The gravity track of Higgs inflation

(work with Vera-Maria Enckell, Kari Enqvist, Eemeli Tomberg and Lumi-Pyry Wahlman)

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Also starring

- Eemeli Tomberg today here at 16.45: **Primordial black holes from Higgs inflation?**
- Lumi-Pyry Wahlman on Wednesday in A315 at 14.15: **Can we Probe Gravitational Degrees of Freedom through Inflation?**

Using what you have

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2+\xi h^2}{2}R - \frac{1}{2}g^{\alpha\beta}\partial_\alpha h\partial_\beta h - V(h)\right]$$

$$V(h) = \frac{\lambda}{4}(h^2-v^2)^2 \simeq \frac{\lambda}{4}h^4$$

- Non-minimal coupling $\xi h^2 R$ is the only new dimension 4 operator for the combined Einstein-Hilbert + SM action.
- The coupling ξ is generated by renormalisation, even if it is classically zero.
- Non-minimal coupling enables Higgs inflation, which uses the only known scalar field that may be elementary.
(Bezrukov and Shaposhnikov: 0710.3755)

The many faces of Einstein gravity

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1+\xi h^2}{2} g^{\alpha\beta} R_{\alpha\beta}(\Gamma, \partial\Gamma) - \frac{1}{2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - V(h) \right]$$

- Usually the gravitational degrees of freedom are taken to be the metric and its derivatives.
- In the Palatini formulation, the metric and the connection are independent degrees of freedom.
- In the Einstein-Hilbert case, the metric and the Palatini formulation are equivalent.
- With a non-minimally coupled scalar field, they are different physical theories. (Bauer and Demir: 0803.2664)

To the Einstein frame

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1+\xi h^2}{2}g^{\alpha\beta}R_{\alpha\beta}(\Gamma,\partial\Gamma)-\frac{1}{2}g^{\alpha\beta}\partial_\alpha h\partial_\beta h-V(h)\right]$$

- The Einstein frame is reached with the conformal transformation $g_{\alpha\beta}\to(1+\xi h^2)^{-1}g_{\alpha\beta}$
- In the Palatini case, the conformal transformation does not affect the Ricci tensor.

- To recover canonical kinetic term, define new field χ:
  - metric: $$\frac{\mathrm{d}\chi}{\mathrm{d}h}=\sqrt{\frac{1+\xi h^2+6\xi^2h^2}{(1+\xi h^2)^2}}\simeq\frac{\sqrt{6}}{h}\Rightarrow h\propto e^{\chi/\sqrt{6}}$$
  - Palatini: $$\frac{\mathrm{d}\chi}{\mathrm{d}h}=\sqrt{\frac{1+\xi h^2}{(1+\xi h^2)^2}}\simeq\frac{1}{\sqrt{\xi}h}\Rightarrow h\propto e^{\sqrt{\xi}\chi}$$

- Polynomial potential is transformed into exponential potential.

Higgs potential in Metric vs Palatini

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[R - \frac{1}{2} g^{\alpha\beta} \partial_\alpha \chi \partial_\beta \chi - U(\chi) \right]$$

- We get a different Einstein frame potential depending on the gravitational degrees of freedom:
  - metric: $U(\chi) \equiv \frac{V[h(\chi)]}{[1+\xi h(\chi)^2]^2} \simeq \frac{\lambda}{4\xi^2}(1 - 2e^{-\frac{2}{\sqrt{6}}\chi})$
  - Palatini: $U(\chi) \simeq \frac{\lambda}{4\xi^2}(1 - 8e^{-2\sqrt{\xi}\chi})$
- The potential is exponentially flat.

Predictions of Higgs inflation on the plateau

- On the exponentially flat plateau, we get:
  - metric: $n_s = 1 - \frac{2}{N}$, $r = \frac{12}{N^2}$, $\frac{U}{\epsilon} = \frac{\lambda}{3\xi^2} N^2$

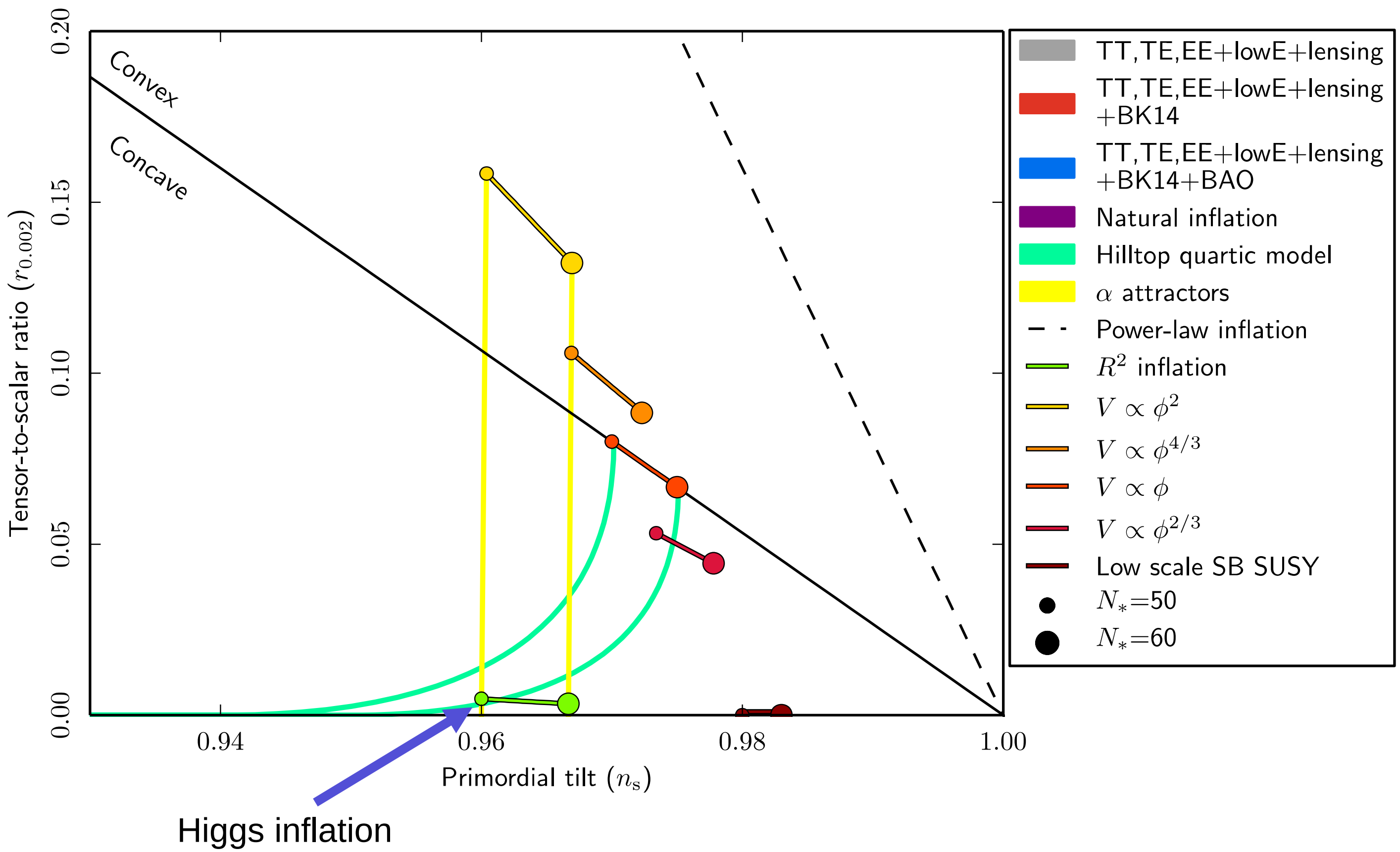
$$n_s = 0.96 \ , \ r = 5 \times 10^{-3} , \ \xi = 4 \times 10^4 \sqrt{\lambda}$$

  - Palatini: $n_s = 1 - \frac{2}{N}$, $r = \frac{2}{\xi N^2}$, $\frac{U}{\epsilon} = \frac{2\lambda}{\xi} N^2$

$$n_s = 0.96, \ r = \frac{8 \times 10^{-4}}{\xi} = \frac{8 \times 10^{-14}}{\lambda} \ , \ \xi = 10^{10} \lambda$$

- Reheating is fixed: *N*≈50. (Figueroa et al: 1504.04600, Rubio and Tomberg: 1902.10148)

The data likes Higgs inflation



Planck: arXiv:1807.06211

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Metric vs Palatini: R^2 term

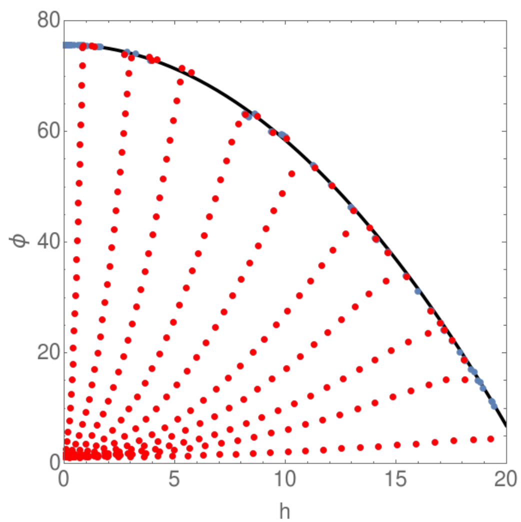
$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2}(1+\xi h^2) g^{\alpha\beta} R_{\alpha\beta} + \alpha R^2 - \frac{1}{2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - V(h) \right]$$

- Loop corrections generate an R^2 term in the action.

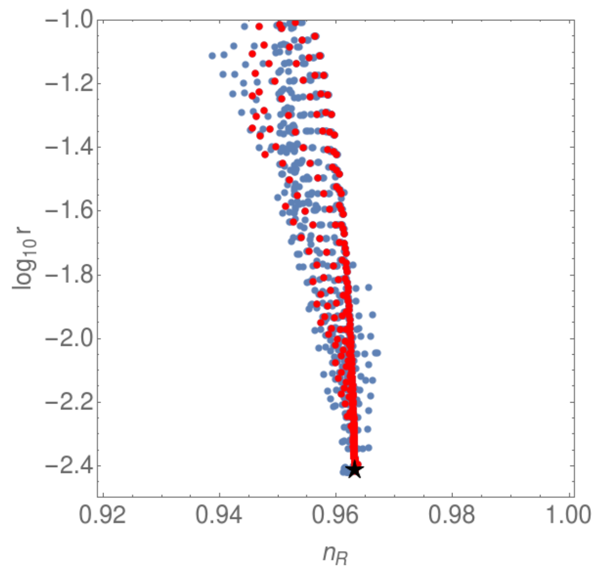
- Its effect is completely different in the metric and in the Palatini formulation.

R^2 à la metric: two-field model

- In the metric case, the R^2 term adds a scalar degree of freedom, so we get a two-field model. (Wang and Wang: 1701.06636, Ema: 1701.07665, Zhang, Huang and Sasaki: 1712.09896, He, Starobinsky and Yokoyama: 1804.00409, Gundhi and Steinwachs: 1810.10546, Enckell, Enqvist, SR, Wahlman: 1812.08754)
- The R^2 term destabilises Higgs inflation. (Enckell, Enqvist, SR, Wahlman: 1812.08754)



Trajectories in field space.



Results for observables.

R^2 à la Palatini: saving your favourite model

$$U(\chi) = \frac{V[h(\chi)]}{[1+\xi h(\chi)^2]^2}$$

- In the Palatini case, the new scalar field does not get a kinetic term and can be integrated out. (Enckell, Enqvist, SR, Wahlman: 1810.05536)

- The action reduces to

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial^\alpha\chi\partial_\alpha\chi + \frac{\alpha}{2}(1+8\alpha U)(\partial^\alpha\chi\partial_\alpha\chi)^2 - \frac{U}{1+8\alpha U}\right] .$$

- The only effect is to change the tensor power spectrum, giving $r \to r/(1+8\alpha U)$.

- This can be used to rescue any scalar field model where *r* was excluded by the data.

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Features unique to Palatini

- In the metric case, the metric and the Riemann tensor are the only geometrical tensors.
- In the Palatini case, we also have the non-metricity tensor $\nabla_\gamma g_{\alpha\beta}$.
- Now there are many more scalars, with no correspondence in the metric case.

Kinetic terms for the metric

- We have new kinetic terms, such as

$$h\nabla_\alpha h\nabla_\beta g^{\beta\alpha} \qquad g_{\alpha\beta}\nabla_\gamma g^{\gamma\alpha}\nabla_\delta g^{\delta\beta}$$

- Mixing of Higgs and metric kinetic terms sources non-metricity (SR: 1811.09514).

- Going to the Einstein frame and canonical field now has more possibilities:

$$\frac{\mathrm{d}\chi}{\mathrm{d}h} = \pm\sqrt{K(h)} = \pm\sqrt{\frac{\sum_{n=0}^{6} c_n h^{2n}}{\sum_{n=0}^{7} d_n h^{2n}}}$$

- By tuning the constants, we can generate an inflection point, α-attractor or the potentials

$$U \propto 1 - a\chi^2, U \propto 1 - a\chi^{-2/3}, U \propto \chi^2, U \propto \chi^{4/3}$$

Higgs as a window to gravity

- Higgs inflation is a conservative possibility using only known degrees of freedom.
- Different formulations of general relativity become inequivalent theories when the matter couples to the connection.
- Higgs opens a window to gravitational degrees of freedom.
- In the Palatini case, r is suppressed.
- In the metric case, R^2 term leads to a complicated two-field model. In the Palatini case, it is harmless.
- In the Palatini case, new gravitational terms can completely change the effective potential. (More to come: teleparallel, Ashtekar, ...)

R^2 à la metric: two-field model details

- In the metric case, the R^2 term adds a scalar degree of freedom, so we get a two-field model. (Wang and Wang: 1701.06636, Ema: 1701.07665, Zhang, Huang and Sasaki: 1712.09896, He, Starobinsky and Yokoyama: 1804.00409, Gundhi and Steinwachs: 1810.10546, Enckell, Enqvist, SR, Wahlman: 1812.08754)
- The action becomes

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{1}{2} R - \frac{\varphi + \xi h^2 + 6\xi^2 h^2}{2(\varphi + \xi h^2)^2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - \frac{3\xi h}{(\varphi + \xi h^2)^2} g^{\alpha\beta} \partial_\alpha h \partial_\beta \varphi \right.$$
$$\left. - \frac{3}{4(\varphi + \xi h^2)^2} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - \hat{V}(h, \varphi) \right]$$

$$\hat{V}(h, \varphi) = \frac{\lambda}{4} \frac{(h^2 - v^2)^2}{(\varphi + \xi h^2)^2} + \frac{1}{8\alpha} \frac{(\varphi - 1)^2}{(\varphi + \xi h^2)^2}$$

Kinetic terms for the metric

- Considering terms with only up to 2 derivatives (and demanding the equations of motion can be derived without adding boundary terms), the action is (SR: 1811.09514)

$$Q_{\alpha\beta\gamma} \equiv \nabla_\alpha g_{\beta\gamma}$$

$$Q_\gamma \equiv g^{\alpha\beta} Q_{\alpha\beta\gamma}$$

$$\hat{Q}_\alpha \equiv g^{\beta\gamma} Q_{\alpha\beta\gamma}$$

$$\begin{aligned}
S = \int \mathrm{d}^4x \sqrt{-g} \bigg[&\frac{1}{2} F(h) g^{\alpha\beta} R_{\alpha\beta}(\Gamma, \partial\Gamma) - \frac{1}{2} K(h) g^{\alpha\beta} \nabla_\alpha h \nabla_\beta h - V(h) \\
&+ A_1(h) \nabla_\alpha h \nabla_\beta g^{\beta\alpha} + A_2(h) g^{\alpha\beta} g_{\gamma\delta} \nabla_\alpha h \nabla_\beta g^{\gamma\delta} + B_1(h) g^{\alpha\beta} g_{\gamma\delta} g_{\epsilon\eta} \nabla_\alpha g^{\gamma\epsilon} \nabla_\beta g^{\delta\eta} \\
&+ B_2(h) g_{\gamma\delta} \nabla_\alpha g^{\beta\gamma} \nabla_\beta g^{\alpha\delta} + B_3(h) g_{\alpha\beta} \nabla_\gamma g^{\gamma\alpha} \nabla_\delta g^{\delta\beta} \\
&+ B_4(h) g^{\alpha\beta} g_{\gamma\delta} g_{\epsilon\eta} \nabla_\alpha g^{\gamma\delta} \nabla_\beta g^{\epsilon\eta} + B_5(h) g_{\gamma\delta} \nabla_\alpha g^{\alpha\beta} \nabla_\beta g^{\gamma\delta} \bigg] \\
= \int \mathrm{d}^4x \sqrt{-g} \bigg[&\frac{1}{2} F(h) g^{\alpha\beta} R_{\alpha\beta}(\Gamma, \partial\Gamma) - \frac{1}{2} K(h) g^{\alpha\beta} \nabla_\alpha h \nabla_\beta h - V(h) \\
&- A_1(h) \nabla_\alpha h \hat{Q}^\alpha - A_2(h) \nabla_\alpha h Q^\alpha + B_1(h) Q_{\gamma\alpha\beta} Q^{\gamma\alpha\beta} + B_2(h) Q_{\gamma\alpha\beta} Q^{\beta\gamma\alpha} \\
&+ B_3(h) \hat{Q}_\alpha \hat{Q}^\alpha + B_4(h) Q_\alpha Q^\alpha + B_5(h) Q_\alpha \hat{Q}^\alpha \bigg] \; .
\end{aligned}$$

- Non-minimal coupling and kinetic mixing source non-metricity and torsion.

Coordinates in field space

$$U(\chi) = \frac{V[h(\chi)]}{[1+\xi h(\chi)^2]^2}$$

- The action can be simplified by choosing suitable coordinates in field space.

$$h \to \chi(h)$$
$$g_{\alpha\beta} \to \Omega(h)^{-1} g_{\alpha\beta}$$
$$\Gamma^{\gamma}_{\alpha\beta} \to \Gamma^{\gamma}_{\alpha\beta} + \Sigma^{\gamma}{}_{\alpha\beta} = \Gamma^{\gamma}_{\alpha\beta} + g^{\gamma\delta}\left[\Sigma_1(h) g_{\alpha\beta}\partial_\delta h + 2\Sigma_2(h) g_{\delta(\alpha}\partial_{\beta)} h + 2\Sigma_3(h) g_{\delta[\alpha}\partial_{\beta]} h\right]$$

- All effects of the non-minimal connection are mapped onto the potential:

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left[R - \frac{1}{2} g^{\alpha\beta}\partial_\alpha\chi\partial_\beta\chi - U(\chi)\right] .$$

- The field transformation is $\frac{\mathrm{d}\chi}{\mathrm{d}h} = \pm\sqrt{K(h)}$, where *K(h)* is a complicated function.

The Higgs case

- Let us consider the Higgs case with only dimension 4 operators.

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}(f+\xi h^2)g^{\alpha\beta}R_{\alpha\beta}(\Gamma,\partial\Gamma) - kg^{\alpha\beta}\partial_\alpha h\partial_\beta h - \frac{\lambda}{4}h^4 \right.$$
$$- a_1 h\partial_\alpha h\hat{Q}^\alpha - a_2 h\partial_\alpha h Q^\alpha + (b_{10}+b_{11}h^2)Q_{\gamma\alpha\beta}Q^{\gamma\alpha\beta} + (b_{20}+b_{21}h^2)Q_{\gamma\alpha\beta}Q^{\beta\gamma\alpha}$$
$$\left. +(b_{30}+b_{31}h^2)\hat{Q}_\alpha\hat{Q}^\alpha + (b_{40}+b_{41}h^2)Q_\alpha Q^\alpha + (b_{50}+b_{51}h^2)Q_\alpha\hat{Q}^\alpha\right]$$

$$U(\chi) = \frac{V[h(\chi)]}{[1+\xi h(\chi)^2]^2}$$

$$Q_{\alpha\beta\gamma} \equiv \nabla_\alpha g_{\beta\gamma}$$

$$Q_\gamma \equiv g^{\alpha\beta}Q_{\alpha\beta\gamma}$$
$$\hat{Q}_\alpha \equiv g^{\beta\gamma}Q_{\alpha\beta\gamma}$$

- Here *f*, *k*, a_i, b_{i0} and b_{i1} are constants. We get

$$\frac{\mathrm{d}\chi}{\mathrm{d}h} = \pm\sqrt{K(h)} = \pm\sqrt{\frac{\sum_{n=0}^{6} c_n h^{2n}}{\sum_{n=0}^{7} d_n h^{2n}}}$$

- By tuning the constants, we can generate an inflection point, α-attractor or $U \propto 1 - a\chi^2, U \propto 1 - a\chi^{-2/3}, U \propto \chi^2, U \propto \chi^{4/3}$.