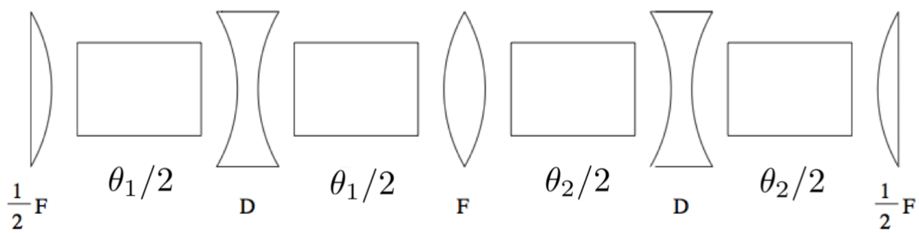
Transverse Beam Dynamics

JUAS 2019 - Tutorial 4

1 Exercise: Dispersion suppressor

In several straight sections of the accelerator, like the ones hosting RF cavities, extraction systems or other devices such as detectors, it is preferable to have no dispersion $\eta(s) = \eta'(s) = 0$. For example, in big colliders, such as the LHC, where small spot sizes are required at the interaction points, the dispersion is reduced to zero at the detector positions. The most common dispersion suppressors consists of two FODO cells of equal length L and equal quadrupole strengths. Bending magnets are placed in the space between the quadrupoles with a different bending field in each FODO. Figure below shows a typical dispersion suppressor.



1. Considering two FODO cells with different total bend angles, $\theta_1 \neq \theta_2$, calculate the relation between the angles θ_1 and θ_2 which must be satisfied to cancel the dispersion at the end of the lattice.

   Hint:

   For each FODO cell, $M_{\text{FODO}} = M_{1/2\text{F}} \cdot M_{\text{dipole}} \cdot M_{\text{D}} \cdot M_{\text{dipole}} \cdot M_{1/2\text{F}}$, in thin-lens approximation we have the following 3×3 matrix:

$$M_{\text{FODO}\ j} = \begin{pmatrix} 1 - \frac{L^2}{8f^2} & L\left(1 + \frac{L}{4f}\right) & \frac{L}{2}\left(1 + \frac{L}{8f}\right)\theta_j \\ -\frac{L}{4f^2}\left(1 - \frac{L}{4f}\right) & 1 - \frac{L^2}{8f^2} & \left(1 - \frac{L}{8f} - \frac{L^2}{32f^2}\right)\theta_j \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} \cos\mu & \beta\sin\mu & \frac{L}{2}\left(1 + \frac{L}{8f}\right)\theta_j \\ -\frac{\sin\mu}{\beta} & \cos\mu & \left(1 - \frac{L}{8f} - \frac{L^2}{32f^2}\right)\theta_j \\ 0 & 0 & 1 \end{pmatrix}$$

   where $j = 1, 2$ (1=first cell, 2=second cell).

   The following condition must be satisfied:

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = M_{\text{suppressor}} \begin{pmatrix} \eta_0 \\ 0 \\ 1 \end{pmatrix} \tag{1}$$

   where η_0 is the initial dispersion (at the middle of the first focusing quadrupole). It can be demonstrated that for a FODO lattice the dispersion has its maximum at the middle of the focusing quadrupole:

$$\eta_0 = \frac{4f^2}{L}\left(1 + \frac{L}{8f}\right)\theta \tag{2}$$

with $\theta = \theta_1 + \theta_2$ the total bend in the suppressor.

2. Obtain the relation between the angles for the cases of phase-advance per cell $\mu = \pi/3$ and $\pi/2$

2 Exercise: Double-Bend Achromat (DBA) lattice

A Double-Bend Achromat (DBA) can be made from two dipoles with a horizontally focusing quadrupole between them. The transfer matrix through the achromat is:

$$M_{\text{DBA}} = M_{\text{bend}} M_{\text{drift}} M_{1/2\text{F}} M_{1/2\text{F}} M_{\text{drift}} M_{\text{bend}}$$

Note that this magnet configuration does not produce vertical focusing, therefore it will not be enough to create a stable lattice. A full DBA typically comprises additional quadrupole doublets before and after the bending section. For sake of simplicity, we will neglect them.

1. Use the thin-lens approximation for quadrupoles and small-angle approximation for bends to find the dispersion in the middle of the quadrupole. Write the focal length in terms of the drift and bend parameters.

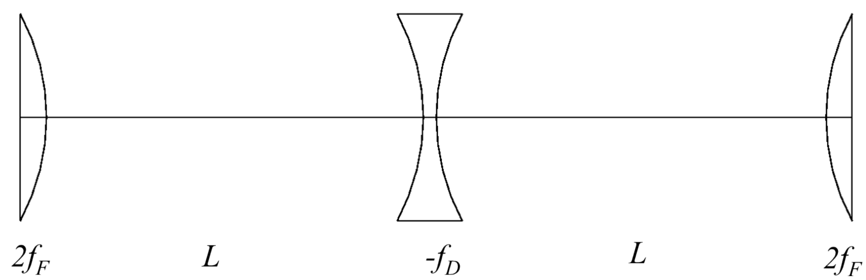
2. Show that the dispersion vanishes after the bend.

3. Compute the parameters L, f for a 10-meter long DBA which bends the beam by an angle of 1 radians. What is the dispersion in the centre? Given a particle with 1% energy deviation, compute the displacement at the centre of the cell.

3 Exercise: Chromaticity in a FODO cell

Consider a ring made of N_{cell} identical FODO cells with equally spaced quadrupoles. Assume that the two quadrupoles are both of length l_q, but their strengths may differ.

1. Calculate the maximum and the minimum betatron function in the FODO cell. (*Use the thin-lens approximations*)



2. Calculate the natural chromaticities for this ring.

3. Show that for short quadrupoles, if $f_F \simeq f_D$,

$$\xi_N \simeq -\frac{N_{cell}}{\pi} \tan\frac{\mu}{2}.$$

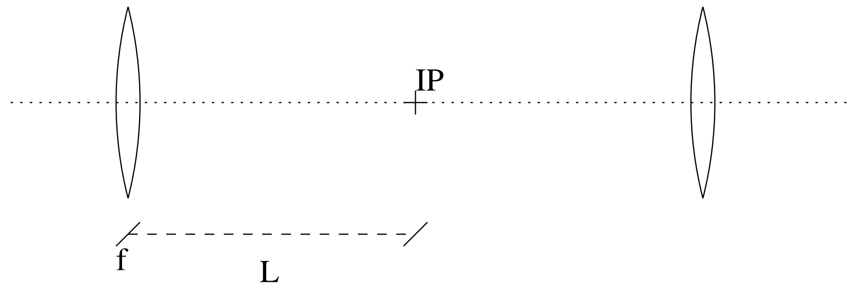
4. Design the FODO cell such that it has: phase advance $\mu = 90$ degrees, a total length of 10 m, and a total bending angle of 5 degrees. What are β_{max}, β_{min}, D_{max}, D_{min}?

5. Add two sextupoles at appropriate locations to correct horizontal and vertical chromaticities. (hints: use 1 sextupole for the horizontal plane and 1 for the vertical plane; do not consider geometric aberrations).

6. If the gradient of all focusing quadrupoles in the ring is wrong by +10%, how much is the tune-shift with and without sextupoles?

4 Exercise: Low-Beta Insertion

Consider the following low-beta insertion around an interaction point (IP). The quadrupoles are placed with mirror-symmetry with respect to the IP:



The beam enters the quadrupole with Twiss parameters $\beta_0 = 20$ m and $\alpha_0 = 0$. The drift space has length $L = 10$ m.

(i) Determine the focal length of the quadrupole in order to locate the waist at the IP.

(ii) What is the value of $\beta^\star$?

(iii) What is the phase advance between the quadrupole and the IP?