Methodical Accelerator Design - Next Generation
Tilt or not Tilt? *Notes about frames, patches and angles.*
ABP-HSS meeting

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PLAN

1. **Overview: Survey, Track, Cofind, Twiss, Match, Plot**
   Unification and (re)use of the command interfaces.

2. **Survey command and Plot command.**
   Geometrical tracking.

3. **Track command and Beam command.**
   Dynamical tracking.

4. **Twiss command and Cofind command.**
   Tracking DA maps, computing optical functions.

5. **Match command.**
   Optimising lattices, functions optima.

Annexes

I. Object model, Commands, Environments.
II. MTable, Sequence, Elements.
III. Symplectic integrators.
IV. DA maps, GTPSA.
V. Linear Algebra.

Clarifications needed here!

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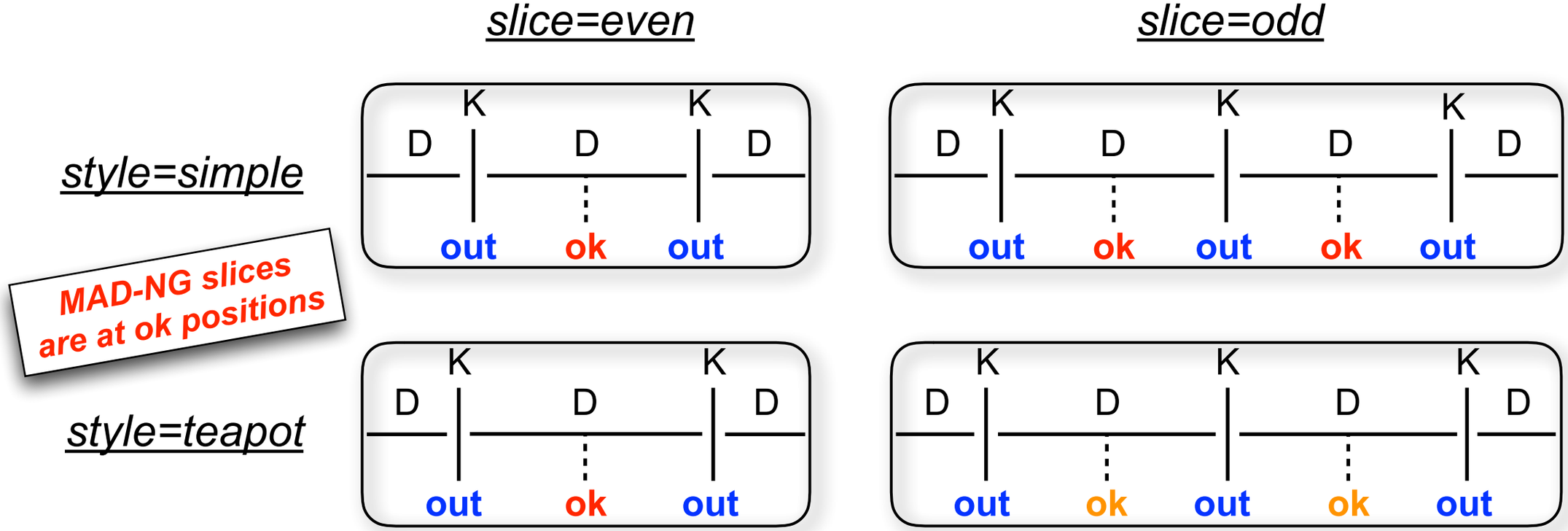
MAD-X consistency

- Recommendations for unified use of MAD-X (main) commands:
  - Use SEQUENCE with *refer=centre* for MAKETHIN compatibility.
  - Use TWISS with *centre=false* for SURVEY compatibility.
  - Use MAKETHIN with *slice>=2* in SBEND for SURVEY and TWISS compatibility at *exit*.
  - Use "**thin lens**" physics at *centre* (even slices) or *exit* of thick elements *(**ok** tags below)*.
    - Install markers on ***ok*** *tags* or *exit* to save values at valid s-positions.



Track: shouldn't be problem as "observe" points are usually at well defined s-positions.
Twiss: save optical functions at MULTIPOLE **out** s-positions, consider results with care.
Space Charge: should take care about "SC kicks" s-positions.

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Example: chicane tilted by 30°

```
n=4 ; el=1.5 ; cl=el/2 ; ang=pi/2 ; tlt=pi/6 ;

seq: sequence, l=(2*n+1)*el ;
!s1: srotation, at=0, angle=-tlt; ! <x,z> only
m1: sbend, at=cl+1*el, l=el, angle=-ang, tilt=+tlt ;
m2: sbend, at=cl+3*el, l=el, angle=+ang, tilt=+tlt ;
m3: sbend, at=cl+5*el, l=el, angle=+ang, tilt=+tlt ;
m4: sbend, at=cl+7*el, l=el, angle=-ang, tilt=+tlt ;
endsequence ;

beam ;
use sequence=seq ;
survey, file="srv_chicane_30.tfs" ;
plot, table=survey, haxis=z, vaxis={x,y} ;
```

patches version

```
! sbend-like patches for survey z->x,y
dl=el*sinc(ang/2) ; ! cord length

pbend: sequence, l=dl ;
ms1: srotation, at= 0, angle=+tlt;
my1: yrotation, at= 0, angle=±ang/2;
my2: yrotation, at=dl, angle=±ang/2;
ms2: srotation, at=dl, angle=-tlt;
endsequence ;
```

pbend replaces sbend in seq

convert to thin lens

In SURVEY, elements change the reference frame, i.e. act as patches: SBEND->angle/2 = YROTATION->angle

```
! thin-sbend for survey z->x,y
select, flag=makethin, slice=3 ;
makethin, sequence=seq, style=simple ;
```

patches version

```
! thin-like patches for survey z->x,y
pbend: sequence, l=el ;
mt1: srotation, at=0     , angle=+tlt ;
my1: yrotation, at=el/6  , angle=±ang/3;
my2: yrotation, at=el/2  , angle=±ang/3;
my3: yrotation, at=5*el/6, angle=±ang/3;
mt2: srotation, at=el    , angle=-tlt ;
endsequence ;
```

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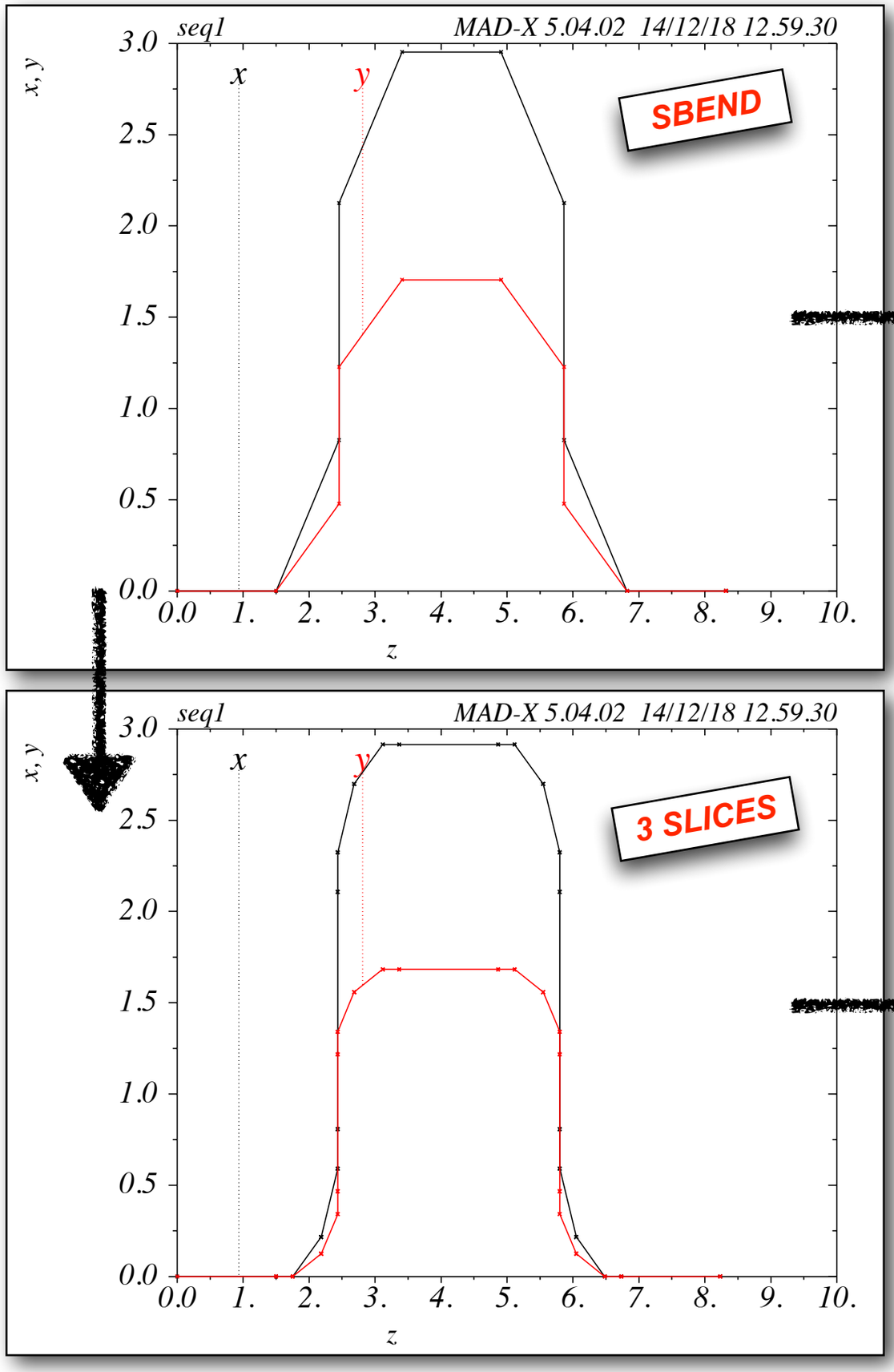
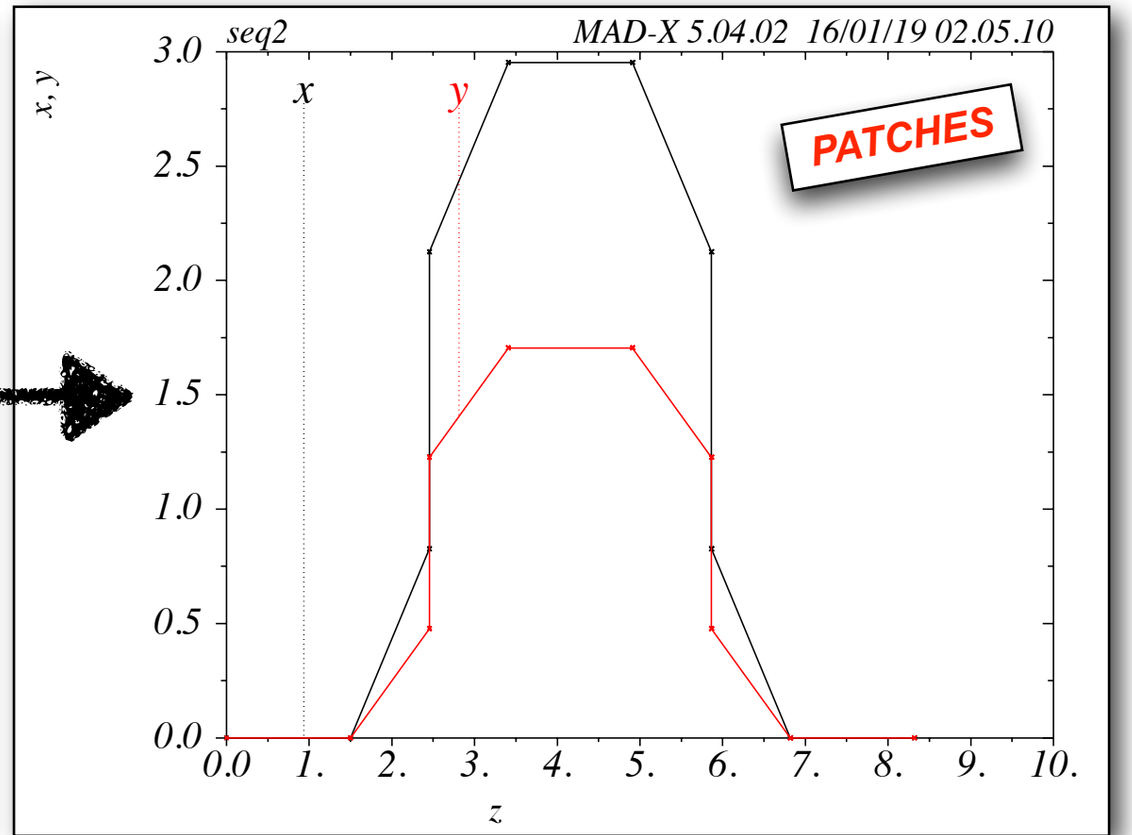
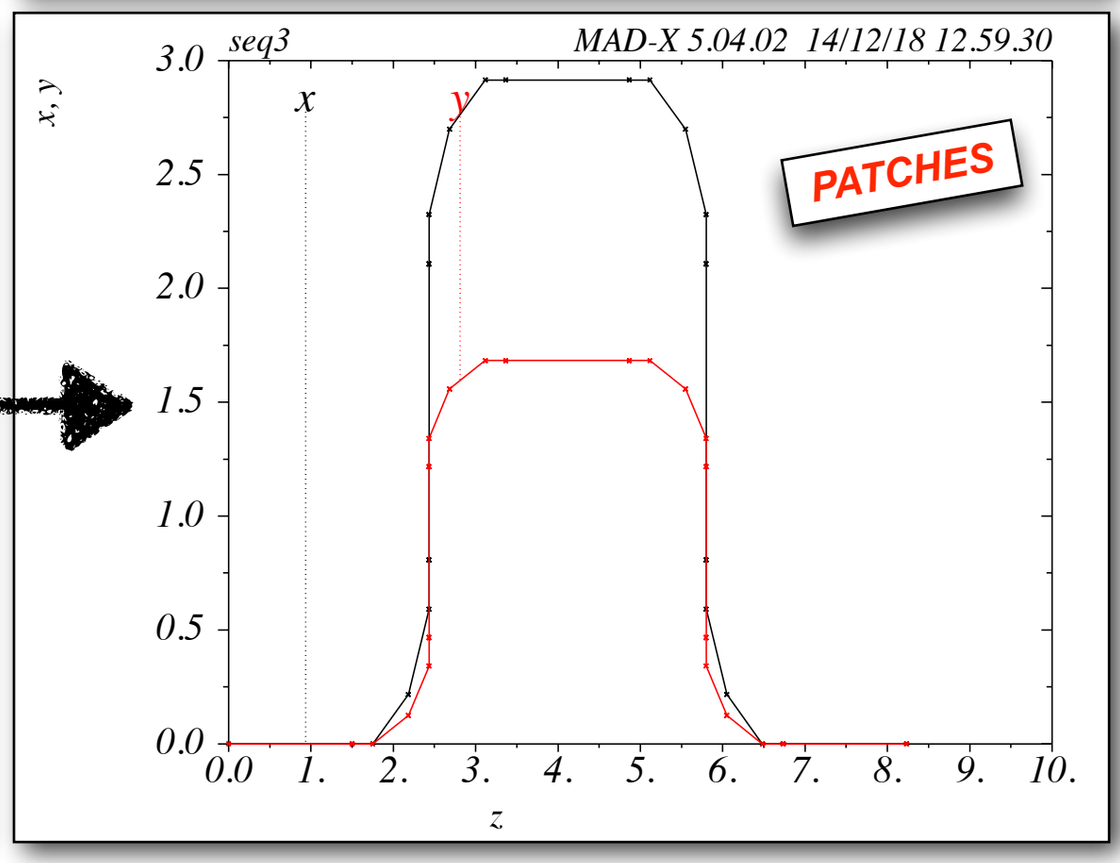
Example: chicane tilted by 30° *(cont'd)*







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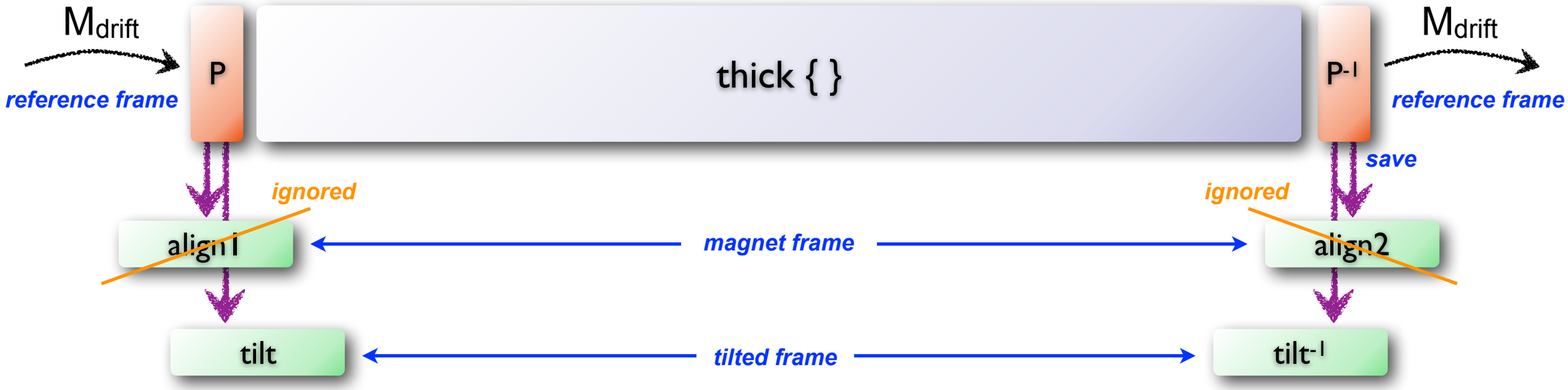
Element tracking - Survey



- ~~**psi**: s-*rotation* from **reference frame** to **magnet frame**.~~
- **tilt**: *s-rotation* from **magnet frame** to **tilted frame**.
- **angle**: *y-rotation* in the **tilted frame.**

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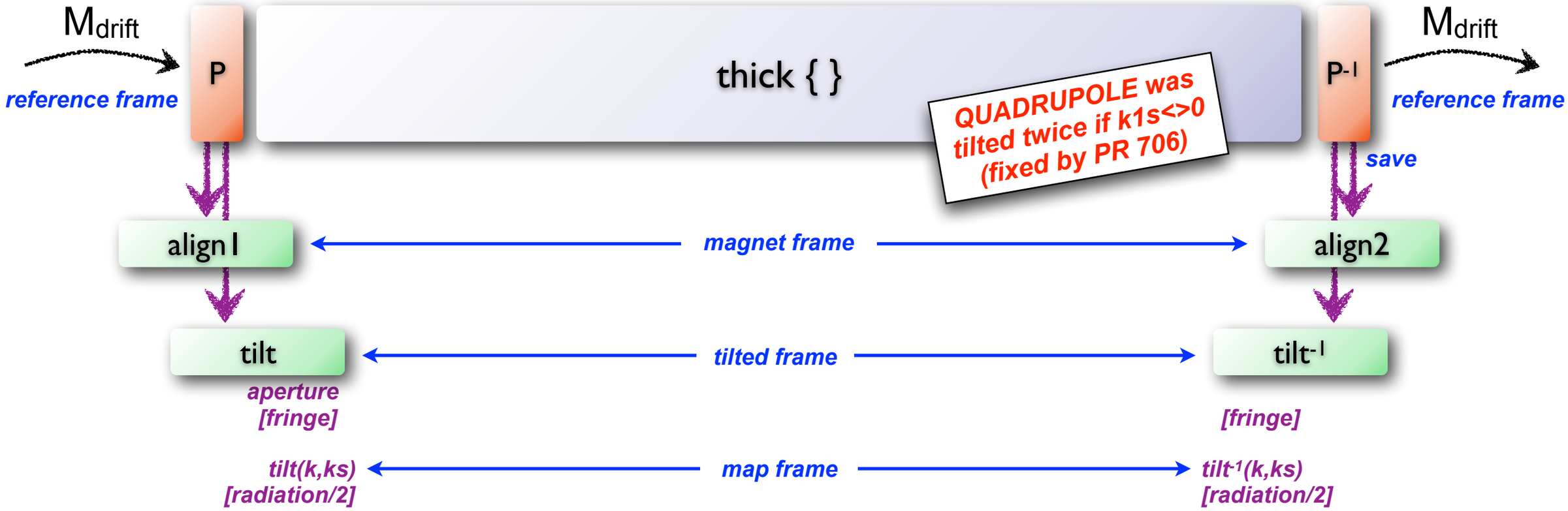
Element tracking - Track and Twiss



- **psi**: s-*rotation* from **reference frame** to **magnet frame**.
- **tilt**: *s-rotation* from **magnet frame** to **tilted frame**.
- **angle**: *y-rotation* in the **tilted frame** (e.g. SBEND).
- *aperture* **tilt**: *s-rotation* in the **tilted frame** (e.g. aperture frame).
- k_i+dknl$_i$/l, ks_i+dksl$_i$/l: magnet main *i*-strengths in the **tilted frame**, i > 0.
- k: magnet main *i*-strength in the (untilted) **map frame**.

Radiation occurs in map frame.

Multipoles and field errors are ignored, except by MULTIPOLE.

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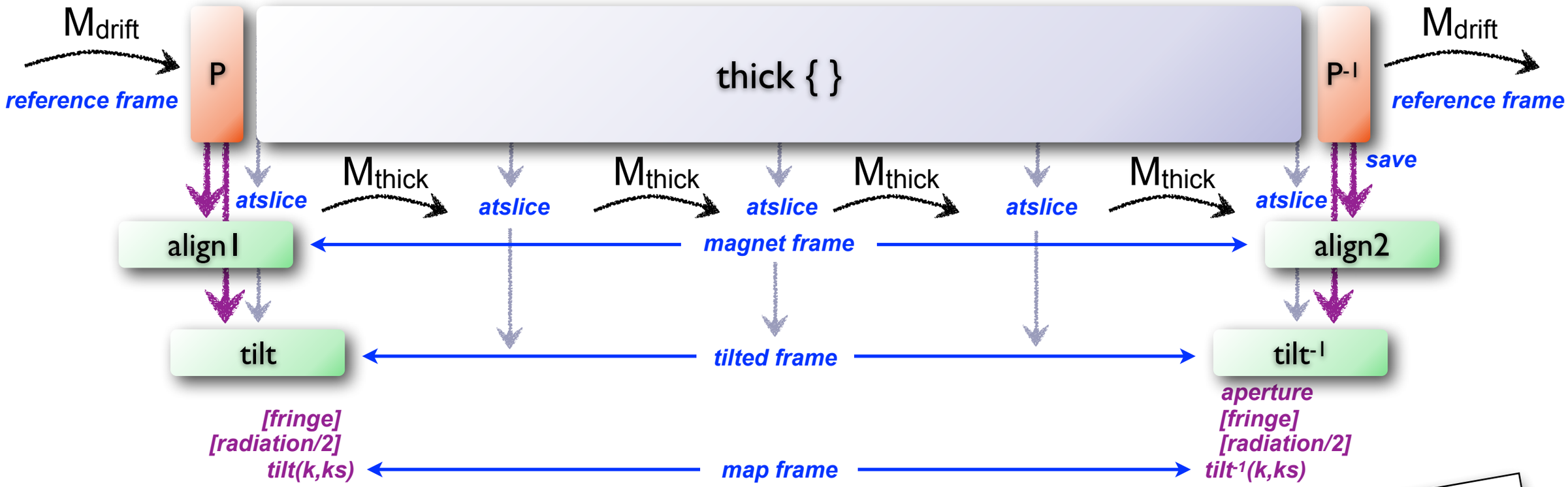
Element tracking with slices - MAD-NG



- **psi**: s-*rotation* from **reference frame** to **magnet frame**.
- **tilt**: *s-rotation* from **magnet frame** to **tilted frame**.
- **angle**: *y-rotation* in the **tilted frame** (e.g. SBEND).
- *aperture* **tilt**: *s-rotation* in the **tilted frame** (e.g. aperture frame).
- $k_i+(knl_i+dknl_i)/l$**,** $ks_i+(ksl_i+dksl_i)/l$: magnet main *i*-strengths in the **tilted frame**, i > 0.
- k: magnet main *i*-strength in the (untilted) **map frame**.
- knl_j+dknl_j**,** ksl_j+dksl_j: magnet integrated multipole strengths (j≠i) in the **tilted frame**.

"atslice" actions and thin subelements act in tilted frame.

Multipoles and field errors are considered.

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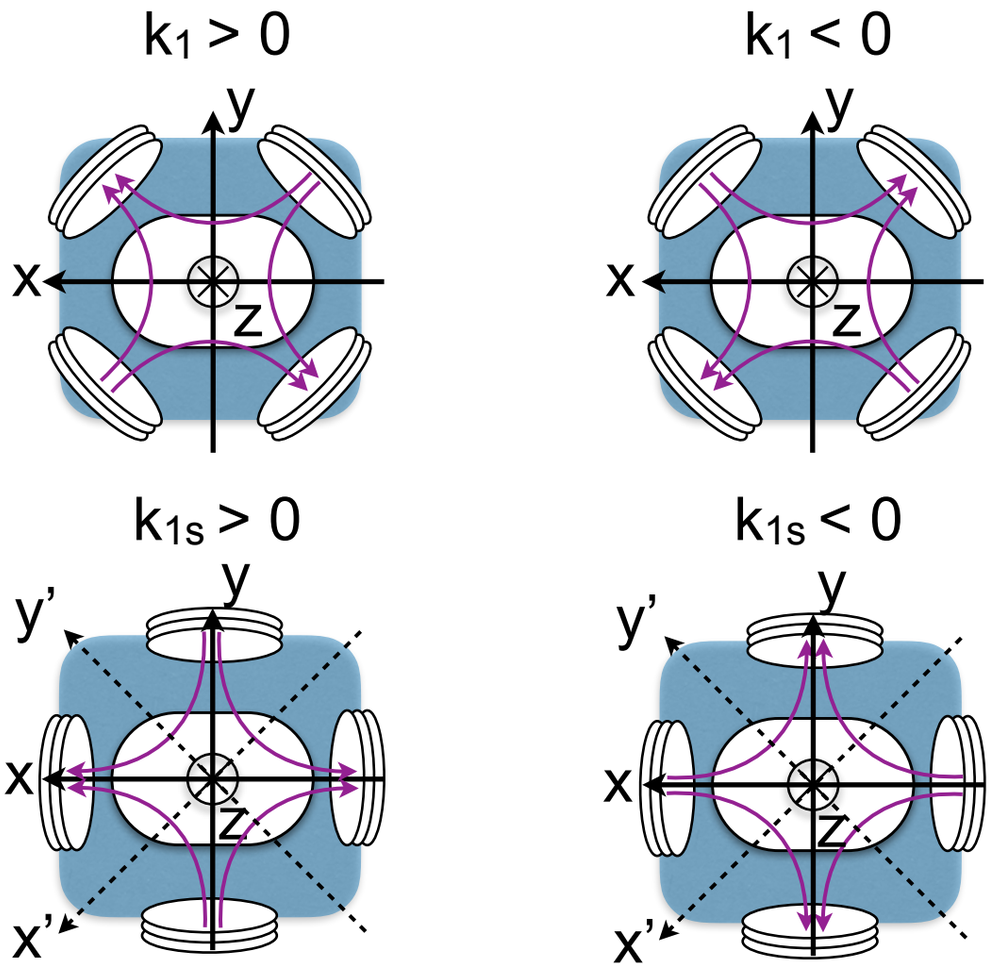
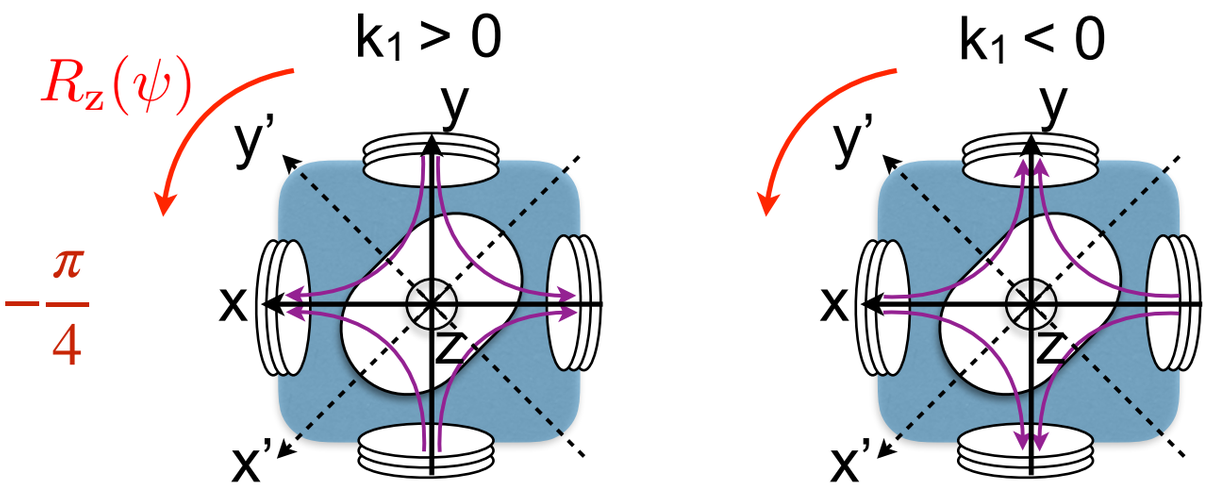
Quadrupole example

Quad:



Skew Quad:



Quad tilt = $-\frac{\pi}{4}$

$$k_1 = \frac{\partial B_y}{\partial x} B_\rho^{-1}$$

$$k_{1s} = \frac{1}{2}\left(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\right) B_\rho^{-1}$$

x,y: magnet frame
x',y': map frame

Aperture checks are in tilted frame.

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From magnet frame to map frame

- Tilt the map by angle α to cancel the tilt ψ and the *i*-skew component ks_i, i > 0.

$$k_m = k_i + (knl_i + dknl_i)/l$$
$$ks_m = ks_i + (ksl_i + dksl_i)/l$$
$$k = \sqrt{k_m^2 + ks_m^2}$$
$$\alpha = \psi - \frac{1}{m} \arctan \frac{ks_m}{k_m}$$

MAD-X ignores knl_i and ksl_i.

- Use the tilted (untitled) map (i.e. only T in TKT model in MAD-NG)

$$\mathbf{x_2} = R_z(\alpha) \cdot \mathcal{M}_\perp(k) \cdot R_z(-\alpha)\, \mathbf{x_1}$$

- MAD-X remarks for Track and Twiss:
  - ➡ Tilted frame and map frame are intermixed, physics must be correct in both.
  - ➡ Thick maps discard integrated strength knl, ksl, $dknl_j$ and $dksl_j$ for j ≠ i (required!).
  - ➡ SBEND and RBEND consider *angle*, k_0+$dknl_0$/l, k_1+$dknl_1$/l [, k_2+$dknl_2$/l]**(twiss only)**.
  - ➡ MULTIPOLE expects knl, ksl, dknl and dksl in the tilted frame.
- MAD-NG remarks:
  - ➡ Tilted frame and (thick untilted) map frame are NOT intermixed, tilted frame is used for everything but T in TKT model, i.e. multipole kicks K are in the **tilted frame**.

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Misalignments and straight element

- EALIGN angles $d\theta$, $-d\phi$ and $d\psi$ are right handed.
  - ➡ $d\theta$ = azimutal angle ($d\theta$ ➠ $-\theta$ of SBEND)
  - ➡ $d\phi$ = elevation angle ($d\phi > 0$ ➠ $dy > 0$)
  - ➡ $d\psi$ = roll angle

Compatible with reversed sequence (e.g. LHCB2).

Angles can be "large" in MAD-NG and computed to full order (as in PTC).

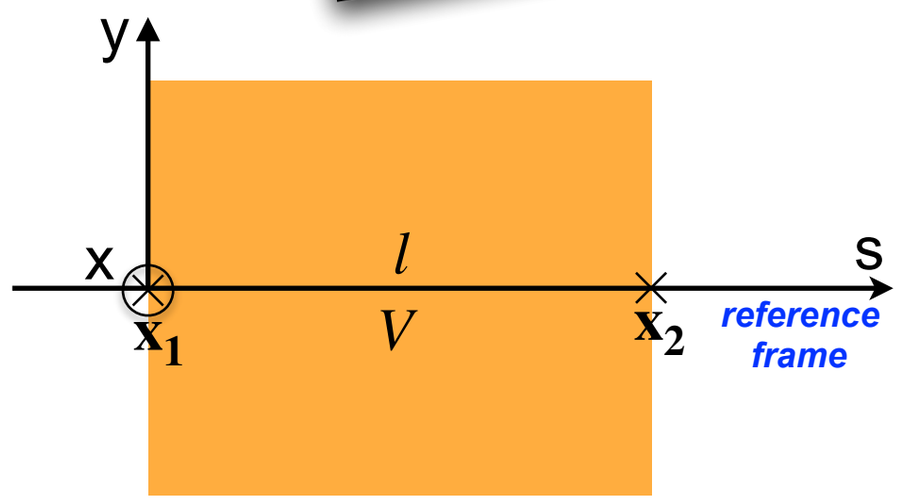
- At **entry**, an element is rotated by $R(d\theta,d\phi,d\psi)$ and translated by $T(dx,dy,ds)$.

$$\mathbf{x_1'} = R\,\mathbf{x_1} + T \qquad \text{(transform the frame)}$$

$$\mathbf{x_1} = R^t(\mathbf{x_1'} - T) \qquad \text{(transform the coordinates)}$$

$$R = R_y(d\theta)\cdot R_x(-d\phi)\cdot R_z(d\psi)$$

$$R^t = R_z(-d\psi)\cdot R_x(d\phi)\cdot R_y(-d\theta) \qquad \text{(inverse)}$$



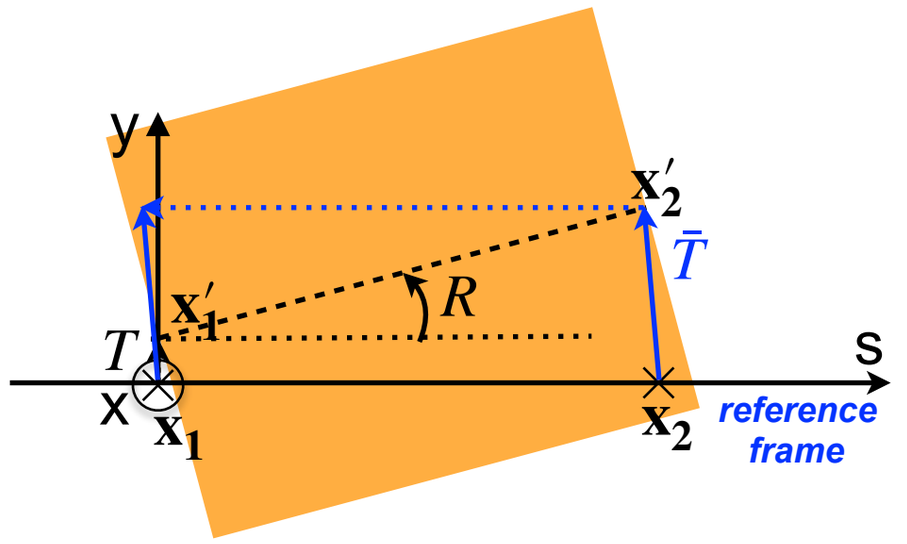
- At **exit**, using $V = \overrightarrow{\mathbf{x_1 x_2}} = [0,0,l]^t$ (displacement) and $W = I$ (rotation) of an element with a length *l*.

$$\mathbf{x_2'} = \bar{R}\,\mathbf{x_2} + \bar{T}$$

$$\mathbf{x_2} = \bar{R}^t(\mathbf{x_2'} - \bar{T})$$

$$\bar{T} = W^t(RV + T - V) = W^t(\overrightarrow{\mathbf{x_1 x_1'}} - \overrightarrow{\mathbf{x_1 x_2}})$$

$$\bar{R} = W^t R W \;;\; \bar{R}^t = W R^t W^t$$



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Misalignments and curved element

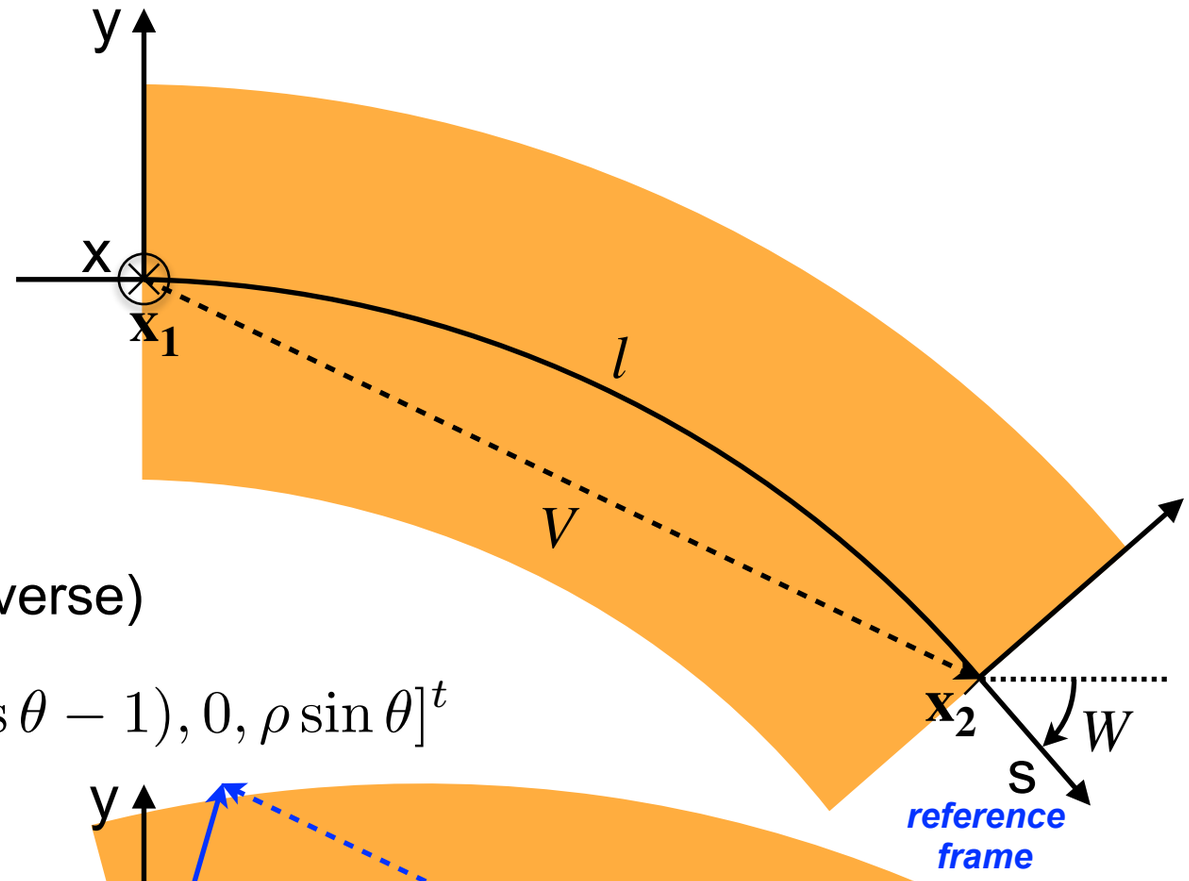
- At **entry**, an element is rotated by *R(dθ,dϕ,dψ)* and translated by *T(dx,dy,ds)*.

$$\mathbf{x_1'} = R\,\mathbf{x_1} + T$$

$$\mathbf{x_1} = R^t(\mathbf{x_1'} - T)$$

$$R = R_y(d\theta)\cdot R_x(-d\phi)\cdot R_z(d\psi)$$

$$R^t = R_z(-d\psi)\cdot R_x(d\phi)\cdot R_y(-d\theta) \qquad \text{(inverse)}$$



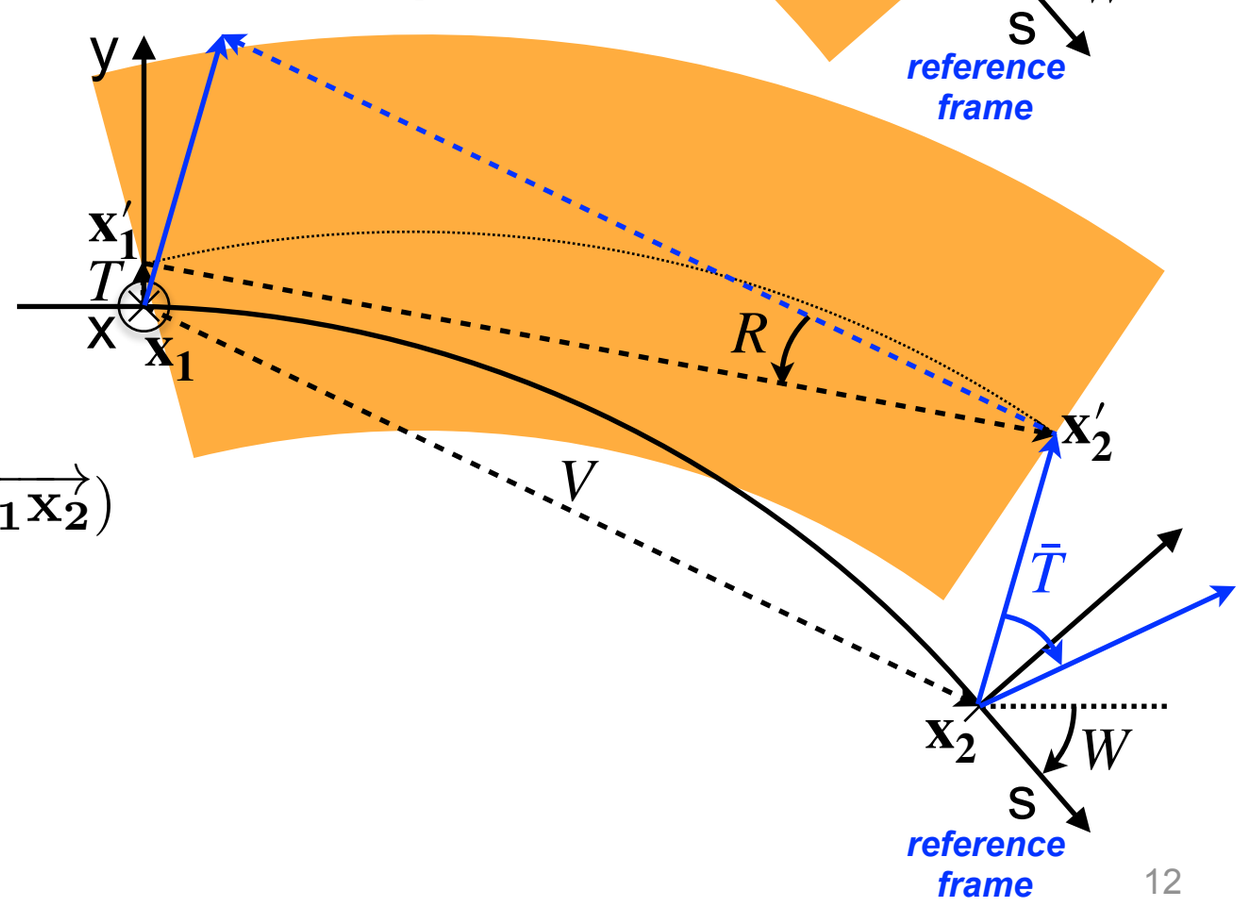
- At **exit**, using $V = \overrightarrow{\mathbf{x_1 x_2}} = R_z(\psi)\cdot[\rho(\cos\theta - 1), 0, \rho\sin\theta]^t$ and $W = R_z(\psi)\cdot R_y(-\theta)\cdot R_z(-\psi)$ of an element with length *l*, angle θ and tilt ψ (i.e. $\rho = l/\theta$).

$$\mathbf{x_2'} = \bar{R}\,\mathbf{x_2} + \bar{T}$$

$$\mathbf{x_2} = \bar{R}^t(\mathbf{x_2'} - \bar{T})$$

$$\bar{T} = W^t(RV + T - V) = W^t(\overrightarrow{\mathbf{x_1 x_2'}} - \overrightarrow{\mathbf{x_1 x_2}})$$

$$\bar{R} = W^t R W \;;\; \bar{R}^t = W R^t W^t$$



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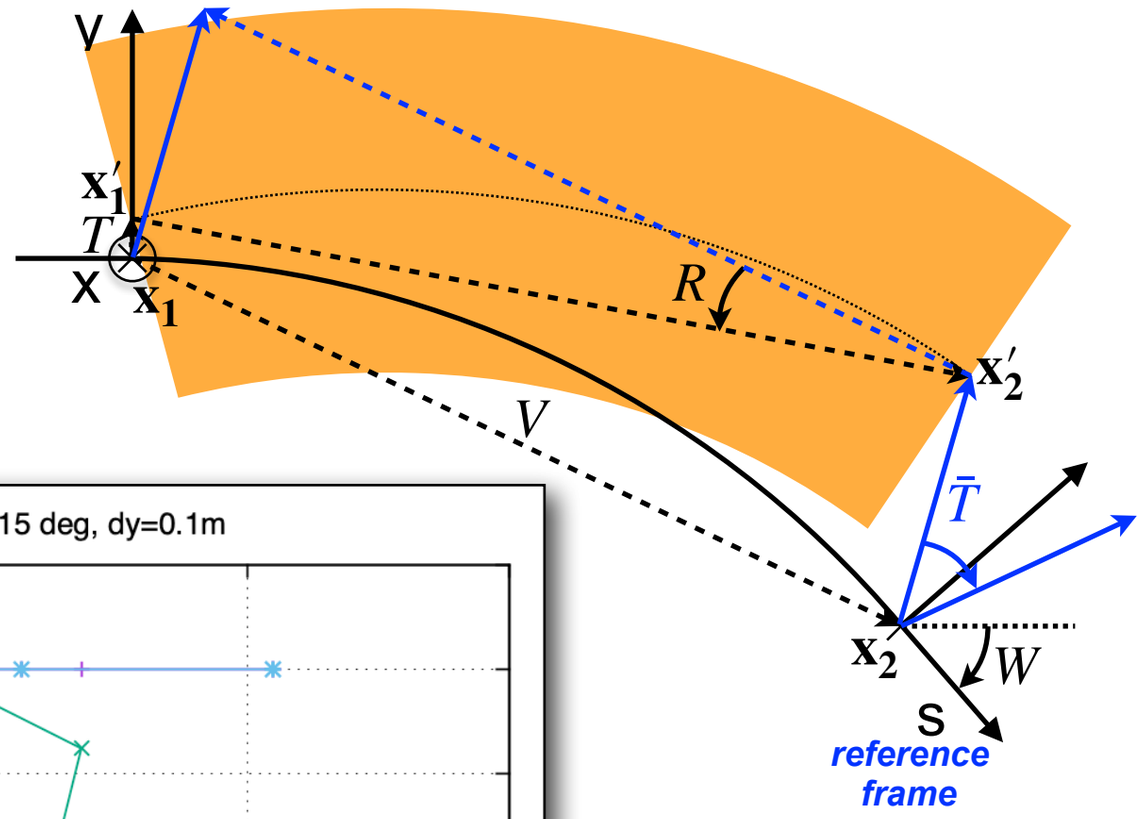
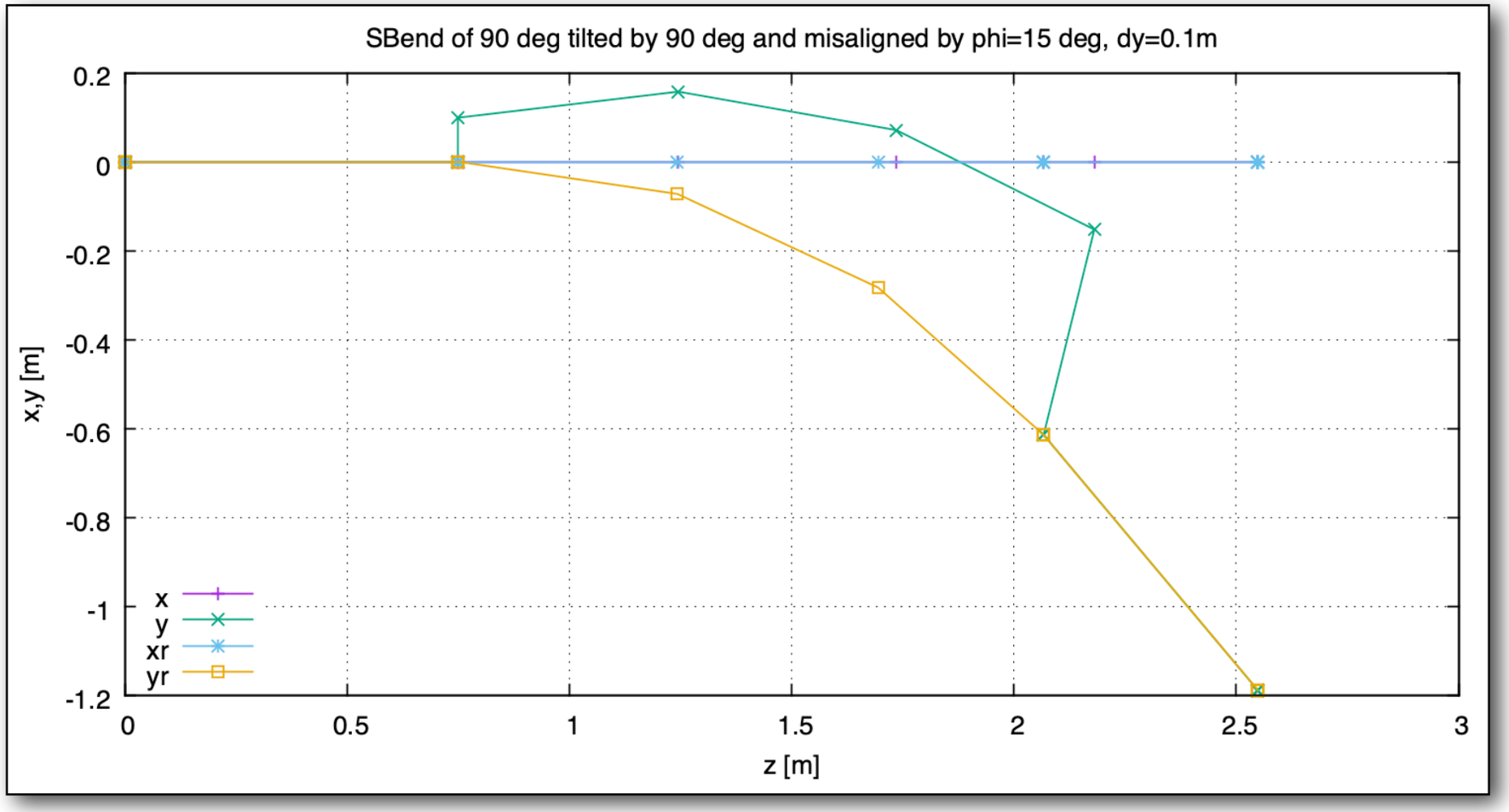
Example: sbend tilted by 90° — dphi 15° dy 0.1m

x, y with misalignments, xr, yr reference frame without misalignment

SBEND angle is ~50 deg.





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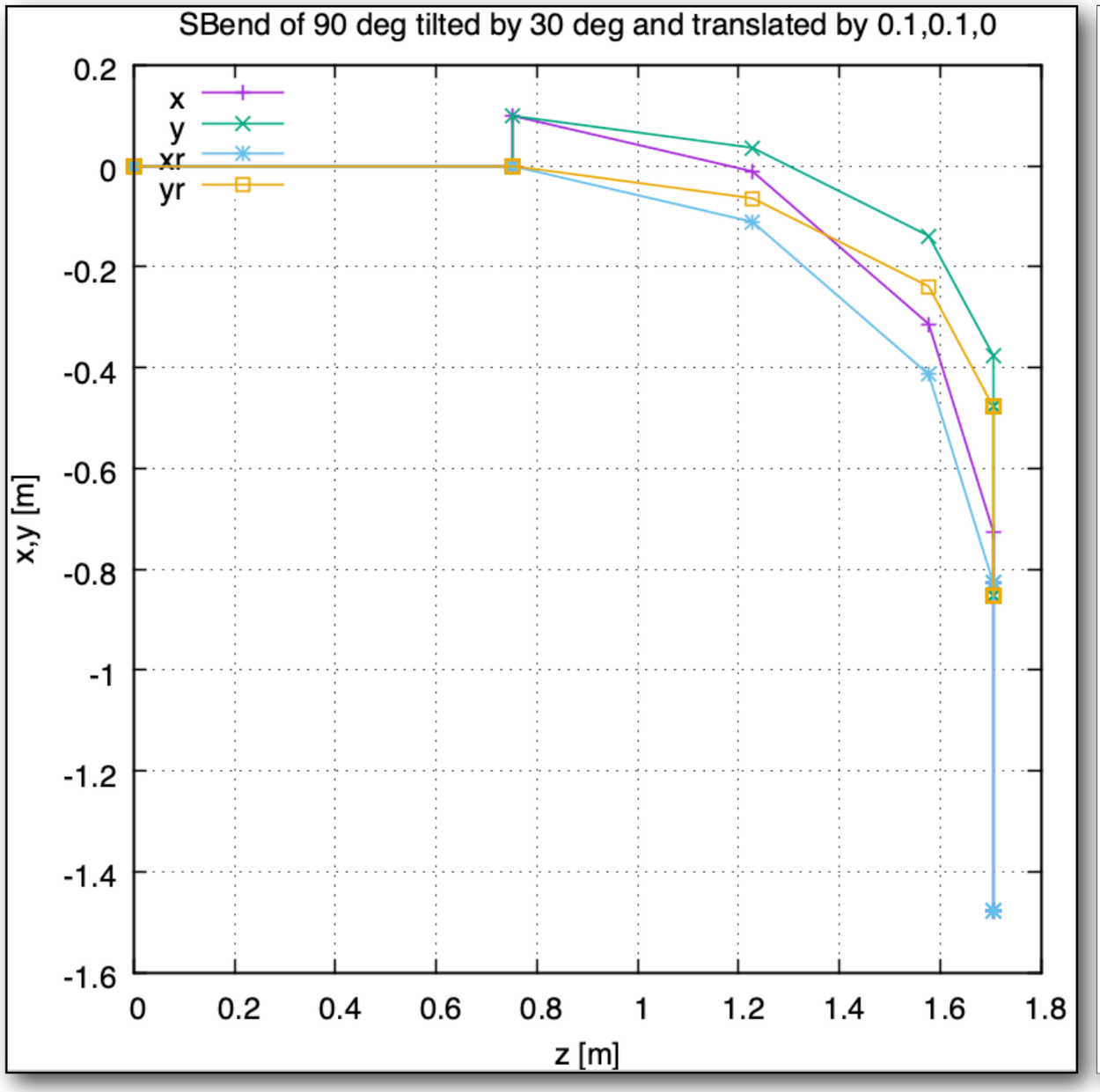
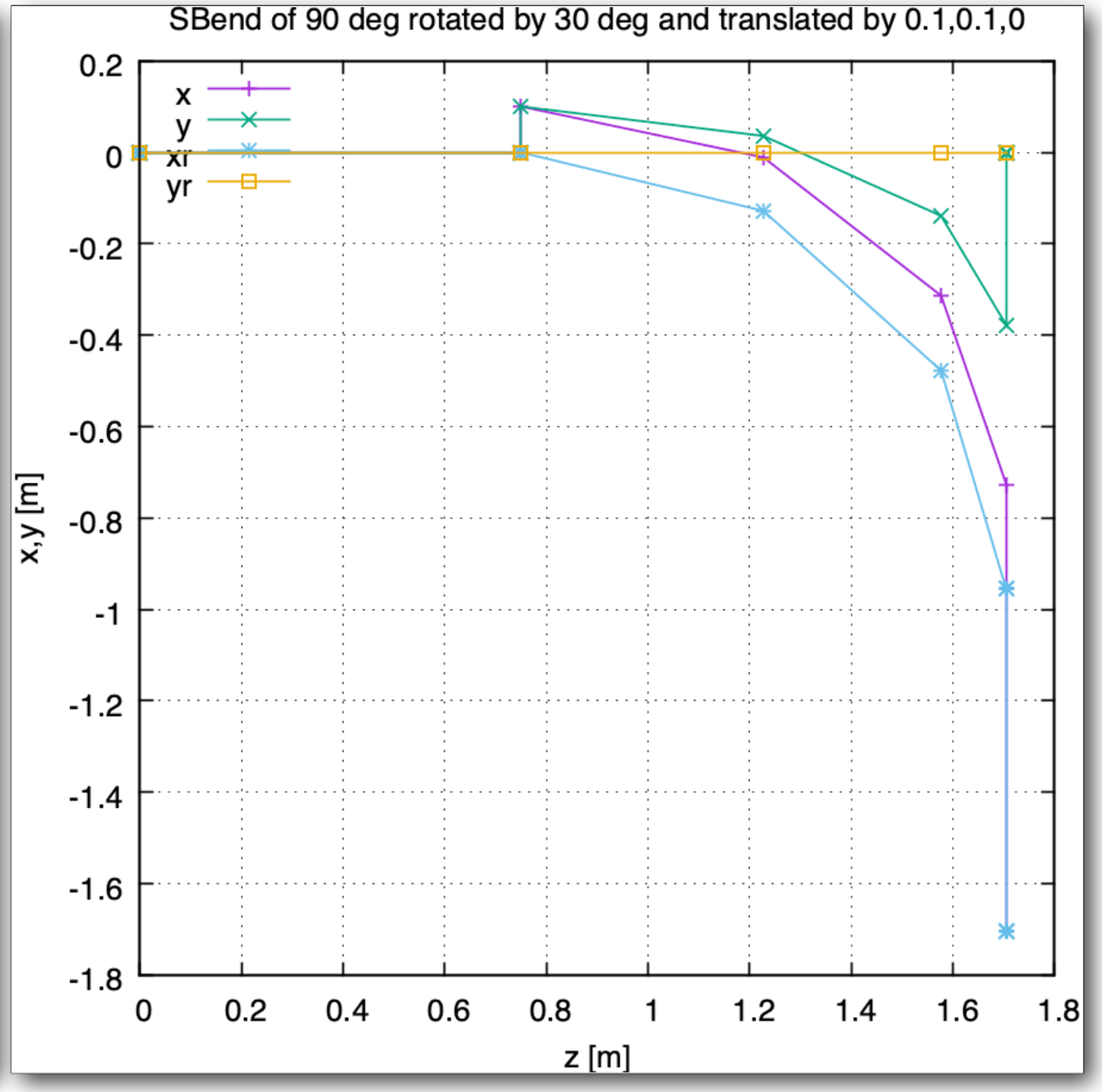
Example: sbend tilted/rotated by 30° with dxdy 0.1m

x, y with misalignments, xr, yr reference frame without misalignment





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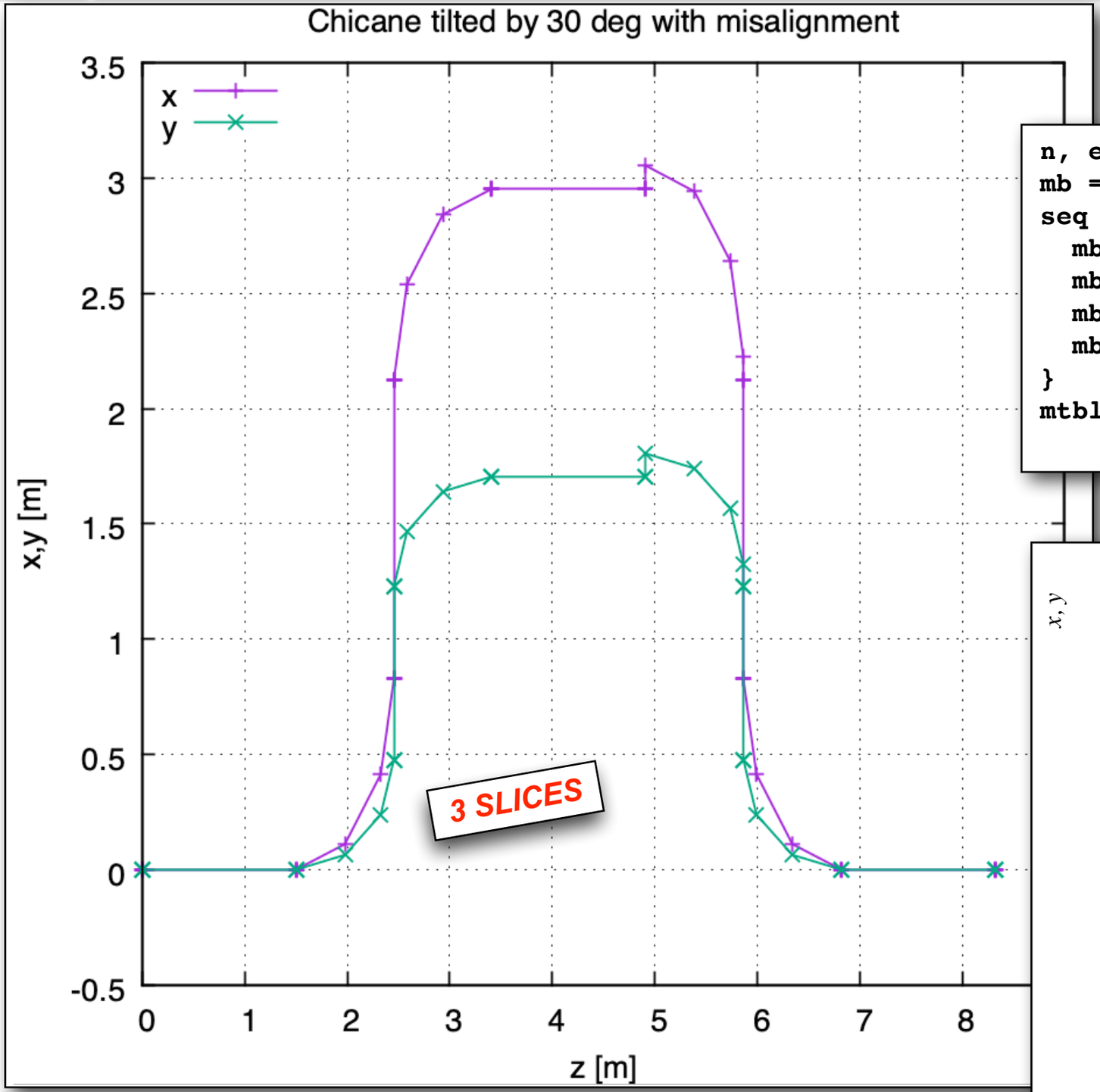
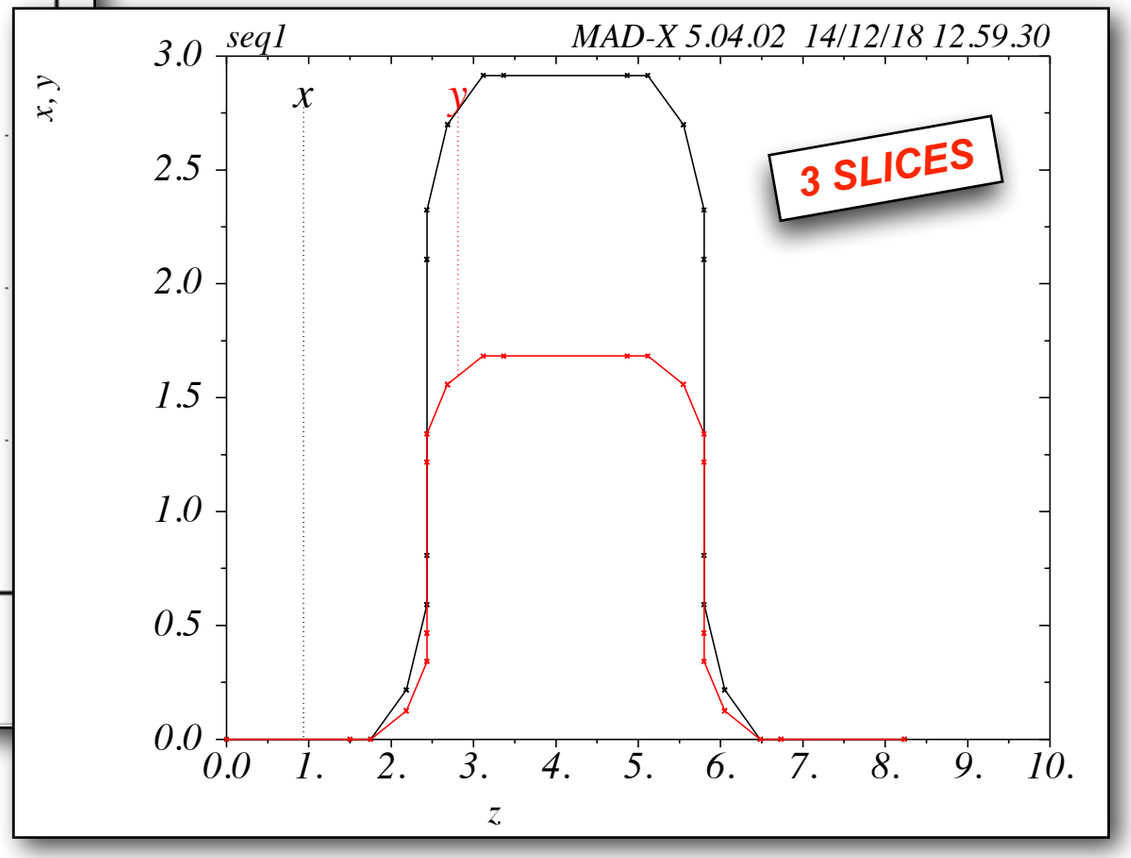
Example: chicane tilted by 30° with dxdy 0.1m




```
n, el, ang, tlt = 5, 1.5, pi/2, pi/6
mb = sbend 'm1' {l=el, tilt=tlt},
seq = sequence 'seq' { l=(2*n-1)*el,
  mb 'm1' {at=1.5*el, angle=-ang},
  mb 'm2' {at=3.5*el, angle= ang},
  mb 'm3' {at=5.5*el, angle= ang, dx=0.1, dy=0.1},
  mb 'm4' {at=7.5*el, angle=-ang},
}
mtbl = survey {sequence=seq,
        nslice=3, atslice=ftrue, misalign=true}
```



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