Wall velocities:
the (often) unknown parameter

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Relics of the electroweak phase transition:
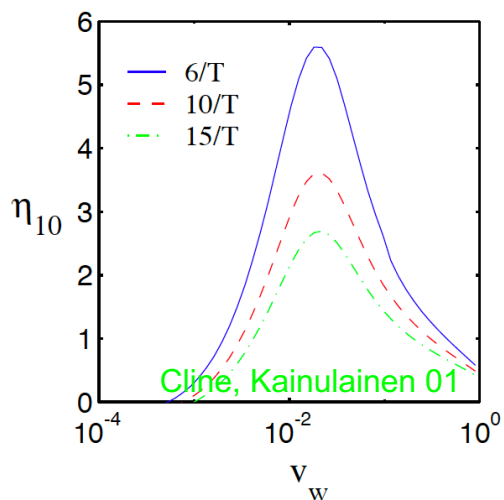
Baryon asymmetry

Transition strength (v/T)

Bubble profile (L_w, etc)
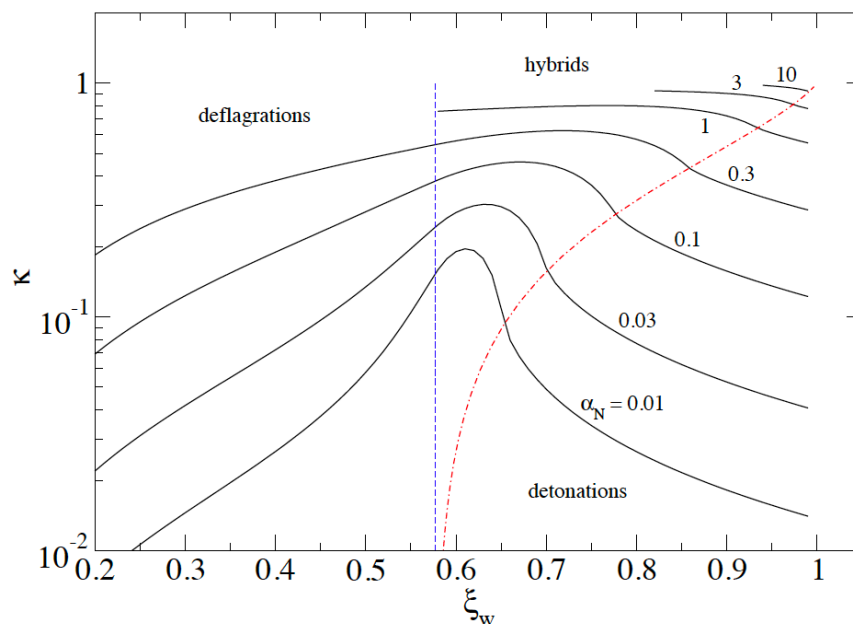
CP violation

Transport coefficients

Bubble velocity

Gravitational waves

Transition strength (energy release)

Bubble separation

Bubble velocity

Cline, Kainulainen 01

Needs subsonic wall

Espinosa, Konstandin, No, Servant 10

Despite its importance, we do know little about wall velocities in interesting situations:

the "Aschenputtel" (Cinderella) of phase transition parameters

Gustave Doré, 1897

Outline

● quick route to a strong phase transition: reduced vacuum depth

● wall velocities in a SM like plasma (but modified Higgs potential)

● Summary & outlook

First order phase transitions

Here for the electroweak phase transition, similar methods for PT's eg. in hidden sectors
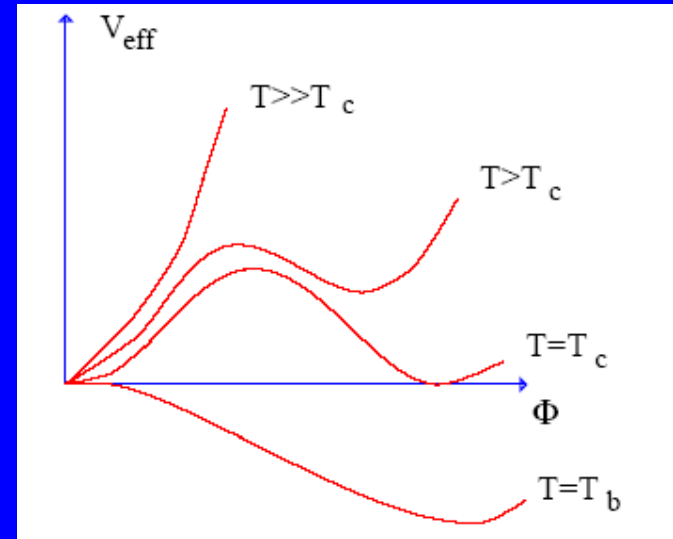
The strength of the PT

Thermal effective potential:

$$V_{\text{eff}}(\phi, T) = (-m^2 + AT^2)\phi^2 - ET\phi^3 + \lambda\phi^4$$

Thermal mass:
symmetry restauration
at high temperature

Cubic term:
bosons only,
induces PT

Useful measure of the <u>strength of the transition</u>: $\xi = \dfrac{v_c}{T_c}$

For strong transitions, $\xi \gtrsim 1$: thermal perturbation theory (1 or 2-loop)

Weak transitions: lattice methods (often 3D), eg. for SM crossover

How to make a strong transition?

1) Add <u>new bosons</u>, coupling sizably to the Higgs (increase E), eg.

● Light stops in the MSSM (now mostly excluded by Higgs properties)

[Carena, Nardini, Quiros,Wagner 2012]

● second Higgs doublet (2HDM)

[eg. Dorsch, SJH, Mimasu, No, 2017

Basler, Muehlleitner, Wittbrodt, 2017

Andersen et al. 2017,…]

● one can also build models relying on singlets, weak triplets, etc.
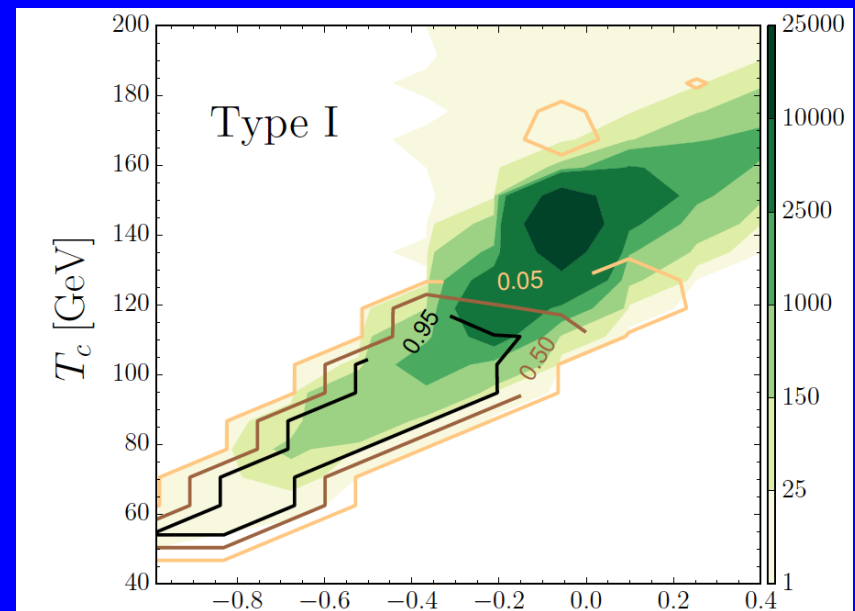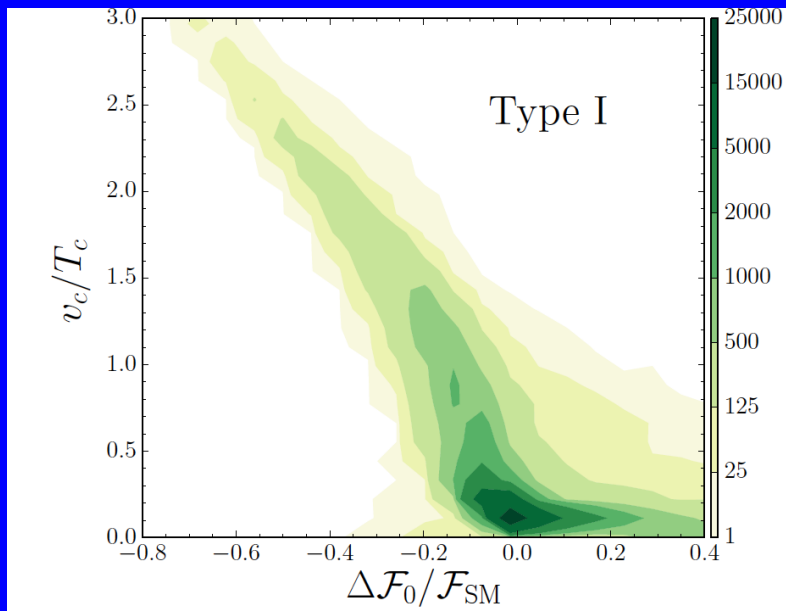
[eg. Niemi, Patel, Ramsey-Musolf, Tenkanen, Weir 2018]

How to make a strong transition?

2) Make the EW minimum less deep (ie. lower T_c, larger v_c/T_c):

a) By bosonic Coleman-Weinberg logs, eg. 2HDM   [Dorsch, SJH, Mimasu, No, 2017]

$$V_1 = \sum_\alpha n_\alpha \frac{m_\alpha^4(h_1, h_2)}{64\pi^2} \left(\log \frac{|m_\alpha^2(h_1, h_2)|}{Q^2} - C_\alpha \right)$$

Dominant effect for strong transitions

How to make a strong transition?

2b) make the EW less deep at <u>tree-level</u>

● include a Φ^6 term in the Higgs potential (a la EFT)

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{1}{8M^2}\phi^6$$

[eg. Chala, Krause, Nardini, 2018]

new term removes the link between the Higgs mass and vacuum depth

● use additional fields, in particular singlets to lower the <u>symmetric phase</u> ("two step transition") ie. broken phase relatively less deep

[eg. Inoue, Ovanesyan, Ramsey-Musolf 2015; Cline, Kainulainen, Tucker-Smith 2017]
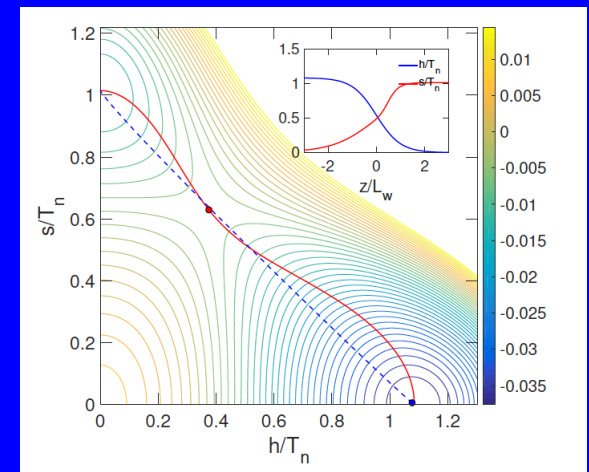
Bubble wall velocity

Types of bubble wall propagation:

deflagration $\xi_w < c_s$ hybrid $\xi_w > c_s$ detonation $\xi_w > c_s$
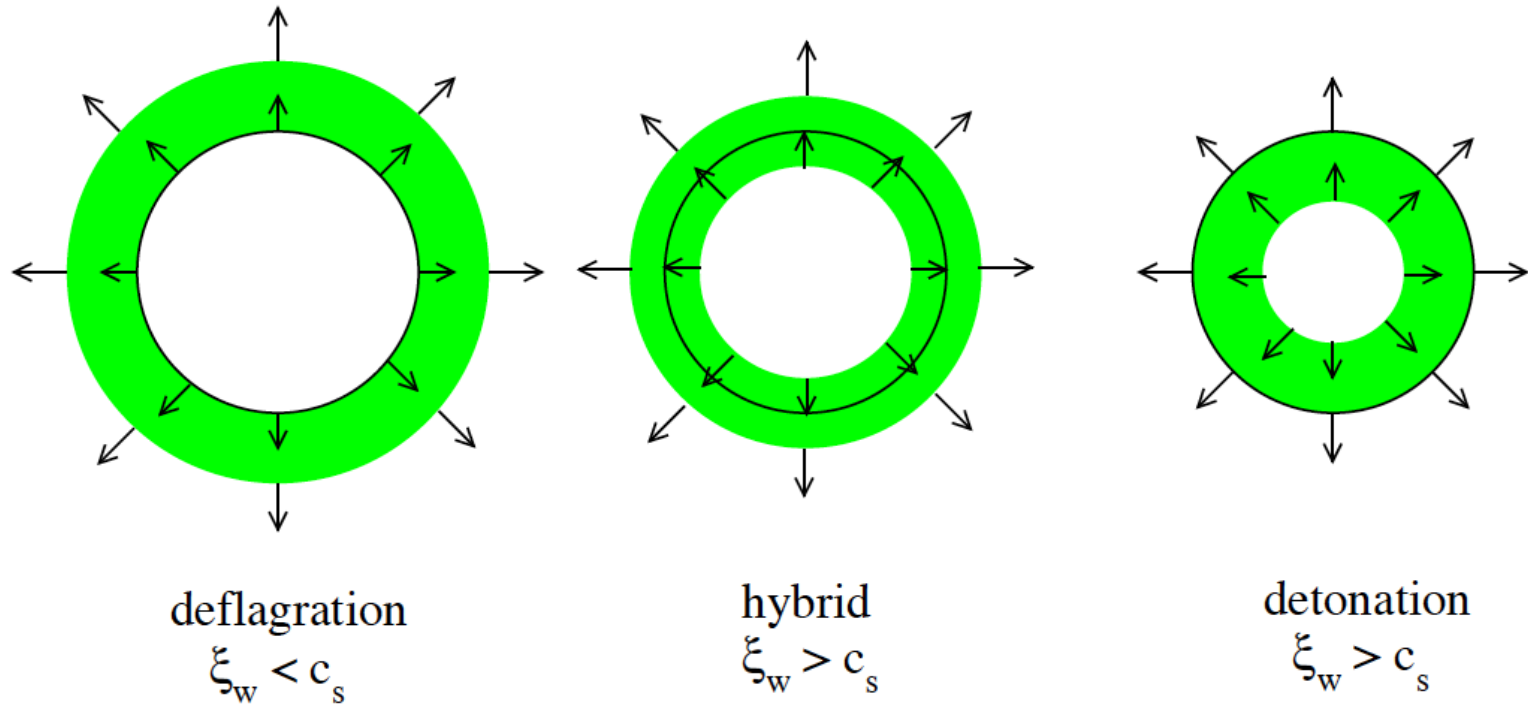
Espinosa, Konstandin, No, Servant '10

Speed of sound $c_s = 1/\sqrt{3}$

What sets the wall velocity?

Wall is accelerated by <u>internal pressure</u>

And slowed down by <u>friction</u> from the surrounding plasma

Often: <u>steady state</u> of constant terminal velocity v_w

Very strong transitions: pressure can win $\Rightarrow$ <u>runaway</u>

<u>in vacuum:</u> walls approach the speed of light as $\gamma \sim R$

How to compute the wall velocity? (1)

Friction comes from <u>departure of equilibrium</u> in the plasma, described by distribution functions $f_i(x,p)=f_{eq}+\delta f_i$

$$p^\mu \partial_\mu f_i = \text{collisions} + \text{forces}$$ (Boltzmann equation)

$$\text{forces} = -\frac{1}{2}\partial_z m_i^2 \partial_{p_z} f_i$$ $m_i(\Phi(z))$

$$\Box\phi + \frac{dV_T}{d\phi} + \sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E}\delta f_i(\vec{p},z) = 0$$ (KG equation)

Coupled set of integro-partial-differential equations

Tedious work to compute the collision terms

Fluid ansatz for the departure from equilibrium:

Wall frame (wall at rest, plasma moves)

$$f_i = \frac{1}{\exp(X_i) \pm 1}$$
$$X_i = \beta_i[u_\mu^i p^\mu + \mu_i]$$

$u_\mu{}^i(z)$ local plasma velocity

$\mu_i(z)$ local chemical potential

$\beta_i = 1/T_i(z)$ local temperature

Linearize, take moments, one arrives ordinary diff. eqs.:

$$av_w\frac{\mu'}{T} + v_w\frac{\delta T'}{T} + \frac{1}{3}v' + F_1 = -\Gamma_{\mu 1}\frac{\mu}{T} - \Gamma_{T1}\frac{\delta T}{T}$$
$$bv_w\frac{\mu'}{T} + v_w\frac{\delta T'}{T} + \frac{1}{3}v' + F_2 = -\Gamma_{\mu 2}\frac{\mu}{T} - \Gamma_{T2}\frac{\delta T}{T}$$
$$b\frac{\mu'}{T} + \frac{\delta T'}{T} + v_wv' + 0 = -\Gamma_v v$$

This requires 2 to 2 processes to be fast to obtain local kinetic equilibrium: check!

Most important in the SM: tops and W's

$$F_1 = -\frac{v_w \ln 2}{9\zeta_3}\frac{(m^2)'}{T^2}, \qquad F_2 = -\frac{v_w\zeta_2}{42\zeta_4}\frac{(m^2)'}{T^2}$$

Literature:

Moore, Prokopec '95: formalism, SM: $v_w \sim$ -0.35 - 0.45

John, Schmidt '00: MSSM: $v_w \sim$ 0.05

Bödeker, Moore '09, '17: "runaway"

SJH, Sopena '11, '13: MSSM, Φ^6 (effective friction parameter)

Konstandin, Nardini, Rues: Quantum-Boltzmann, SM, Φ^6

Kozaczuk '15: Higgs+singlet

Dorsch, SJH, Konstandin, No '16: 2HDM (deflagrations, effective friction parameter, baryogenesis and gravitiational waves at the same time?)

"Runaway": Bödeker, Moore '09, '17
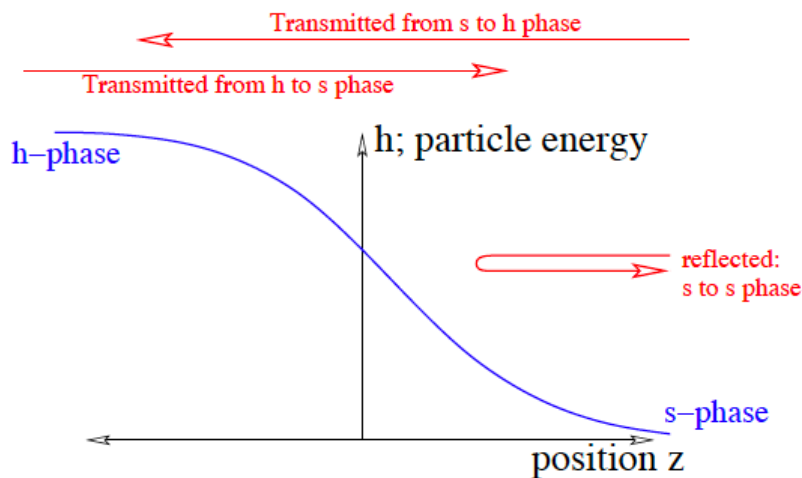
Highly relativistic wall: neglect particle interactions

Use equilibrium distributions in front of the wall

Momentum transfer to the wall from kinematics $\bar{p}_z = \sqrt{p_z^2 - m_b^2 + m_s^2}$

$$\frac{F}{A} = \sum_a (m_a^2(h_2) - m_a^2(h_1)) \int \frac{d^3p}{(2\pi)^3 2E_{p,h_1,a}} f_a(p, \text{in}) + \mathcal{O}(1/\gamma^2)$$

This is <u>finite</u>, so given sufficient pressure, the wall can <u>keep accelerating</u>

Particle production leads to a term which grows as γ, so

$$\gamma \sim 1/\alpha$$

(needs gauge bosons)

Bubble wall velocity
for a SM-like plasma

(With Glauber Dorsch and Thomas Konstandin 2018)

Aim:

make <u>maximal use</u> of the computation of friction in the <u>SM case</u> :

<u>tops quarks plus W bosons</u>

To avoid the integro-diff structure: make a <u>tanh ansatz</u> for the field

This leaves only two <u>free parameters</u>: wall velocity and wall thickness L_w

$$\Box\phi + \frac{dV_T}{d\phi} + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E} \delta f_i(\vec{p}, z) = 0$$

$$\frac{\Delta V_T}{T^4} = f$$

$$-\frac{2}{15(TL_w)^2}\left(\frac{\phi_0}{T}\right)^3 + \frac{W}{T^5} = g$$

$$W \equiv -\int_0^{\phi_0} \frac{dV_T}{d\phi}(2\phi - \phi_0)d\phi$$

ϕ_0 vev in the broken phase

W related to pressure gradient and L_w

The remaining terms

$$f = T^{-4} \int dz \frac{d\phi}{dz} \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E} \delta f_i(\vec{p}, z) \,,$$

$$g = T^{-5} \int dz \frac{d\phi}{dz} (2\phi - \phi_0) \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E} \delta f_i(\vec{p}, z)$$

Obtained from solving the fluid system and depend on

ϕ_0 /T, velocity and wall thickness, and transport rates

So the wall velocity will depend on $\{\Delta V, W, T, \phi_0\}$

More precisely three dimensionless ratios of these

Idea:

W can be eliminated by using the information from bubble nucleation

The critical bubbles satisfies

$$\frac{d^2\phi}{d\rho^2} + \frac{2}{\rho}\frac{d\phi}{d\rho} = \frac{dV}{d\phi}$$

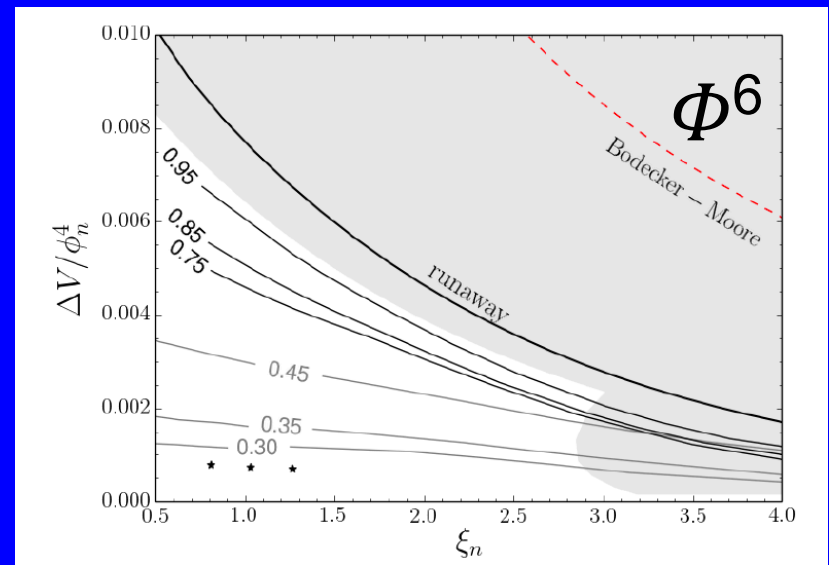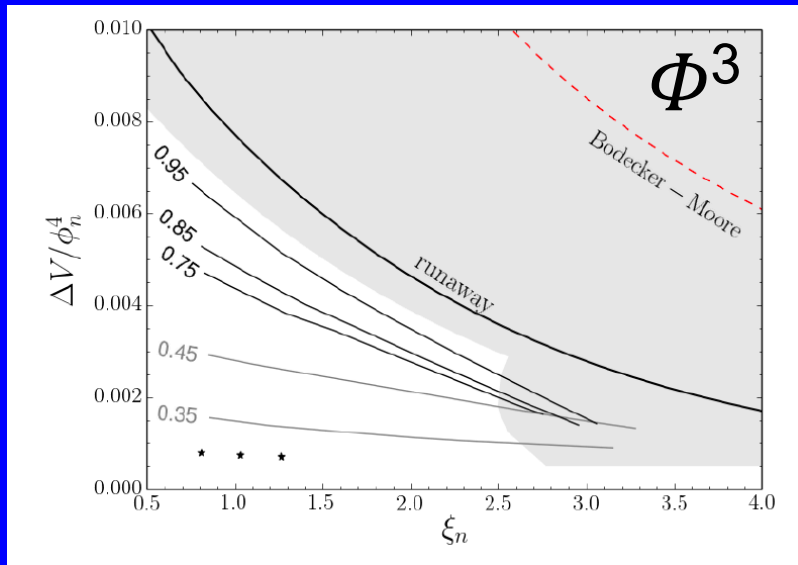with energy $S_3/T \sim 135$ at the temperature of the transition

Define X

$$\frac{S_3}{\phi_0} = \frac{W^{3/2}\phi^{1/2}}{\Delta V^2} \times X(W/\Delta V/\phi_0)$$

So X is approximately constant

and we can eliminate W and

v_{w} will depend only on $\{\Delta V/\phi_0^4, \phi_0/T\}$
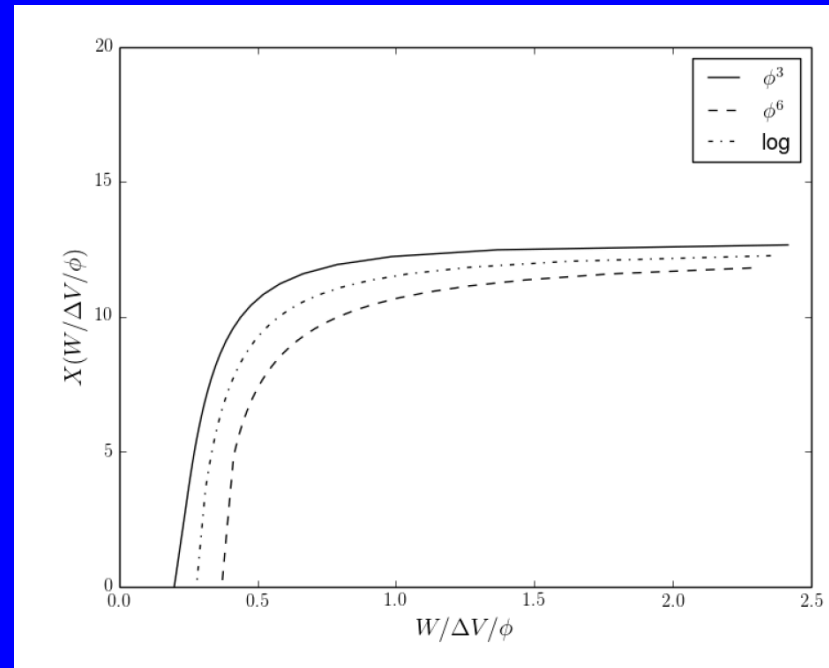
Results for the wall velocity:

In both models the wall velocity agrees when written in terms of the new variables!

Easy to use: compute $\{\Delta V/\phi_0^4, \phi_0/T\}$

and read off the wall velocity from the plot above

Drawback: only covers the case of SM friction, but with modified scalar potential

Summary

1) Strong phase transitions can be engineered by tuning the <u>depth of the electroweak minimum</u>

2) The bubble wall velocity is a <u>key parameter</u> of the transition (eg. for gravitational waves, baryon asymmetry), but not known in many situations

3) Simple criterion for highly relativistic bubbles: "runaway"

4) We have provided <u>user friendly results</u> to compute the wall velocity in situations where the <u>friction is SM-like</u>, but the scalar potential is not

5) Outlook: studies for general cases would be highly interesting!

Status of baryogenesis in the 2HDM

[Dorsch, SJH, Konstandin, No, 2016]

Key progress: computation of the <u>bubble velocity</u>, which needs to be subsonic for successful baryogenesis via <u>diffusion</u>
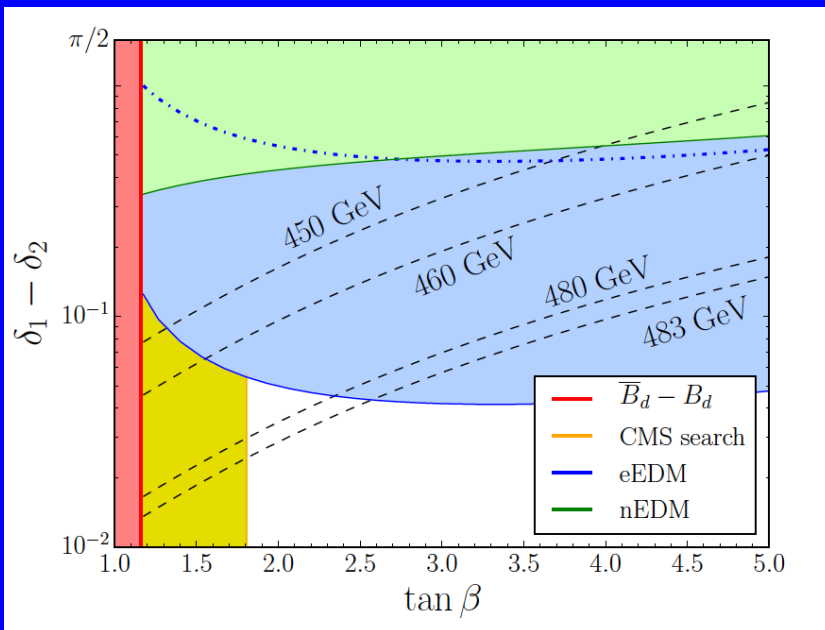
True for even very strong transitions
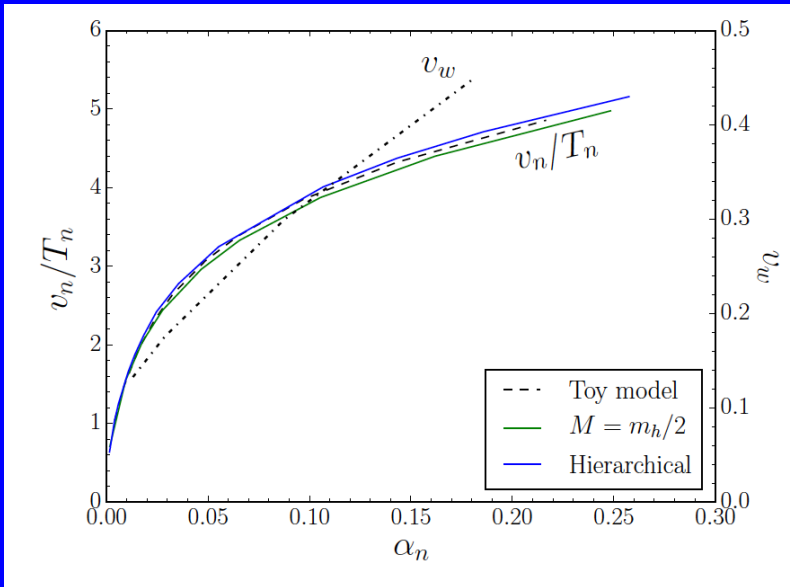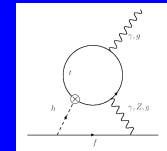
Only one phase: baryon asymmetry makes a definite prediction for EDMs

Improved bound on the electron EDM by ACME

$$|d_e^{\text{ACME}}| < 8.7 \times 10^{-29} \, e \cdot \text{cm}$$
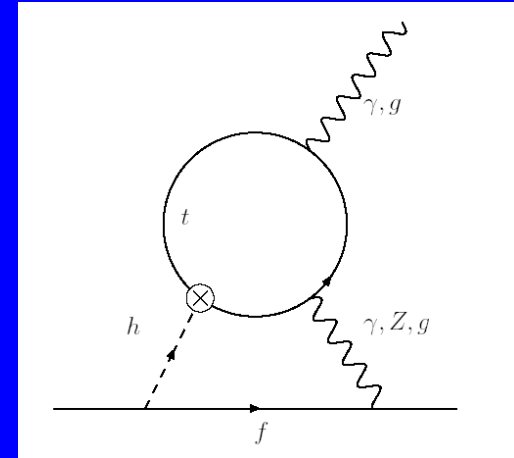
Baryogenesis now tightly constrained but still possible (uncertainties?)

Remarks:

- The EDMs in 2HDMs are of Barr-Zee type

- The baryon asymmetry scales as

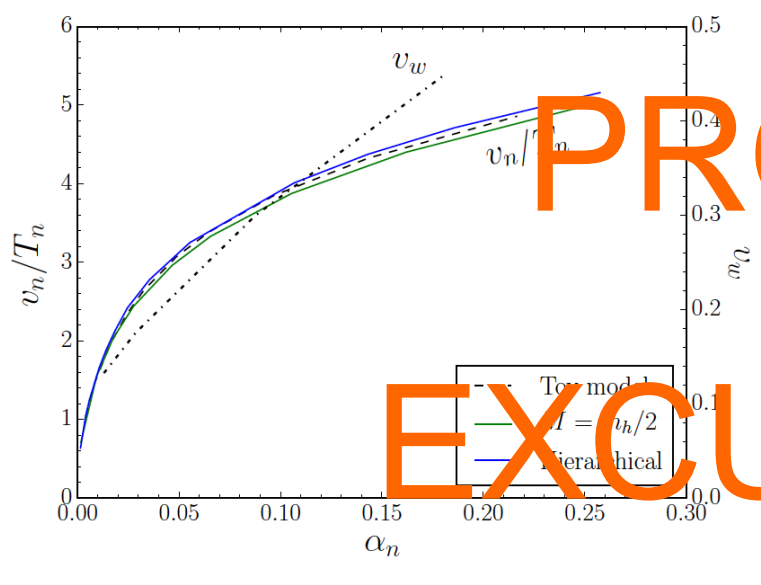$$\eta \sim \frac{\delta}{L_w T_n} \left(\frac{v_n}{T_n}\right)^2 \frac{1}{1 + \tan^2 \beta}$$

  so needs a strong transition with a thin wall and small tan β

- Even though the transition is very strong, $v_n/T_n \sim 4$, the wall still moves subsonic (deflagration) because of strong Higgs self couplings

Status of baryogenesis in the 2HDM
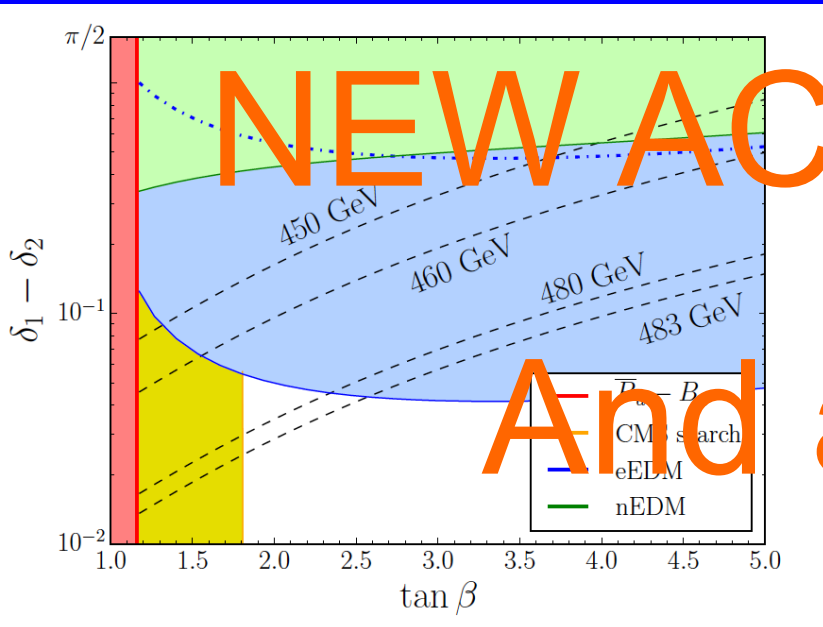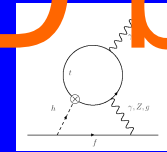
[Dorsch, SJH, Konstandin, No, 2016]

Key progress: computation of the bubble

velocity, which needs to be subsonic for

Successful baryogenesis via diffusion

True for even very strong transitions

Only one phase: baryon asymmetry makes a definite prediction for EDMs

Improved bound on the electron EDM by ACME

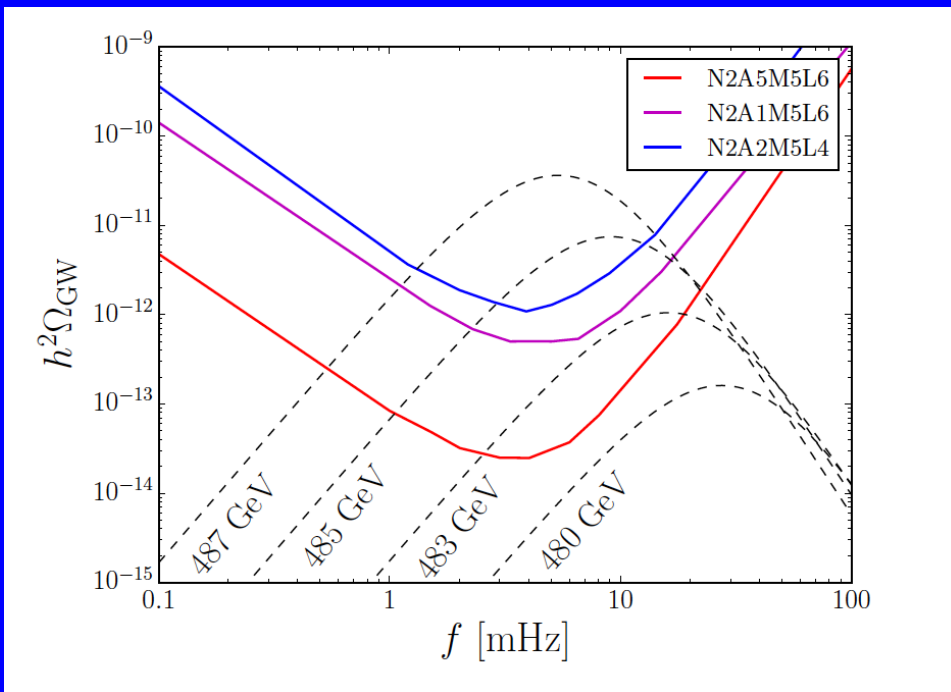$$|d_e^{\mathrm{ACME}}| < 8.7 \times 10^{-29}\ e \cdot cm$$

Baryogenesis now tightly constrained but still possible (uncertainties?)

PROBABLY EXCULDED bY

NEW ACME RESULT

And also LHC

GWs in the 2HDM

Consider the 2HDM from the first part:

[Dorsch, SH, Konstandin, No '16]

One can at the same time have successful baryogenesis and observational GWs:

m_{A^0} [GeV]	T_n	v_n/T_n	$L_w T_n$	$\Delta\Theta_t$	α_n	β/H_*	v_w
450	83.665	2.408	3.169	0.0126	0.024	3273.41	0.15
460	76.510	2.770	2.632	0.0083	0.035	2282.42	0.20
480	57.756	3.983	1.714	0.0037	0.104	755.62	0.30
483	53.549	4.349	1.556	0.0031	0.140	557.77	0.35
485	50.297	4.668	1.441	—	0.179	434.80	0.45
487	46.270	5.120	1.309	—	0.250	306.31	$\approx c_s$

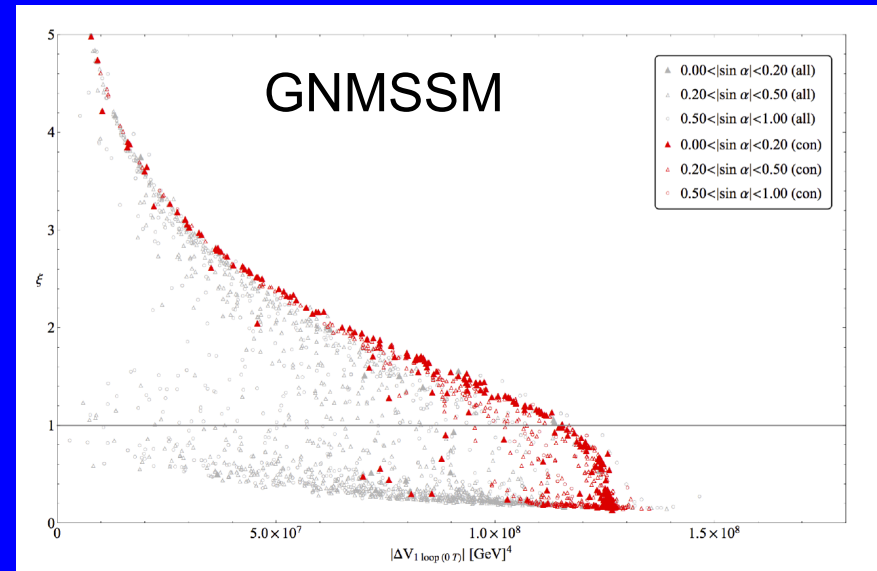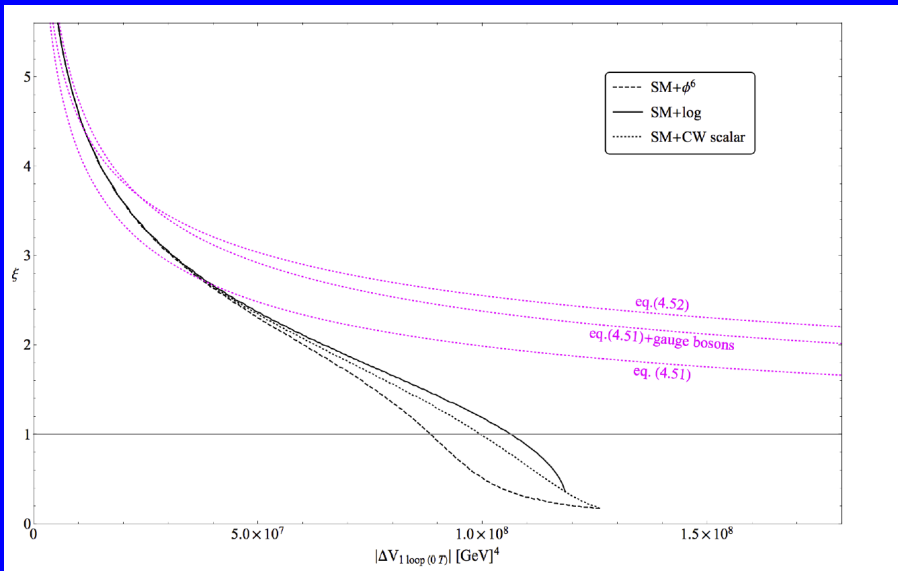In the 2HDM the GW frequency is one to two orders of magnitude larger (same α)

Deflagrations!

Turbulence?

vacuum energy: general models

Consider the T=0 depth of the EM minimum: [Harman S.H. '15]

$$\Delta V_{1 \text{ loop (0T)}} = V_{1 \text{ loop (0T)}}\big|_{\text{broken}} - V_{1 \text{ loop (0T)}}\big|_{\text{symmetric}}$$

$$= V_{1 \text{ loop (0T)}}(v, v_S) - V_{1 \text{ loop (0T)}}(0, \tilde{v}_S)$$

Strong transitions are entirely fixed by ΔV (once the Higgs SM-like)