# Transverse Beam Dynamics

JUAS 2019 - Tutorial 2

## 1 Exercise: Particle momentum, geometry of a storage ring and thin lenses

The LHC storage ring at CERN collides proton beams with a maximum momentum of $p = 7$ TeV/c per beam. The main parameters of this machine are:

| | | |
|---|---|---|
| Circumference | $C_0 = 26658.9$ m | |
| Particle momentum | $p = 7$ TeV/c | |
| Main dipoles | $B = 8.392$ T | $l_B = 14.2$ m |
| Main quadrupoles | $G = 235$ T/m | $l_q = 5.5$ m |

1. Calculate the magnetic rigidity of the design beam, the bending radius of the main dipole magnets in the arc and determine the number of dipoles that is needed in the machine.
2. Calculate the k-strength of the quadrupole magnets and compare its focal length to the length of the magnet. Can this magnet be treated as a thin lens?

## 2 Exercise: Stability condition

Consider a lattice composed by a single 2 meters long quadrupole, with $f = 1$ m

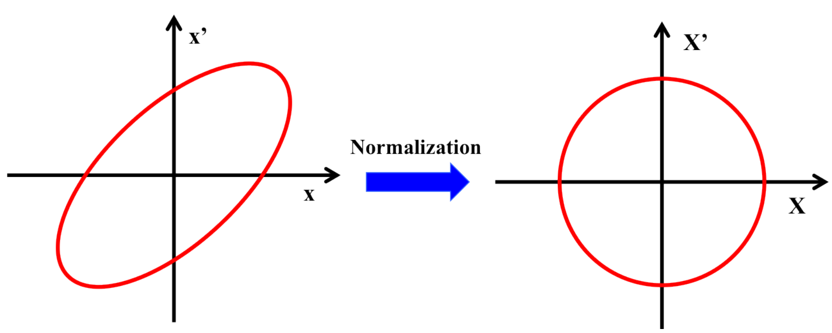
- Prove that if the quadrupole is defocusing, then a lattice is not stable
- Prove that if the quadrupole is focusing, then the lattice is stable

## 3 Exercise: Normalised phase space

Let us consider the following phase space vector: $(x, x')$. The transformation to a *normalised phase space* $(X, X')$ is given by:

$$\begin{pmatrix} X \\ X' \end{pmatrix} = \begin{pmatrix} 1/\sqrt{\beta_x} & 0 \\ \alpha_x/\sqrt{\beta_x} & \sqrt{\beta_x} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$

The normalisation process of the phase space is illustrated in the figure below:




If we know that the transfer matrix between two points 1 and 2 (with phase advance $\phi_x$ between them) in the phase space $(x, x')$ is given by:

$$M_{1\to 2} = \begin{pmatrix} \sqrt{\frac{\beta_{x2}}{\beta_{x1}}}(\cos\phi_x + \alpha_{x1}\sin\phi_x) & \sqrt{\beta_{x1}\beta_{x2}}\sin\phi_x \\ \frac{(\alpha_{x1}-\alpha_{x2})\cos\phi_x - (1+\alpha_{x1}\alpha_{x2})\sin\phi_x}{\sqrt{\beta_{x2}\beta_{x1}}} & \sqrt{\frac{\beta_{x1}}{\beta_{x2}}}(\cos\phi_x - \alpha_{x2}\sin\phi_x) \end{pmatrix}$$

Obtain the transfer matrix between two points 1 and 2 in the normalised phase space.

## 4 Exercise: beam size and luminosity

An $e^+e^-$ collider has an interaction Point (IP) with $\beta_x^* = 0.5$ m and $\beta_y^* = 0.1$ cm. The peak luminosity available by a $e^+e^-$ collider can be written as:

$$L = \frac{N_\mathrm{b} N_{e^-} N_{e^+} f_\mathrm{rev}}{4\pi\sigma_x^*\sigma_y^*}\ [\mathrm{cm}^{-2}\mathrm{s}^{-1}]$$

where $N_\mathrm{b} = 80$ is the number of bunches per beam (we assume the same number of bunches for both the $e^-$ and the $e^+$ beams), $N_{e^-} = N_{e^+} = 5 \times 10^{11}$ is the number of particles per bunch (we assume the same number for both $e^-$ and $e^+$ bunches), and $f_\mathrm{rev}$ is the revolution frequency. The horizontal and vertical normalised beam emittances are respectively: $\epsilon_{x,N} = 2.2$ mm and $\epsilon_{y,N} = 4.7$ $\mu$m.

- Compute the revolution frequency $f_\mathrm{rev}$, knowing that the circumference is 80 km and that the beam moves nearly at the speed of light
- Calculate the beam transverse beam sizes $\sigma_x^*$ and $\sigma_y^*$ at the IP, and the luminosity $L$ for two different beam energies: 45 GeV and 120 GeV
- What are the beam divergences (horizontal and vertical) at the IP for the 45 GeV case?
- What is the value of the betatron function at position $s = 0.5$ m from the IP?