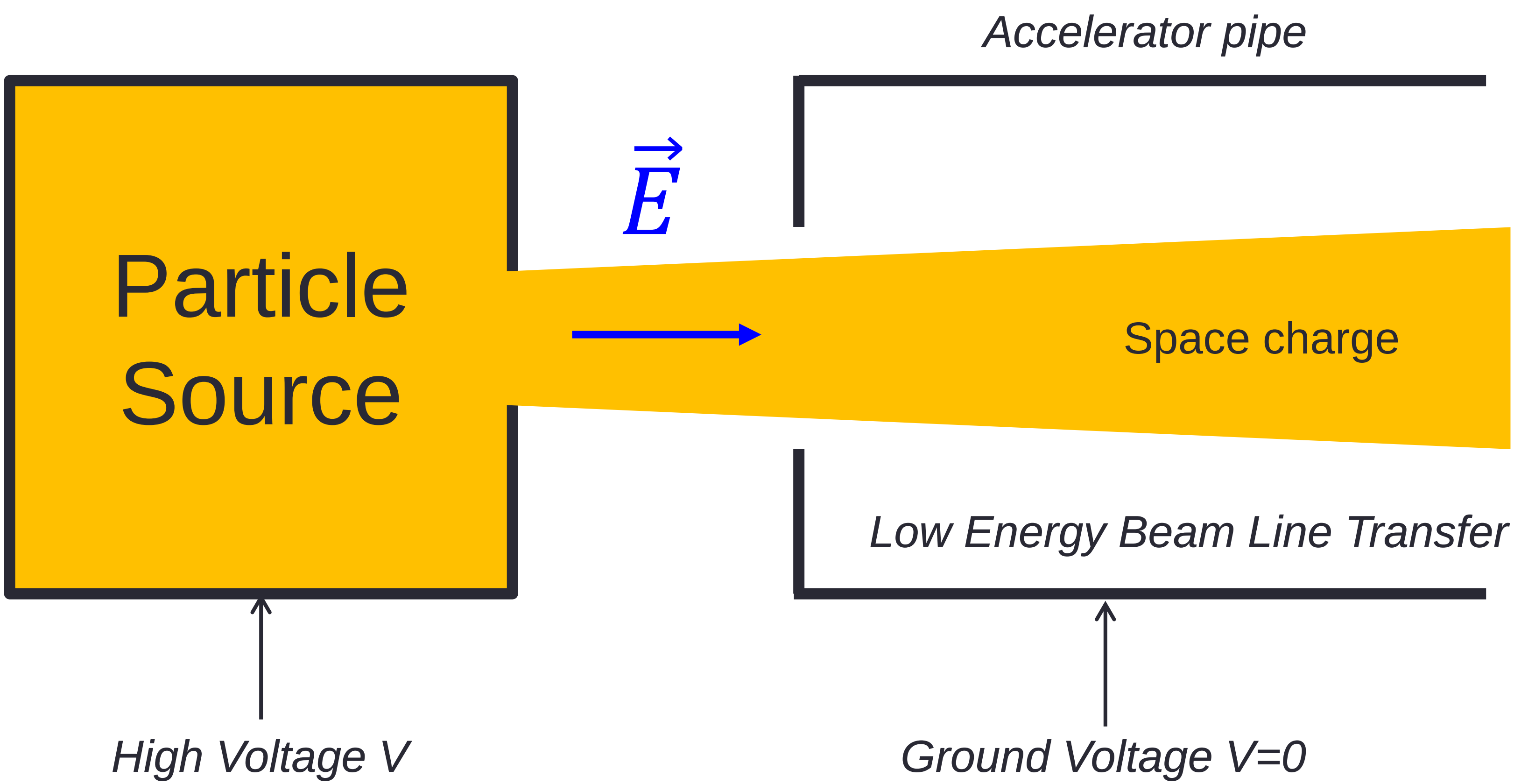
TUTORIAL ON PARTICLE BEAM FORMATION

Revision 1

Basics of particle extraction from a source

- Usually a static electric field accelerate the particles



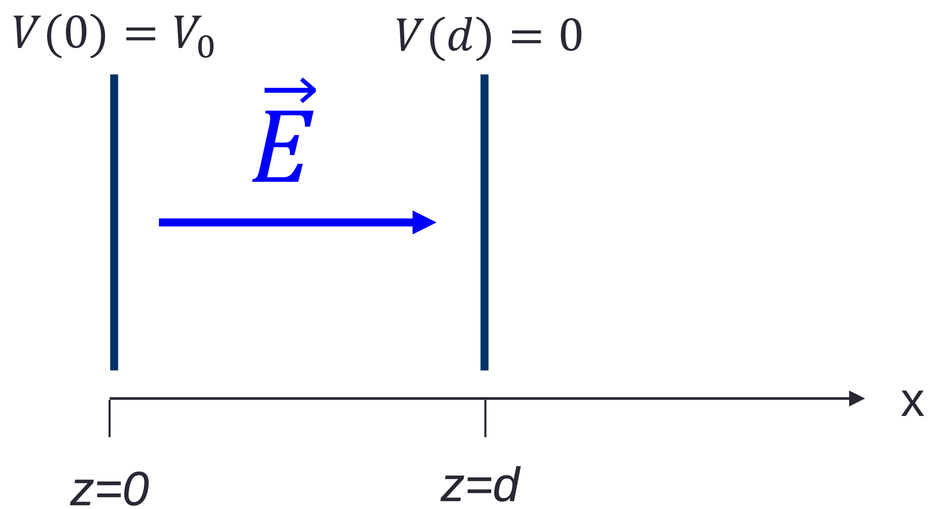
Exercise 1, particle extraction

- Assuming a 1D problem, demonstrate from $\Delta V = \frac{\rho}{\epsilon_0} = 0$ that :

$$\vec{E} = \frac{V_0}{d}\vec{z}$$

$$V(z) = V_0 \frac{(d - z)}{d}, z \leq d$$

$$V(z) = 0, z > d$$

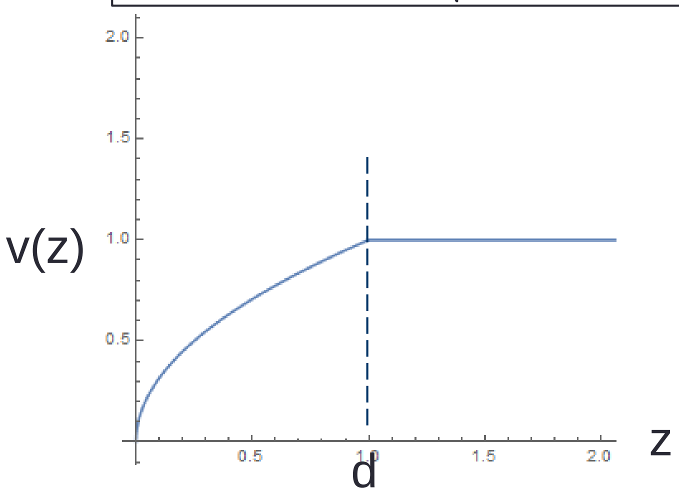


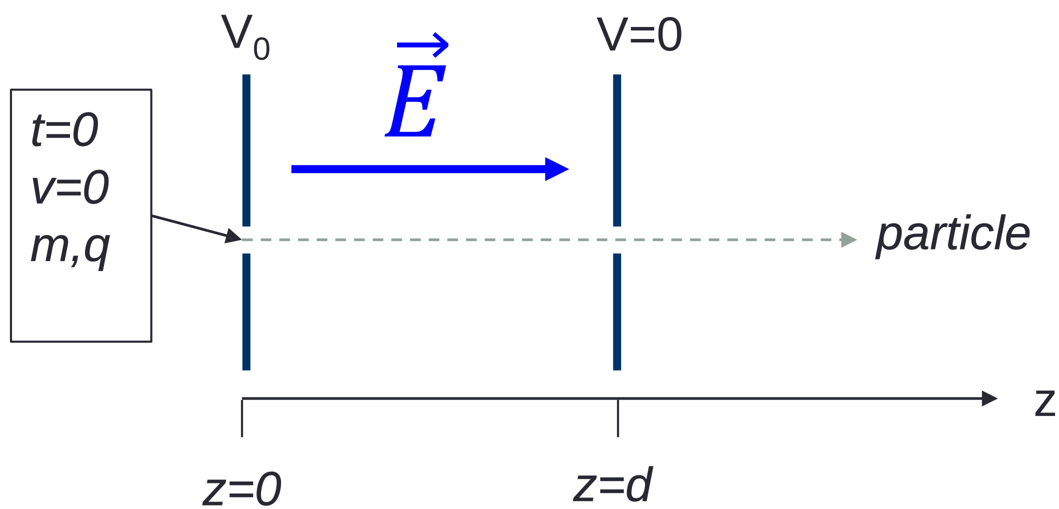
Exercise 2, particle extraction

- Using Newton second's law, derive the evolution of velocity of a particle with charge q , mass m versus x in the accelerating gap:

$$v(z) = \sqrt{\frac{2qV_0 z}{md}}, z \leq d$$

$$v(z) = v(d) = \sqrt{\frac{2qV_0}{m}}, z > d$$



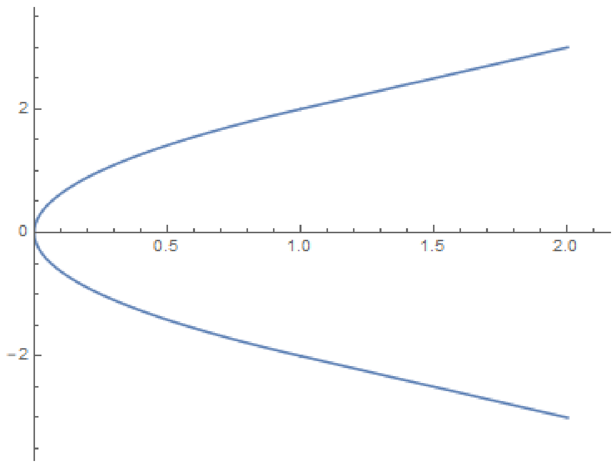


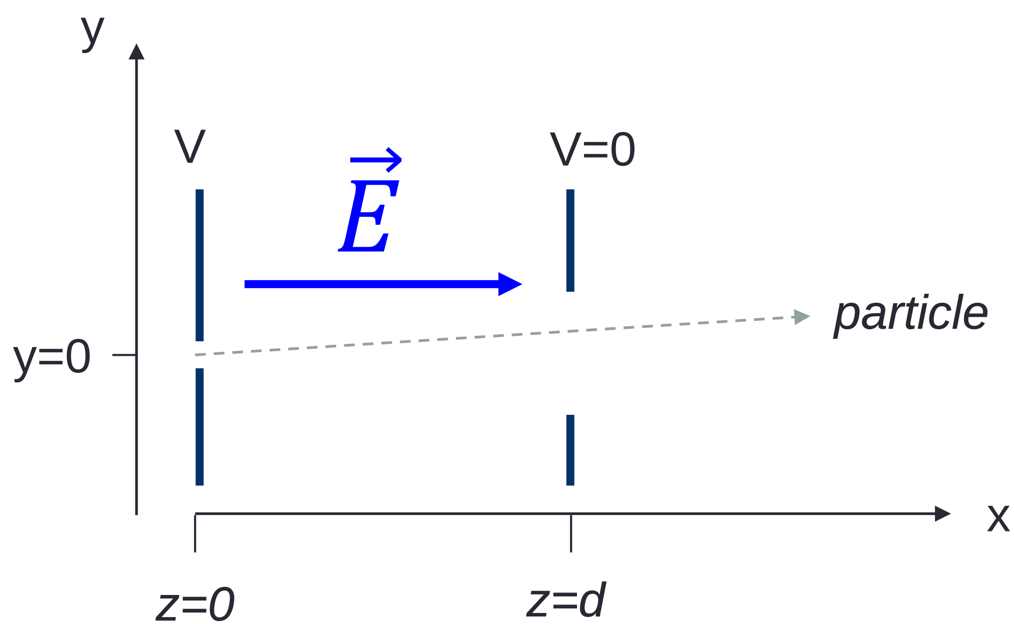
Exercise 3, particle extraction

- A transverse velocity is added: $\overrightarrow{v_y} = v_y\vec{y}$, notation: $v_z = v = v_z(d)$
- Show that :

$$y(z) = \frac{v_y}{v} 2\sqrt{dz}, z \le d$$

$$y(z) = \frac{v_y}{v}(z + d), z > d$$

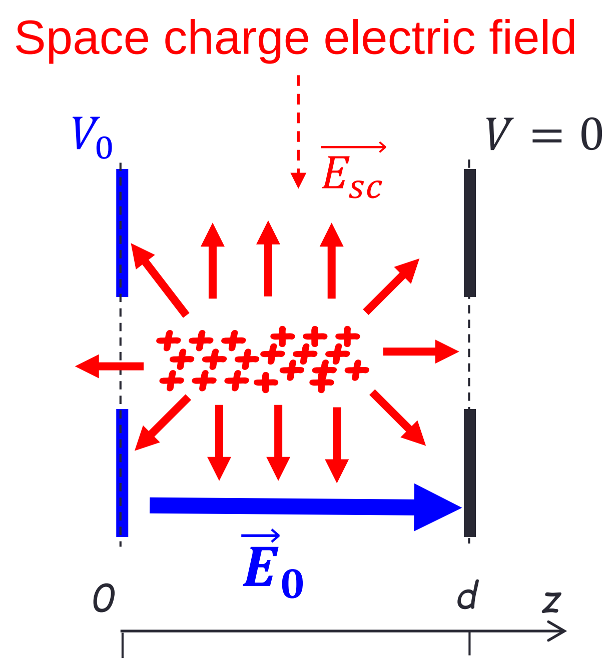




Reduce beam divergence
=> increase v
=> Increase high voltage V

Exercise 4 : The Child Langmuir Law (1/3)

- Space charge limitation of particle extraction
  - Assume a 1D extraction or particles (mass m, charge q)
  - Assume a charge density of particles $\rho(z)$ in the gap
  - The particles create a space charge electric fields $\overrightarrow{E_{sc}}$ which perturbates the main Electric field $\overrightarrow{E_0} = \frac{V_0}{d}\vec{z}$: $\vec{E} = \overrightarrow{E_0} + \overrightarrow{E_{sc}}$
  - $\Delta V = -\frac{\rho}{\epsilon_0}$
  - $div\,\vec{J} = \frac{\partial \rho}{\partial t}$
  - $\vec{J} = \rho\vec{v}$
  - **4.1** using the conservation of energy, show that : $v(z) = \sqrt{\frac{2q(V_0 - V(z))}{m}}, 0 \le z \le d$



Exercise 4 : The Child Langmuir Law (2/3)

- **4.2** Assuming a stationary problem, show that:

$$J(z) = J(0) = J_0 = \rho(z)v(z)$$

- **4.3** Show that $V(z)$ is solution of:

$$\frac{d^2V(z)}{dz^2} = -\frac{J_0}{\epsilon_0}\sqrt{\frac{m}{2q}}\left(V_0 - V(z)\right)^{-1/2}$$

- we set $\mathrm{k} = \frac{J_0}{\epsilon_0}\sqrt{\frac{m}{2q}}$ and $\mathrm{U(z)} = V_0 - V(z)$, the equation writes:

$$\frac{d^2U}{dz^2} = kU^{-1/2}$$

- **4.4** Introducing the Electric Field $E(z)$ and using $\vec{E} = -\vec{\nabla}V$, show that :

$$\frac{d^2U}{dz^2} = E.\frac{dE}{dU} \quad and \quad EdE = kdU/\sqrt{U}$$

Exercise 4 : The Child Langmuir Law (3/3)

- **4.5** Deduct that :
- $E^2(z) - E^2(0) = 4k\left(\sqrt{U(z)}\right), 0 \leq z \leq d$
- **4.6** The condition for the particle extraction to be blocked by space charge is reached when the space charge density in the gap is so high that it screens the electric field at $z = 0$, $E(0) = 0$. Then, available particles are no more accelerable. Integrate 4.5 from z=0 to z=d to find:
- $\frac{4}{3}[U(d)]^{\frac{3}{4}} = \sqrt{4k}d$
- **4.7** Deduct that the current density J_0 blocking the particle extraction is such that:

$$\boldsymbol{J_0 = \frac{4\epsilon_0}{9}\sqrt{\frac{2q}{m}}\frac{V_0^{\frac{3}{2}}}{d^2}}$$

- THIS IS THE CHILD LANGMUIR CURRENT DENSITY LIMIT