Introduction for Magnets

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JUAS
Archamps

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2. Fundamentals 1: Maxwell and friends
3. Fundamentals 2: harmonics

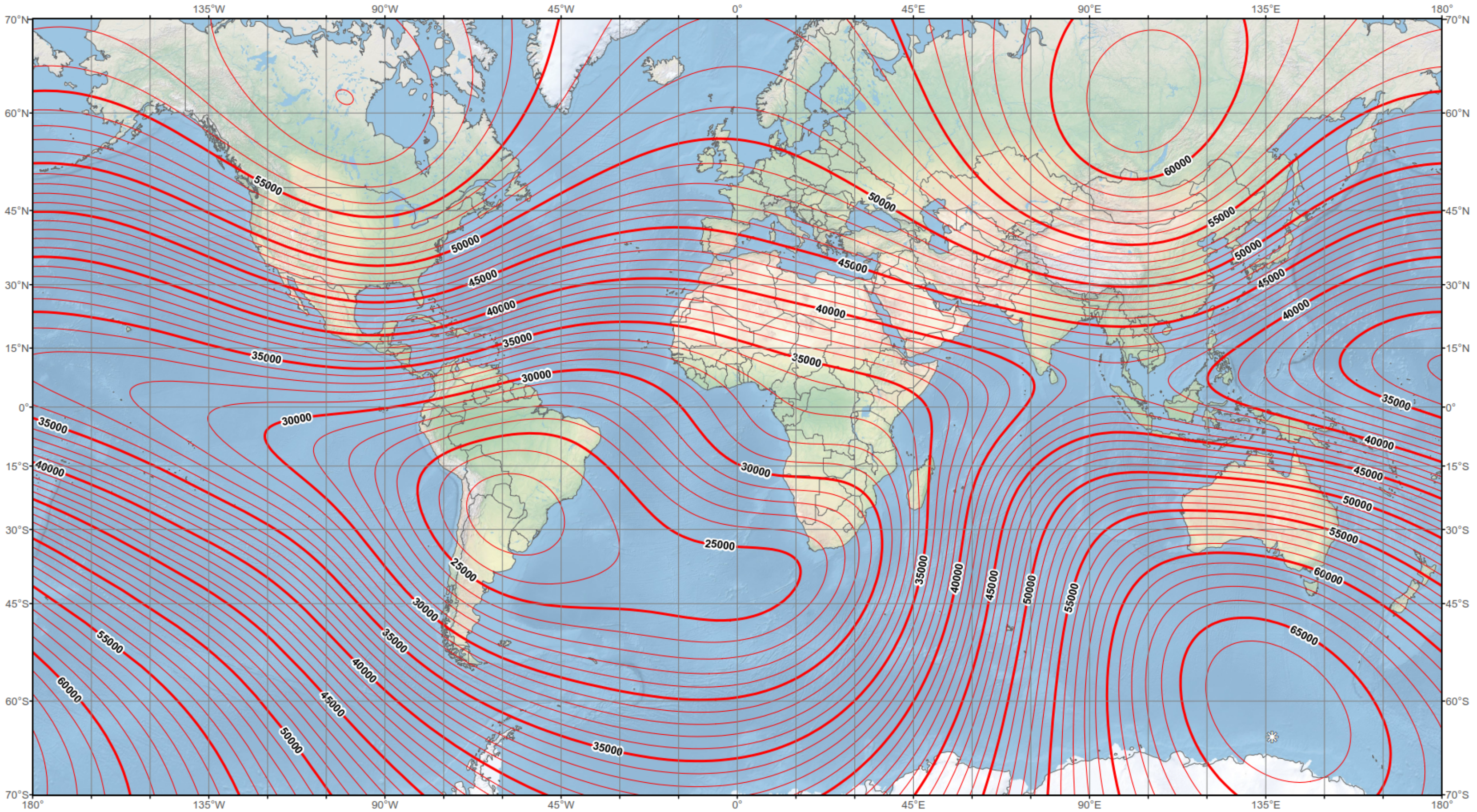
This lecture is a based on previous lectures by Attilio Milanese and Davide Tommasini

Earth magnetic field

In Archamps, on 30/01/2019, the (estimated) magnetic field (flux density) is

$|B| = 47447\ \mathrm{nT} = 0.047447\ \mathrm{mT} = 4.7447 \cdot 10^{-5}\ \mathrm{T} \approx 0.5\ \mathrm{Gauss}$

US/UK World Magnetic Model - Epoch 2015.0
Main Field Total Intensity (F)



Main Field Total Intensity (F)
Contour interval: 1000 nT.
Mercator Projection.
✲: Position of dip poles

Map developed by NOAA/NGDC & CIRES
http://ngdc.noaa.gov/geomag/WMM
Map reviewed by NGA and BGS
Published December 2014

Magnet types, functional view

We can classify magnets based on their geometry (that is, what they do to the beam)

dipole	bend
quadrupole	focus
sextupole	Chromatic effects
octupole	damping
kicker / septum	Injection - extraction
solenoid	focus
combined function bending	Bend and focus
corrector	Correct errors
skew magnet	coupling
undulator / wiggler	Synchrotron light

Magnet types, technological view

We can also classify magnets based on their technology

electromagnet

permanent magnet

iron dominated

coil dominated

normal conducting (resistive)

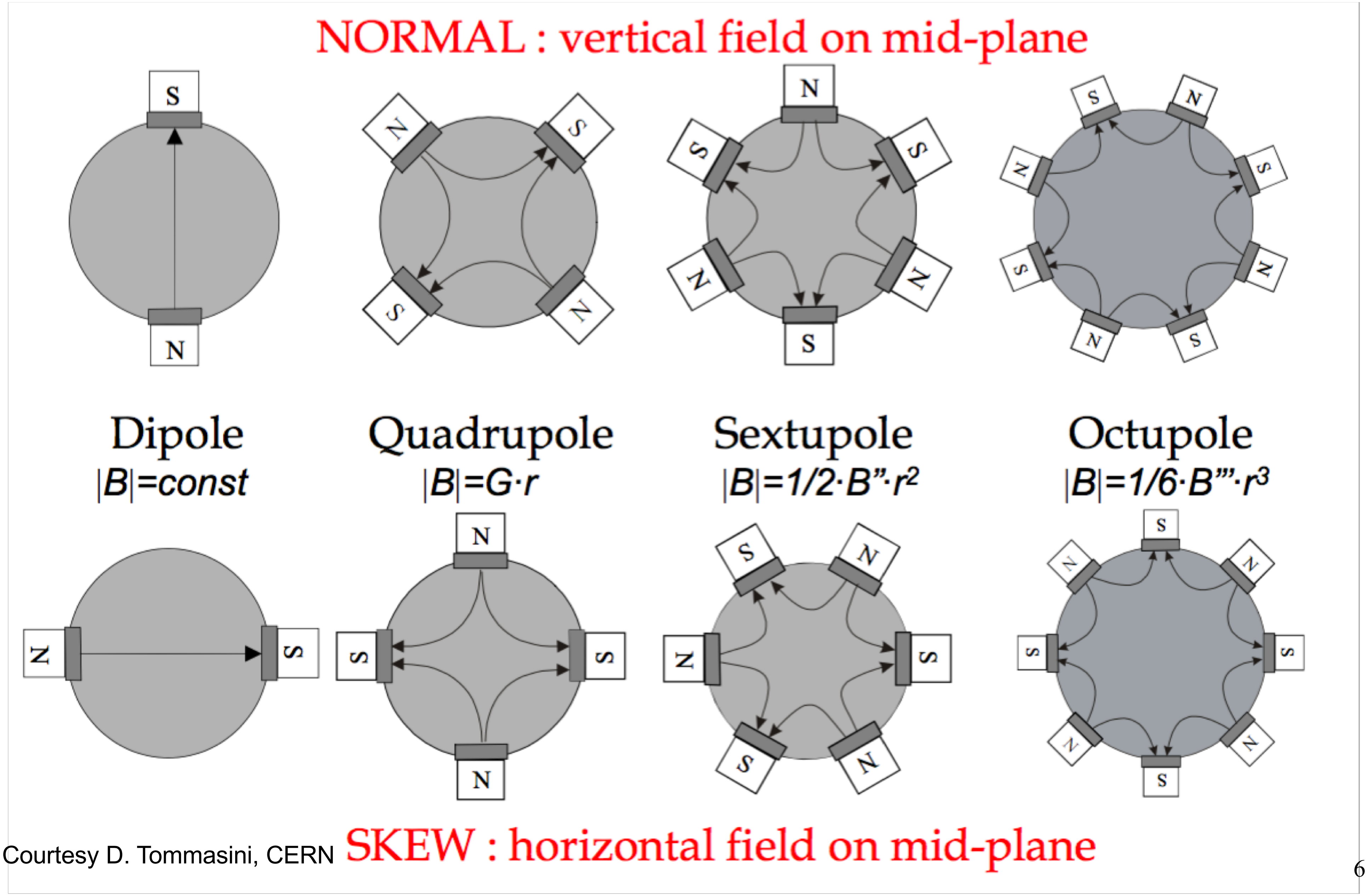
superconducting

static

cycled / ramped slow pulsed

fast pulsed

Types of iron dominated, resistive magnet fields for accelerators


NORMAL : vertical field on mid-plane

Dipole
$|B|=const$

Quadrupole
$|B|=G\cdot r$

Sextupole
$|B|=1/2\cdot B''\cdot r^2$

Octupole
$|B|=1/6\cdot B'''\cdot r^3$

Courtesy D. Tommasini, CERN

SKEW : horizontal field on mid-plane

Types of superconducting magnet fields for accelerators

a “pure” multipolar field can be generated by a specific coil geometry

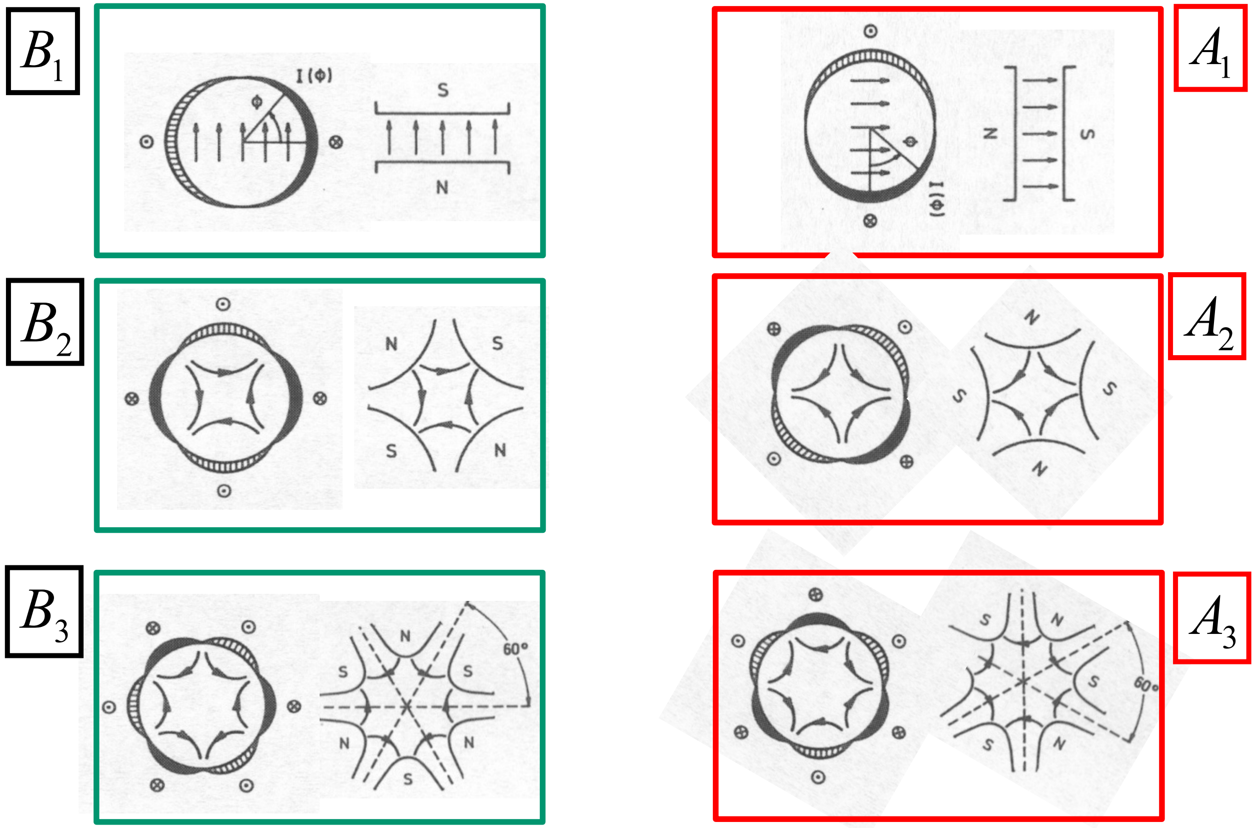
normal

skew

dipole n=1

quadrupole n=2

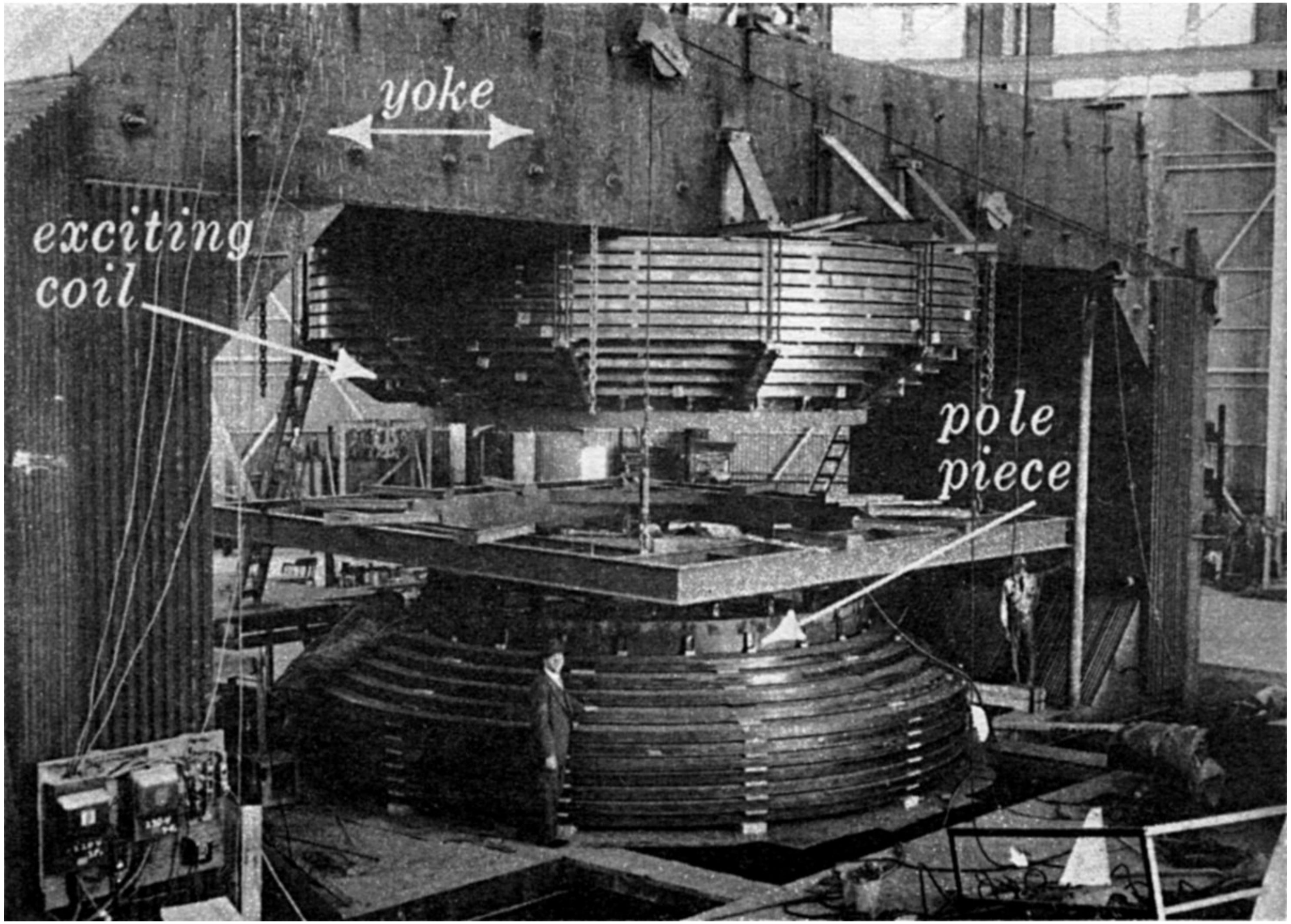
sextupole n=3



Courtesy P. Ferracin, CERN

Early Cyclotron

The 184’’ (4.7 m) cyclotron at Berkeley (1942)

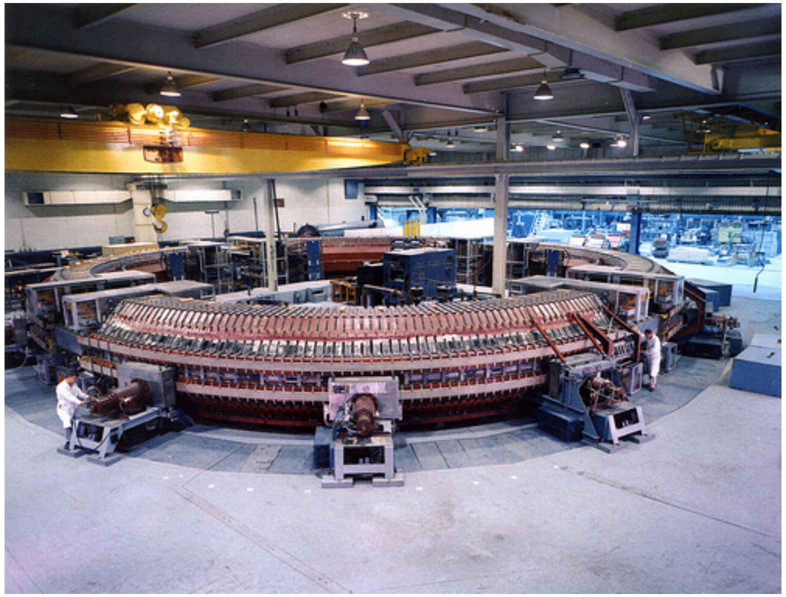


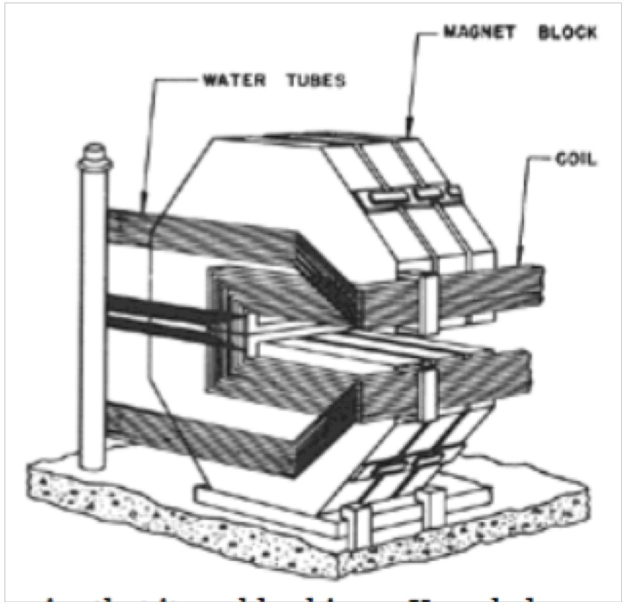
Courtesy A. Milanese, CERN

Some early synchrotron magnets (early 1950-ies)

Bevatron
(Berkeley)
1954, 6.2 GeV

Cosmotron
(Brookhaven)
1953, 3.3 GeV
Aperture:
20 cm x 60 cm



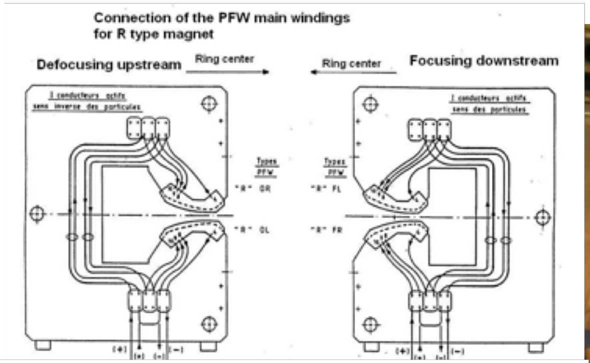


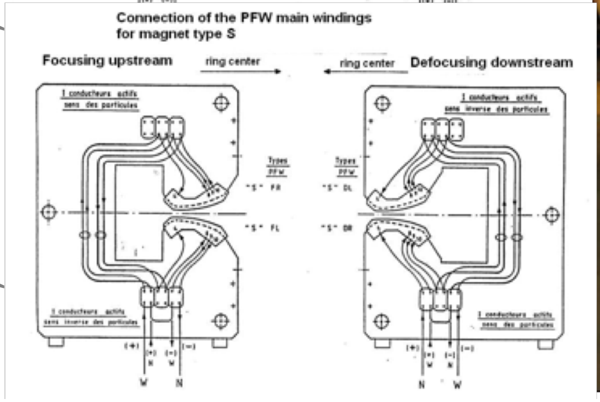
PS combined function dipole (1959)

Magnetic field:	
at injection	147 G
for 24.3 GeV	1.2 T
maximum	1.4 T
Weight of one magnet unit	38 t

Gradient @1.2 T : 5 T/m

Equipped with pole-face windings for higher order corrections





Water cooled Al race-track coils

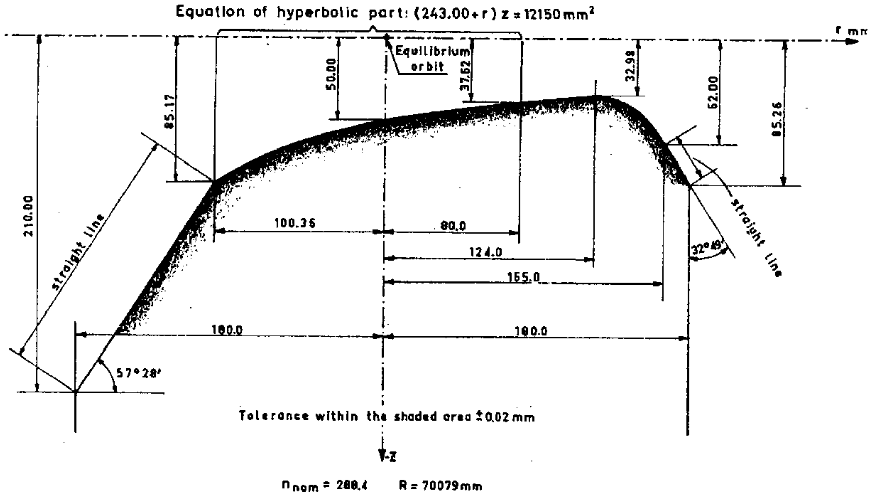


Fig. 9: Final pole profile.

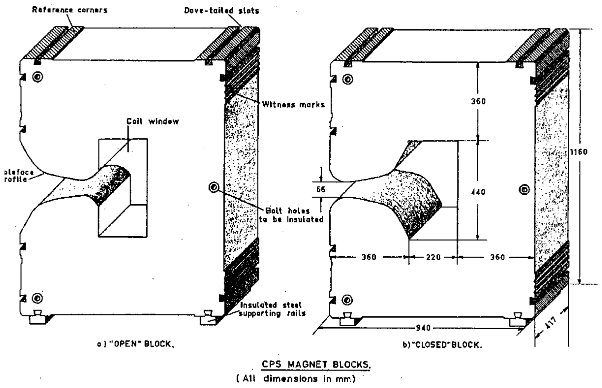
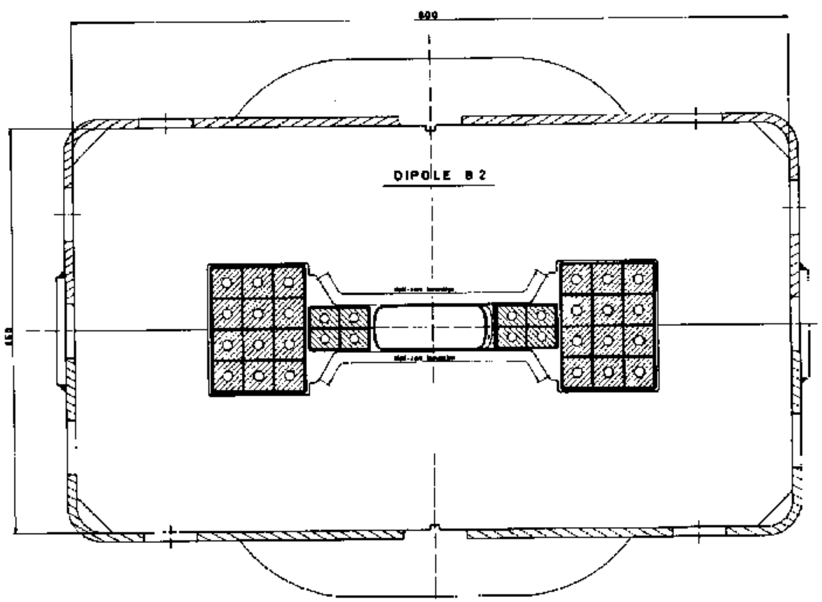


Fig. 12: Final form of the magnet blocks.

Courtesy D. Tommasini, CERN

dipole magnet : SPS dipole (1975)



H magnet type MBB

B = 2.05 T

Coil : 16 turns

I_{max} = 4900 A

Aperture = 52 × 92 mm^2

L = 6.26 m

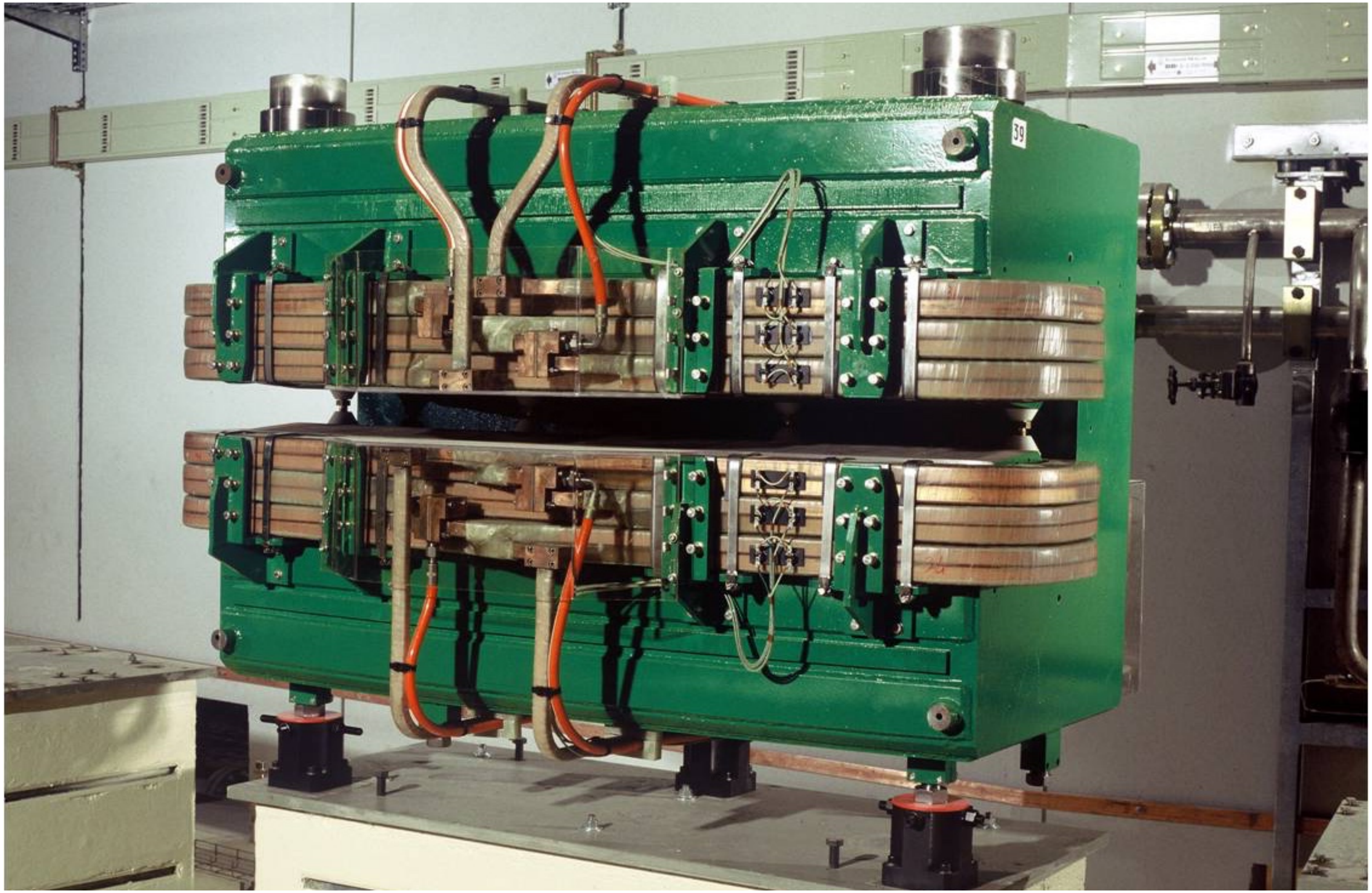
Weight = 17 t

SPS main dipole

These are main quadrupoles of the SPS at CERN: 22 T/m × 3.2 m

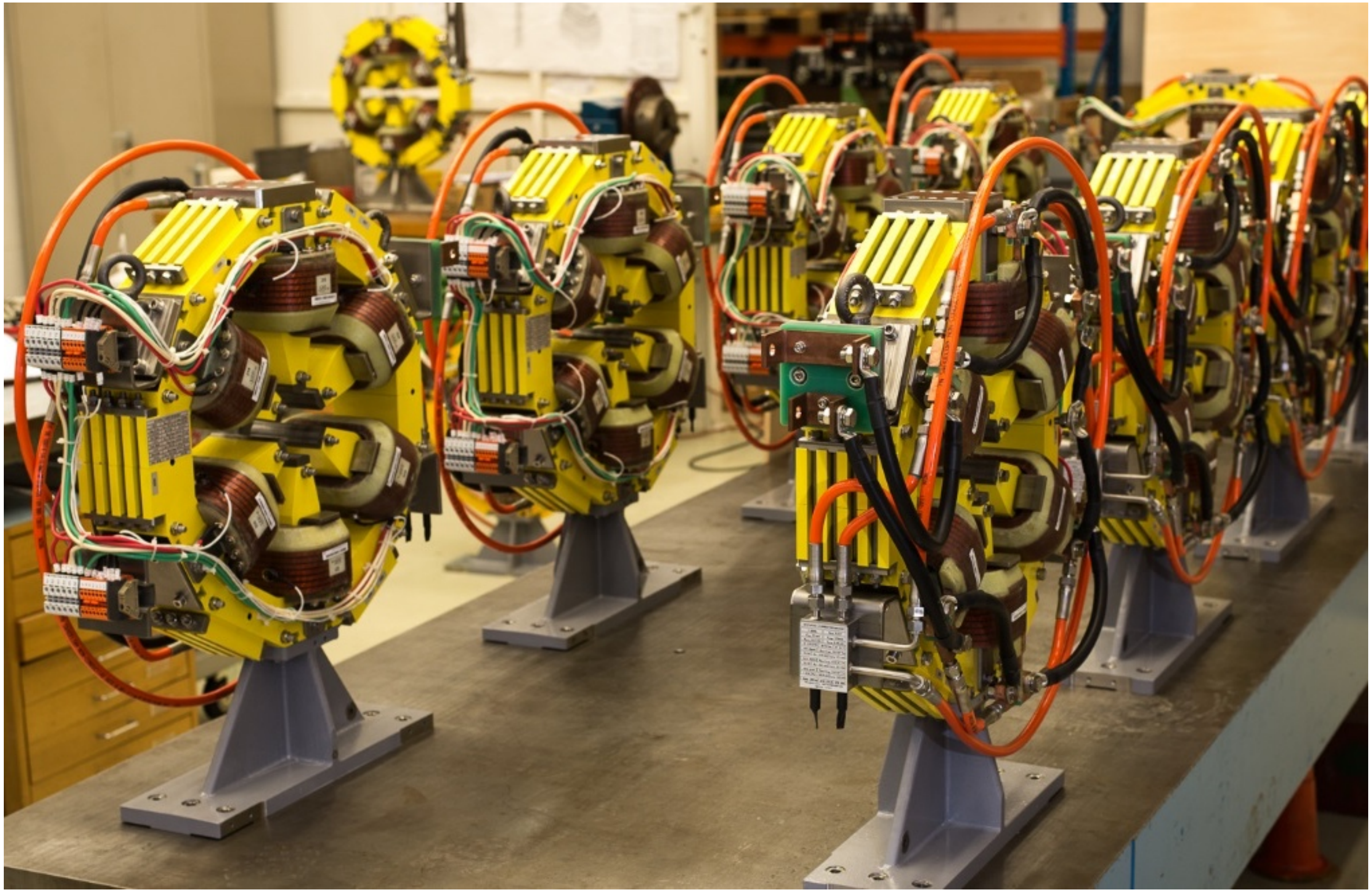
Elettra combined function magnet

This is a combined function bending magnet of the ELETTRA light source



SESAME sextupoles

These are sextupoles (with embedded correctors) of the main ring of the SESAME light source



Beam Transfer line magnets: Castor and Cesar

1977: Very first SC magnets at CERN in an SPS beam line

- CESAR dipole: aperture 150 mm, B=4.5 T
  l = 2 m
- CASTOR quadrupole

Both use a monolithic conductor would into a cosΘ coil

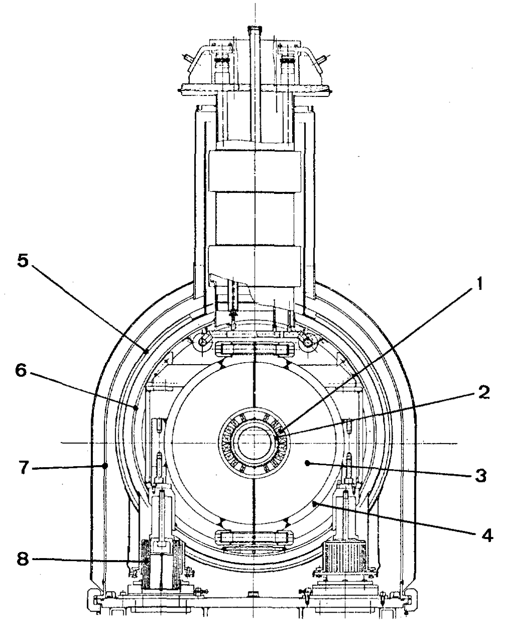
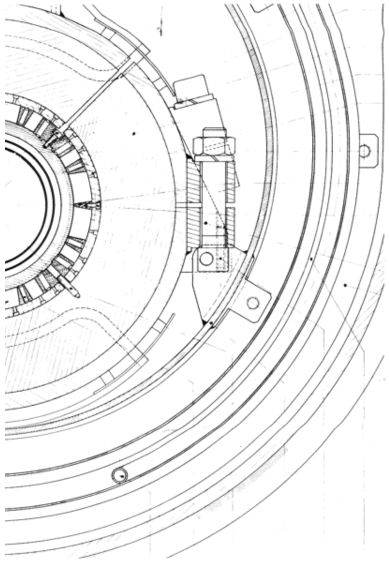
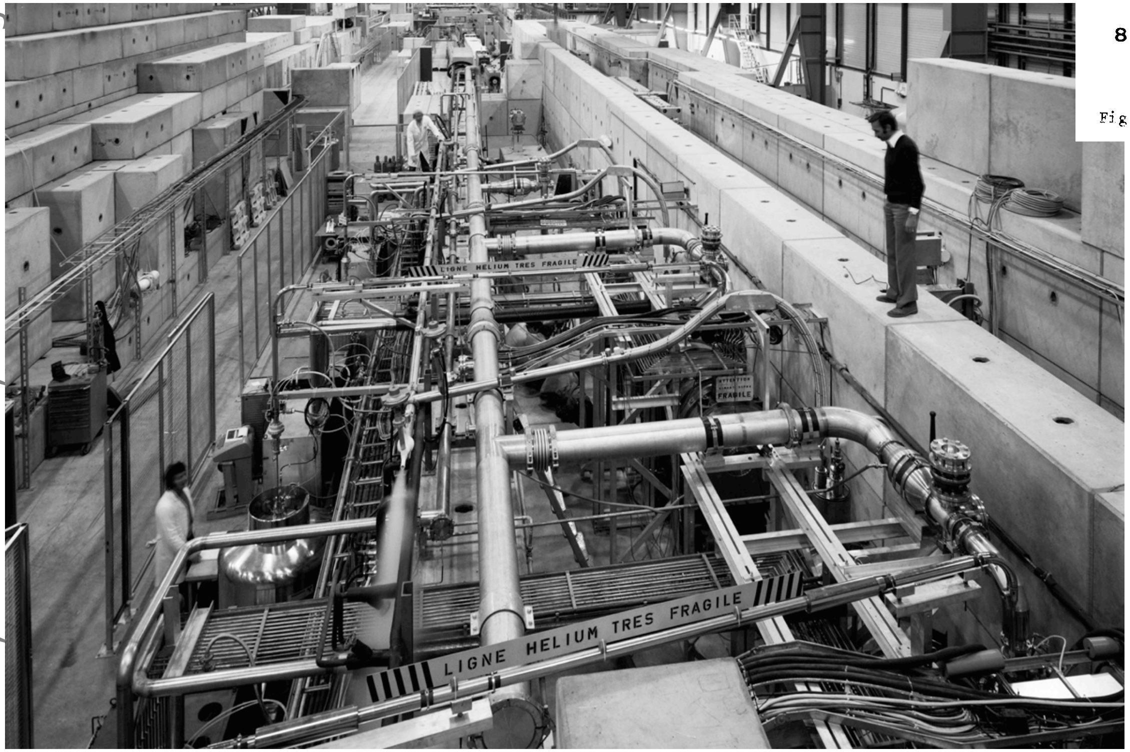
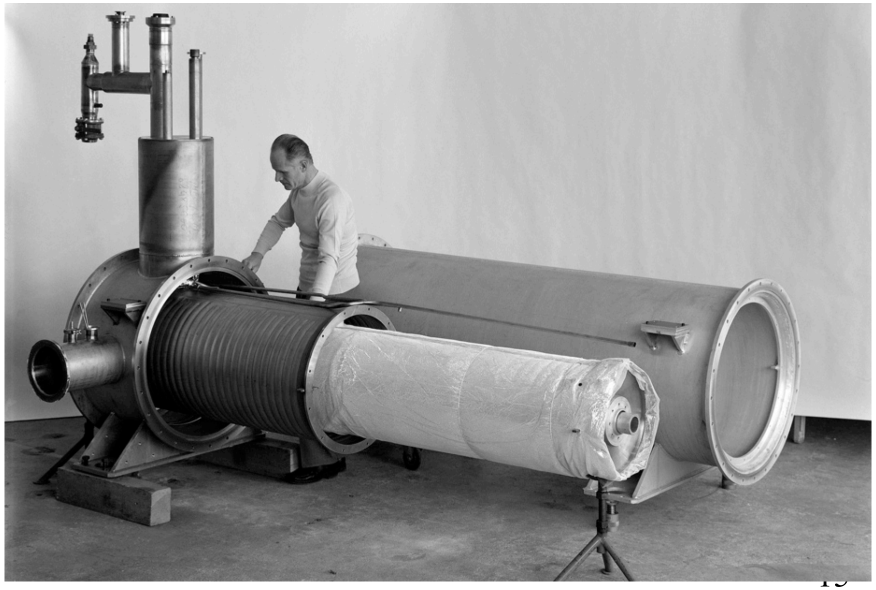


Fig.1. Magnet cross section.

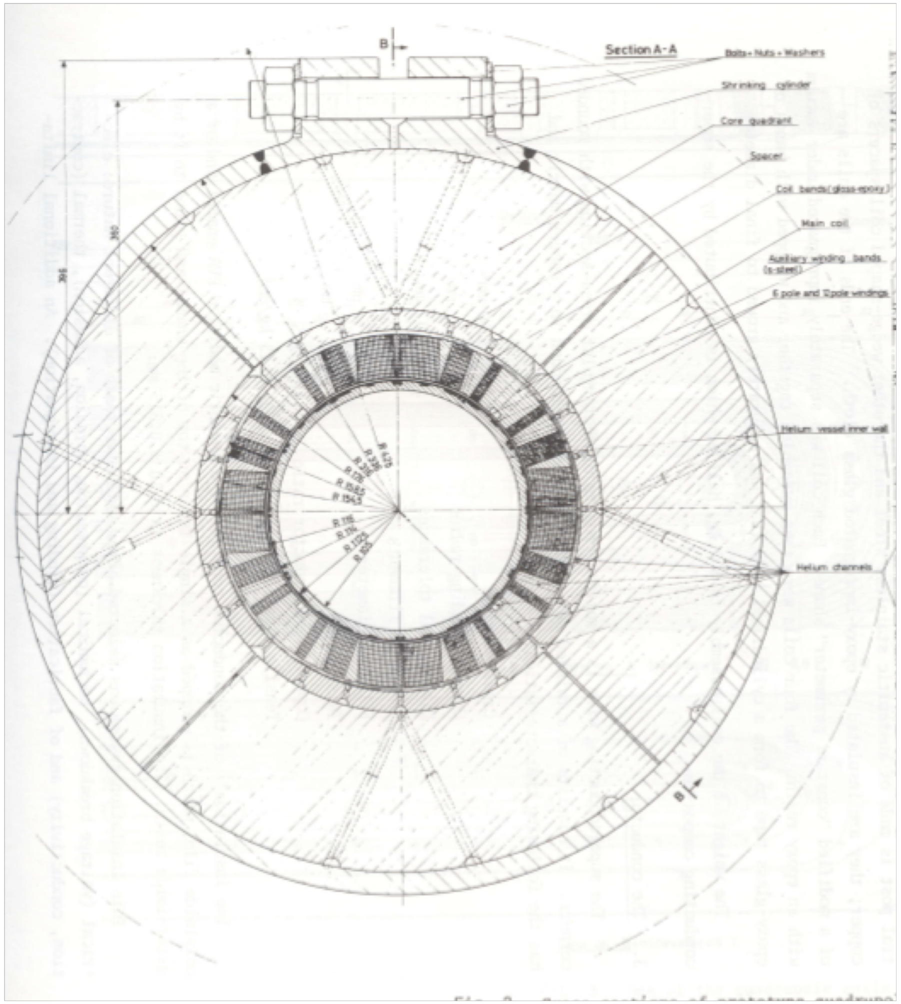


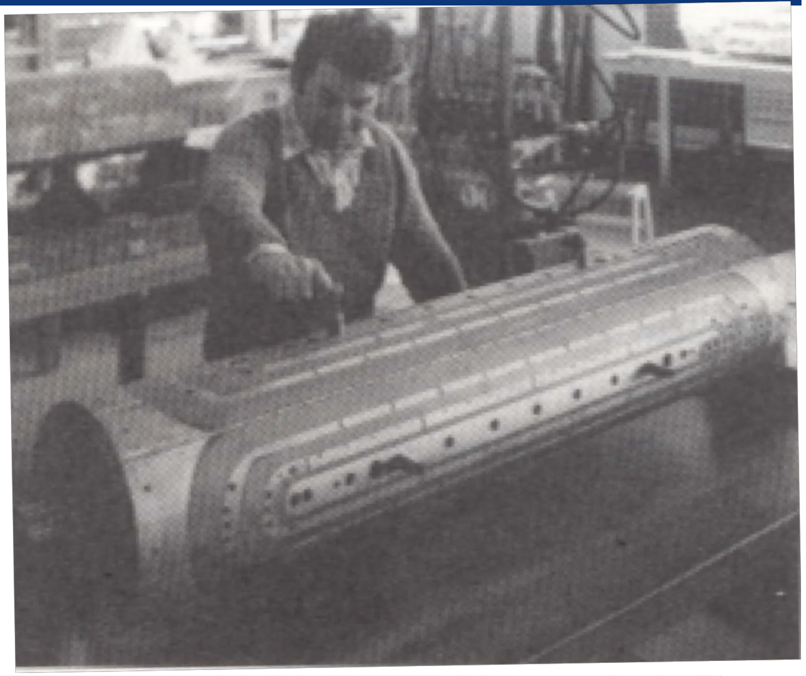


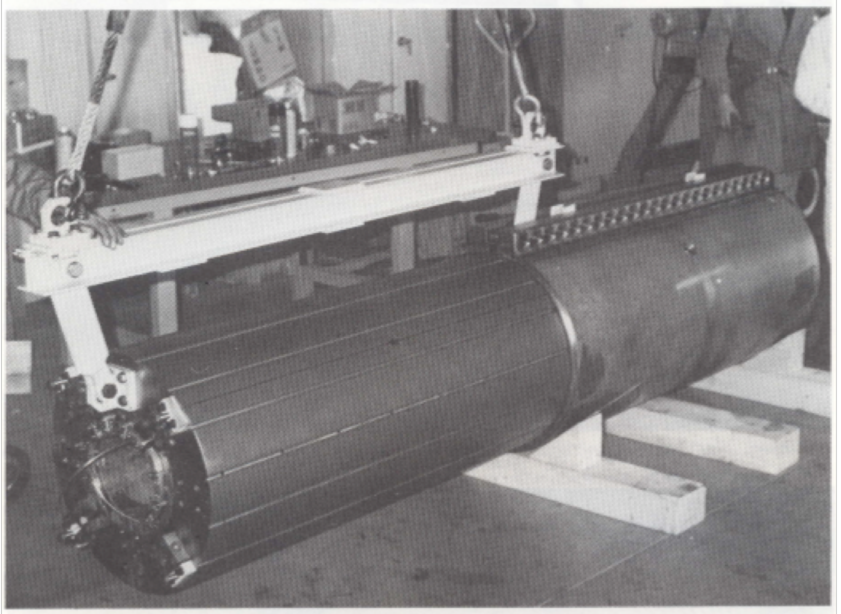


ISR Insertion quadrupole

- Nb-Ti monolitic conductor
- fully impregnated coil
- Prestress from yoke + shell

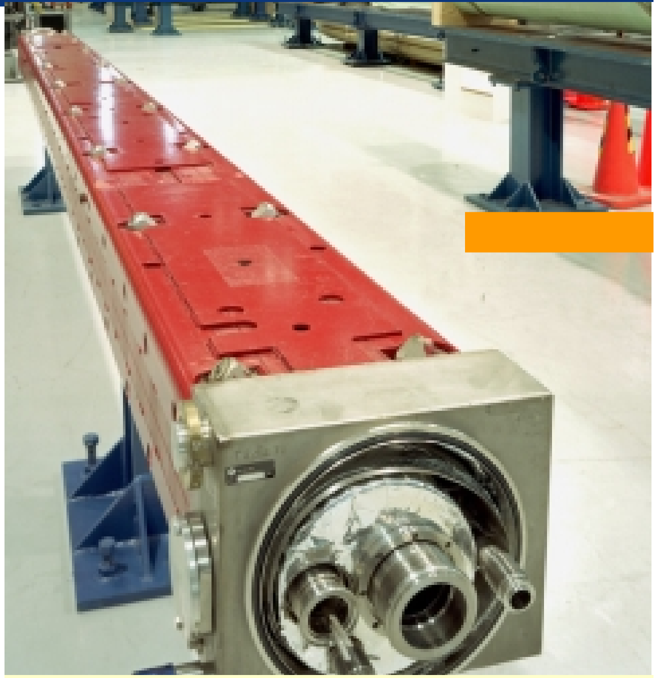




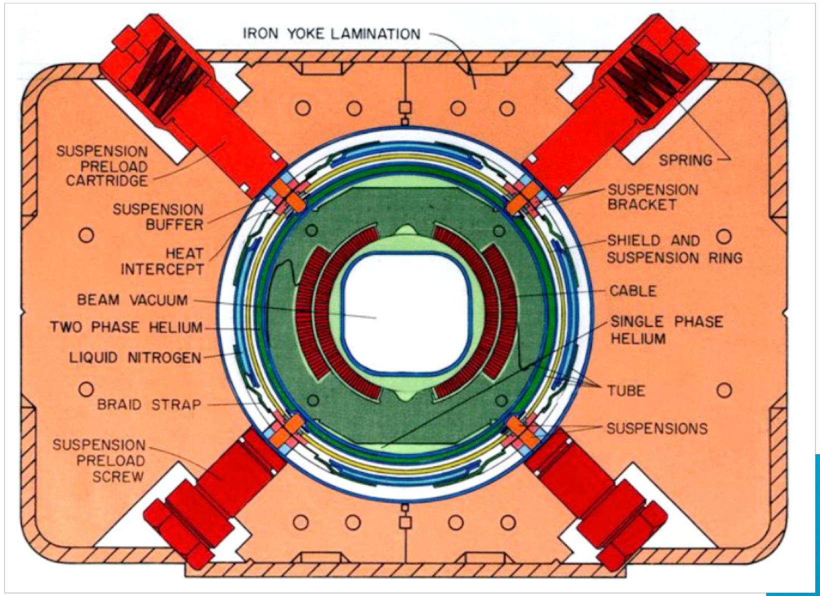


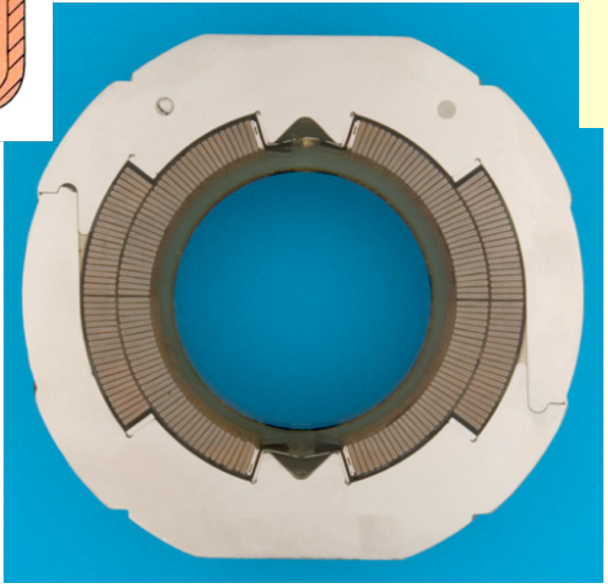
Tevatron proton-antiproton ring

- Nb-Ti conductor at 4.2 K
- Collars for prestress
- warm iron



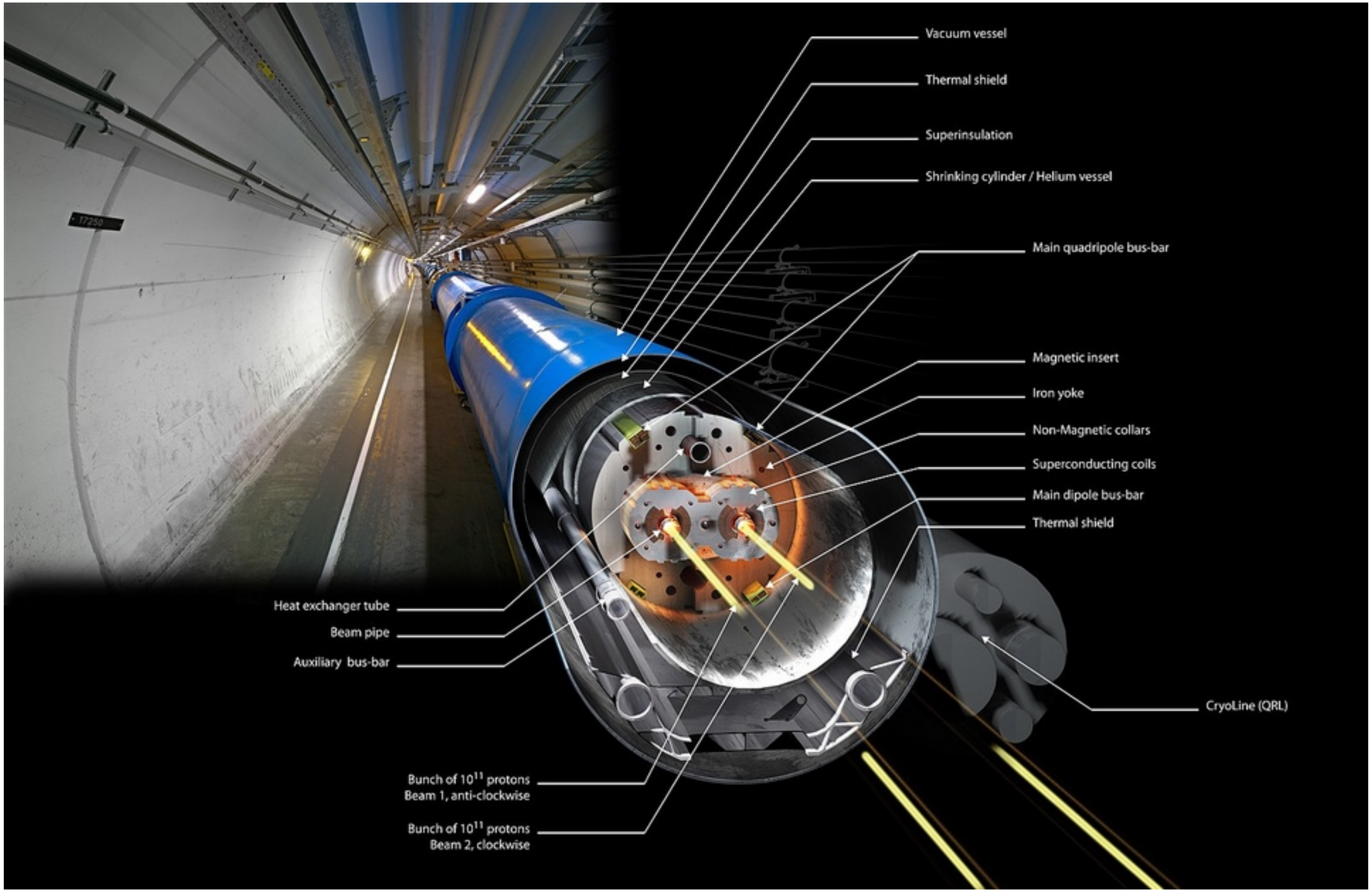
Tevatron dipoles: 4.2 T
single aperture, warm yoke





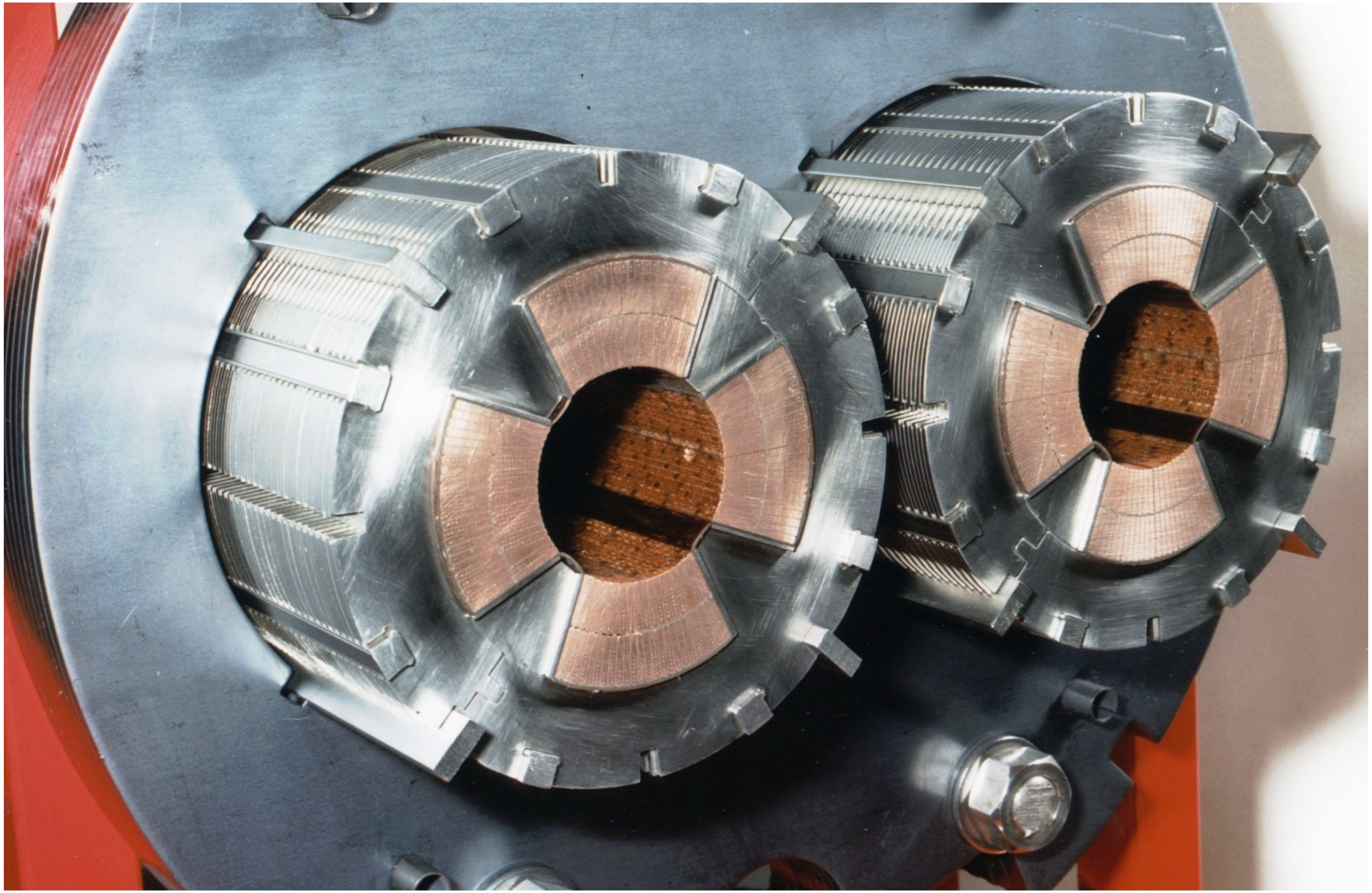
LHC dipole

This is a main dipole of the LHC at CERN: 8.3 T × 14.3 m



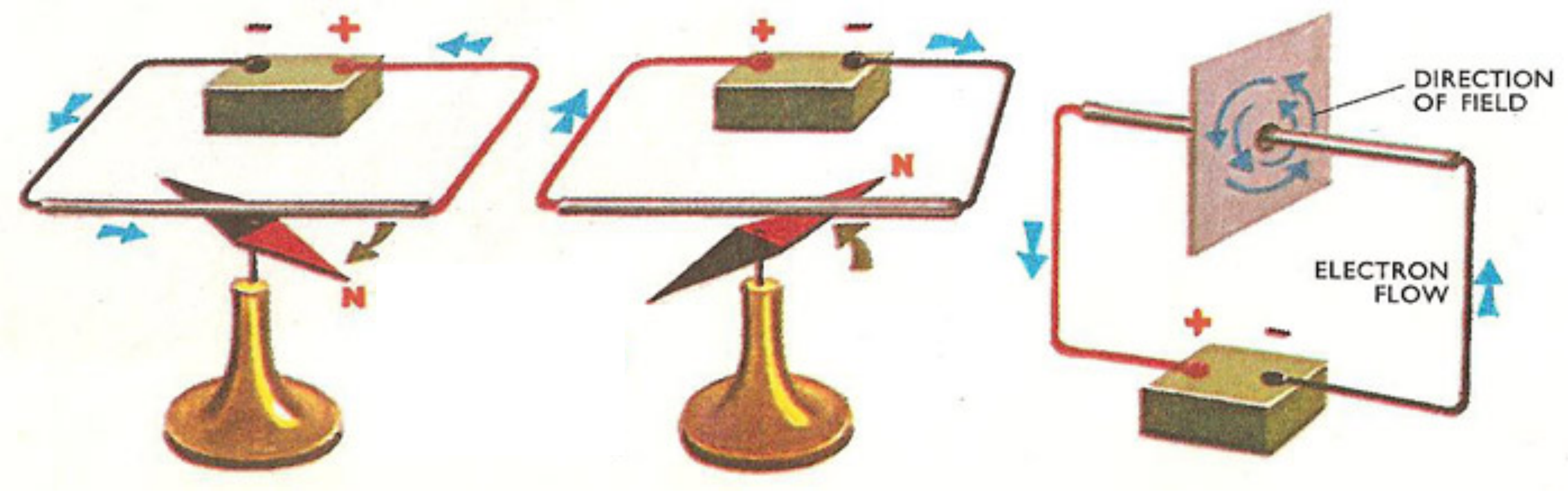
LHC main quadrupole

This is a cross section of a main quadrupole of the LHC at CERN:
223 T/m × 3.2 m



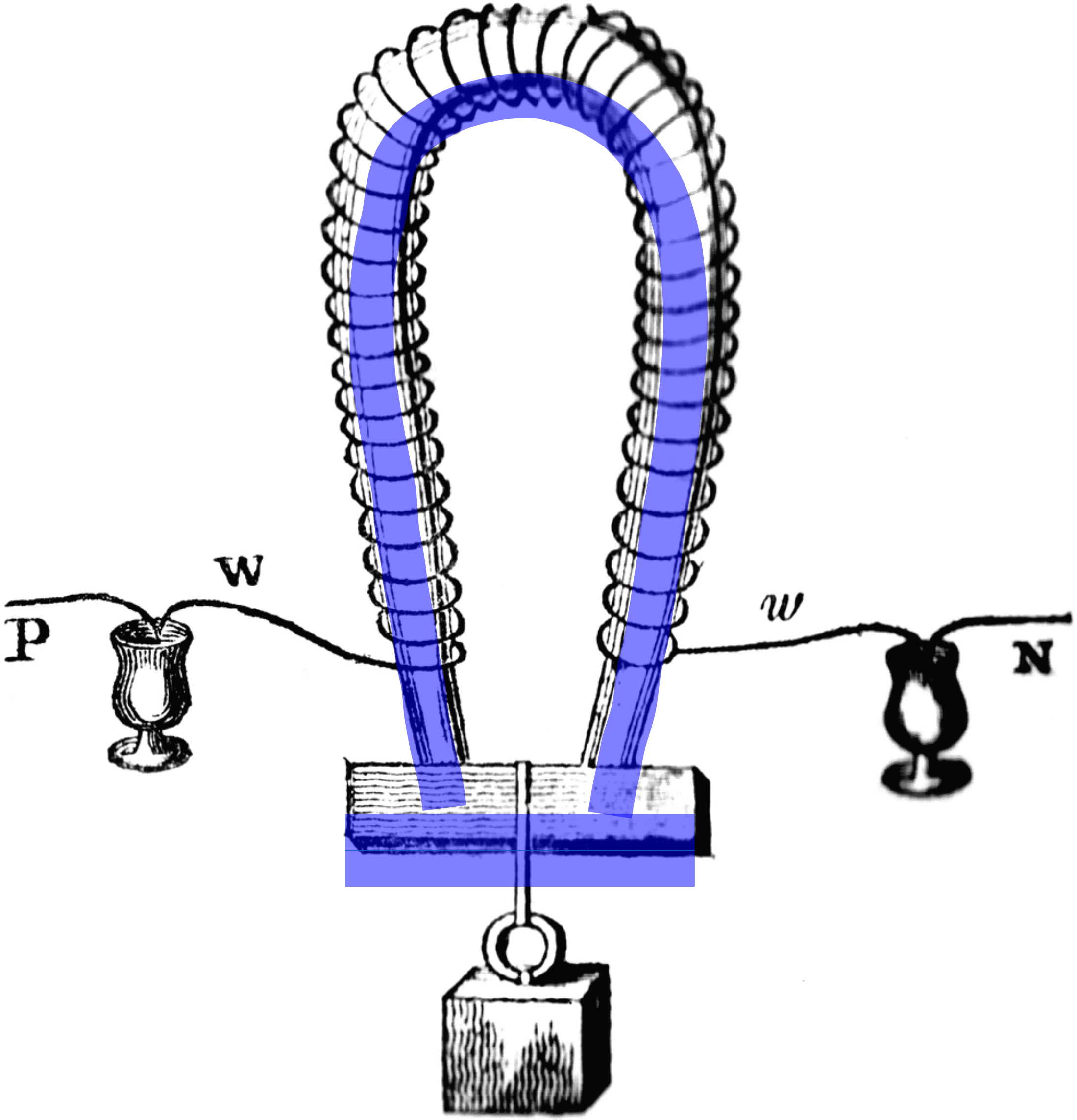
Electro-magnetism

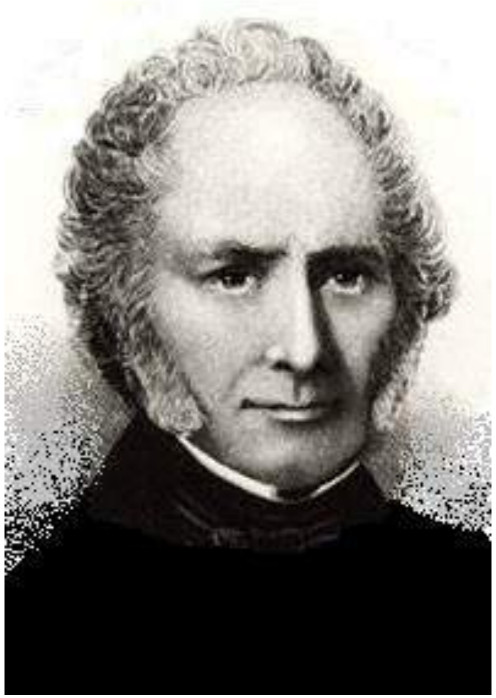
Ørsted showed in 1820 that electricity and magnetism were somehow related



Electromagnet

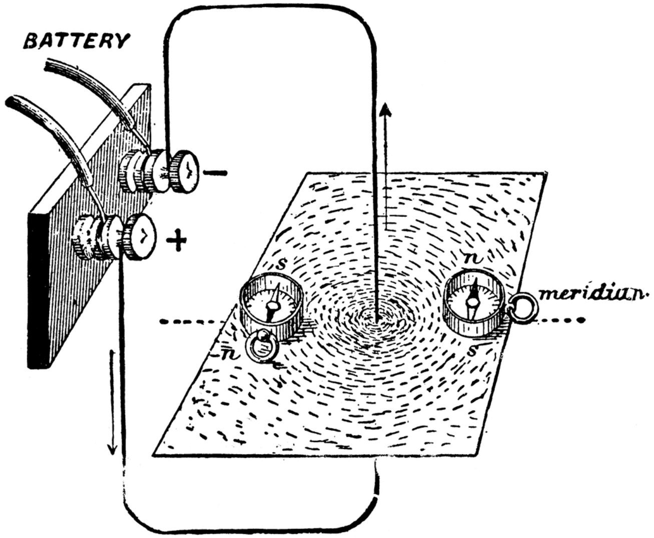
The first electromagnet was built in 1824 by Sturgeon



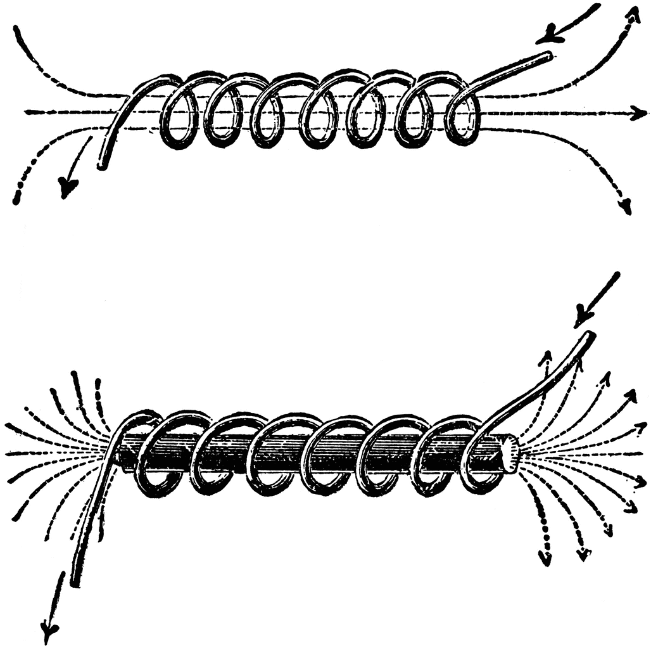


Basic magnet type

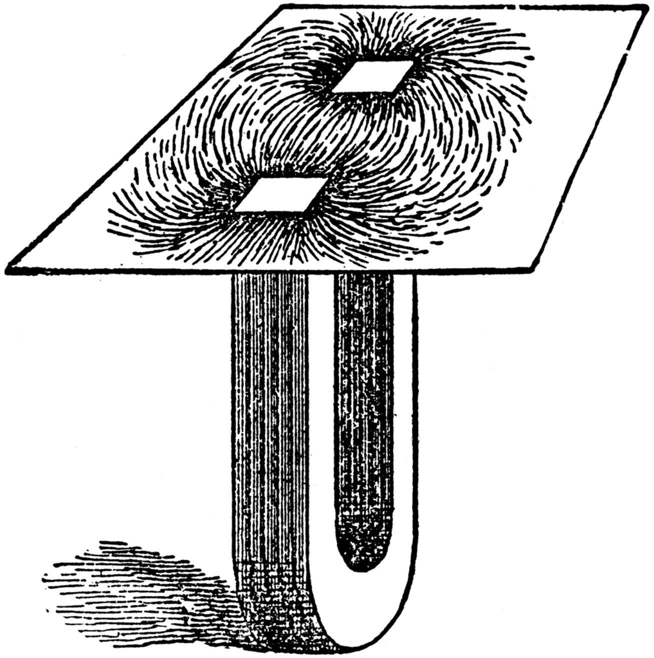
Our magnets work on a few basic principles (steady state only)



an electrical current induces a magnetic effect



some materials (e.g. iron) greatly enhance these effects



some other materials produce these effects even without electrical currents

1. Introduction

2. Fundamentals 1: Maxwell and friends

3. Fundamentals 2: harmonics

So, how do we properly describe all this? I

Maxwell Equations

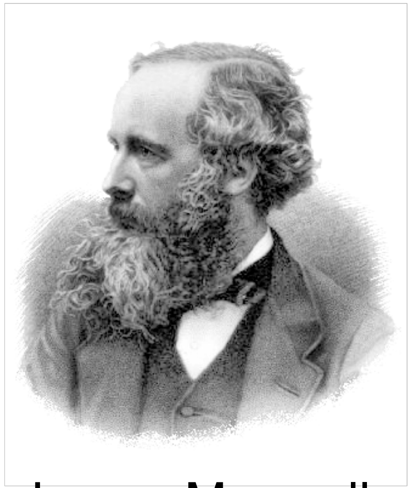
Integral form		Differential form
$\oint \vec{H} d\vec{s} = \int_A \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) d\vec{A}$	Ampere's law	$rot\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$
$\oint \vec{E} d\vec{s} = -\frac{\partial}{\partial t} \int_A \vec{B}\, d\vec{A}$	Faraday's equation	$rot\vec{E} = -\frac{\partial \boldsymbol{\vec{B}}}{\partial t}$
$\int_A \vec{B}\, d\vec{A} = 0$	Gauss's law for magnetism	$div\vec{B} = 0$
$\int_A \vec{D}\, d\vec{A} = \int_V \rho\, dV$	Gauss's law	$div\vec{D} = \rho$

With:

$$\vec{B} = \mu\vec{H} = \mu_0(\vec{H} + \vec{M})$$

$$\vec{D} = \varepsilon\vec{E} = \varepsilon_0(\vec{E} + \vec{P})$$

$$\vec{J} = \kappa\vec{E} + J_{imp.}$$



James Maxwell
1831 – 1879

So, how do we properly describe all this? II

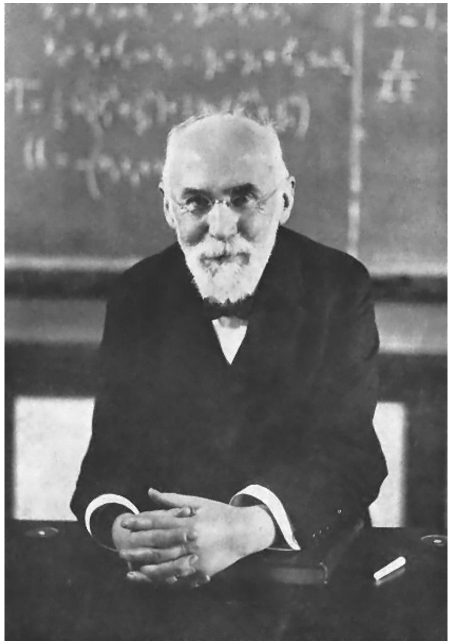
Lorentz force

$$\overrightarrow{F_m} = q\left(\vec{v}\times\vec{B}\right)$$

for charged beams

$$\overrightarrow{F_m} = I\vec{\ell}\times\vec{B}$$

for conductors



Hendrik Lorentz
1853 –1928

Nomenclature

B	flux density magnetic field B field magnetic induction	T (Tesla)
H	magnetic field magnetic field strength H field	A/m (Ampere/m)
μ_0	permeability of vacuum	$4\pi \cdot 10^{-7}$ H/m (Henry/m)
μ_r	relative permeability	dimensionless
μ	permeability, $\mu = \mu_0 \mu_r$	H/m

Magnetostatics

Let's have a closer look at the 3 equations that describe magnetostatics

Gauss law of magnetism	(1) $\mathrm{div}\,\vec{B} = 0$	always holds
Ampere's law with no time dependencies	(2) $\mathrm{rot}\,\vec{H} = \vec{J}$	holds for magnetostatics
Relation between $\vec{H}$ field and the flux density $\vec{B}$	(3) $\vec{B} = \mu_0 \mu_r \vec{H}$	holds for linear materials

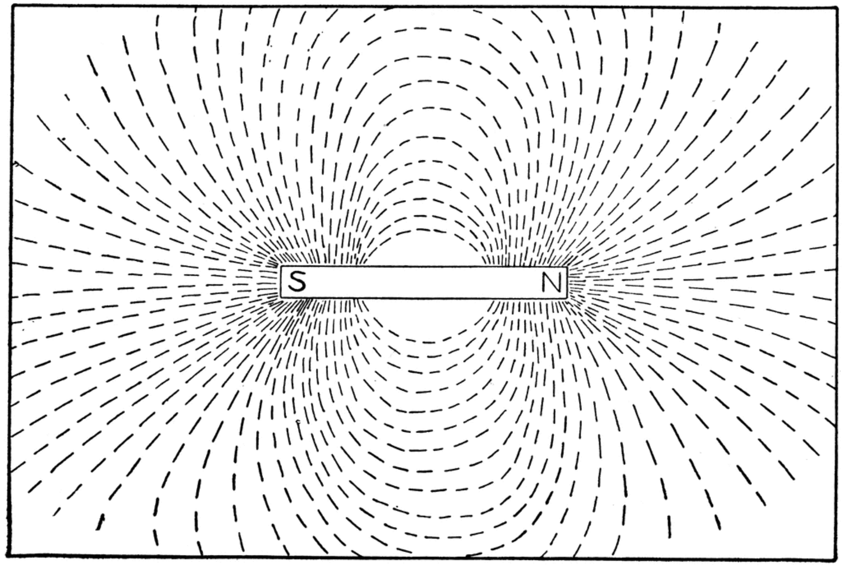
Divergence free fields

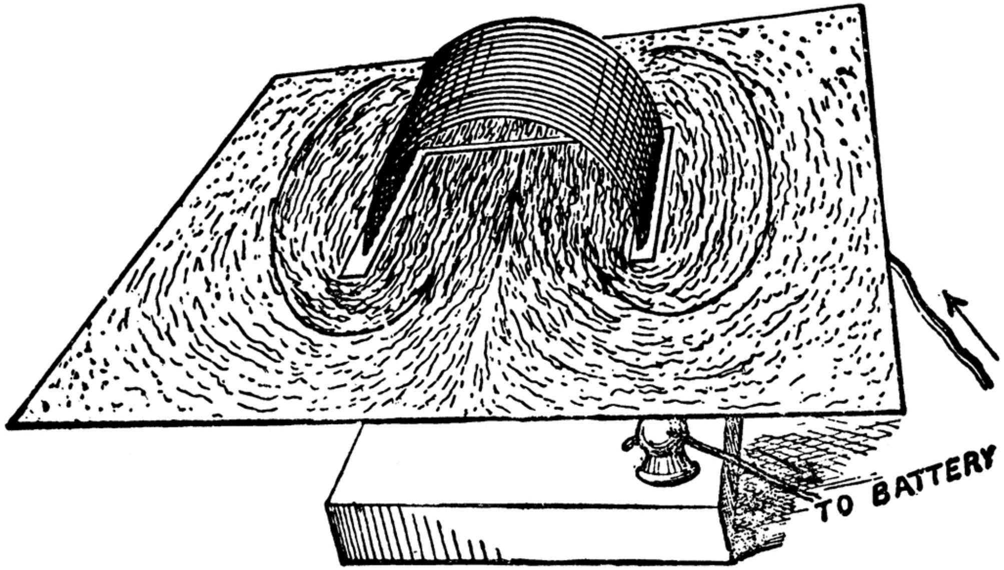
Gauss law of magnetism:
the magnetic flux tubes wrap around, with neither sources nor sinks

$$\mathrm{div}\,\vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

$$\oiint \vec{B} \cdot \overrightarrow{dS} \underset{\uparrow}{=} \iiint \mathrm{div}\,\vec{B}\,dV = 0$$

divergence / Gauss theorem





Electrical currents generate magnetic fields

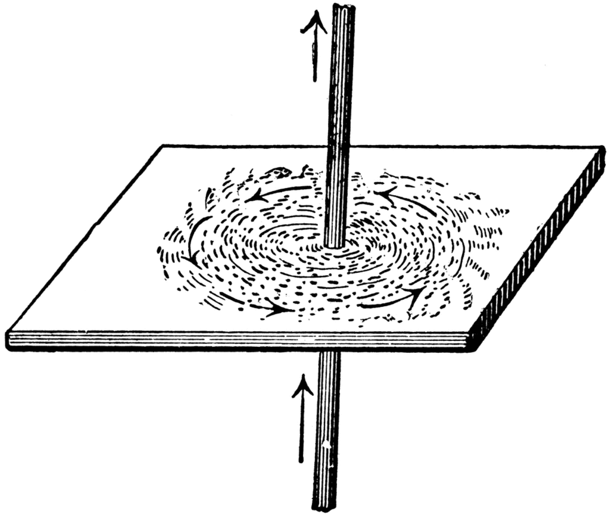
Ampere's law:

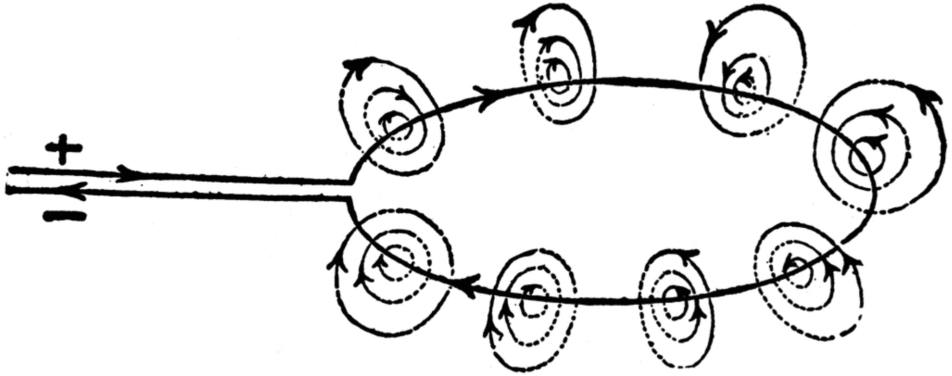
electrical currents generate ("stir up") a magnetic field

$$\text{rot}\,\vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\overrightarrow{i_x} + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\overrightarrow{i_y} + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\overrightarrow{i_z} = \vec{J}$$

$$\oint \vec{H}\cdot\overrightarrow{dl} \underset{\uparrow}{=} \iint \text{rot}\,\vec{H}\,dS = \iint \vec{J}\,dS = NI$$

Kelvin–Stokes theorem





Law of Biot & Savart

From Ampere's law without time dependencies and Gauss law we can derive the Biot & Savart law

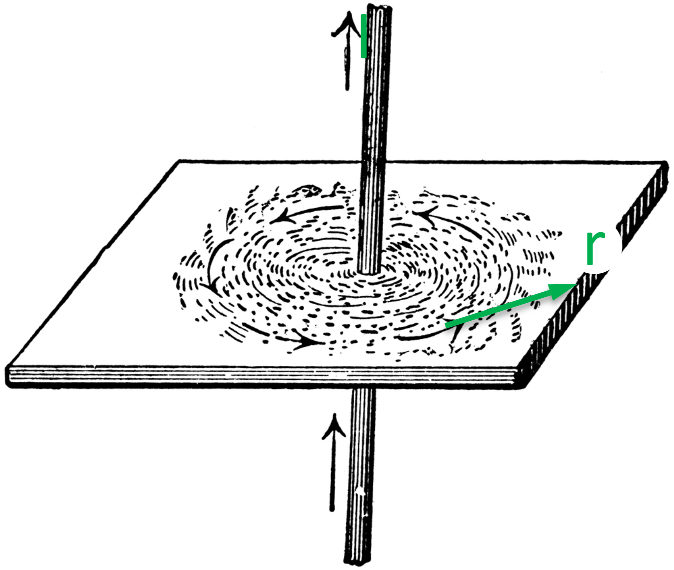
$\oint \vec{H} \cdot \vec{dl} = I$ ->

H $(2\pi r) = I$ ->

$$H = \frac{I}{2\pi r}$$

$$\vec{B} = \mu_0 \vec{H}$$

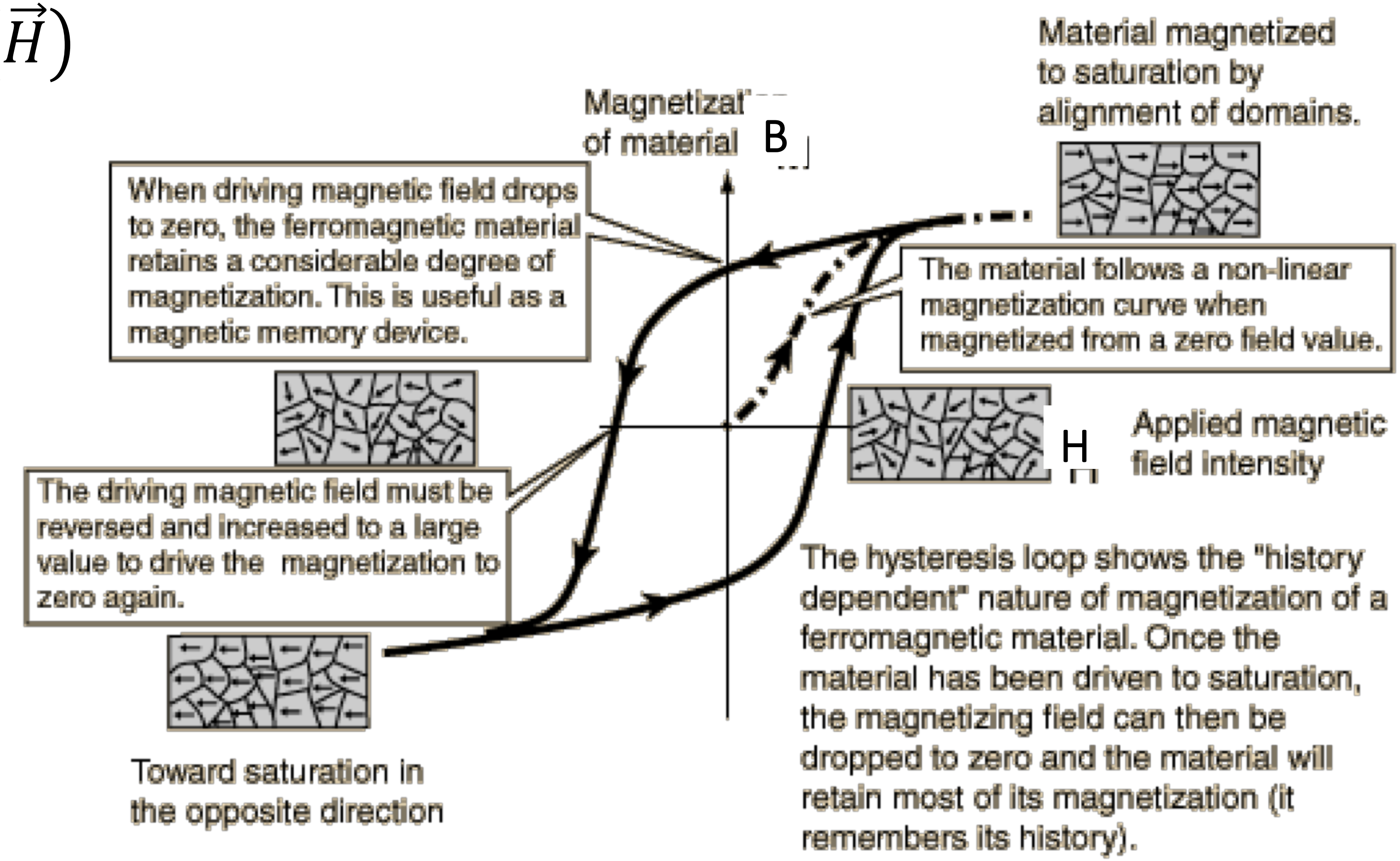
-> $B = \mu_0 H = \frac{\mu_0 I}{2\pi r}$



Non-linear materials - magnetisation

In a nonlinear material (with for ex. saturation and hysteresis), the constitutive law becomes more complex

$$\vec{B} = \mu_0 \vec{f}(\vec{H})$$



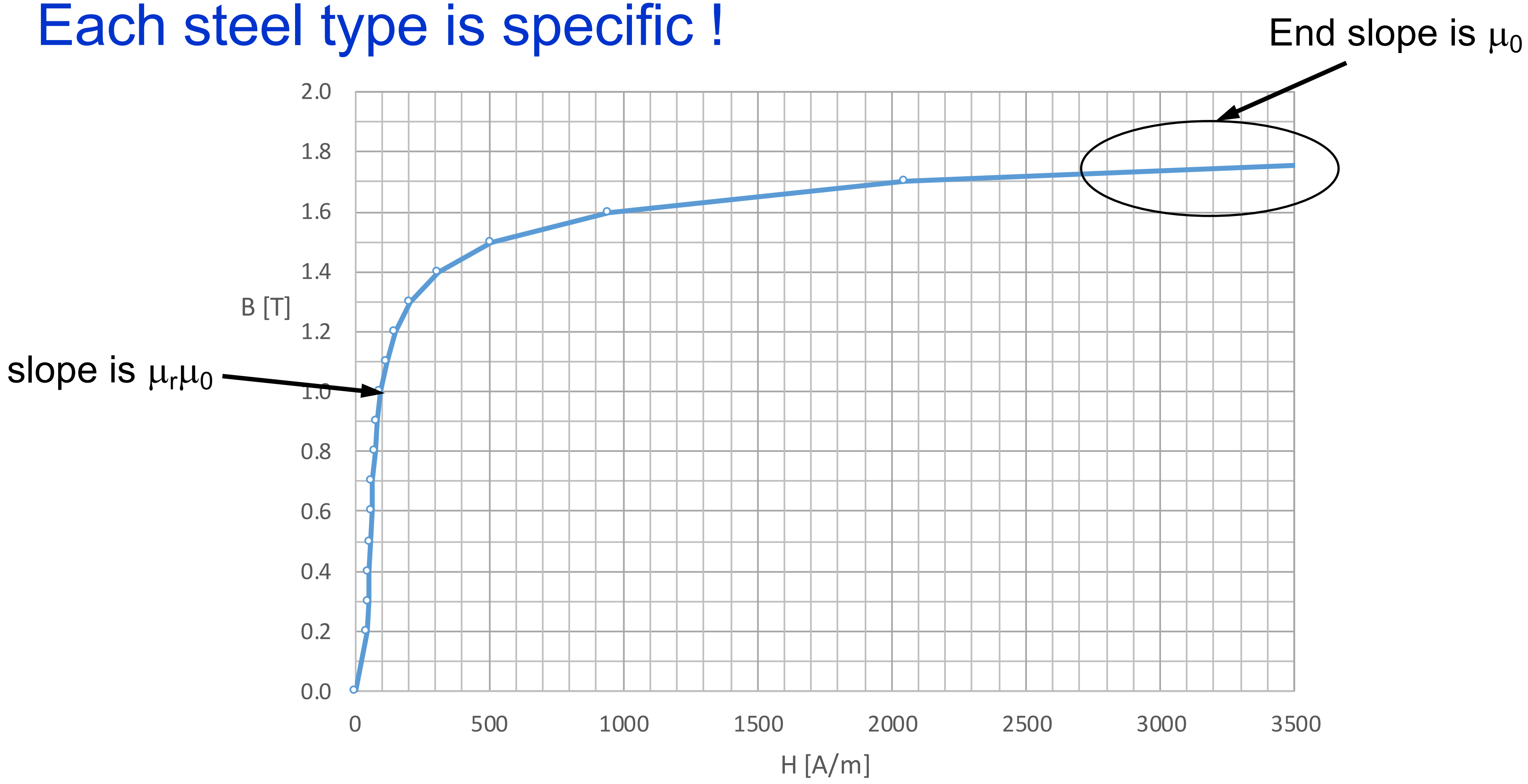
The hysteresis loop shows the "history dependent" nature of magnetization of a ferromagnetic material. Once the material has been driven to saturation, the magnetizing field can then be dropped to zero and the material will retain most of its magnetization (it remembers its history).

Non-linear materials: BH curves

In most of our simulations we use a simple BH model for the material: this is a typical curve for an electrical steel.

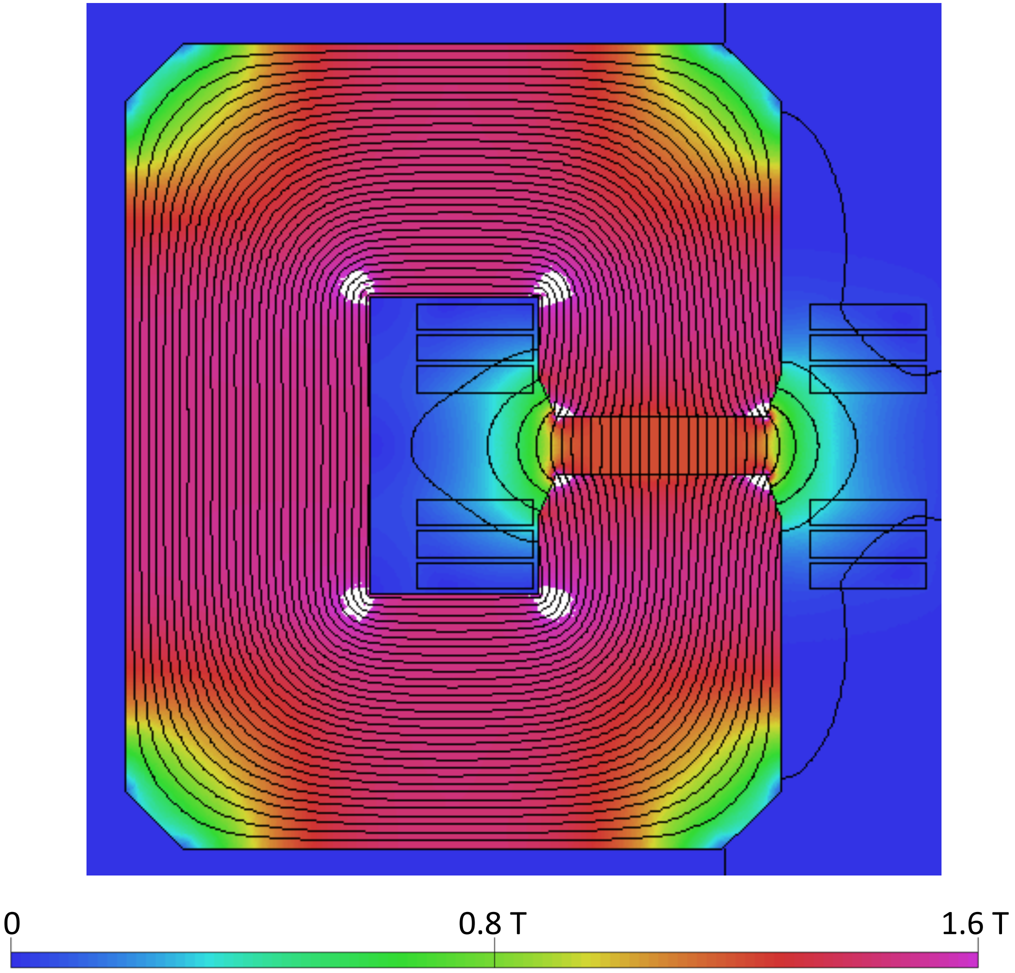
The flattening-off is called "saturation"

Each steel type is specific !



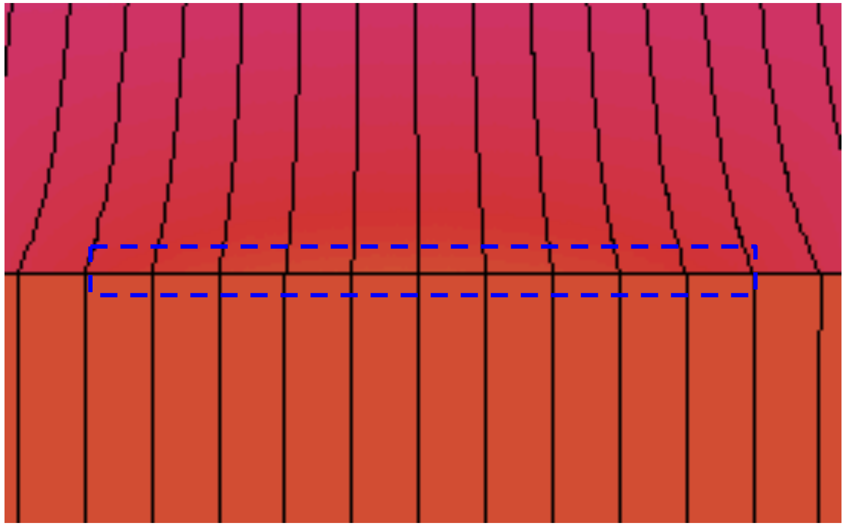
Field in a magnet with a steel yoke I

Now, why do the flux lines come out perpendicular to the iron?



Field in a magnet with a steel yoke II

Because they obey to Maxwell!



iron $\mu_r \gg 1$

air $\mu_r = 1$

$$H_{\parallel,\,\mathrm{air}} = H_{\parallel,\,\mathrm{iron}}$$

$$B_{\parallel,\,\mathrm{air}} = \frac{B_{\parallel,\,\mathrm{iron}}}{\mu_{r,\mathrm{iron}}} \approx 0$$

$$B_{\perp,\,\mathrm{air}} = B_{\perp,\,\mathrm{iron}}$$

Vector potential $\vec{\mathrm{A}}$

This is an “advanced introduction”, so let’s introduce the vector potential (3D)

Definition: $\vec{B} = \mathrm{rot}\,\vec{A}$

In magnetostatics, we can combine Eqs. 1 to 3 in a more compact form (3D)

$$\left.\begin{array}{l} \mathrm{div}\,\vec{B} = 0 \\ \mathrm{rot}\,\vec{H} = 0 \\ \vec{B} = \mu_0 \vec{H} \end{array}\right\} \quad \nabla^2 \vec{A} = \vec{0}$$

holds for magnetostatics and in air

In 2D this becomes a scalar Laplace equation

$$\nabla^2 A_z = 0 \qquad \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = 0$$

holds for magnetostatics and in air

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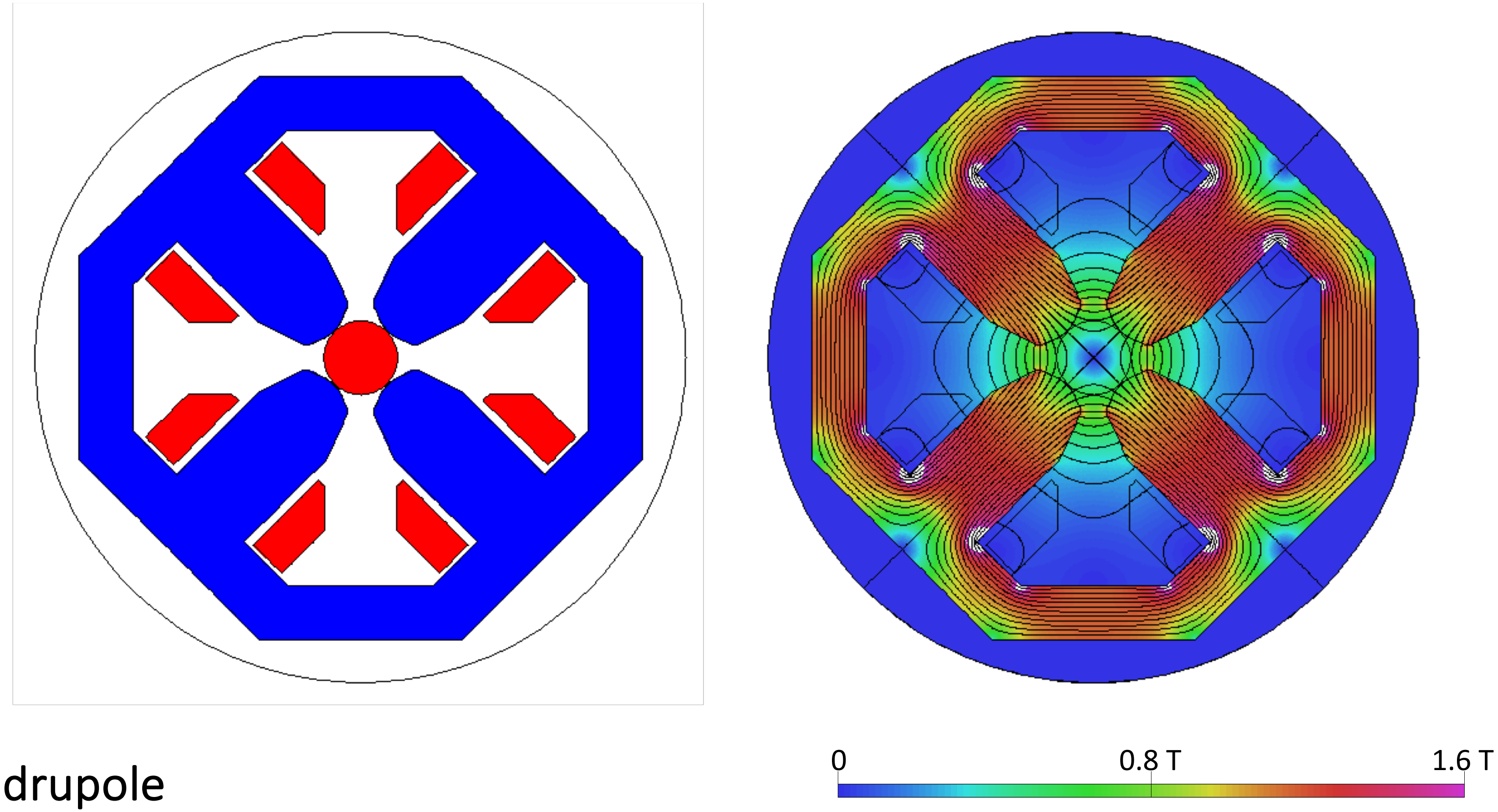
Multipoles I, quadrupole

We look at the 2D first: how can we conveniently describe the field in the aperture, for ex. in a quadrupole?

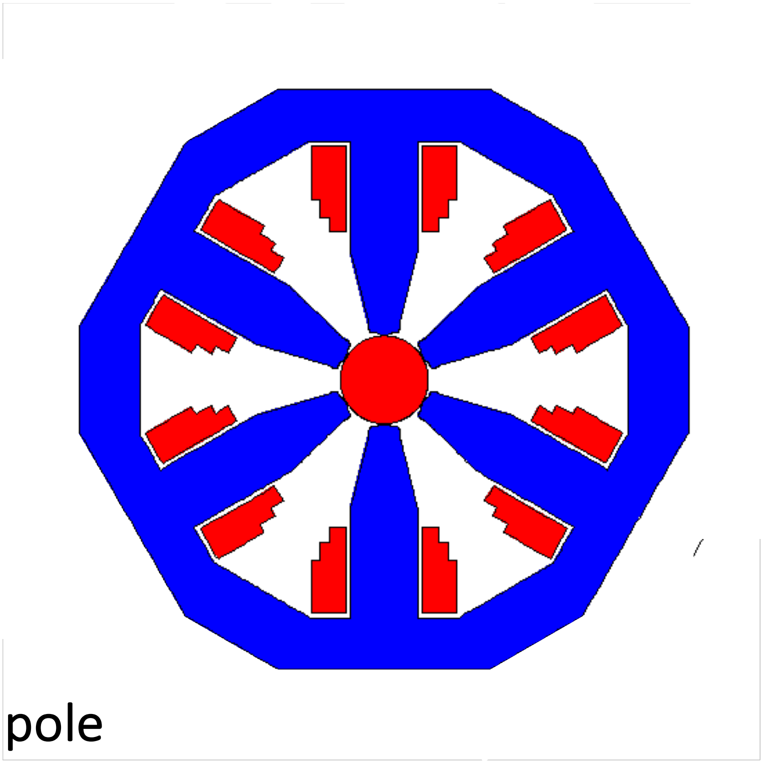


SESAME quadrupole

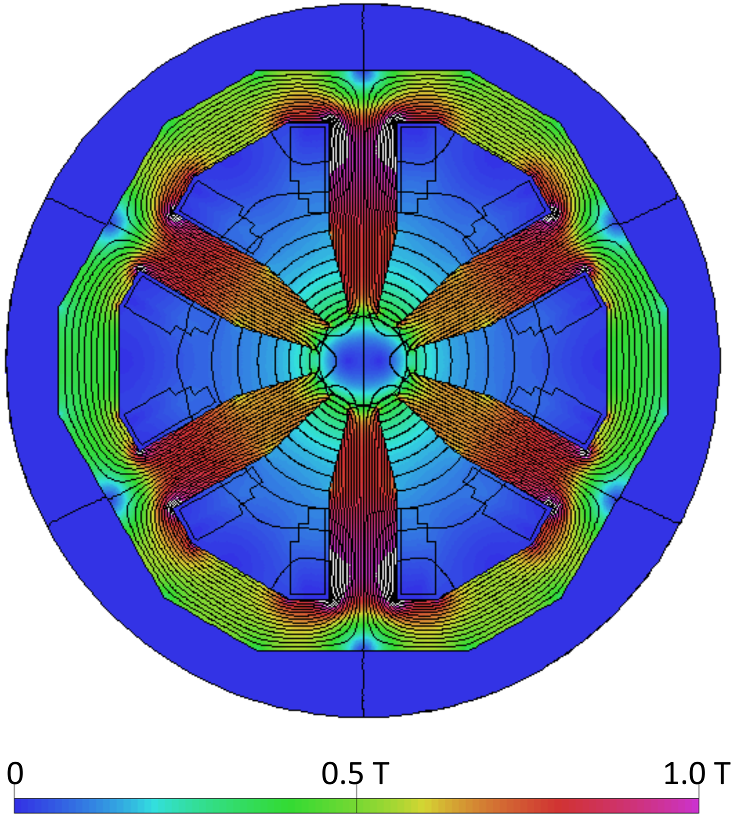
B_{pole} = 0.6 T

Multipoles III, sextupole

And in another resistive magnet, with a different configuration?

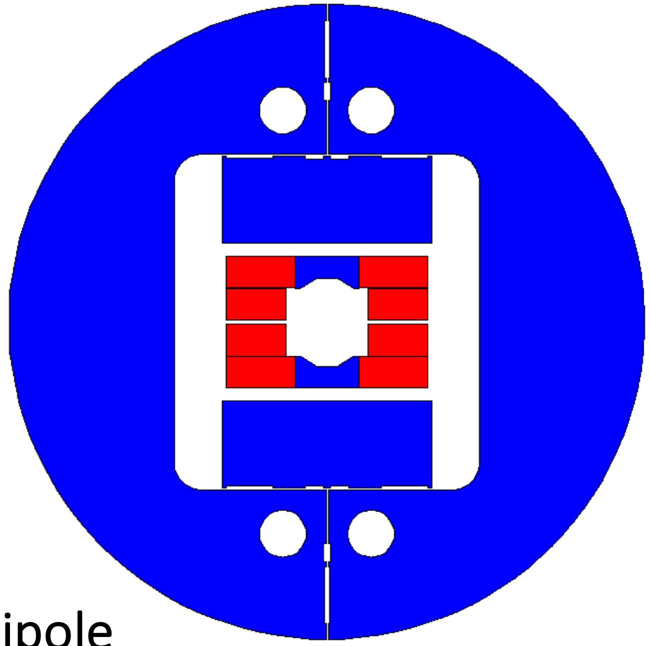


SESAME sextupole
+ vertical dipole corrector

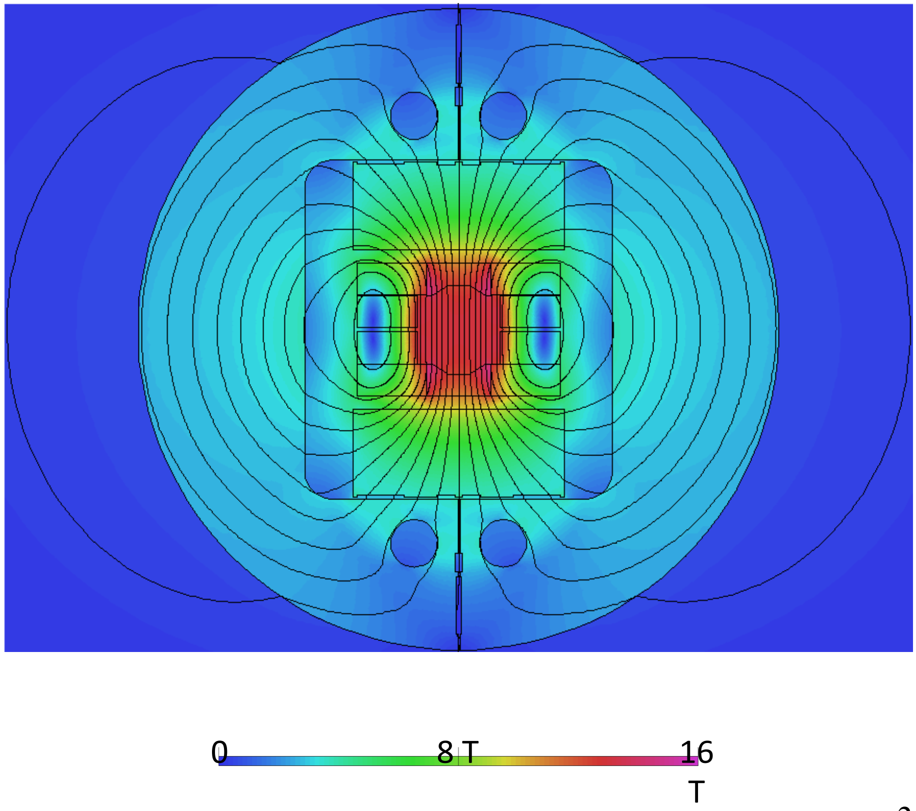


Multipoles IV, Superconducting dipole

Can the same formalism also describe the field in the aperture of a superconducting dipole?

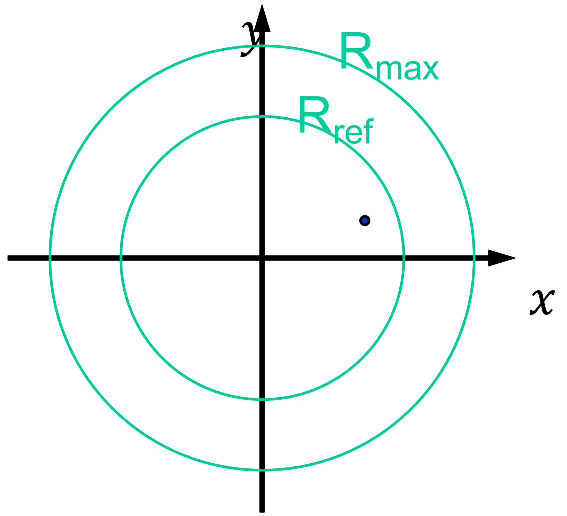


FRESCA2 dipole
13 T



Multipoles V, harmonic expansion

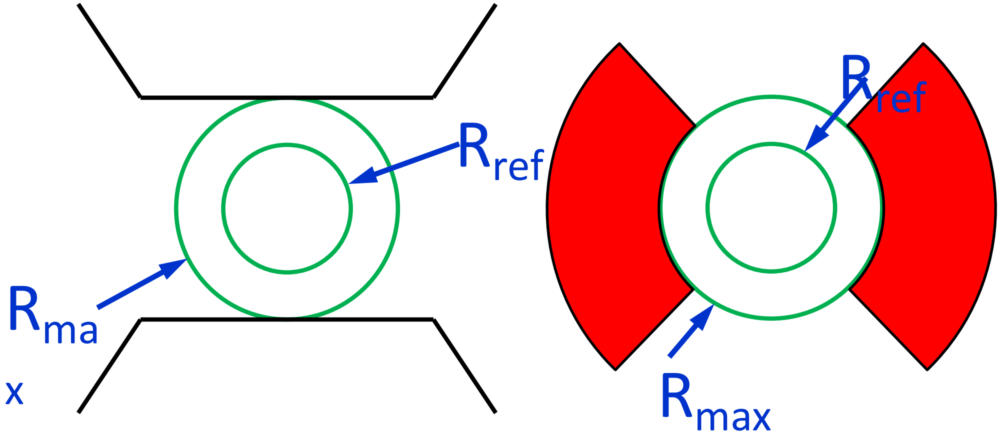
The solution is a harmonic (or multipole) expansion, describing the field (within a circle of validity) with scalar coefficients



$$B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{R_{ref}} \right)^{n-1} \qquad (4)$$

with: $z = x + iy = re^{i\theta}$

This decomposition has two characteristic radii: R_{ref} and R_{max}

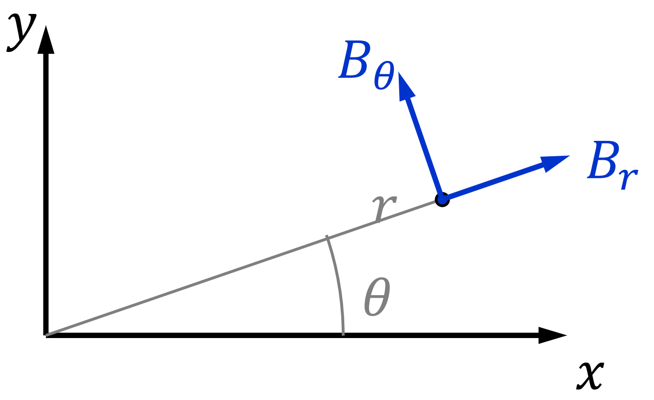


Multipoles VI, cylindrical coordinates

Expanding Eq. 4 in terms of radial and tangential components, we find sin and cos terms

$$B_r = \sum_{n=1}^{\infty} \left(\frac{r}{R_{ref}}\right)^{n-1} [B_n \sin(n\theta) + A_n \cos(n\theta)]$$

$$B_\theta = \sum_{n=1}^{\infty} \left(\frac{r}{R_{ref}}\right)^{n-1} [B_n \cos(n\theta) - A_n \sin(n\theta)]$$



Multipoles VII, normalized coefficients

In most cases, there is a main fundamental component, to which the other terms are normalized

take: (4)

$$B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{R_{ref}}\right)^{n-1}$$

define:

$$b_n = 10000 \frac{B_n}{B_N} \qquad a_n = 10000 \frac{A_n}{B_N}$$

hence:

$$B_y(z) + iB_x(z) = \underset{\uparrow\ \text{field strength}}{B_N} \underbrace{\sum_{n=1}^{\infty} \frac{b_n + ia_n}{10000} \left(\frac{z}{R_{ref}}\right)^{n-1}}_{\uparrow\ \text{field shape}}$$

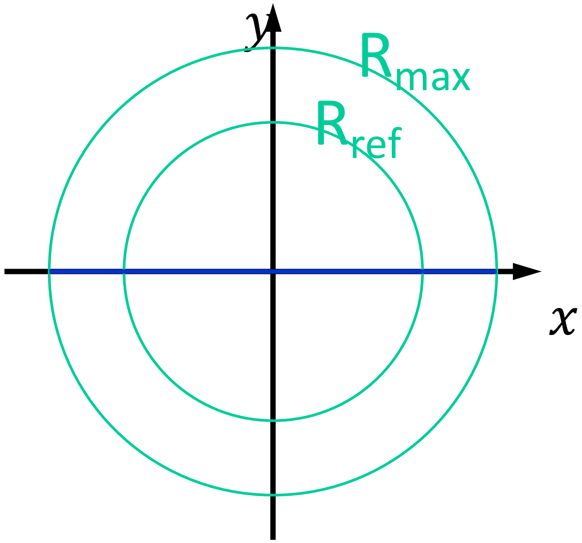
NB. The multipole coefficients b_n and a_n dimensions are referred to as "units"

Multipoles VIII, midplane field

Another useful expansion derived from Eq. 4 is that of B_y and B_x on the midplane, i.e. at y = 0

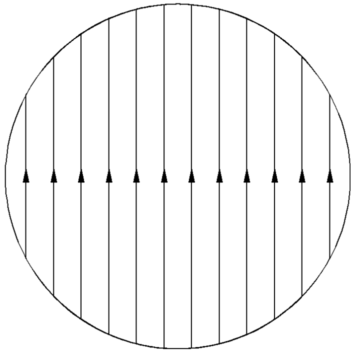
$$B_y(x) = \sum_{n=1}^{\infty} B_n \left(\frac{x}{R_{ref}}\right)^{n-1} = B_1 + B_2 \frac{x}{R_{ref}} + B_3 \left(\frac{x}{R_{ref}}\right)^2 + \cdots$$

$$B_x(x) = \sum_{n=1}^{\infty} A_n \left(\frac{x}{R_{ref}}\right)^{n-1} = A_1 + A_2 \frac{x}{R_{ref}} + A_3 \left(\frac{x}{R_{ref}}\right)^2 + \cdots$$

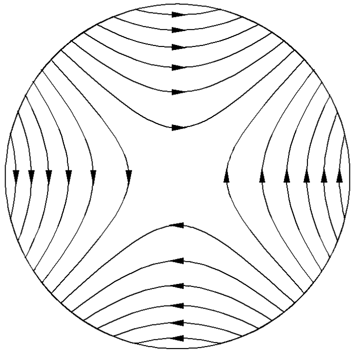


Multipoles IX, multipole fields

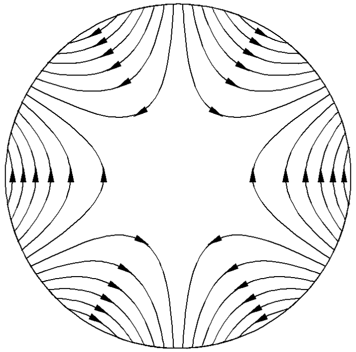
Each multipole corresponds to a field distribution: adding them up, we can describe everything (this is nicely compatibly with Maxwell)



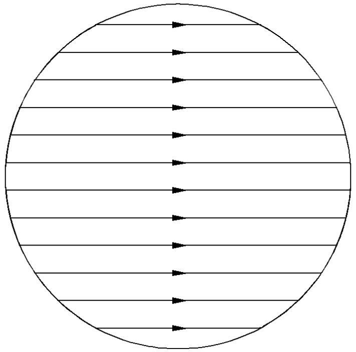
B_1: normal dipole



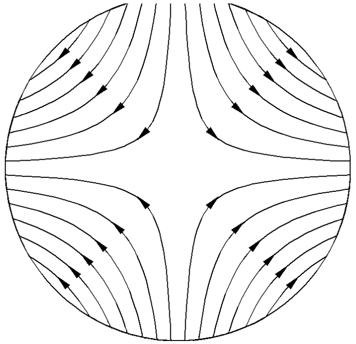
B_2: normal quadrupole



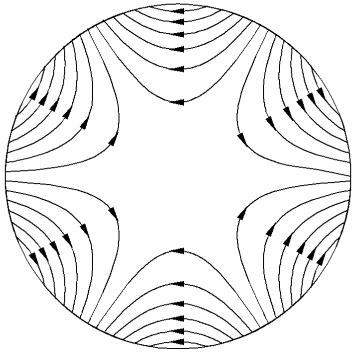
B_3: normal sextupole



A_1: skew dipole



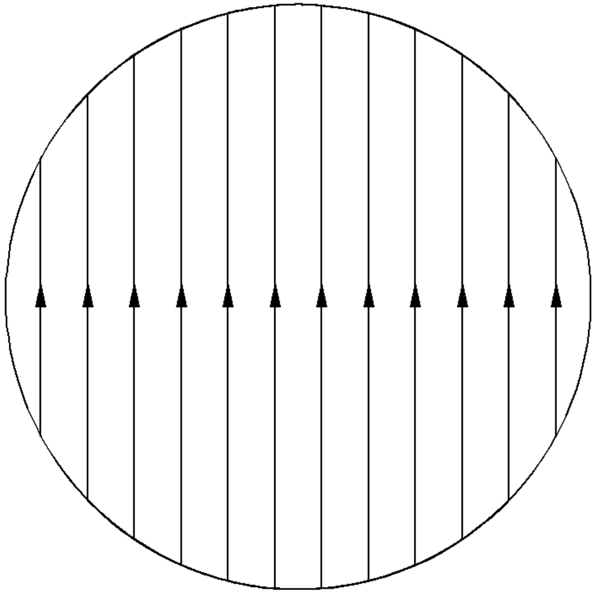
A_2: skew quadrupole

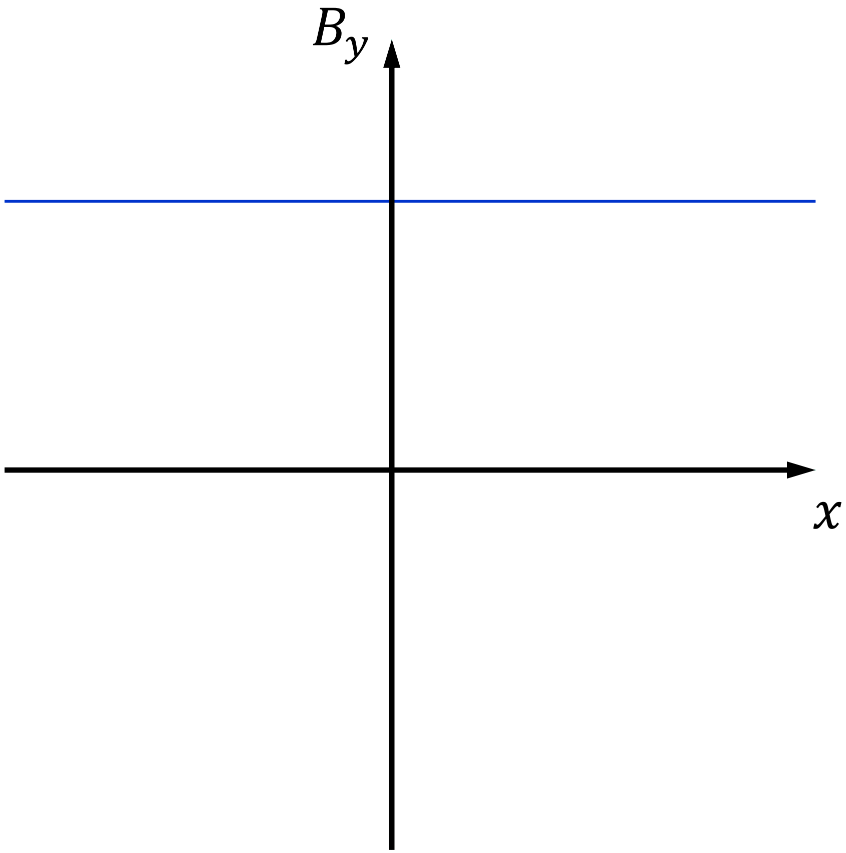


A_3: skew sextupole

Multipoles X, dipole field

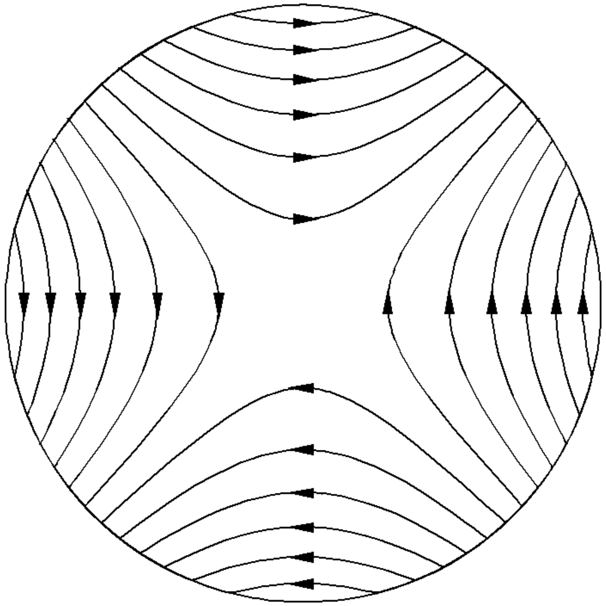
B_1 is the normal dipole

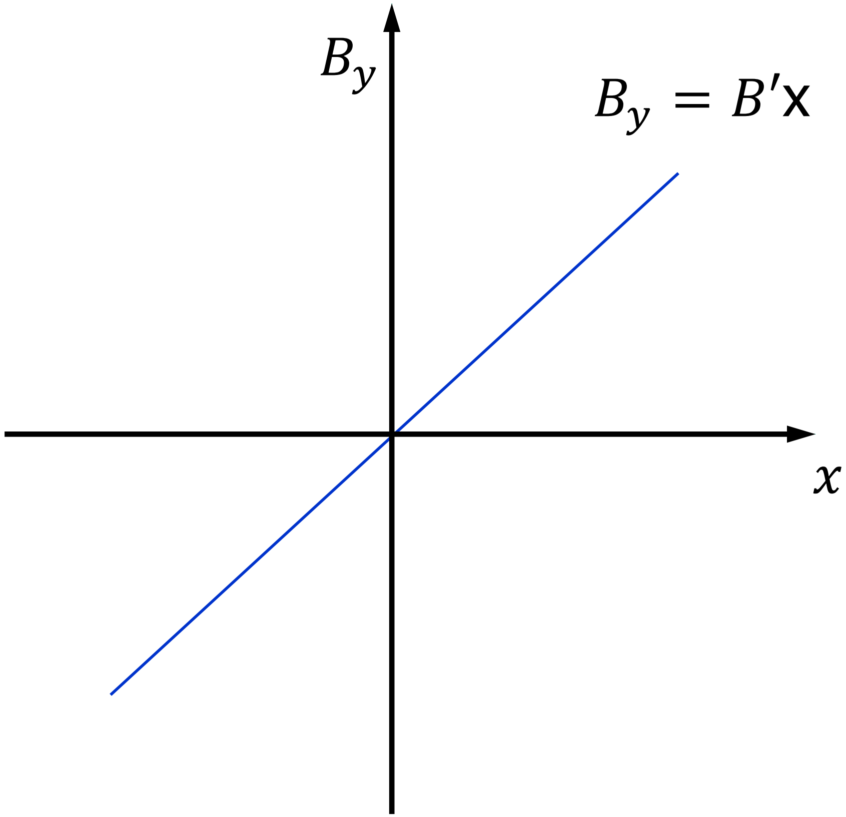




Multipoles XI, quadrupole field

B_2 is the normal quadrupole



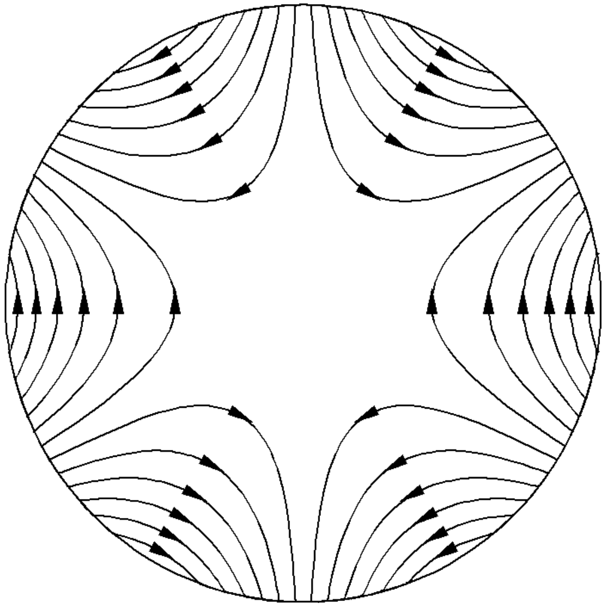


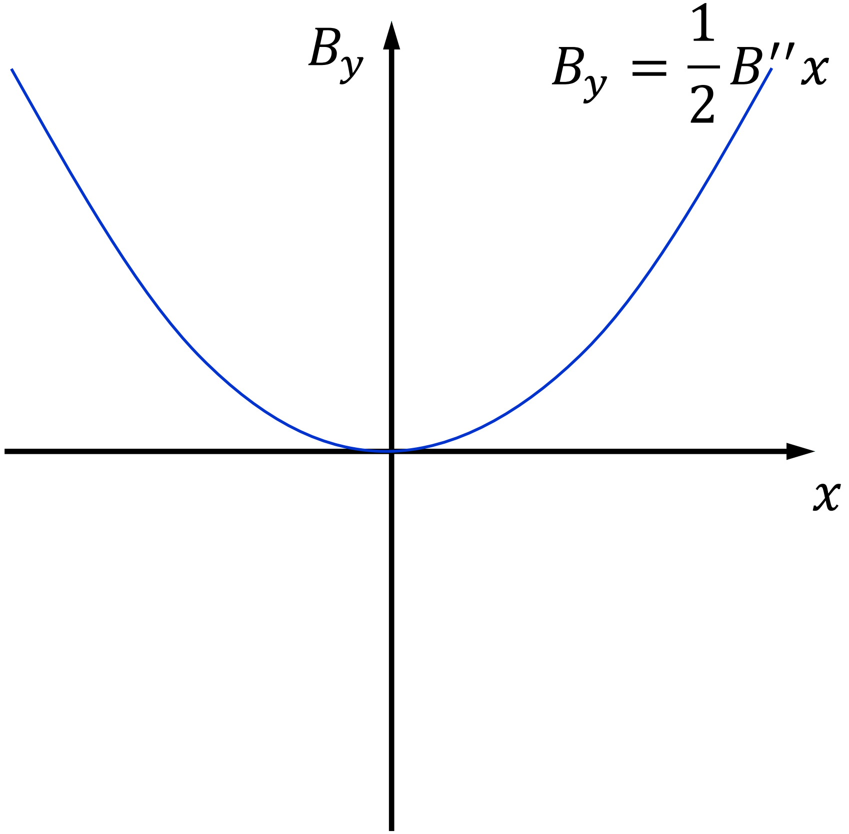
gradient: $G = \frac{B_2}{R} = \frac{\partial B_y}{\partial x} = B'$

field on the pole tip: $B_{pole} = B' R_{pole}$

Multipoles XII, sextupole field

B$_3$ is the normal sextupole



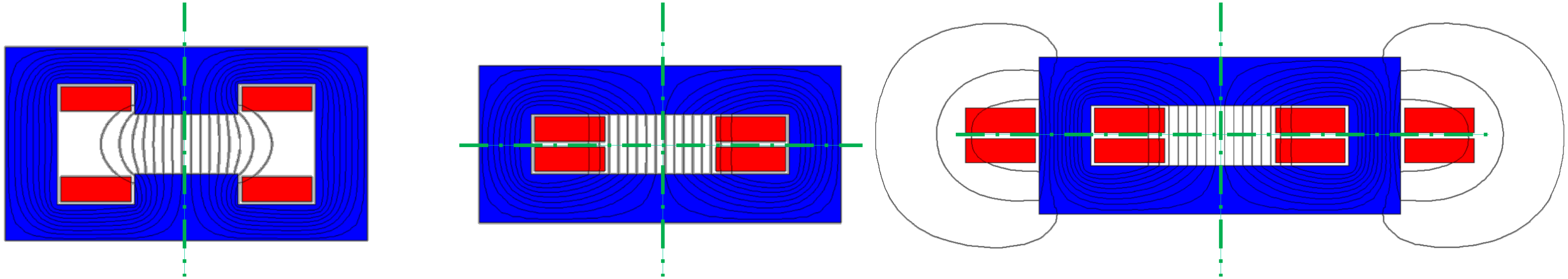


gradient: $$B'' = \frac{\partial^2 B_y}{\partial x^2} = \frac{2B_3}{R^2}$$

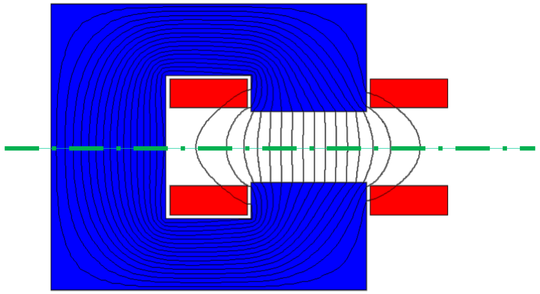
field on the pole tip: $$B_{pole} = \frac{1}{2} B'' R_{pole}$$

Multipoles XIII, allowed multipoles

The allowed / not-allowed harmonics refer to the terms that shall / shall not cancel out thanks to design symmetries



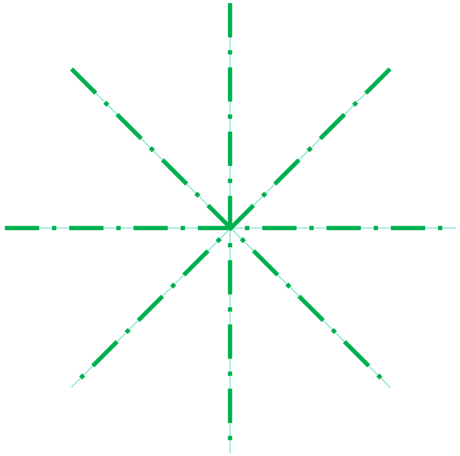
fully symmetric dipoles: only B_1, b_3, b_5, b_7, b_9, etc.



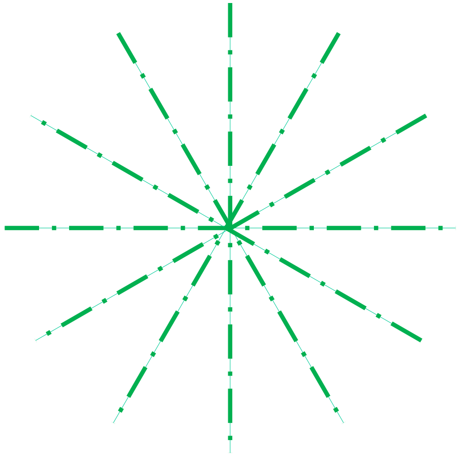
half symmetric dipoles: B_1, b_2, b_3, b_4, b_5, etc.

Multipoles XIV, allowed multipoles

These are the allowed harmonics for fully symmetric quadrupoles and sextupoles



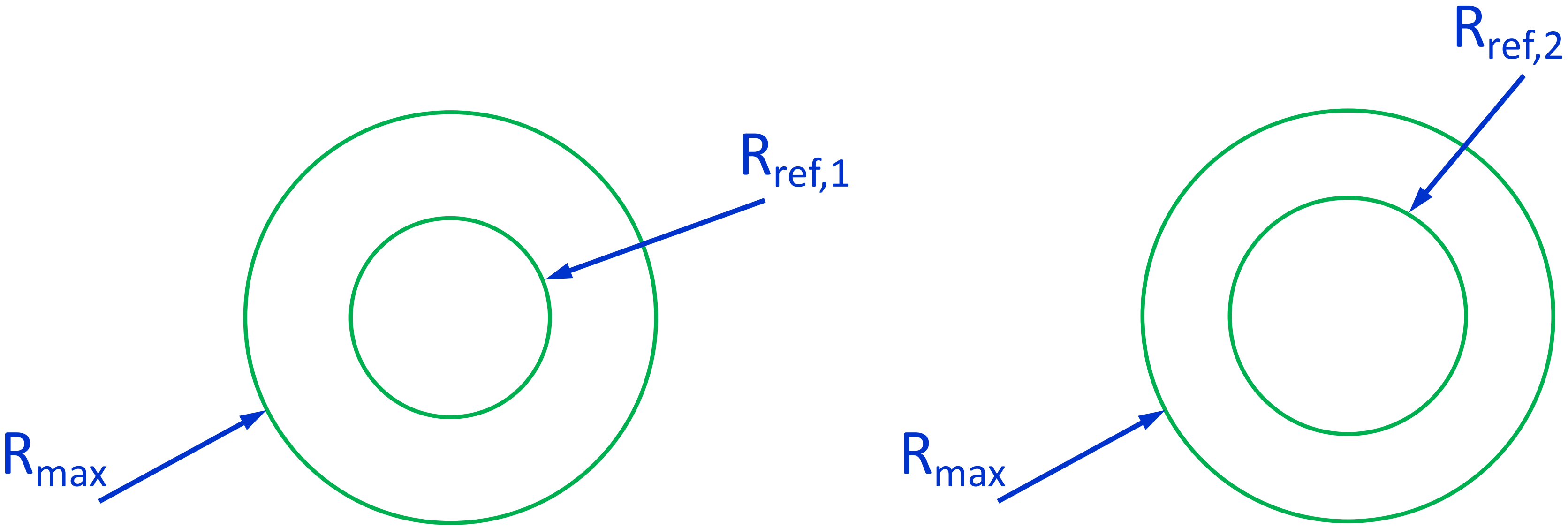
fully symmetric quadrupoles: B_2, b_6, b_{10}, b_{14}, b_{18}, etc.



fully symmetric sextupoles: B_3, b_9, b_{15}, b_{21}, etc.

Multipoles XV, scaling

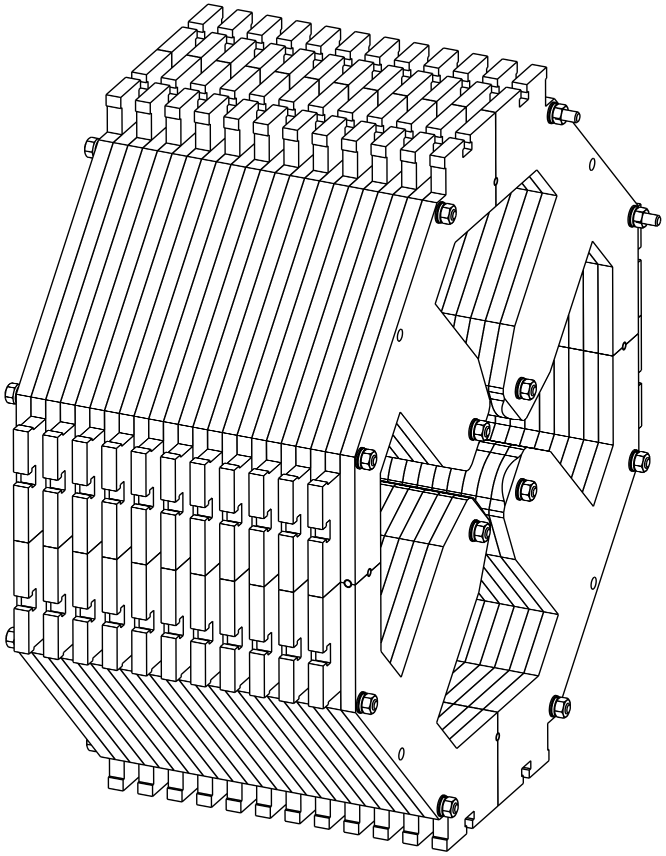
We can change R_{ref} and scale up (or down) the harmonics



$$B_{n,2} = B_{n,1}\left(\frac{R_{ref,2}}{R_{ref,1}}\right)^{n-1} \qquad b_{n,2} = b_{n,1}\left(\frac{R_{ref,2}}{R_{ref,1}}\right)^{n-N}$$

Multipoles XVI, example

Let's have a look at a real case: the measurements of 33 quadrupoles built for SESAME



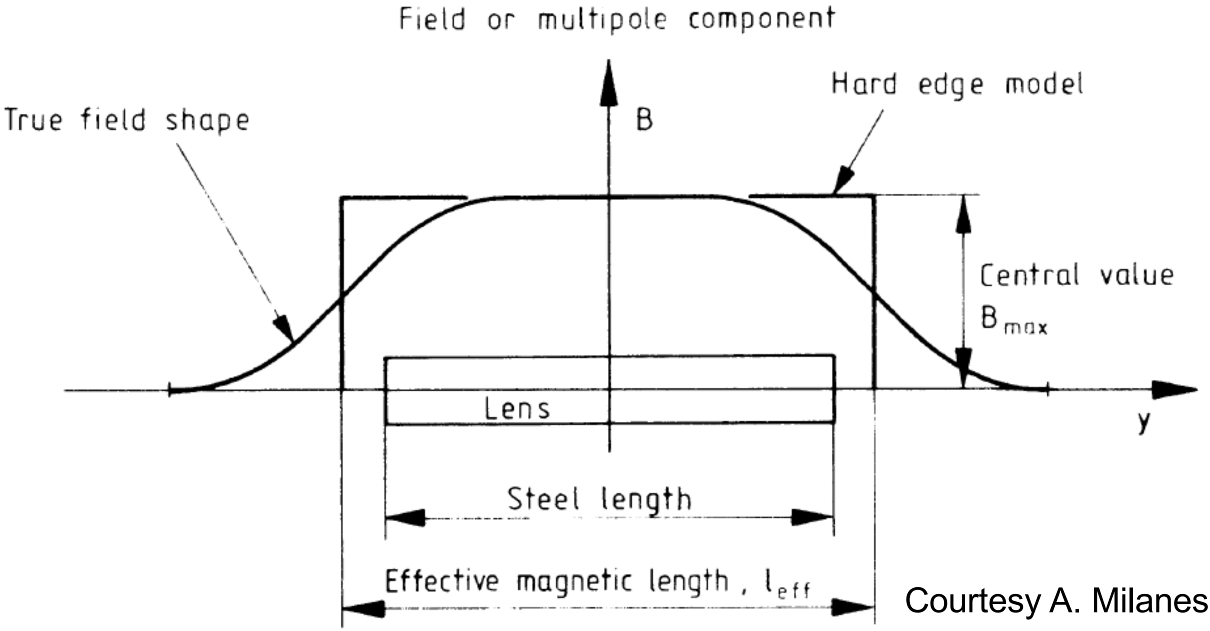
SESAME QF

mean ± rms	QF @ 250 A
b_3	-0.2 ± 0.8
a_3	-0.1 ± 0.9
b_4	0.3 ± 0.4
a_4	-0.3 ± 0.1
b_5	0.0 ± 0.1
a_5	0.0 ± 0.1
b_6	-0.1 ± 0.1
b_{10}	-0.3 ± 0.0
b_{14}	0.3 ± 0.0

harmonics in 10^{-4} at 24 mm radius

Magnetic Length

In 3D, the longitudinal dimension of the magnet is described by the magnetic length



Courtesy A. Milanese, CERN

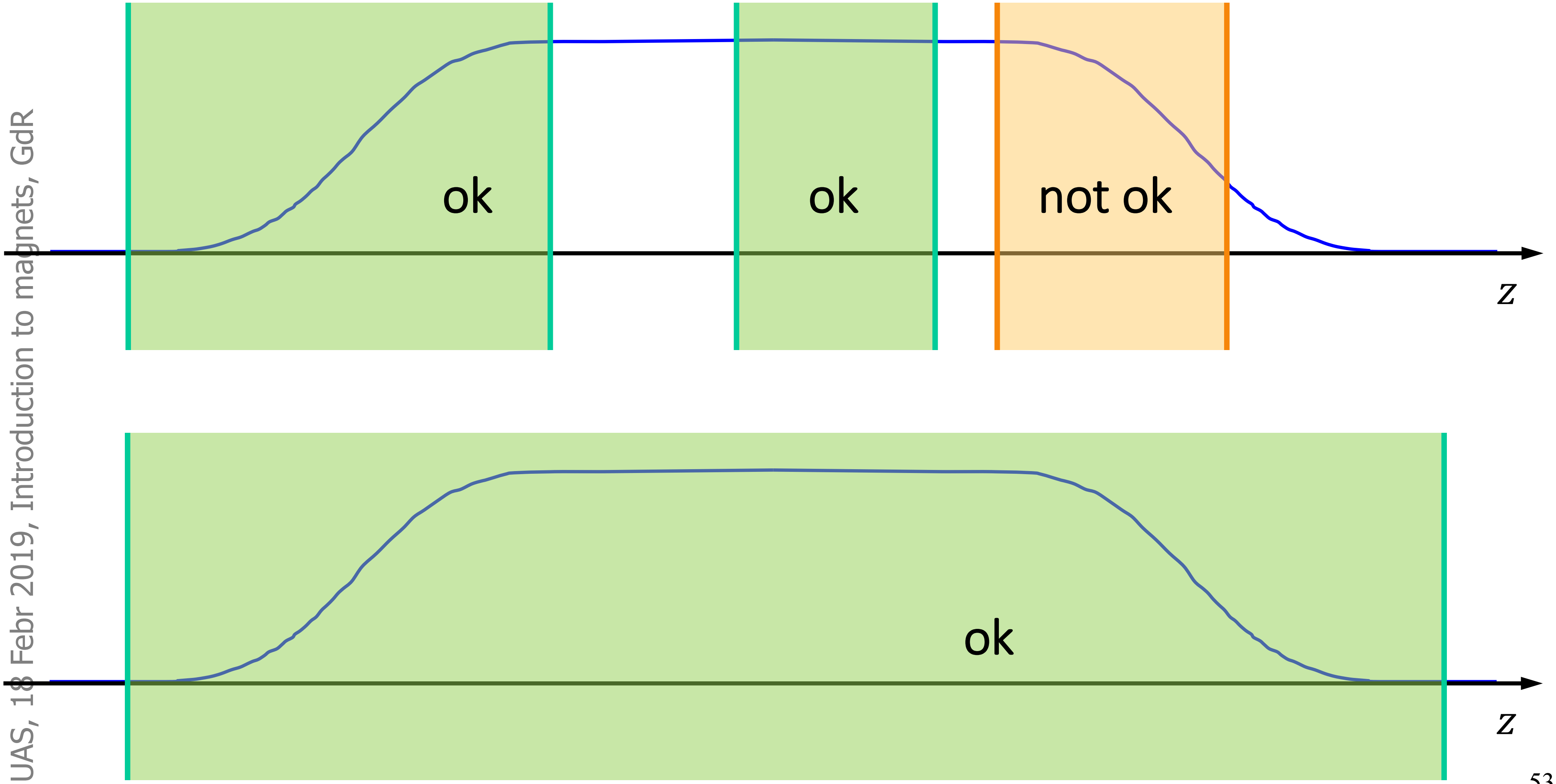
$$l_m B_0 = \int_{-\infty}^{\infty} B(z)dz$$

magnetic length L_{mag} as a first approximation in an irn dominated magnet :

- For dipoles $L_{mag} = L_{yoke} + d$ — d = pole distance
- For quadrupoles: $L_{mag} = L_{yoke} + r$ — r = radius of the inscribed circle between the 4 poles

Multipoles along a magnet

This 2D decomposition holds also for the integrated 3D field, as long as at the start / end B is constant along z

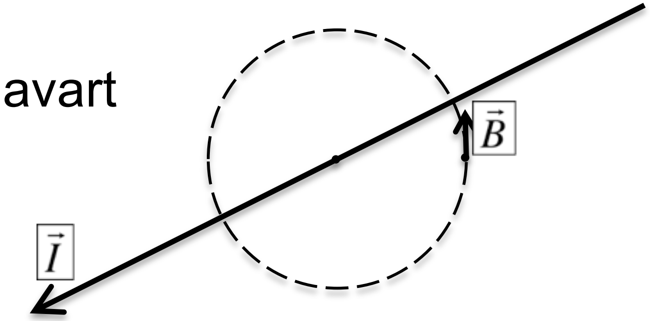


Magnetic fields, order of magnitudes

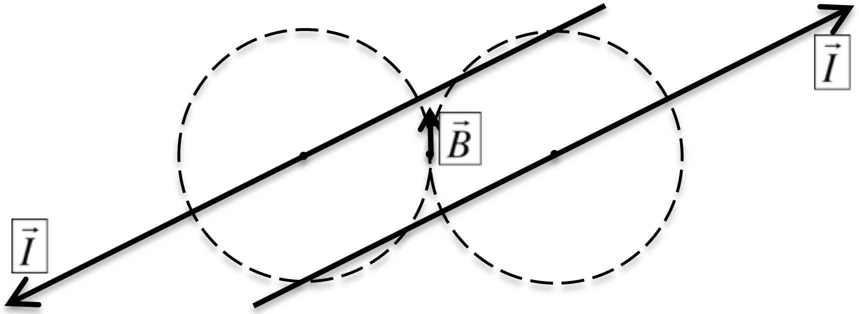
From Ampere's law with no time dependencies

(Integral form) $$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_{encl.}$$

We can derive the law of Biot and Savart



$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\varphi}$$



If you wanted to make a *B* = 1.5 T magnet with just two infinitely thin wires placed at 100 mm distance in air one needs :
I = 187500 A

- To get reasonable fields (*B* > 1 T) one needs large currents
- Moreover, the field homogeneity will be poor

Iron dominated magnets, simple example

With the help of an iron yoke we can get fields with less current

$$\oint_C \vec{H}\cdot d\vec{l} = N\cdot I$$

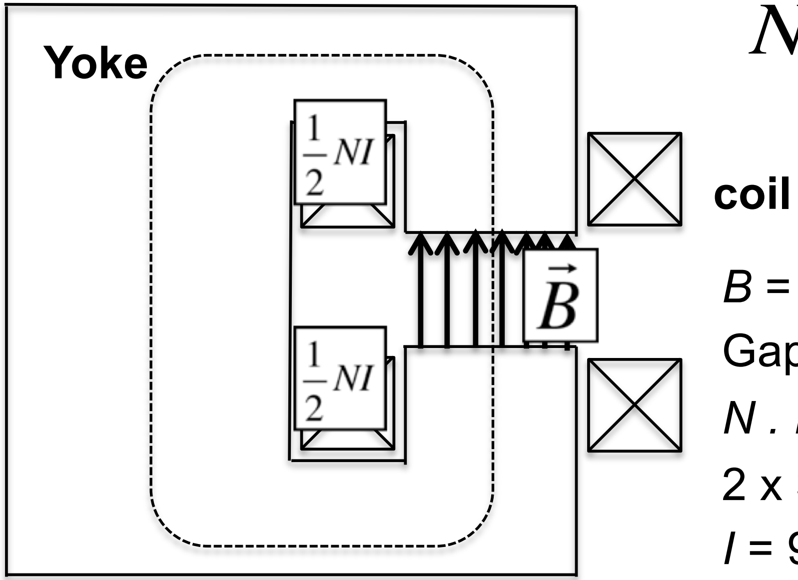
$$N\cdot I = H_{iron}\cdot l_{iron} + H_{airgap}\cdot l_{airgap} \Rightarrow$$

$$N\cdot I = \cancel{\frac{B}{\mu_0\mu_r}\cdot l_{iron}} + \frac{B}{\mu_0}\cdot l_{airgap} \Rightarrow$$

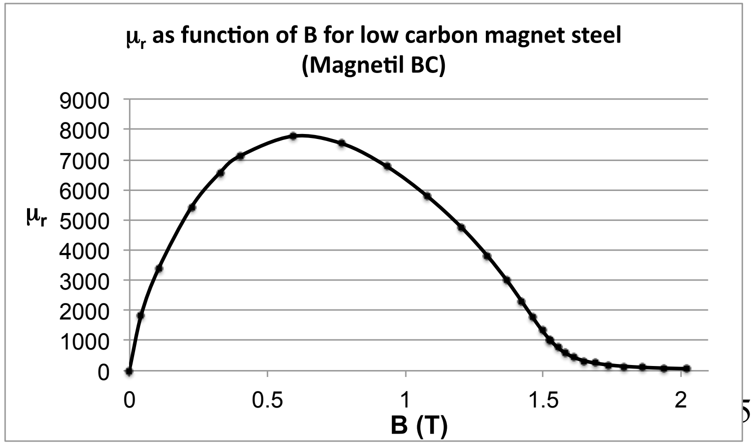
$$N\cdot I = \frac{l_{airgap}\cdot B}{\mu_0}$$

This is valid as $\mu_r >> \mu_0$ in the iron : limited to B < 2 T

Example: C shaped dipole for accelerators



B = 1.5 T
Gap = 50 mm
N . I = 59683 A
2 x 30 turn coil
I = 994 A
@5 A/mm^2, 200 mm^2
14 x 14 mm Cu

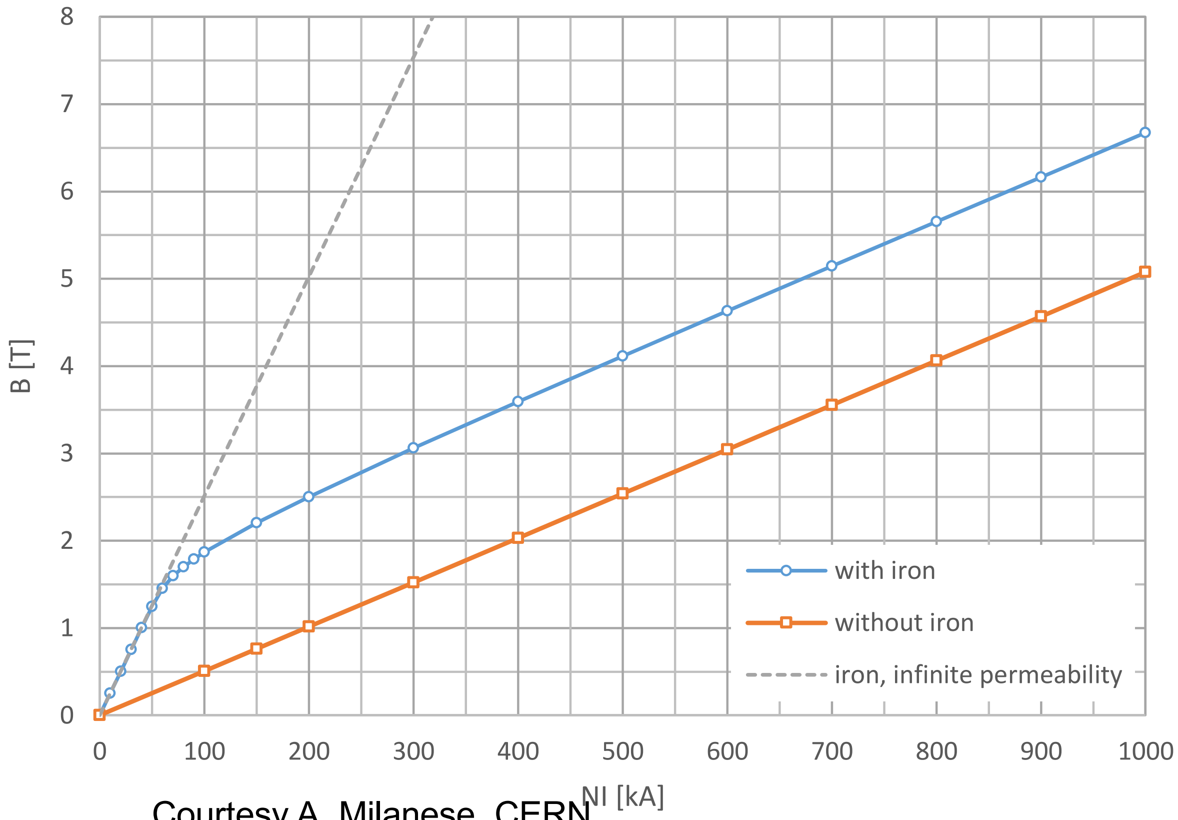


Comparison : iron magnet and air coil

Imagine a magnet with a 50 mm vertical gap (horizontal width ~100 mm)
Iron magnet wrt to an air coil:

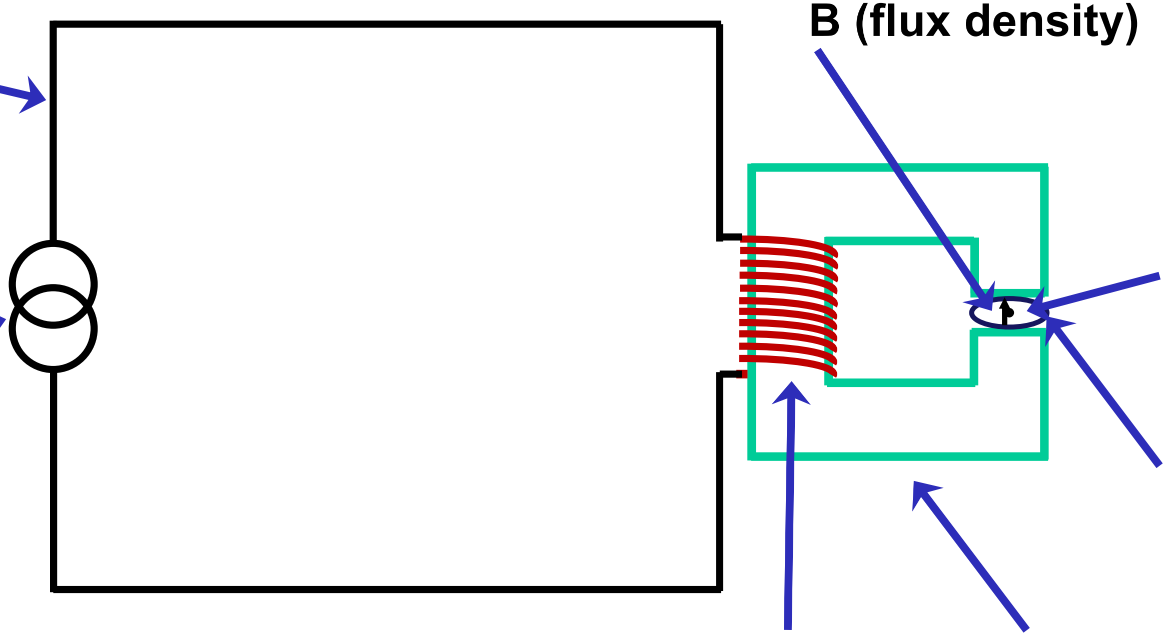
- Up to 1.5 T we get ~6 times the field
- Between 1.5 T and 2 T the gain flattens of : the iron saturates
- Above 2 T the slope is like for an air-coil: currents become too large to use resistive coils

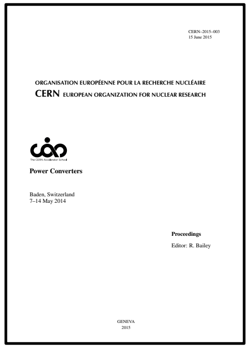
These two curves are the transfer functions – B field vs. current – for the two cases



Courtesy A. Milanese, CERN

Magnets in an accelerator: power convertor and circuit





CERN–2015–003
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ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE
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The CERN Accelerator School

Power Converters

Baden, Switzerland
7–14 May 2014

Proceedings

Editor: R. Bailey

GENEVA
2015

- B field stability in time: ~10^{-5} - 10^{-6}
- Typical R of a magnet ~20mΩ - 60mΩ
- Typical L of a magnet ~20mH – 200mH
- Powering cable (for 500A): Cu 250 mm^2 (Cu: 17 nΩ.m) R = 70 μΩ/m, for 200m: R= 13mΩ
- Take a typical rise time 1s

Then the Power Convertor has to Supply : 0-500 A with a stability of a few ppm.

Voltage up to 40 V (resitive)

And 100 V (inductive)

Acknowledgement

This lecture is a based on previous lectures by Attilio Milanese and Davide Tommasini

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