Consistent use of Effective potentials

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The Effective Potential

Perturbative expansion of the potential

$$V_{\text{eff}}(\phi) = V_0(\phi) + V_1(\phi) + V_2(\phi) + \dots$$

1-loop contribution

$$V_1(\phi) \sim \sum_{\Psi} (-1)^{2s_\Psi}(2s_\Psi + 1) \int d^3k \frac{1}{2} \omega_\Psi(\phi, k)$$

$$\omega_\Psi^2(\phi, k) = \vec{k}^2 + M_\Psi^2(\phi),$$

$$V_1(\phi) \sim \sum_{\Psi} (-1)^{2s_\Psi}(2s_\Psi + 1) M_\Psi^4(\phi) \left(\log\left(\frac{M_\Psi^2(\phi)}{\mu^2} \right) + \text{const} \right)$$

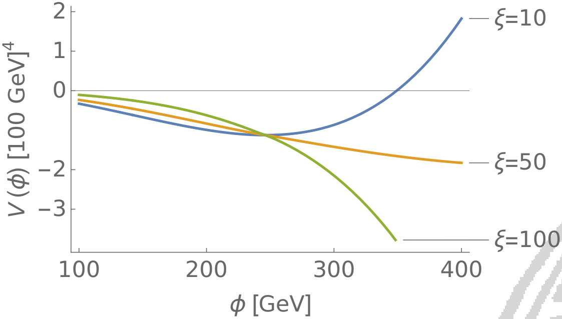
Gauge dependence

Typical gauge fixing

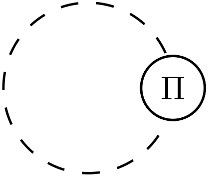
$$\mathscr{L}_{g.f.} = -\frac{1}{2\xi}(\partial_\mu A^\mu)^2$$

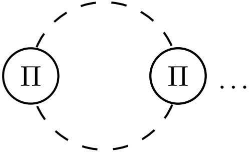
$$\mathscr{L}_{g.f.} = -\frac{1}{2\xi}(\partial_\mu A^\mu + \xi\phi\chi)^2$$

Landau gauge $\xi = 0$



Example of IR divergences





$$\int \frac{d^4k}{(2\pi)^4}\left(\frac{\Pi(k^2)}{k^2 - G(\phi)}\right) \qquad \int \frac{d^4k}{(2\pi)^4}\left(\frac{\Pi(k^2)}{k^2 - G(\phi)}\right)^2 \ \ldots$$

$$\sim G(\phi) \qquad \sim \log G(\phi)$$

Divergence at $\phi = \phi_0$:

$$\partial V_0(\phi)|_{\phi=\phi_0} = 0$$
$$G(\phi)|_{\phi=\phi_0} = 0$$

IR divergences

Higher order divergences

$$V_3(\phi) \sim \log G(\phi),$$
$$V_4(\phi) \sim \frac{1}{G(\phi)} + \log G(\phi)$$
$$V_5(\phi) \sim \frac{1}{G(\phi)^2} + \frac{1}{G(\phi)} + \log G(\phi)$$
$$\vdots$$

Worst divergences at L Loops (in Landau gauge)

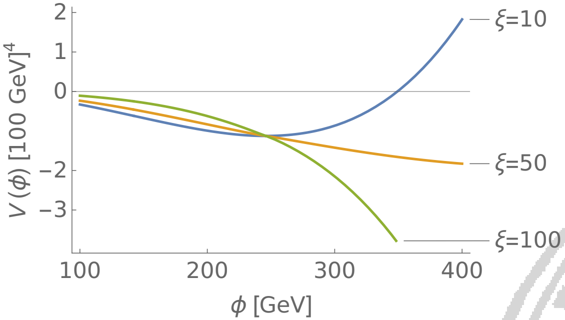
$$V_L(\phi) \sim G^{3-L}(\phi) + \dots$$

Nielsen identity

Gauge dependence

$$\left(\xi \frac{\partial}{\partial \xi} + C(\phi,\xi)\frac{\partial}{\partial \phi}\right) V_{\text{eff}}(\phi,\xi) = 0$$

$$\xi \frac{\partial}{\partial \xi}\phi^{\text{min}}(\xi) = C(\phi^{\text{min}},\xi)$$



Perturbative expansion

The effective potential is gauge invariant at its extrema

$$\frac{\partial}{\partial\phi} V_{\text{eff}}(\phi,\xi)|_{\phi=\phi^{\text{min}}} \equiv \partial V_{\text{eff}}(\phi,\xi)|_{\phi=\phi^{\text{min}}} = 0$$

$$V_{\text{eff}}(\phi,\xi) = V_0(\phi) + \hbar V_1(\phi,\xi) + \hbar^2 V_2(\phi,\xi) + \dots$$

$$\phi^{\text{min}} = \phi_0 + \hbar\phi_1 + \hbar^2\phi_2 + \dots$$

Position of the extrema order-by-order in $\hbar$

$$\mathscr{O}(\hbar^0): \ \partial V_0|_{\phi=\phi_0} = 0,$$

$$\mathscr{O}(\hbar^1): \ \left(\phi_1\partial^2 V_0 + \partial V_1\right)|_{\phi=\phi_0} = 0$$

$$\mathscr{O}(\hbar^2): \ \left(\frac{\phi_1^2}{2!}\partial^3 V_0 + \phi_2\partial^2 V_0 + \phi_1\partial^2 V_1 + \partial V_2\right)\Bigg|_{\phi=\phi_0} = 0$$

$$\vdots$$

$\hbar$-expansion of the effective potential

The effective potential evaluated at its extrema

$$V_{\text{eff}}\big|_{\phi=\phi^{\text{min}}} = \left(V_0(\phi) + \hbar V_1(\phi,\xi) + \hbar^2 V_2(\phi,\xi) + \ldots \right)\Bigg|_{\phi^{min}=\phi_0+\hbar\phi_1+\ldots}$$

Expansion order-by-order in $\hbar$

$$V_{\text{eff}}\big|_{\phi=\phi^{\text{min}}} = \left[V_0 + \hbar V_1 + \hbar^2 \left(V_2 + \phi_1 \partial V_1 + \frac{\phi_1^2}{2} \partial^2 V_0 \right) + \ldots \right]_{\phi=\phi_0}$$

Example of IR Divergence Cancellation

At the L^{th} order in $\hbar$

$$V_{\text{eff}}\big|_{\phi=\phi^{min}} = \dots \hbar^L \left(V_L + \phi_1 \partial V_{L-1} + \frac{\phi_1^2}{2!}\partial^2 V_{L-2} + \dots \right)\Bigg|_{\phi=\phi_0} + \dots$$

$$V_L\big|_{\phi\approx\phi_0} \sim G^{3-L}(\phi)$$

$$\partial^n V_L\big|_{\phi\approx\phi_0} \sim G^{3-L-n}(\phi)$$

$$V_L + \dots + \frac{\phi^{L-1}}{(L-1)!}\partial^{L-1} V_1 \propto G(\phi)^{L-3} \sum_{i=0}^{L-1} \frac{(-1)^i}{(L-1-i)!\,i!} = 0$$

$V_{\text{eff}}\big|_{\phi=\phi^{min}}$ is Finite & Gauge invariant order-by-order

Summary

- The effective potential is, in general, both IR divergent and Gauge dependent
- Both problems can be solved by using a consistent power counting, see 1810.01416 for more details