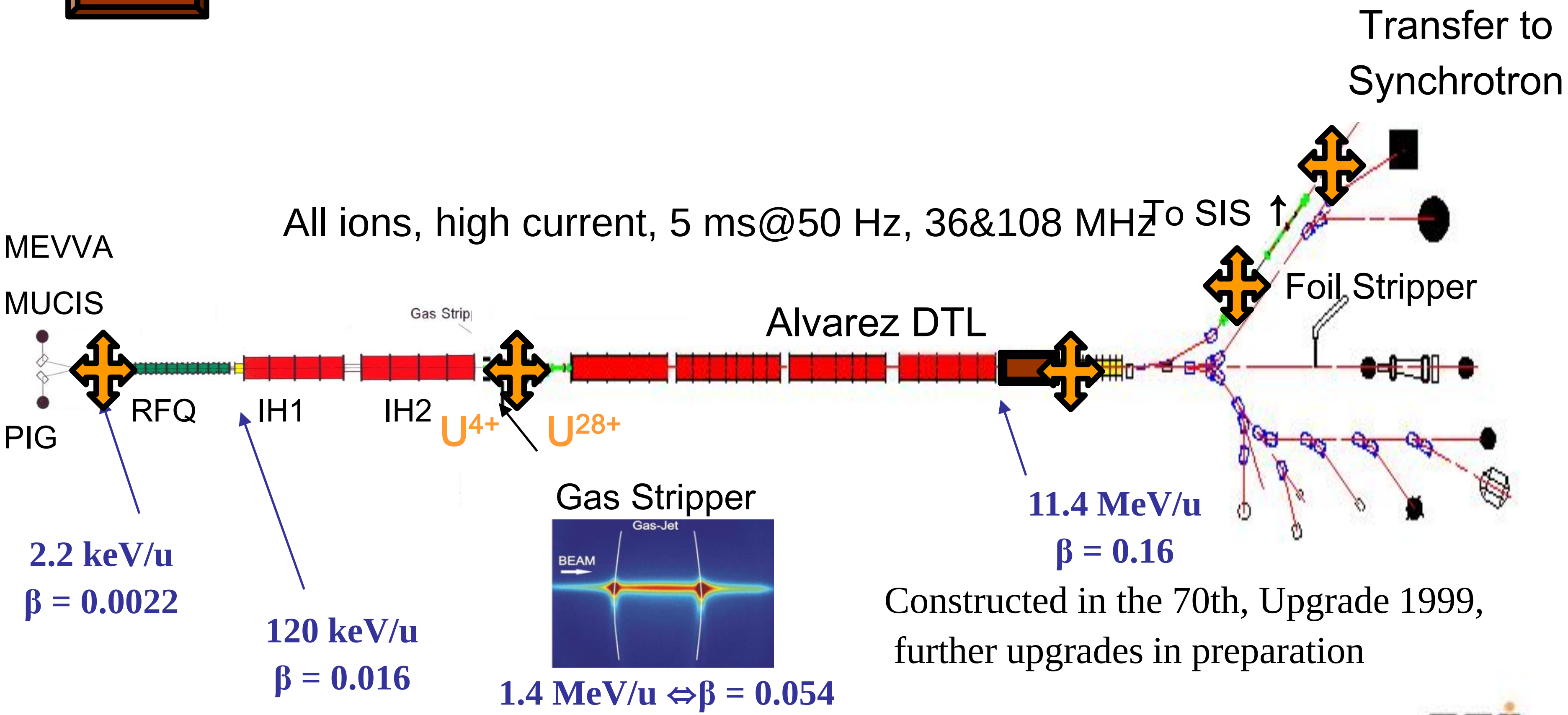
Important remark: For a synchrotron with a *stable beam storage*,

width measurement is sufficient using $x_{rms} = \sqrt{\varepsilon_{rms} \cdot \beta}$

Appendix GSI Ion LINAC: Emittance Measurement Devices

Slit Grid Emittance: Standard device, total 9 device

Pepper-pot Emittance: Special device, total 1 device



The basic vector is 6 dimensional:

$$\vec{x}(s) = \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix} = \begin{pmatrix} \text{hori. spatial deviation} \\ \text{horizontal divergence} \\ \text{vert. spatial deviation} \\ \text{vertical divergence} \\ \text{longitudinal deviation} \\ \text{momentum deviation} \end{pmatrix} = \begin{pmatrix} [\text{mm}] \\ [\text{mrad}] \\ [\text{mm}] \\ [\text{mrad}] \\ [\text{mm}] \\ [‰] \end{pmatrix}$$

The transformation from a location s_0 to s_1 is given by the Transfer Matrix R

$$\vec{x}(s_1) = \mathrm{R}(s) \cdot \vec{x}(s_0) = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \\ l_0 \\ \delta_0 \end{pmatrix}$$

Remark: At ring accelerator a comparable (i.e. a bit different) matrix is called **M**