Transverse Beam Dynamics

JUAS 2019 - Tutorial 1

1 Exercise: Wien Filter

A Wien Filter is a device that allows to select particles in a beam according to their velocity.

1. Write down the expression of the Lorentz force.
2. How should we orient an electric field $\vec{E}$ if we want to compensate the force of an uniform magnetic field $\vec{B}$?
3. Assuming the magnetic field is 2 mT, what would be the required electric field (V/m) to select protons travelling with a velocity of $0.15c$?
4. Assuming that the particles move along the z axis and $\vec{B} = \begin{pmatrix} 0, & B_y, & 0 \end{pmatrix}$, write the equations of motion.
5. [Optional] Could we use a Wien filter with a neutral beam (eg. neutron)? What other techniques could be employed to create a velocity filter?

2 Exercise: Understanding the phase space concept

1. Phase Space Representation of a Particle Source:
   - Consider a source at position s_0 with radius w emitting particles. Make a drawing of this setup in the configuration space and in the phase space. Which part of the phase space can be occupied by the emitted particles?
   - Any real beam emerging from a source like the one above will be collimated. This can be modelled by assuming that a distance d away from the source there is an iris with opening radius $R = w$. Draw this setup in the configuration space and in the phase space. Which part of the phase space is occupied by the beam, right after the collimator?
2. Sketch the emittance ellipse of a particle beam in:

   (I) horizontal x-x' phase space at the position of a transverse waist,

   (II) when the beam is divergent, and

   (III) when the beam is convergent.

3 Exercise: Local radius, rigidity

We wish to design a proton ring with a radius of $R = 200$ m. Let us assume that only 50% of the circumference is occupied by bending magnets:

- What will be the local radius of bend ρ in these magnets if they all have the same strength?
- If the kinetic energy of the protons is 2 GeV, calculate the beam rigidity $B\rho$ and the field in the dipoles.

4 Exercise: Thin-lens approximation

1. Compute and compare the matrices of the thick and thin-lens approximation of a quadrupole with $k_q = 0.01$ m^{-2}; $L_q = 5.5$ m
2. Verify if the stability condition is valid
3. Would the answer be the same, if the quadrupole was defocusing?

5 Exercise: Hill's equation

Solve the Hill's equation:

$$y'' + k(s)y = 0$$

by substituting:

$$y = A\sqrt{\beta(s)}\cos[\phi(s) + \phi_0] \text{ with } \phi' = \tfrac{1}{\beta(s)}, \text{ and where } A \text{ and } \phi_0 \text{ are constants,}$$

demonstrating that a necessary condition is:

$$\tfrac{1}{2}\beta\beta'' - \tfrac{1}{4}\beta'^2 + k(s)\beta^2 = 1$$