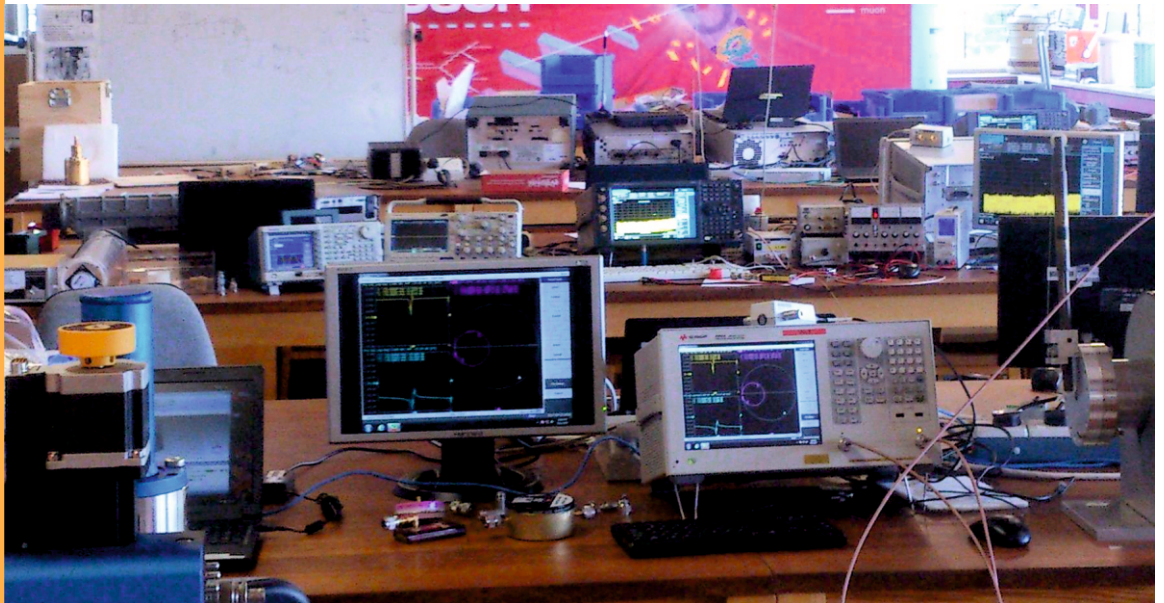


H. Damerau
P. Kowina
C. Völlinger
M. Wendt



RF - Lab

Instructions



Niels Bohr Institute
of the University of Copenhagen

Slangørup - Denmark
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RF Measurements Techniques

Suggested RF measurements – overview

Spectrum Analyzer test stand 1

- ◆ Measurements of several types of modulation (AM FM PM) in the time and frequency domain.
- ◆ Superposition of AM and FM spectrum (unequal carrier side bands).
- ◆ Concept of a spectrum analyzer: the superheterodyne method. Practice all the different settings (video bandwidth, resolution bandwidth etc.). Advantage of FFT spectrum analyzers.

Spectrum Analyzer test stand 2

- ◆ Measurement of the TOI point of some amplifier (intermodulation tests).
- ◆ Concept of noise figure and noise temperature measurements, testing a noise diode, the basics of thermal noise.
- ◆ EMC measurements (e.g.: analyze your cell phone spectrum).
- ◆ Nonlinear distortion in general concept and application of vector spectrum analyzers, spectrogram mode.
- ◆ Measurement of the RF characteristic of a microwave detector diode (output voltage versus input power... transition between regime output voltage proportional input power and output voltage proportional input voltage).

Spectrum Analyzer test stand 3

- ◆ Concept of noise figure and noise temperature measurements, testing a noise diode, the basics of thermal noise.
- ◆ Noise figure measurements on amplifiers and also attenuators.
- ◆ The concept and meaning of ENR numbers.
- ◆ Noise temperature of the fluorescent tubes in the room using a satellite receiver

Network Analyzer test stand 1

- ◆ Calibration of the Vector Network Analyzer.
- ◆ Navigation in the Smith Chart.
- ◆ Application of the triple stub tuner for matching.
- ◆ Measurements of the light velocity using a trombone (constant impedance adjustable coax line) in the frequency domain.
- ◆ N-port (N=1...4) S-parameter measurements for different reciprocal and non-reciprocal RF-components.
- ◆ Self-made RF-components: Calculate build and test your own attenuator in a SUCO box (and take it back home then).

Network Analyzer test stand 2

- ◆ Time Domain Reflectometry (TDR) using the synthetic pulse method: direct measurement of coaxial line characteristic impedance.
- ◆ Measurements of the light velocity using a trombone (constant impedance adjustable coax line) in the time domain.
- ◆ 2-port measurements for active RF-components (amplifiers): 1 dB compression point (power sweep).
- ◆ Beam transfer impedance measurements with the wire (button PU, stripline PU.)
- ◆ Coupling impedance measurement with a stretched-wire setup

Network Analyzer test stand 3

- ◆ Measurements of the characteristic cavity features (Smith Chart analysis).
- ◆ Cavity perturbation measurements (bead pull).
- ◆ Perturbation measurements using rectangular wave guides.

Network Analyzer test stand 4

- ◆ Measurements of the cavity driven by an amplifier (critical coupling, transfer from input coupler to cavity probes, tuning of resonance frequency).
 - ◆ Direct RF feedback (decrease and increase of cavity impedance).
 - ◆ Transfer function of periodic filters.
 - ◆ Cavity with 1-turn delay feedback (effect on cavity impedance at specific frequencies).
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- ◆ Invent your own experiment!

Measurements with Spectrum Analyzer test stand 1

Suggested topics:

- Becoming familiar with the spectrum-analyzer
- Measurements of several types of modulation (AM FM) in the time and frequency domain.
- Superposition of AM and FM spectrum (unequal carrier side bands).
- Concept of a spectrum analyzer: the superheterodyne method. Practice all the different settings (video bandwidth, resolution bandwidth etc.). Advantage of FFT spectrum analyzers

Equipment:

SA (Keysight N9020B), 2 HP 8657B, HP 8656B (G.1.1.007, G.1.1.006), and Agilent E8257D signal generators, oscilloscope (1.1.001), mixer (2.3.011), combiner (2.2.008), DC-block

- ◆ Set the analyzer to the Spectrum Analyzer mode and enable the spectrum analyzer measurements: Press **Mode, Spectrum Analyzer**.
- ◆ Display the spectrum of RF signals present in the classroom (using a short wire as an antenna).
- ◆ Measure the spectrum of an output signal from a signal generator (CW mode, no modulation) and look for second and third harmonics. How can you discriminate against SPA input mixer-related harmonics? (Do not exceed 0 dBm generator output power.)

Amplitude Modulation (AM) and Frequency Modulation (FM) in the time domain

- ◆ Connect output of HP8656B signal generator to the oscilloscope (set proper input termination, proper scale, and trigger mode to normal and proper trigger level).
- ◆ On generator set the carrier frequency to 0.1 MHz and amplitude to 0dBm. Set oscilloscope to see sine wave.
- ◆ Play with AM modulation using internal 1 kHz and 400 Hz oscillators changing the modulation depth from 0% to 100%.
- ◆ At 100% modulation depth change time base of oscilloscope to 1ms/div to see nice AM envelopes (in Acquire menu set **peak detection**).
- ◆ Change the modulation depth for 400 Hz and 1 kHz AM and observe the results.
- ◆ Change time base back to resolve carrier frequency and repeat last four points for FM modulation changing the maximal peak deviation in the range 0-200kHz.
- ◆ Switch on AM and FM simultaneously and observe the results.

Measurements of AM and FM in frequency domain

- ◆ Set carrier frequency to 10 MHz (do not exceed 0 dBm generator output power).
- ◆ Connect HP 8657 to SPA via DC-Block
- ◆ Set the analyzer to the Spectrum Analyzer mode and enable the spectrum analyzer measurements: Press **Mode, Spectrum Analyzer**.
- ◆ Preset the analyzer: Press **Mode Preset**.
- ◆ Set the analyzer center frequency and span to adequate values.
- ◆ Change RBW and observe behaviour of the signal width and noise floor (what is an influence on the measurement time? What is advantageous in FFT analyzers).
- ◆ Measure AM and FM with 1 kHz internal source changing AM modulation depth and peak deviation for FM same way as before in time domain tests. (Set proper frequency span and RBW).
- ◆ Change the carrier frequency to 100 MHz (do not forget to change center frequency on SPA). Compare the results with previous point.

Let us try to measure modulation at higher frequencies using external mixer (2.3.011)

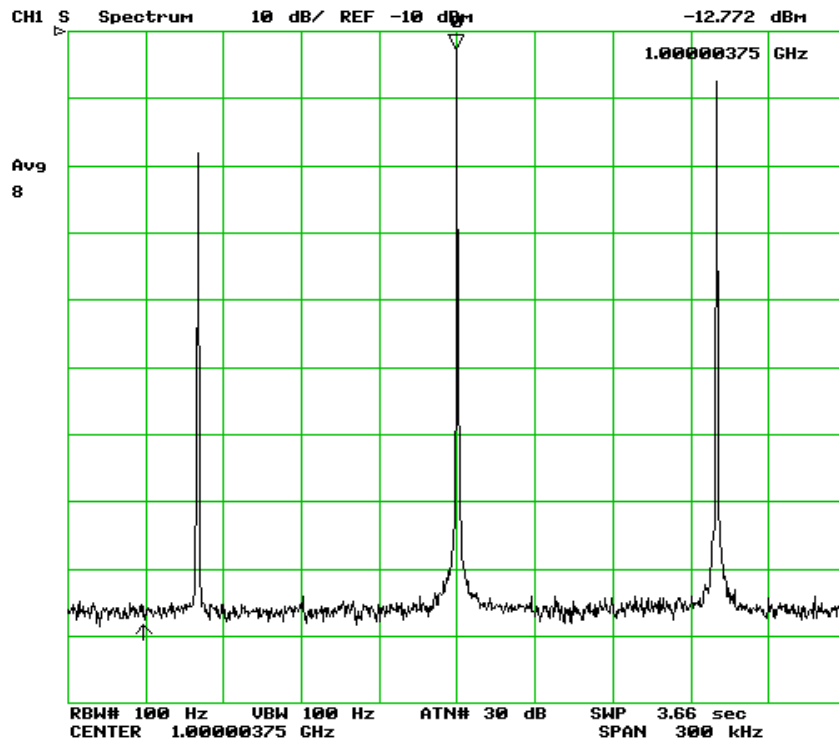
- ◆ First synchronize both signal generators: Switch off all modulations, set signal carrier frequency to 10 MHz and connect them to the oscilloscope. Synchronize both generators using BNC time base I/O on the generators back plane.
- ◆ Set one generator to 1 GHz and the other one to 1 MHz (do not exceed 0 dBm generator output power).
- ◆ Connect both generators to SPA via mixer connecting signals to appropriate mixer ports: what is your **Local Oscillator**, what is the signal you want to send and what is the resulting RF?
Set properly the analyzer center frequency, span and RBW.
- ◆ Change amplitude of modulation (do not exceed 0 dBm generator output power), change the carrier frequency, change modulation frequency and observe the results on SPA. What might be a reason for the changes of the sidebands high when changing modulation frequency below 100 kHz?
- ◆ Let us see how different frequencies can be sent in the same AM channel: On the generator that is used to send modulating signal switch FM modulation with internal 1kHz source. Changing peak deviation in the range 0-200 kHz observe modulated side bands.
- ◆ Now you can send your 1 GHz radio signal over 10 cm of air using two short wires as an antenna.

Superposition of AM and FM spectrum

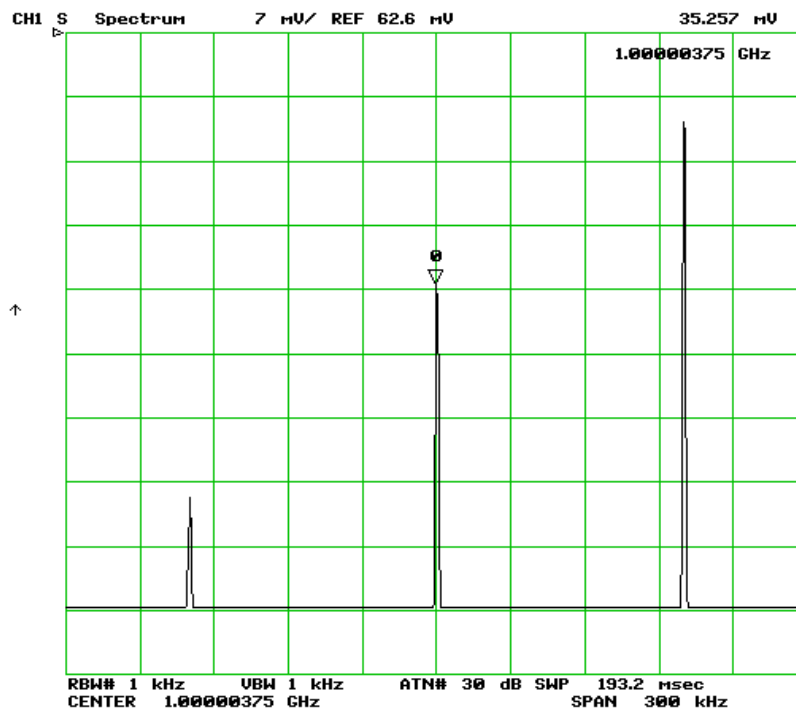
In this exercise, we will use HP 8656B as an external generator and we will do AM and FM modulation using internal mixer of HP 8657B.

- ◆ Set carrier frequency of HP 8656B to 0.1 MHz and amplitude to 0 dBm.
- ◆ Connect RF signal to **MOD** input (front panel) of 8657B.
- ◆ Set carrier frequency of HP 8657B to 1GHz and connect its output to SPA.
- ◆ Switch AM modulation with external source: **ext AM**. Vary the modulation depth from 0-100% and observe results.
- ◆ Switch off AM **ext AM off** and switch on **ext FM**. Vary peak deviation from 0-200kHz and observe results.

- ◆ Switch on simultaneously AM and FM modulations with external source. Vary AM modulation depth and FM peak deviation to reach maximal difference between left and right side bands. You should obtain similar picture to that presented below.



In the linear representation, it looks like that:



Build you own superheterodyne receiver

Let us assume we have a system that allows us to measure only up to 100 MHz. But we want to measure signal with a frequency of 1 GHz. This expansion of our measurement bandwidth can be reached by applying superheterodyne principle.

- ◆ Preset SPA and set **stop frequency** to 100 MHz (let us assume it for a while as an upper limit for our system).
- ◆ Set one HP generator to 1 GHz and and the other to 950 MHz (do not exceed 0 dBm generators output power)
- ◆ Connect generators via mixer to SPA. What is now yor LO, RF and IF? (You can use lecture printouts).
- ◆ Tune LO to get your peak in the position you want.
- ◆ In the spectrum, you see some higher harmonics. What can be a reason for that?
- ◆ Measure AM and FM modulation using internal source in HP 8657B (set proper span and RBW to be able to resolve close laying side bands for 1 kHz modulation).

Measurements with Spectrum Analyzer test stand 2

Suggested topics:

- Becoming familiar with the spectrum-analyzer
- Measurement of the SOI and TOI point of some amplifier (intermodulation tests).
- EMC measurements (e.g.: analyze your cell phone spectrum).
- Nonlinear distortion in general concept and application of vector spectrum analyzers, spectrogram mode.
- Measurement of the RF characteristic of a microwave detector diode (output voltage versus input power... transition between regime output voltage proportional input power and output voltage proportional input voltage).

Equipment:

SA (Keysight N9020B), 2 HAMEG synthesizers (G.1.1.017 and G.1.1.018), power supplier (G.1.010), multimeter, adjustable attenuator (2.2.004), Anzac combiner (2.2.008), amplifiers (DUT) (3.1.003 and others)

- ◆ Set the analyzer to the Spectrum Analyzer mode and enable the spectrum analyzer measurements: Press **Mode, Spectrum Analyzer**.
- ◆ Display the spectrum of RF signals present in the classroom (using a short wire as an antenna).
- ◆ Measure the spectrum of an output signal from a signal generator (CW mode, no modulation) and look for second and third harmonics. How can you discriminate against SPA input mixer-related harmonics? (Do not exceed 0 dBm generator output power.)
- ◆ Measure 10 frequency points (100 MHz – 1 GHz) manually. Measure the frequency response of an amplifier (scalar network analyzer mode) using sweep function in the generator.
- ◆ Measure the 1 dB compression point (i.e. small signal gain reduction by 1 dB due to the beginning of saturation effects of the amplifier under test) at three different frequencies (low, mid-band, and high).
- ◆ Measure the second-order intercept point (non-linear products at sum and difference frequency of the two input signals; both input signals (= tones) should have equal amplitude. To increase measurement accuracy balance both generators using adjustable attenuator (select suitable frequencies for input signals in order to be able to display the sum and difference frequencies). Plot the results on the graph.

- ◆ Measure at three different amplitude levels; watch out for second and third order generator harmonics; you may use low-pass filters at the input.
- ◆ Measure the Third-Order Intercept (TOI) point (use two frequencies about 50 MHz apart but both within the amplifier bandwidth). The IM3 products appear separated by the frequency difference from each tone.

Third-Order Intermodulation Distortion

Two-tone, third-order intermodulation distortion is a common test in communication systems.

When two signals are present in a non-linear system, they can interact and create third-order intermodulation distortion products that are located close to the original signals. These distortion products are generated by system components such as amplifiers and mixers. This procedure tests a device for third-order intermodulation using markers. Two sources are used, one set to 325 MHz and the other to 375 MHz. The distortion from this setup may be better than the specified performance of the analyzer; however, it is useful for determining the TOI performance of the source/analyzer combination. In some cases, the two sources may interact and produce intermodulation distortion. The separation between sources can be improved by inserting of the fixed attenuators at the outputs of the sources. After the performance of the source/analyzer combination has been verified, the device-under-test (DUT) (for example, an amplifier) should be inserted between the combiner output and the analyzer input.

Third Order Intercept (TOI) is defined as the absolute power level at which the third-order distortion products intercept the level of two equal level test signals.

$$TOI = \frac{2 \times Ampl_{signal A} - Ampl_{distortion product A} + Ampl_{signal B}}{2}$$

The frequency of distortion product A is

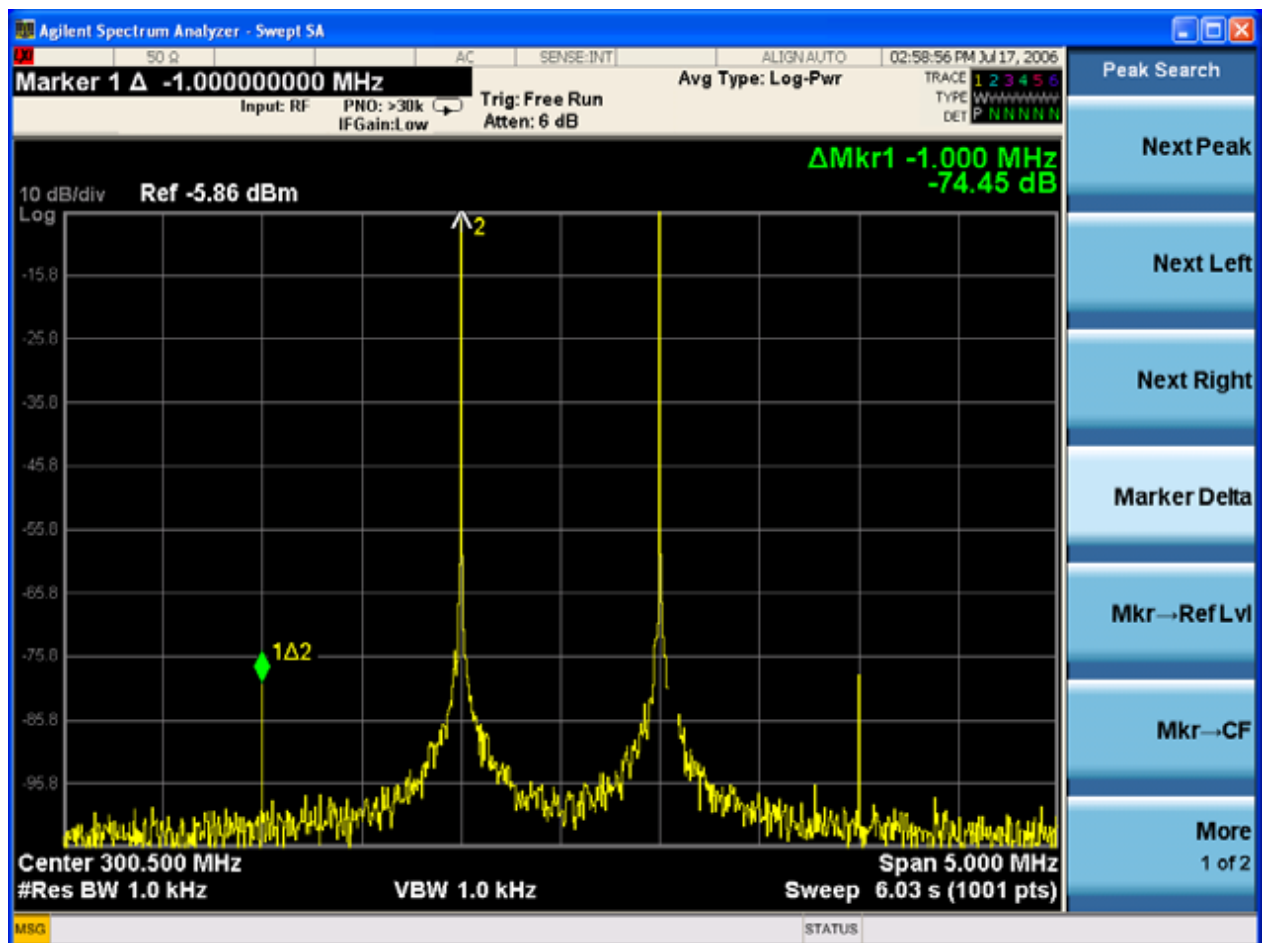
$$Freq_{distortion product A} = 2 \times Freq_{signal A} - Freq_{signal B}$$

Procedure for the measurement is the following (copied form Agilent N9020A/N9010A Spectrum Analyzer Mode Measurement Guide):

- ◆ Set the sources as follows: Set one signal generator to 325 MHz. Set the other source to 375 MHz. This produces a frequency separation of 50 MHz. Set the sources equal in amplitude as measured by the analyzer (you can increase the balancing using adjustable attenuator applied to on of source). You can also increase measurements quality by keeping the signals as close as possible to the top of the screen.
- ◆ Set the analyzer to the Spectrum Analyzer mode and enable the spectrum analyzer measurements: Press **Mode, Spectrum Analyzer**.
- ◆ Preset the analyzer: Press **Mode Preset**.
- ◆ Set the analyzer center frequency and span: Press **FREQ Channel, Center Freq, 350, MHz**.
Press **SPAN X Scale, Span, 100, MHz**.
- ◆ Set the analyzer detector to Peak:
Press **Trace/Detector, Detector, Peak**.
- ◆ Set the mixer level to improve dynamic range:
Press **AMPTD Y Scale, Attenuation, Max Mixer Lvl, -10, dBm**. The

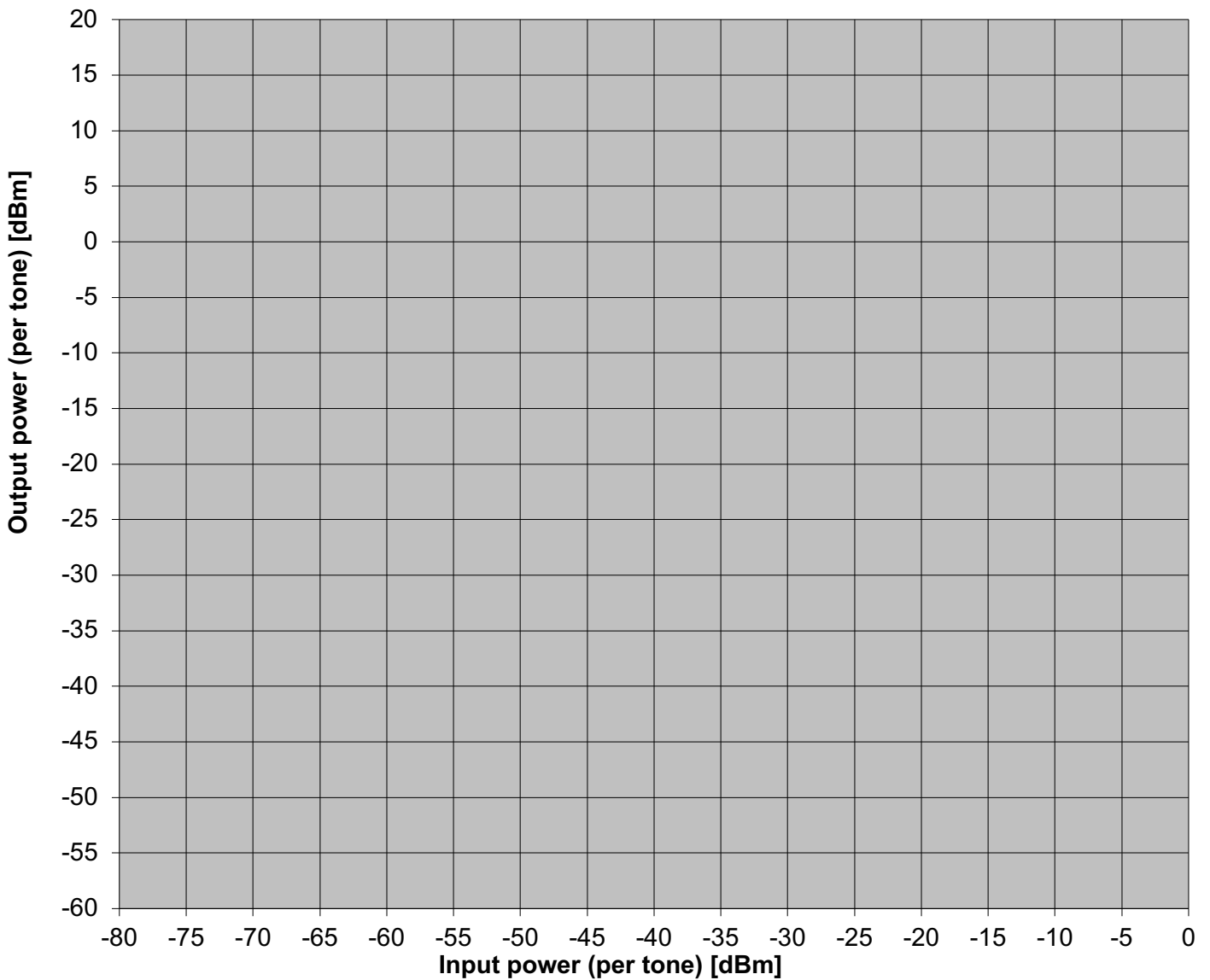
analyzer automatically sets the attenuation so that a signal at the reference level has a maximum value of -10 dBm at the input mixer.

- ◆ Move the signal to the reference level: Press **Peak Search, Mkr →, Mkr → Ref Lvl**.
- ◆ Reduce the RBW until the distortion products are visible: Press **BW, Res BW, ↓**. You should obtain similar plot as bellow (note: frequency, span RBW etc. differ from yours).
- ◆ Activate the second marker and place it on the peak of the distortion product closest to the marker test signal using the **Next Right** key (if the first marker is on the right-hand test signal) or **Next Left** key (if the first marker is on the left-hand test signal): Press **Peak Search, Marker Delta, Next Left** or **Next Right** (as appropriate).
- ◆ Measure the other distortion product: Press **Marker, Normal, Peak Search, Next Peak**.
- ◆ Activate the second marker and place it on the peak of the distortion product closest to the marked test signal using the **Next Right** key (if the first marker is on the right-hand test signal) or the **Next Left** key (if the first marker is on the left-hand test signal) (see Figure below): Press **Marker, Normal, Marker Delta, Next Left** or **Next Right** (as appropriate).
- ◆ Repeat measurements at three different amplitude levels. Present your results in the graph.



There is an additional method to check if distortion origin is DUT or spectrum analyzer itself. An easy way is an increase of the input attenuation. If the displayed results of TOI measurements remain constant then the value is the result of the device under test. If not – the measured value is dominated by internal imperfection of spectrum analyzer. In this case continue to increase attenuation until the measurement result becomes independent on the attenuator settings.

For further reading see printouts: “RF Measurement Concepts”, p.8 and p.18.



EMC measurements

Equipment:

SA (Keysight N9020B), HP 11940A and HP11941 close field probes (1.2.001 and 1.2.002),

HP 8747F dual gain probe amplifier (1.1.002), Kethley multimeter

- ◆ Analyze your cell phone spectrum.
- ◆ Try to find other signal of distortion in the environment.
- ◆ Build your own antenna and try to receive any signal

Measurement of the RF characteristic of a microwave detector diode

Equipment:

SPA, HAMEG synthesizer (G.1.1.017), RF-detector (1.2.007), Tektronix oscilloscope TDS3014 (G.1.1.002)

- ◆ Measure power of you signal generators using a scope (below 100 MHz). Read the value from scope for 50 Ω and 1 M Ω . Explain the difference.
- ◆ Measure DC response of a diode detector between -10dBm and +10dBm for 50 MHz and 500 MHz: are you in the linear or square law region?

For more information see printouts: “RF Measurement Concepts”, p. 2.

Measurements with Spectrum Analyzer test stand 3

Suggested topics:

- Becoming familiar with the spectrum-analyzer
- Concept of noise figure and noise temperature measurements, testing a noise diode, the basics of thermal noise.
- Noise figure measurements on amplifiers and also attenuators.
- The concept and meaning of ENR numbers.
- Noise temperature of the fluorescent tubes in the room using a satellite receiver.

Becoming familiar with the spectrum-analyzer

Equipment:

SA (Keysight N9020B), Tektronix AFG 3102 generator (G.1.1.019)

- ◆ Set the analyzer to the Spectrum Analyzer mode and enable the spectrum analyzer measurements: Press **Mode, Spectrum Analyzer**.
- ◆ Display the spectrum of RF signals present in the classroom (using a short wire as an antenna).
- ◆ Measure the spectrum of an output signal from a signal generator (CW mode, no modulation) and look for second and third harmonics. How can you discriminate against SPA input mixer-related harmonics? (Do not exceed 0 dBm generator output power.)

Noise and noise figure measurements

Equipment:

SA (Keysight N9020B), noise diode, power supplier (G.1.011), multimeter, (DUT): amplifiers (3.1.004, 3.1.009 etc.)

- ◆ Make sure to be familiar with the most important functions of the spectrum analyzer (frequency setting, resolution bandwidth (RBW), video bandwidth (VBW), amplitude scale)
- ◆ With resolution BW = 1 MHz, Start = 10 MHz, Stop = 1000 MHz, Video BW = 100 Hz, input attenuator = 0 dB display the baseline and read the power. How many dB is it above the thermal noise floor (thermal noise at 290 K = -174 dBm/Hz)?
- ◆ For the same settings as in point 2, now connect the solid-state noise source (to be powered with +28Volt DC via rear BNC connector) to the SPA input. The ENR (excess noise ratio) of this device is close to 16dB or a factor of 40 in spectral power

density as compared to thermal noise of a common 50 Ohm load. Use the table below to correct for absolute power reading. Note that for absolute power measurements with the spectrum analyzer close to the noise floor the reading is too high by the amount indicated in the right column. The analyzer should be set for this measurement in “sample” mode and not be in “peak hold” mode, which may be a default setting.

Error due to analyzer internal noise

Measured noise level in dB relative to internal noise	Error in measured noise [dB]
20	0.04
15	0.14
10	0.46
9	0.58
8	0.75
7	0.97
6	1.26
5	1.65
4	2.20
3	3.02
2	4.33
1	6.87

From R. A. Witte Spectrum & Network Analyzer Measurements

- ◆ Load the noise measurement option software to the SPA (if available). Otherwise skip the next 5 points
- ◆ Calibrate the SPA with preamplifier. Record the measured noise figure of the system (SPA + preamplifier) from the reading on the CRT after calibration.
- ◆ Measure the gain and noise figure of some amplifiers. Convert noise figure into noise measure.
- ◆ Measure the gain and noise figure of some attenuators.
- ◆ Measure the noise figure of two amplifiers in cascade by the method described in point 5. Measure the noise figure of an attenuator the same way.
- ◆ Connect to the input of the preamplifier with a coax cable of 1-2 meter length terminated by a short or open. Discuss the results observed (frequency range 10 MHz - 1 GHz). Also use a 50 Ohm load or a triple stub tuner. Try to tune the 50 Ohm input termination into optimum noise source match using the triple stub tuner.
- ◆ If the noise measurement option is not available, connect a preamplifier to the SPA (input attenuator = 0dB) and record the two traces "noise source on" and noise source off". Calculate from those traces the NF of the DUT. Convert noise figure into noise measure.

Noise temperature of the fluorescent tubes in the room using a satellite receiver.

WARNING! Use DC-Block to protect SA from DC!

Equipment:

SA (Keysight N9020B), LNB, satellite receiver, satellite signal splitter, eventually Sonoma instruments signal amplifier (1.1.001), N-type DC-block, 75Ω BNC cable, matching pad 75Ω/50Ω, multimeter (G.1.1.003)

- ◆ (as a 18/16V power supplier),
- ◆ Connect LNB to a satellite receiver via signal splitter. Satellite receiver will be used as a 16/18V power supplier (LNB splitters and receivers were tested in different configurations; recommended sets are marked with labels “sat1” and “sat2”).
- ◆ Second output of the splitter connect using 75Ω cable, matching pad and DC-block to SPA.

WARNING! Do not forget about DC-Block to protect SA from DC!

Some useful equations for noise figure evaluation

There is frequently confusion how to handle the dB (deci-Bel). The dB is used to describe some power ratio and thus it is a dimensionless unit. As the power dissipated in some resistor is proportional to the square of the voltage or square of the current one may also take the ratio of these quantities into account. The dB is also used to describe absolute signal levels, but then there must be an additional letter to indicate which reference one refers to e.g. + 10 dBm (=10 milliWatt) is a power level of 10 dB above 1 milli-Watt (+20 dBm=100milliWatt).

$$a[dB] = 10 \cdot \log\left(\frac{P_1}{P_2}\right) = 20 \cdot \log\left(\frac{V_1}{V_2}\right)$$

$$10^{\frac{a[dB]}{10}} = \frac{P_1}{P_2} \quad 10^{\frac{a[dB]}{20}} = \frac{V_1}{V_2}$$

The term “noise-figure” and “noise-factor” are used to describe the noise properties of amplifiers. F is defined as signal to noise (power) ratio and the input of the DUT versus signal to noise power ratio at the output. F is always >1 for linear networks i.e. the signal to noise ratio at the output of some 2-port or 4- pole is always more or less degraded. In other words, the DUT (which may be also an amplifier with a gain smaller than unity i.e. an attenuator) is always adding some of its own noise to the signal.

NF[dB] is called “noise figure”

F[linear units of power ratio] sometimes noise factor

NF[dB] = 10 log F[linear units]

$$F[\text{linear unit}] = \frac{ENR[\text{linear unit}]}{Y[\text{linear unit}] - 1} = \frac{T_{ex}}{T_0 \cdot (Y - 1)} \quad \text{with } T_{ex} = T_H - T_0$$

ENR stand for excess noise ratio delivered by the noise diode and tells us how much “warmer” than room temperature the noise diode appears. For an ENR of 16 dB this amounts roughly a factor of 40 in power or 40 times 300 K which is 12000 K.

The quantity “Y” is the ratio of noise power densities measured on the SPA between the settings: noise source on and noise source off.

As shown in the equations below, also the gain of the DUT can be found from the two readings on the SPA. Thus, one can measure simultaneously gain and noise figure.

$$Y = \frac{\text{measured DUT output power (density) with noise source = hot}}{\text{measured DUT output power (density) with noise source = cold}}$$

$$ENR[\text{linear unit}] = \frac{(T_H - T_0)}{T_0}$$

$$ENR[\text{dB}] = 10 \cdot \log\left(\frac{(T_H - T_0)}{T_0}\right)$$

$$G_{(DUT)}[\text{lin}] = \frac{N(\text{SPA} + \text{DUT}, \text{Diode on})[\text{lin}] - N(\text{SPA} + \text{DUT}, \text{Diode off})[\text{lin}]}{N(\text{SPA}, \text{Diode on})[\text{lin}] - N(\text{SPA}, \text{Diode off})[\text{lin}]}$$

N = noise power measured on the SPA for e.g. 1 MHz resolution bandwidth

$$F_{total}[\text{linear units}] = F_1[\text{linear units}] + \frac{F_2[\text{linear units}] - 1}{G_1[\text{linear units}]} + \dots$$

Measurements with Network Analyzer test stand 1

Suggested topics:

- Calibration of the Vector Network Analyzer.
- Navigation in the Smith Chart using 3-stub tuner and trombone (constant impedance adjustable coax line)
- Application of the triple stub tuner for matching.
- Measurements of the velocity of light in the frequency domain using a trombone.
- N-port ($N=1\dots 4$) S-parameter measurements for different reciprocal and non-reciprocal RF-components.
- Self made RF-components: Calculate build and test your own attenuator in a SUCO box (and take it back home then).

Equipment:

VNA (Keysight N5231A), N-type calibration kit, 3-stub tuner (2.2.002), different Tees, 3x trombone (constant impedance adjustable coax line) (2.2.001), SUCO boxes with different lamped elements.

Calibration of the network analyzer

- ◆ Now lets start. **PRESET** the instrument with the green button. The VNA starts with full range sweep, CH1 active in S_{11} mode with logarithmic magnitude (log mag) and reference level at 0dB (REF).
- ◆ Dial a frequency span from 0.5 to 50 MHz using the Start and Stop controls.

Set start frequency to 0.5 MHz: **START; 0.5; M/u**

Set stop frequency to 50 MHz: **STOP; 50; M/u**

- ◆ Define the calibration kit (it has to be redone each time you press **PRESET**)
- ◆ In sweep menu select the number of point to 1601.
- ◆ Calibrate S_{11} and S_{22} using enhanced response calibration (not full two port) in the **CAL** menu with the reference plane at the end of the cable. Typical RF cables are equipped with N-connectors type M (male) on either side. Thus, the calibration kit elements to be attached at the end of such a cable are N-connectors type F (female). At the screen of the instrument during the calibration procedure you will be asked, which type of calibration kit element you are using. Keep in mind that the type of calibration is defined for the type of connector you are doing the calibration on (i.e. the end of the cable), but NOT for the calibration kit element itself!

By the way: The term N for the type of cable connector we are using here has its historical roots in the word Navy since this kind of connector was first used by the US–Navy more than 50 years ago. The frequently applied competitor, the BNC connector comes from the same shop (**B**ajonnet **N**avy **C**onnecter). But the BNC has much lower performance as compared to the N-connector.

- ◆ Now follow the menu on the calibration page and attach the open, short load as requested by the system. For the through use I-connector. Don't forget to confirm that the calibration is done –otherwise it will be not stored and whole procedure has to be repeated.
- ◆ For very critical and accurate measurements you may calibrate the system using the trace average function, but this rather time-consuming. However, you may reduce the IF bandwidth from 3 KHz (standard setting) to 100 Hz and watch the difference.
- ◆ As a next step select in the **display** menu select the number of traces 2. For trace 2 we are measuring now in transmission from port 1 to port 2 i.e. S_{21} .
- ◆ Connect the end of the cable where you just did the S_{11} calibration to port 2 of the VNA

Now you are well prepared to measure simultaneously some DUT in reflection and transmission with a calibrated system. As DUT you can use the 10 Ohm resistor in a SUCO box (blue box). This DUT contains a simple 10 Ohm carbon resistor between the inner conductors of the input and output connector. You can check now up to which frequency this test-object looks like a pure resistor and what kind of parasitic effects show up. Despite the fact that the geometrical length of the DUT is just a few cm you will see already at 50 MHz a considerable inductive component. The free space wavelength of 50 MHz amounts to 6 meter!

- ◆ At the end of the cable from port 1, attach the DUT and terminate the open connector of the DUT with a short (from the cal kit).
- ◆ For Trace 1 select as display format the Smith Chart ($R+jX$ as you want to observe impedance) and look at the readout of the marker on top of the screen and you get a reading of the resistance and inductance of the lumped resistor. Which readout changes significantly versus frequency? Discuss the results.
- ◆ Exchange the blue test box against another one, look to the results in different formats, operate the round dial button to measure at different frequencies.

Now you may try a further method to measure frequency dependent complex impedance of the 10 Ω test box (leaving the DUT connected to the cable to port 1 with a short at its end). The instrument provides a conversion menu that allows to display directly from the S_{11} measurement the real and imaginary part of the DUT's complex impedance as a function of frequency. This kind of conversion is not only possible from the reflection but also the transmission measurement. Try both techniques and discuss the reason for possible discrepancies. Play with the electrical delay correction in the transmission type test. How can you explain the (of course unphysical) negative real parts for certain electrical delay settings?

Demonstration of calibration effectiveness

- ◆ Display the locus of S_{11} of a 25 Ohm DUT the Smith-Chart for the frequency range 600-1200 MHz after having done a reflection calibration. This 25 Ohm DUT may be realised by using two 50 Ohm terminations and a coaxial T-piece (2.2.005) to connect them “ in parallel ”.
- ◆ Now produce a severe generator mismatch with another coaxial T-piece inserted at port 1 between port 1 and the coax cable. The open port of this T-piece will be terminated with another 50 Ohm load. In this configuration, we have artificially modified the generator impedance from about 50 Ohm to a value around 25 Ohm.
- ◆ As a next step perform the usual open, short, load calibration at the end of your (say 1 meter long) test cable. Reconnect the 25 Ohm DUT at the end and display its characteristic in the Smith Chart. You should not be able now to see any significant different to the result found in step 1.

Sometimes one is not really sure that having done a certain calibration procedure in particular in reflection has really been done well. There are many possible ways to make a mistake and an independent cross-check is desirable. A rather sensitive test consists in connecting a short piece of coaxial cable with its end left open or shorted and then display the reflection

coefficient in the Smith-Chart. In case of a poor calibration you may find the locus of S_{11} exceeding the boundary of the Smith-Chart which would only be valid for an active element with a reflection coefficient larger than unity. For a passive device, this kind of negative resistance response is impossible and indicates a calibration or display error. However, in the past reflection amplifiers were in use for special applications (certain parametric amplifiers or negative resistance devices containing tunnel diodes).

Navigation in the Smith Chart using 3-stub tuner and trombone

- ◆ Activate 2nd trace for S_{11} and display it in Smith Chart G+jY format. You will see absolutely the same locus as for 1st trace. However, the grid consists now superimposed conductance circles.
- ◆ Then insert the triple stub tuner (2.2.002), between the end of the calibrated test cable and the 25 Ohm load. Set marker to 1 GHz (You may also change centre frequency to 1GHz and span to 20 MHz to show only a small part of the locus on the Smith Chart. In this case carry out full 1-port calibration).
- ◆ By (cut and try method) adjusting the length of the three stubs try to achieve an impedance match ($S_{11}=0$) around 1 GHz.
- ◆ Exchange 3-stub tuner with trombone (2.2.001). Change the length of the trombone and observe the results.

See also printouts: “RF Engineering Basic Concepts: The Smith Chart”, p. 4.

Measurements of the light velocity in the frequency domain using a trombone

- ◆ Short the end of trombone using short standard from calibration kit.
- ◆ Adjust the length of trombone to reach short ($\rho = -1$) at 1 GHz. (You might also use port extension function). Measure the length of trombone.
- ◆ Now change the length of trombone by $\lambda/4$ i.e. such, that the short is transformed to open.
- ◆ Measure again the length of the trombone and calculate the velocity of light.

In a free space: $c = \lambda / T = \lambda * f$.

Build you own 3-stub tuner.

- ◆ Use 3 trombones and connect them with N-type tees. Leave ends of trombone opened and changing the length of trombones try to reach matching at 1GHz. Do the same with trombones shorted on its ends.
Note: not all T-like components are just normal tees. Some of them consist resistors or are build such that they are rather impedance transformers.
- ◆ You may also build a 3-stub tuner in the micro strip technology.

N-port ($N=1\dots4$) S-parameter measurements for different reciprocal and non-reciprocal RF-components.

Equipment:

VNA (Keysight N5231A), N-type calibration kit, different reciprocal and non-reciprocal N-ports, soldering iron, multimeter

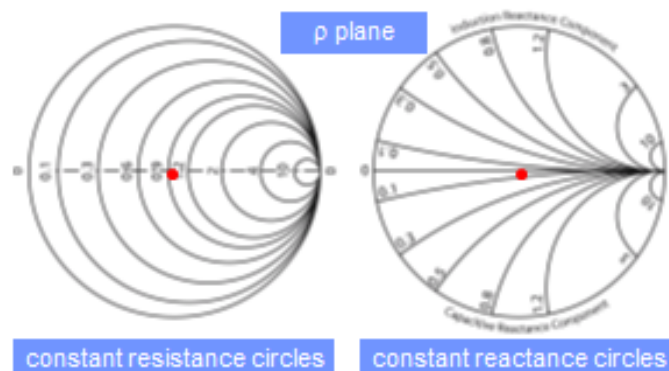
- ◆ Set a frequency range from 10 MHz to 15 GHz. Set sweep mode to log. freq. Carry out full 4-port calibration. Though it is time consuming, it will be helpful during measurements of 4-port components.
- ◆ Measure remaining 2-ports, 3-ports, and 4-port. (Be aware that all component output should be properly terminated). What is special about circulator?
- ◆ Try to recognize the elements marked as UCO

Calculate build and test your own attenuator in a SUCO box (and take it back home then).

- ◆ Calculate a T attenuator using a calculator program installed on the laptop and verify results using a multimeter
- ◆ Build your attenuator in SUCO box and test it.

The Smith Chart - construction

- ◆ A very basic use of the Smith Chart is to graphically convert values of ρ into z and vice versa. To this purpose in the ρ plane a grid is drawn that allows to find the value of z at a given point ρ .
- ◆ An important property of conformal mappings is that general circles are mapped to general circles. Straight lines are considered as circle with infinite radius
- ◆ Below the loci of constant resistance and constant reactance are drawn in the ρ plane
- ◆ The origin of the ρ plane is marked with a red dot, the diameter of the largest circle is $|\rho|=1$



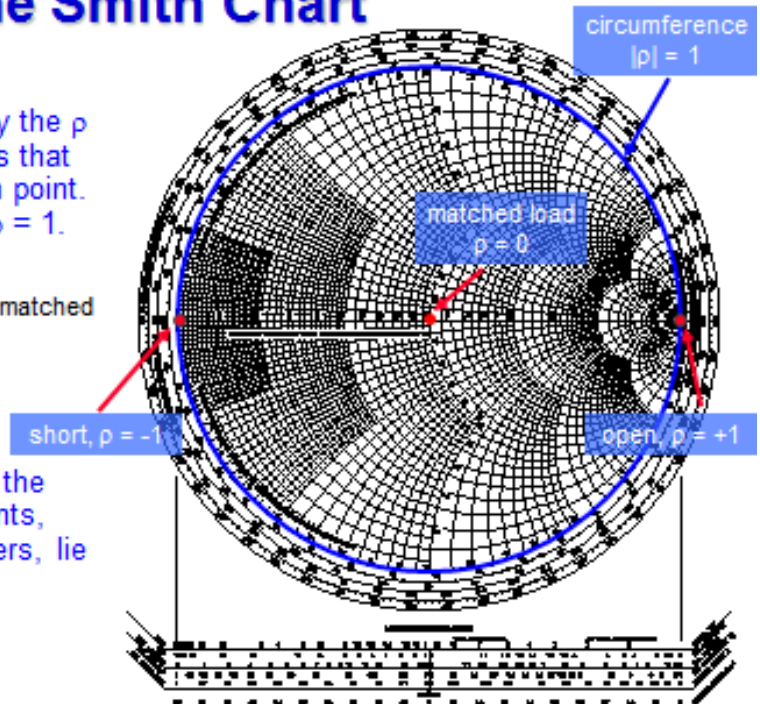
CAS, Trondheim, August 2013

RF Basic Concepts, Caspers, Kowina

1

The Smith Chart

- ◆ The Smith Chart is simply the ρ plane with overlaid circles that help to find the z for each point. The radius in is general $\rho = 1$.
- ◆ Important points:
 - Center of the Smith Chart: matched load. $\rho = 0, z = 1$
 - Open circuit: $\rho = +1, z = \infty$
 - Short circuit: $\rho = -1, z = 0$
- ◆ Lossless elements lie on the circle $|\rho|=1$; active elements, such as reflection amplifiers, lie outside this circle.



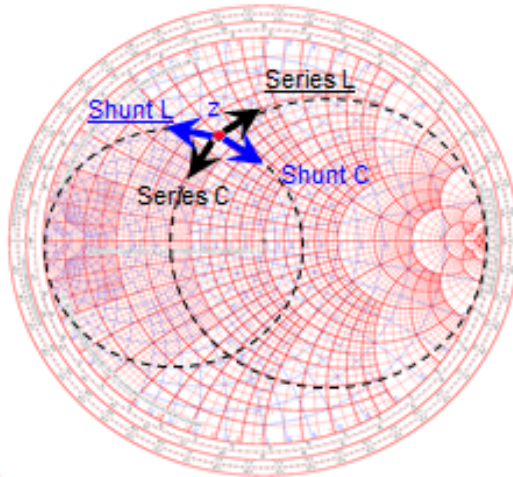
CAS, Chios, September 2011

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2

Navigation in the Smith Chart 1

- ◆ When a lossless element is added *in series* to an impedance z , one moves along the constant resistance circles ($R = \text{const}$)
- ◆ When a lossless element is added *in parallel (shunt)* to an impedance z , one moves along the circles conductance ($G = \text{const}$)



	Up	Down
Black circles	Series L	Series C
Blue circles	Shunt L	Shunt C

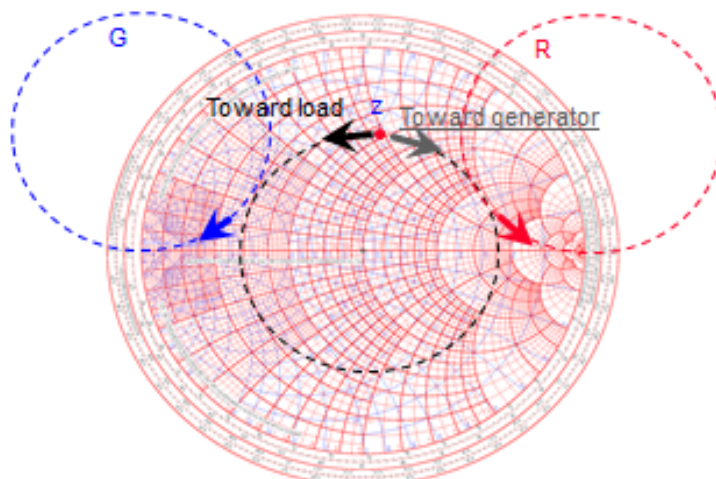
CAS, Trondheim, August 2013

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3

Navigation in the Smith Chart 2

- ◆ An ideal lossless transmission line only changes the phase of $\rho \Rightarrow$ a transmission line gives a rotation about the center of the Smith Chart
- ◆ For a line of length $\lambda/4$ we get a rotation by 180 degrees \Rightarrow a short circuit is converted into an open circuit and vice versa. Such a line is called $\lambda/4$ transformer.



Red arcs	Resistance R in series
Blue arcs	Conductance G in parallel
Con-centric circle	Transmission line going Toward load Toward generator

CAS, Trondheim, August 2013

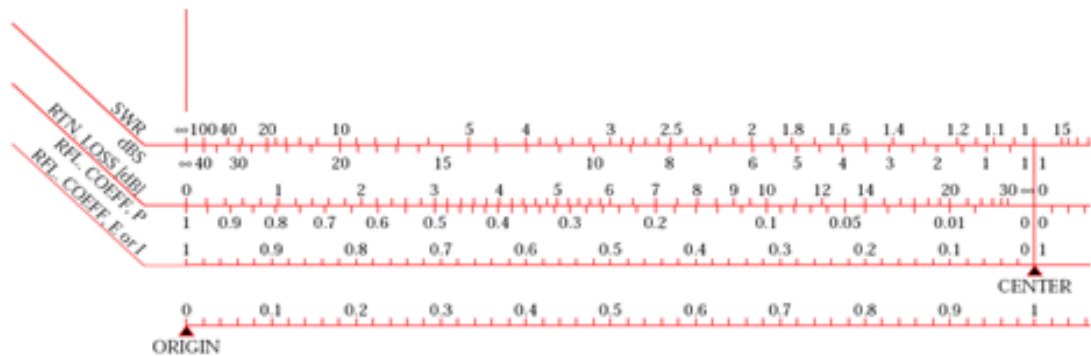
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4

What about all these rulers below the Smith chart (1)

How to use these rulers:

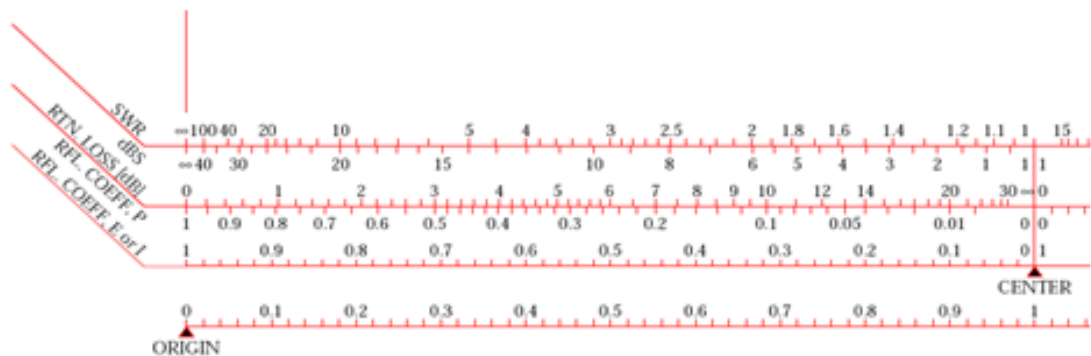
You take the modulus of the reflection coefficient of an impedance to be examined by some means, either with a conventional ruler or better take it into the compass. Then refer to the coordinate denoted to CENTER and go to the left or for the other part of the rulers (not shown here in the magnification) to the right except for the lowest line which is marked ORIGIN at the left.



Example

What about all these rulers below the Smith chart (2)

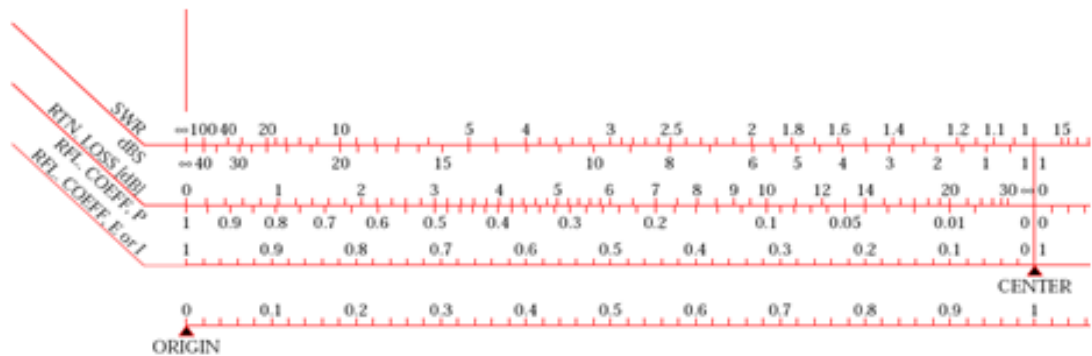
First ruler / left / upper part, marked SWR. This means VSWR, i.e. Voltage Standing Wave Ratio, the range of value is between one and infinity. One is for the matched case (center of the Smith chart), infinity is for total reflection (boundary of the SC). The upper part is in linear scale, the lower part of this ruler is in dB, noted as dBS (dB referred to Standing Wave Ratio). Example: SWR = 10 corresponds to 20 dBS, SWR = 100 corresponds to 40 dBS [voltage ratios, not power ratios].



Example

What about all these rulers below the Smith chart (3)

Second ruler / left / upper part, marked as RTN.LOSS = return loss in dB. This indicates the amount of reflected wave expressed in dB. Thus, in the center of SC nothing is reflected and the return loss is infinite. At the boundary we have full reflection, thus return loss 0 dB. The lower part of the scale denoted as RFL.COEFF. P = reflection coefficient in terms of POWER (proportional $|\Gamma|^2$). No reflected power for the matched case = center of the SC, (normalized) reflected power = 1 at the boundary.



Example

CAS, Trondheim, August 2013

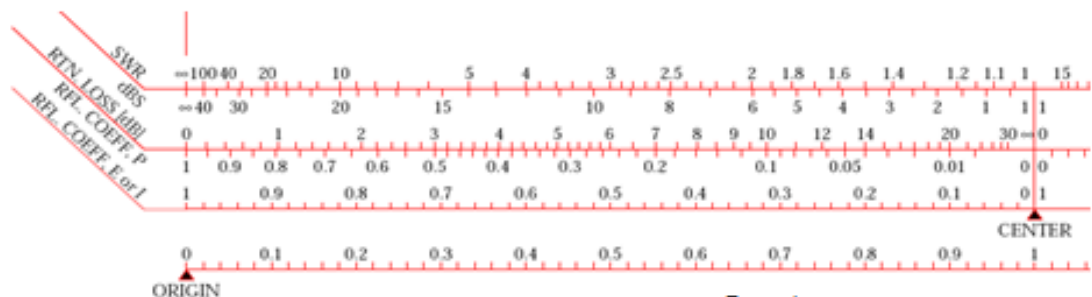
RF Basic Concepts, Caspers, Kowina

7

What about all these rulers below the Smith chart (4)

Third ruler / left, marked as RFL.COEFF,E or I = gives us the modulus (= absolute value) of the reflection coefficient in linear scale. Note that since we have the modulus we can refer it both to voltage or current as we have omitted the sign, we just use the modulus. Obviously in the center the reflection coefficient is zero, at the boundary it is one.

The fourth ruler has been discussed in the example of the previous slides: Voltage transmission coefficient. Note that the modulus of the voltage (and current) transmission coefficient has a range from zero, i.e. short circuit, to +2 (open = $1+\Gamma$ with $\Gamma=1$). This ruler is only valid for $Z_{load} = \text{real}$, i.e. the case of a step in characteristic impedance of the coaxial line.



Example

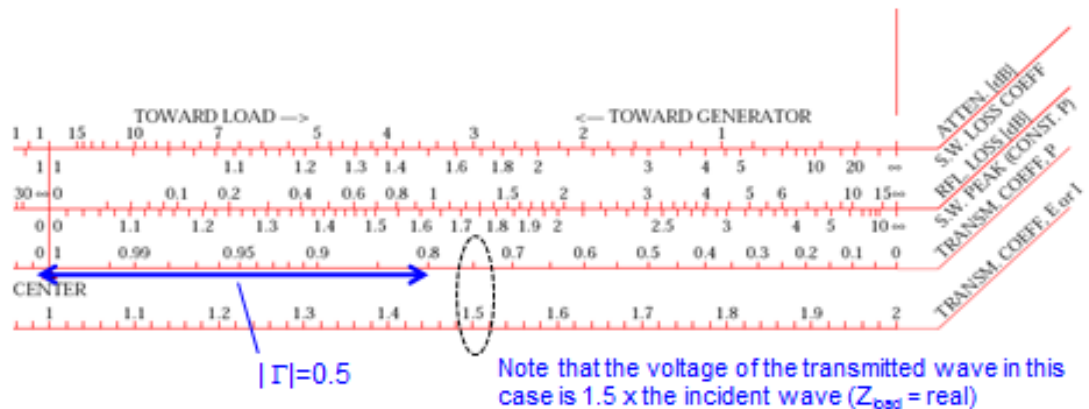
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8

What about all these rulers below the Smith chart (5)

Third ruler / right, marked as TRANSM.COEFF.P refers to the transmitted power as a function of mismatch and displays essentially the relation $P_t = P_i(1 - |\Gamma|^2)$. In the center of the SC full match, all the power is transmitted. At the boundary we have total reflection and e.g. for a Γ value of 0.5 we see that 75% of the incident power is transmitted.



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Example

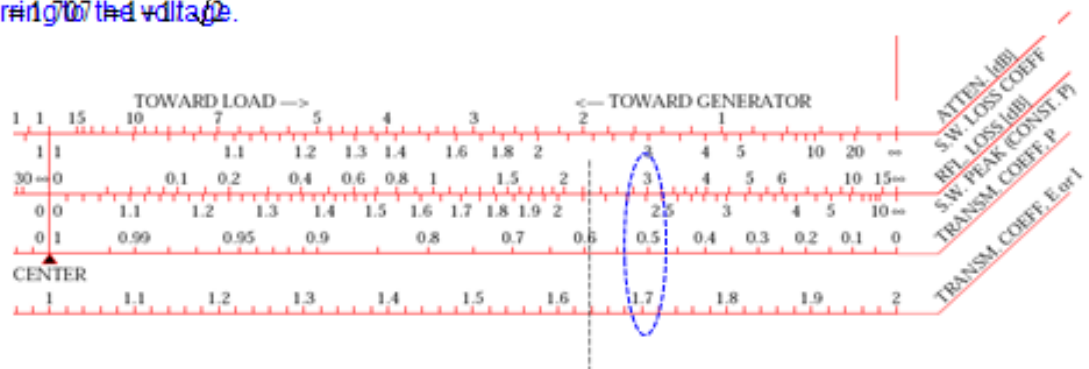
9

What about all these rulers below the Smith chart (6)

Second ruler / right / upper part, denoted as RFL.LOSS in dB = reflection loss. This ruler refers to the loss in the transmitted wave, not to be confounded with the return loss referring to the reflected wave. It displays the relation $P_t = P_i(1 - |\Gamma|^2)$ in dB.

Example: $|\Gamma| = 1/\sqrt{2} = 0.707$, transmitted power = 50% thus loss = 50% = 3dB.

Note that in the lowest ruler the voltage of the transmitted wave ($Z_{load} = \text{real}$) would be if referring to the voltage.



Example

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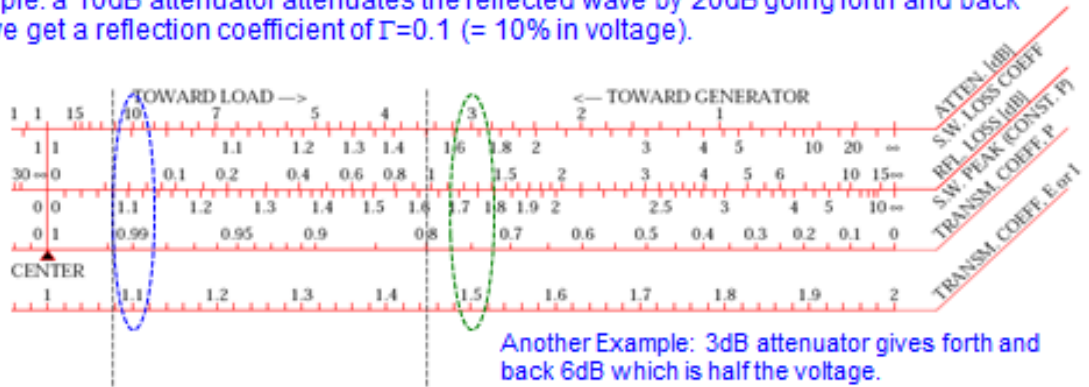
10

What about all these rulers below the Smith chart (7)

First ruler / right / upper part, denoted as ATTEN. in dB assumes that we are measuring an attenuator (that may be a lossy line) which itself is terminated by an open or short circuit (full reflection). Thus the wave is travelling twice through the attenuator (forward and backward). The value of this attenuator can be between zero and some very high number corresponding to the matched case.

The lower scale of ruler #1 displays the same situation just in terms of VSWR.

Example: a 10dB attenuator attenuates the reflected wave by 20dB going forth and back and we get a reflection coefficient of $\Gamma=0.1$ (= 10% in voltage).



Measurements with Network Analyzer test stand 2

Suggested topics:

- 2-port measurements for active RF-components (amplifiers): 1 dB compression point (power sweep).
- Time Domain Reflectometry using synthetic pulse: direct measurement of coaxial line characteristic impedance.
- Measurements of the light velocity using a trombone (constant impedance adjustable coax line) in the time domain.
- Beam transfer impedance measurements with the wire (button PU, stripline PU.)

Amplifier measurements

Equipment:

VNA (Keysight E5071C), 3.5mm - type calibration kit, power supplier (G.1.1.008), 20dB N-Type attenuator, 30 dB SMA attenuator, Amplifier (DUT) (3.1.006)

If you use another amplifier you may have to adapt certain measurement parameters.
For background material see printouts: “Network Analysis” , p.19.

Whenever you measure medium or high power amplifiers, be sure that the power level cannot destroy the input of the VNA. For example, even measuring the input impedance of an amplifier may destroy the VNA, when the amplifier produces parasitic (self) oscillations.

- ◆ Preset the Vector Network Analyzer (VNA)
- ◆ To protect the vector network analyser (VNA) against overload from the amplifier output, start with the following set-ups:
Output Power: -10dBm
Attenuator port 1: external 20 dB; this leads to an input power of -30 dBm for the amplifier
Never remove the fixed 30 dB attenuator from the output of the amplifier. Assume that its attenuation is exactly 30.0 dB, constant over the complete frequency range.
- ◆ Set the start frequency to 400kHz, the stop frequency to 2.5 GHz. Carry out a extended response calibration for S_{21} and S_{11} using the cables with SMA connectors and SMA calibration kit. (You have to specify which calibration kit type are you presently using).
- ◆ Insert amplifier without DC supply. IN to port 1 and OUT to port 2.
- ◆ Measure the transmission-coefficient from port1 to port2 $\implies S_{21}$ not S_{12} !! Display the response using the auto scale function; Select trace averaging with an averaging

factor of 10; Measure the response (in this case the isolation) at 1 GHz; produce a hard copy (plotter) of the screen.

- ◆ Reduce the IF bandwidth (IF-BW) to 100Hz; compare the result with the copy done before
- ◆ Go back to IF-BW=3000Hz and connect or turn on the DC voltage of 15V to the amplifier. Use the full screen display for a single channel; determine the small signal gain of the amplifier at 1 GHz with marker1. Produce a hardcopy
- ◆ Measure the 3 dB bandwidth (and also 1 dB BW) of the amplifier (determine the frequencies, where the gain is 3dB/1dB lower than at 1 GHz) using marker functions. Use bandwidth function in **search marker** menu. Hardcopy!
- ◆ Test other possibilities to determine the 1 dB and 3 dB bandwidth of this amplifier
- ◆ Determine the 1 dB compression point of the amplifier at 1 GHz. Go via **SWEEP** to CW frequency and power sweep. Use marker functions. Hardcopy!
- ◆ Measure the frequency range over which the gain compression of 1 dB occurs, first in the specified frequency range of the amplifier using the CW mode at the lower, mid-band and upper frequency of the amplifier. Apply the display features of the for this measurement and the marker search function. Hardcopy!
- ◆ Now return to the frequency sweep display, set a suitable range (response calibration) to cover the 3 dB bandwidth of the amplifier and store the response trace for a small input signal (small enough that the output power is well below the 1 dB compression point). Select a vertical resolution of 1 dB/div and an adequate reference level to position the trace approximately in the middle of the screen.
- ◆ Now increase the input power until the readout is approximately -1dB below the previous trace. Note, that the 1 dB compression level is frequency dependent. Copy the result.
- ◆ Preset the VNA, set the frequency range and carry out a S_{11} one PORT calibration. Measure the SWR (standing wave ratio) of the input of the amplifier in the specified frequency range. Determine the maximum SWR (or VSWR-voltage standing wave ratio)
- ◆ Measure the deviation from linear phase and the group delay in transmission.

Time domain measurements

Equipment:

VNA, N - type calibration kit, blue boxes with different lumped elements

More information see printouts: "RF Measurement Concepts", p. 9.

When using the time domain option of the vector-network-analyser we have the "low-pass" mode available or the "band-pass" mode. The low-pass mode can only be used for equidistant sampling in the frequency domain (equidistant with respect to DC), since the Fourier Transform of a repetitive sequence of pulses has a line spectrum with equidistant spacing of the lines including the frequency zero. This implies that for a given frequency range and number of data points the instrument must first work out the exact frequencies for the low-pass-mode (done by using the soft-key: set frequency Low-pass). Once these frequencies are defined, calibration can be applied. For a linear time-invariant system frequency and time domain measurements are basically completely equivalent (except for signal to noise ratio issues) and may be translated mutually via the Fourier transform. Note that the Fourier transform of a spectrum with

constant density over a given frequency range (rectangular spectrum) has a $\sin(f)/f$ characteristic in the time domain. This characteristic shows undesired "side-lobes" and thus an (amplitude) weighting function [=window] is applied in the frequency domain before entering the FFT. This weighting function is typically \sin^2 or Gaussian and helps to suppress strongly side-lobes in the time-domain. Within the low-pass mode we can use the pulse and step function respectively. The step function is nothing else than the integral over the pulse response. When using the gating function, keep in mind that gating is a non-linear operation and thus gating may generate artificially frequency components which were not present before gating. In the band-pass mode, the spectral lines (frequency domain data points) need no longer be equidistant to DC but just within the frequency range of interest. The corresponding time-domain response for the same bandwidth is twice as long as in the low-pass mode and we get in general complex signals in the time domain. These complex signals are equivalent to the I and Q signals (I = in phase and Q = quadrature) often found in complex mixer terminology. They can be directly displayed using soft-keys "real" or "imaginary" in the format menu. The real part is equivalent to what one would see on a fast scope i.e. an RF signal with a Gaussian envelope. The meaning of the time-domain band-pass mode response in linear magnitude format is the "modulus of the complex envelope [$\text{SQRT}\{(re^j(t)+imag^j(t))\}$] of a carrier modulated signal". Note that the time domain mode can also be applied for CW excitation from the VNA but then to analyse a slowly time variant response of the DUT (up to the IF bandwidth of 3 kHz).

To start with the time-domain option, follow the next instructions:

- ◆ Preset the instrument, dial a frequency range of 300 KHz to 3 GHz, 801 data points) and go into the time-domain, low-pass mode, step function
- ◆ The VNA sets by this operation a frequency range, which is required for the low-pass mode Fourier -transform-calculation. Check the frequency reading.
- ◆ Now you have to calibrate S_{11} as you did before, with the only difference, that you have to use the "OPEN" from the cal-kit, as you are now measuring up to about 3GHz.
- ◆ (Refer to the above descriptions of calibrating S_{11} in case you do not remember)
- ◆ Read out the pulse amplitude with the end connector of the cable open.
- ◆ Now connect a SHORT and read the signal.
- ◆ Discuss the meaning of the sign from the readout for the reflected wave.
- ◆ Connect the 10 Ω test box with a SHORT at the end to the THRU.
- ◆ Can you calculate the resistance from the readout?
- ◆ Remember the definition of S_{11} and which simple formula you have to apply:

$\rho = (Z-Z_c)/(Z+Z_c)$; this is the reflection coefficient as seen in the reference plane of the DUT Z_c stands for the characteristic impedance of the cable and usually amounts to 50 Ω

Note that the reflection coefficient is often denoted as Γ , r and also ρ .

- ◆ Repeat the experiment with the 100 Ω blue box using a SHORT at the end.
- ◆ Look at the 12(18) pF and 100pF capacitor boxes (end=open). Discuss the traces and remember that it is a single step response. The two numbers (12 and 18) for the capacitance indicate that a 12 pF capacitor mounted inside the test box returns a total capacity of 18 pF due to the connector feed-throughs and other parasitic capacitances.

- ◆ Apply now in the calibrated reference plane instead of the capacitor a 25Ω DUT, using two 50Ω loads connected in parallel via a coaxial T-piece
- ◆ Use an appropriate vertical scale factor to obtain a good resolution on the screen
- ◆ Store the result to the memory and display memory and data.
- ◆ Put a SHORT on the end of the calibrated cable (instead of the previously used 25Ω) and compare the data- with the memory- trace. Plot the results and discuss them.
- ◆ You may also try building a simple notch filter by attaching the T-piece to your calibrated reference plane and a 50Ω load at one connector of the T-piece and an open [or shorted] stub (say about 1 meter of cable) at the other connector.
- ◆ If you are interested to go further repeat all the time domain measurements mentioned above also in the pulse-mode (low-pass) instead of the step-mode.
- ◆ You can also measure velocity of light in time domain using trombone.

There is a wide range of application for this synthetic pulse time-domain technique. A VNA in the time-domain low-pass step mode has a very similar range of applications as a sampling scope (Fig.1 and Fig.2). However, it must always be kept in mind, that carrying out a measurement in the frequency domain and then going via FFT or similar into the time-domain implies strict linearity of the DUT. Thus, a transient on a non-linear system such as the onset of oscillations on some microwave oscillator with active elements after turn-on of the supply voltage would not return meaningful results when using the technique mentioned above. The dynamic range of a typical sampling scope is limited at about 60 dB with a maximum input signal of 1 Volt and a noise floor around 1 mV. The VNA can easily go beyond 100 dB for the same maximum level of the input signal of about +10 dBm. Both instruments are using basically the same kind of detector, either a balanced mixer (4 diodes) or the sampling head (2 or 4 diodes), but the essential difference is the noise floor and the average power arriving at the receiver. In the case of the VNA we have a CW signal with bandwidth of a few Hz and thus can obtain with appropriate filtering a very good signal to noise ratio since the thermal noise floor is a -174 dBm/Hz. For the sampling oscilloscope, we get a short pulse with a rather low repetition rate (typically around 100 KHz) and all the energy is spread over the full frequency range (typically 20 GHz bandwidth). With this low average power (around a micro-Watt) the spectral density is orders of magnitude lower than in the case of the VNA and this finally makes the large difference in dynamic range (even without gain switching). Also, the VNA permits in the band-pass to tailor a wide range of band-limited RF-pulses, which would be very tedious with a sampling scope.

Beam transfer impedance measurements with the wire (button PU, stripline PU.)

- ◆ *use instruction by J.Byrd et al. reprinted below.*

What we see on a network analyzer

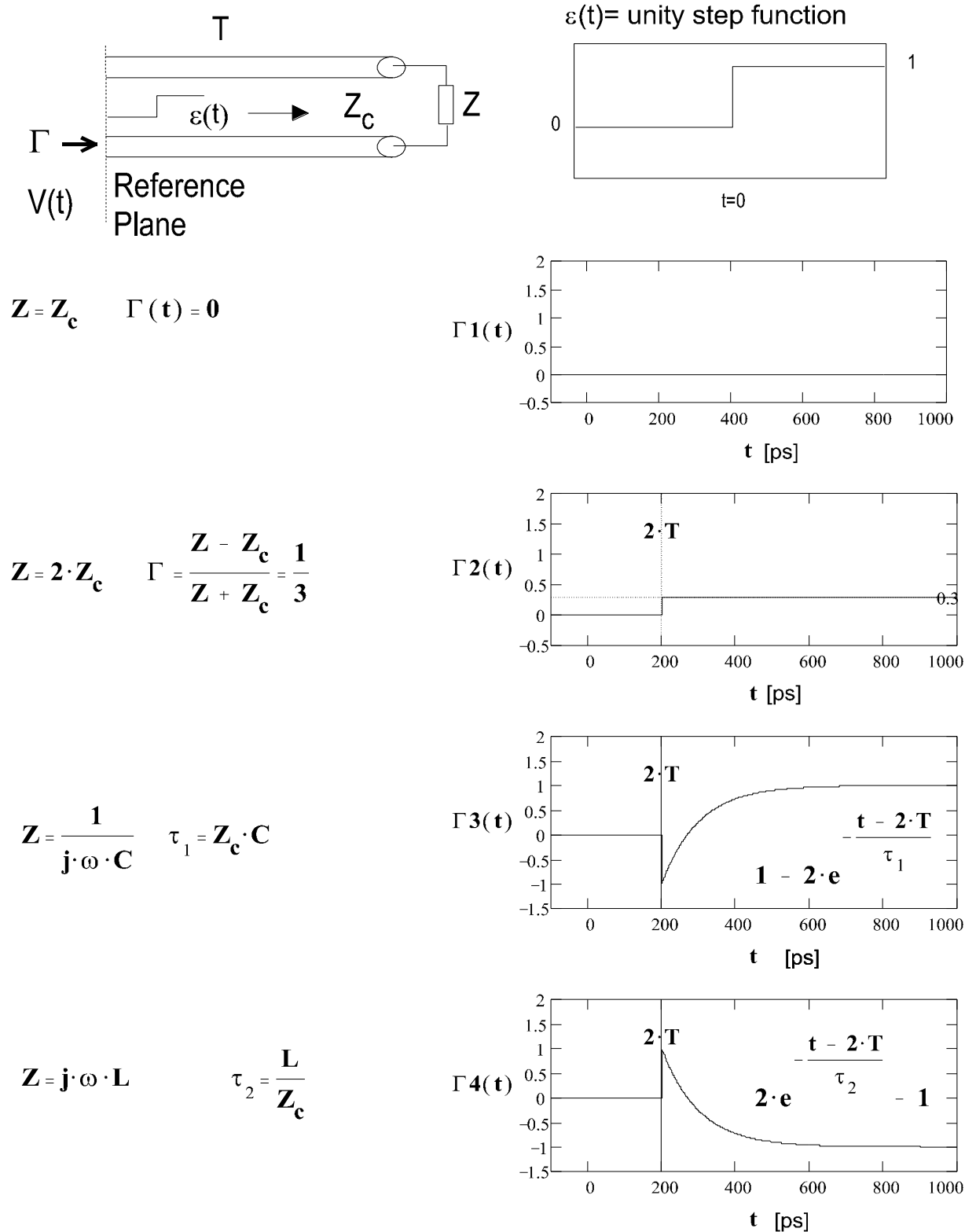
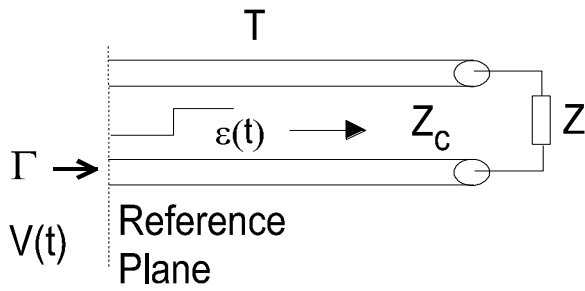


Fig 1 Step-function response for different terminations on a VNA.

What we see on a sampling scope



$$Z = Z_c \quad \Gamma(t) = 0$$

$$V1(t) = \varepsilon(t)$$

$$Z = 2 \cdot Z_c \quad \Gamma = \frac{Z - Z_c}{Z + Z_c} = \frac{1}{3}$$

$$V2(t) = \varepsilon(t) + \Gamma \cdot \varepsilon(t - 2 \cdot T)$$

$$Z = \frac{1}{j \cdot \omega \cdot C} \quad \tau_1 = Z_c \cdot C$$

$$Z = j \cdot \omega \cdot L \quad \tau_2 = \frac{L}{Z_c}$$

$\varepsilon(t) =$ unity step function

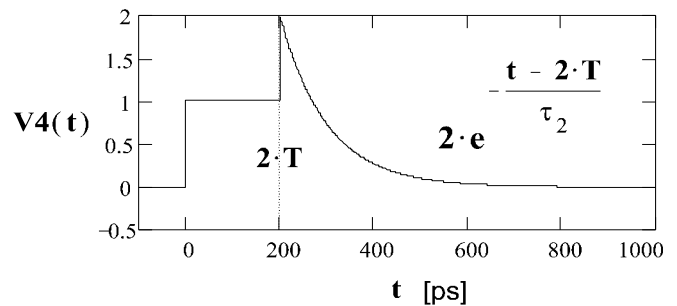
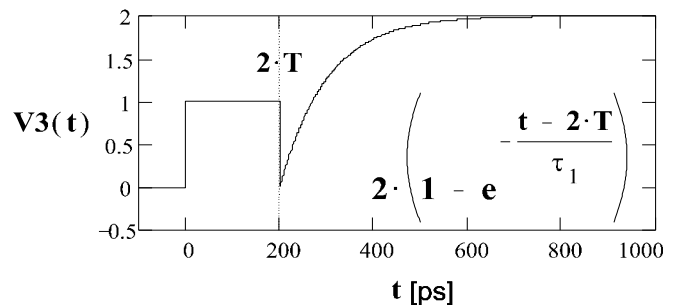
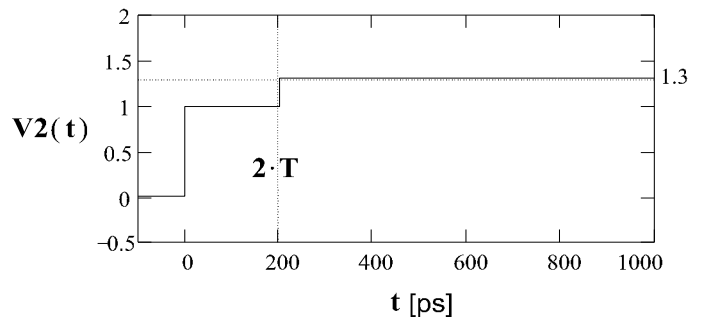
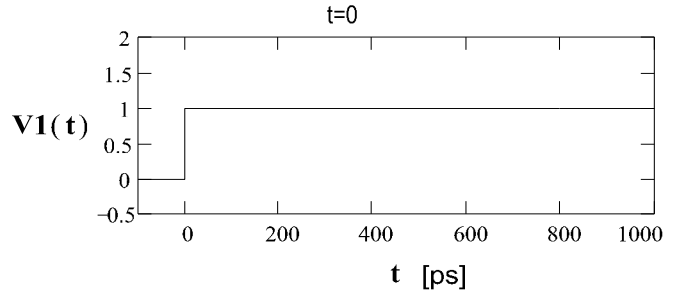
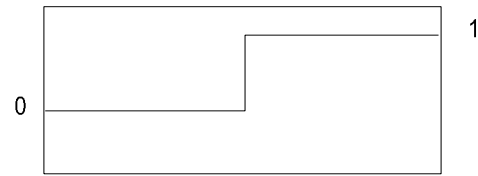
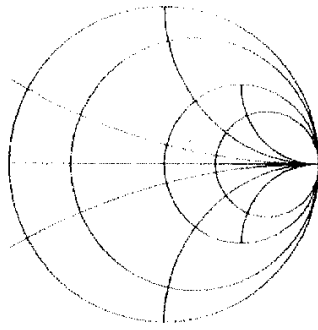


Fig 2 Step-function response for different terminations on a sampling scope (through sampler)



Microwave Measurements Laboratory

Joint US/CERN/Japan
Accelerator School,
Shonan Village
9-18 September, 1996

Stripline electrode measurements

John Byrd, Lawrence Berkeley Laboratory
(adapted from notes by G. Lambertson)

- Introduces the student to the the wire method of impedance measurement.
- Introduces the student to a stripline pickup/kicker.

Introduction

This experiment introduces the wire method of impedance measurement. This is the primary method measuring the impedance of vacuum chamber components. The characteristics of a simple stripline are measured.

Equipment:

Aluminum model stripline shell
wire impedance rig with reference shell
Network Analyzer + calibration kit
2 cables
2 50Ω SMA loads
1 SMA F-F connector (for Thru calibration)

Experiment:

- Set the NWA frequency range from 300 kHz to 2.3 GHz with 801 points. Do a full 2-port calibration (omit isolation calibration.)
- Set up the impedance rig with the stripline. Terminate the stripline terminals. Measure S21 and store the result in memory.
- Replace the stripline with the reference piece and measure S21. Using the math functions on the plot the ratio $S_{21,stripline}/S_{21,ref}$. Calculate Z_B at the two maxima where

$$Z_B = 2R_w \left(\frac{S_{21,ref}}{S_{21,object}} - 1 \right)$$

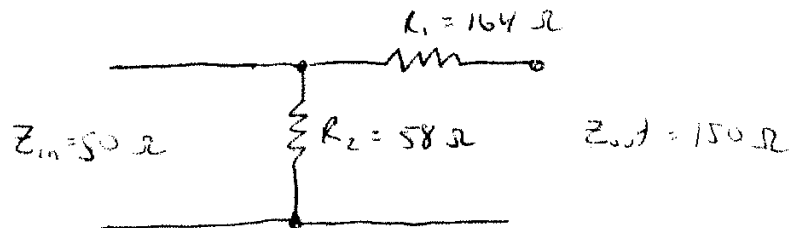
- Move terminal two to the upstream pickup. Measure S21 and plot the ratio of the $S_{21,upstream}/S_{21,ref}$.
 - Calculate $Z_p = A_2 R_0 (S_{21,upstream}/S_{21,ref})$
 - Calculate the effective width of the strip. ($Z_p = R_w \phi_w / 2\pi$)
 - From the frequency of the zeroes in calculate the effective length of the stripline.
- Measure at the downstream pickup port. Why is this not zero? What is the fraction of upstream pickup impedance do you observe?

Background material

See the handout on impedance measurements by Fritz Caspers for more material.

Resistive Matching

In order to make a good measurement of the beam impedance, it is important to match the characteristic impedance of the test setup (usually 50Ω) to the impedance of the coaxial line formed by the wire and device under test. This is usually done by transforming the impedance using smooth tapers. However, tapers are more elaborate mechanically and so for this we are using resistive matching techniques for simplicity. This experiment uses a T-network to match from the 50Ω input impedance to the 150Ω of the wire/pipe coaxial line ($=60 \ln(a/b)$) as shown below. This matches the impedance in both the forward and reverse directions.

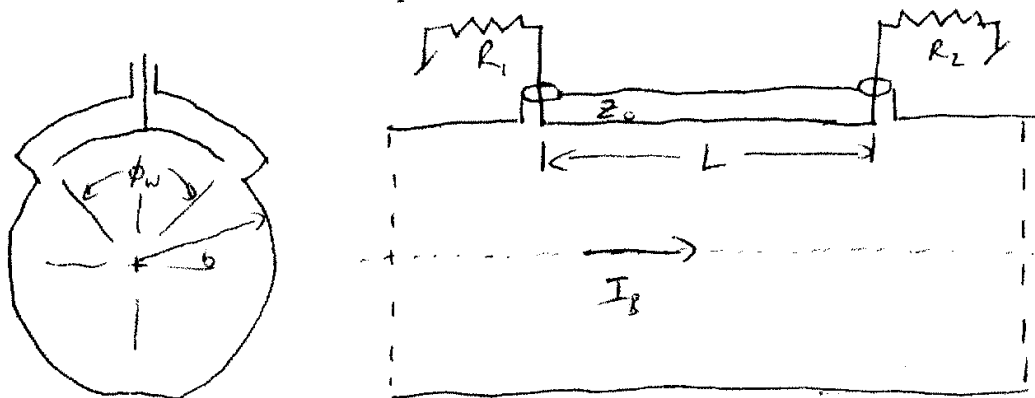


One of the disadvantages of resistive matching is that the signal levels can be considerably attenuated by the resistors, complicating the analysis. However, impedance transformers are somewhat impractical at lower frequencies where resistive matching is ideal. The attenuation for the resistive matching at ports 1 and 2 has been found to be $A_1=0.14$ and $A_2=0.54$.

N.B. It is not necessary to match the impedance in both directions at port 2 since we only need to measure S_{21} (i.e. port 1 to port 2) to determine the impedance. A single series resistor is sufficient to match at port 2 and provides lower attenuation than the series/shunt arrangement shown above.

Striplines

Consider a stripline as shown below. A relativistic charged particle ($v=c$) moves along the center axis with current I_b . As the beam passes the upstream end of the stripline, an image current $-gI_b$ is induced on the inner side of the stripline plate, where g is the fraction of a circle subtended by the plate, and travels along with the beam. An equal opposite current is drawn from the load resistance and is distributed on the outer side of the stripline.



Assuming that the upstream terminal impedance R_1 is matched to the stripline impedance Z_0 , half of the current exits the upstream ($gI_b/2$) terminal and half travels down the stripline with velocity c . As the beam passes the downstream port, the current on the inner electrode plate, $-gI_b$, combines with the current of the outer side of the plate. A current of $-gI_b/2$ travels back towards the upstream port and exits.

The voltage at the upstream port is given by

$$V_1(t) = \frac{1}{2} g R_0 I_b (\delta(t) - \delta(t - 2L/c))$$

where L is the stripline length, and $R_0=R_1=R_2=Z_0$ is the characteristic impedance of the stripline. The voltage as function of frequency is

$$V_1(\omega) = \frac{1}{2} g R_0 I_b (1 - e^{-2j\omega L/c})$$

where $k=\omega/c$. This is shown below.

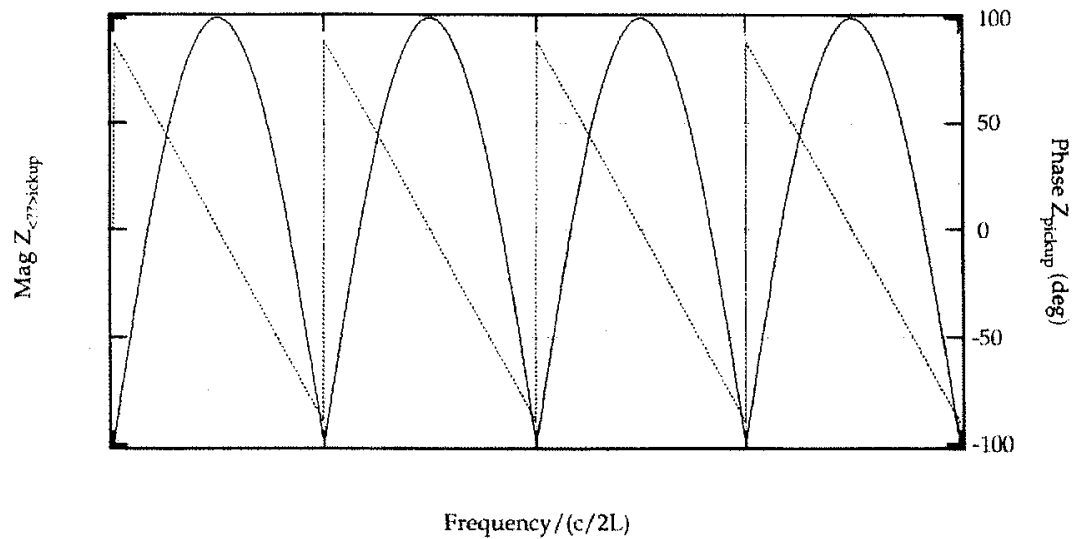


Figure 2. Pickup impedance of a matched stripline. Voltage at upstream port only.

The voltage at the downstream port is zero for ideal matching of Z_0 and the output terminal. In practice it is difficult to get perfect matching and thus signals can also be observed at the downstream port.

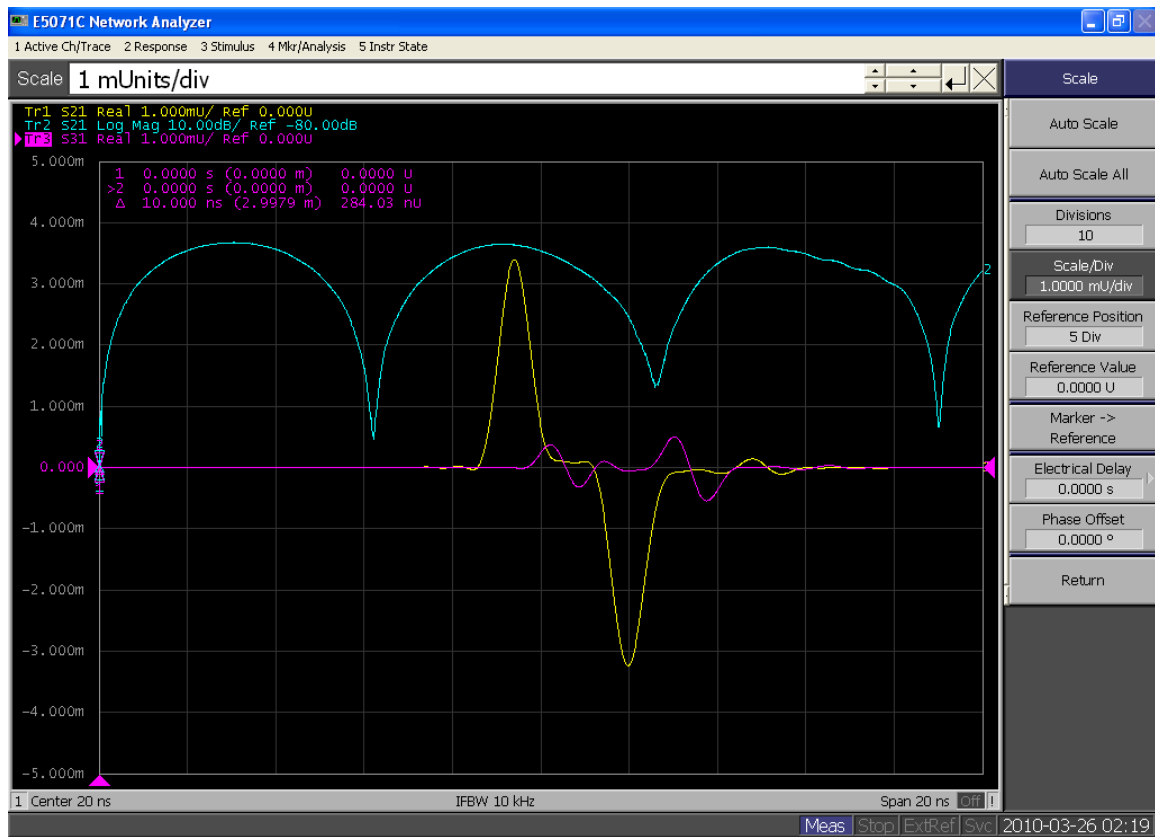


Fig 3 Pickup impedance of a matched stripline (blue trace). Other two traces depict corresponding time domain signals measured in up-stream port (yellow trace) and down-stream port (magenta trace). Try to obtain similar spectra.

Measurement of the Longitudinal Coupling Impedance with a stretched-wire setup

Background Information

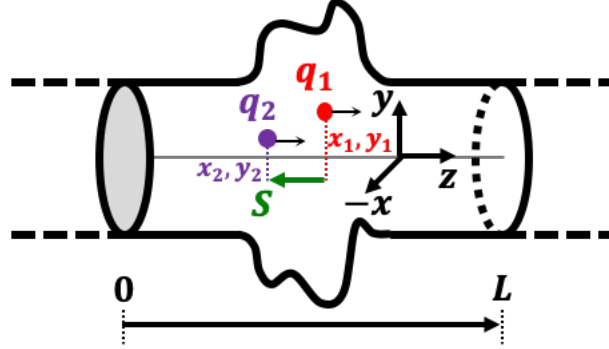


Figure 1. Wake field of a leading point charge q_1 on a trailing point charge q_2 .

If a beam pipe has structures with discontinuities, e.g. cavities, beam instruments, pumping ports, bellows, transitions, tapers, etc., a part of the EM-field of a leading point charge q_1 will generate a *Lorenz* force on a trailing point charge q_2 (see also Fig. 1):

$$\vec{F} = \frac{d\vec{p}}{dt} = q_2(\vec{E} + c_0\vec{e}_z \times \vec{B})$$

given by the wake potential of the structure of length L :

$$\vec{w}(x_1, y_1, x_2, y_2, s) = \frac{1}{q_1} \int_{-\infty \text{ (or } 0)}^{+\infty \text{ (or } L)} dz [\vec{E}(x_2, y_2, z, t) + c_0\vec{e}_z \times \vec{B}(x_2, y_2, z, t)]_{t=(s+z)/c_0}$$

The beam coupling impedance Z is the frequency domain equivalent of the wake potential, defined by the Fourier transformation

$$Z(x_1, y_1, x_2, y_2, \omega) = -\frac{1}{c_0} \int_{-\infty}^{+\infty} ds \vec{w}(x_1, y_1, x_2, y_2, s) e^{-j\omega s/c_0}$$

and can be decomposed in longitudinal Z_{\parallel} and transverse Z_{\perp} components, which are related described by the *Panofsky-Wenzel* theorem.

For many practical problems the longitudinal wake w_{\parallel} , and therefore the longitudinal beam coupling impedance Z_{\parallel} is of primary interest. For resonant structures, e.g. cavities, the longitudinal coupling impedance $Z_{\parallel,i}(\omega_i)$ of each eigenmode ω_i is equivalent to its shunt impedance:

$$R_{sh,i} = Z_{\parallel,i} = \frac{2k_{loss,i} Q_i}{\omega_i}$$

Summing all relevant $R_{sh,i}$, e.g. characterized by a VNA-based bead-pull measurement will result in the total beam coupling impedance Z_{\parallel} , however, might be rather time consuming and may not be applicable for non-resonant, broadband structures.

Stretched-wire Setup to measure the longitudinal Coupling Impedance

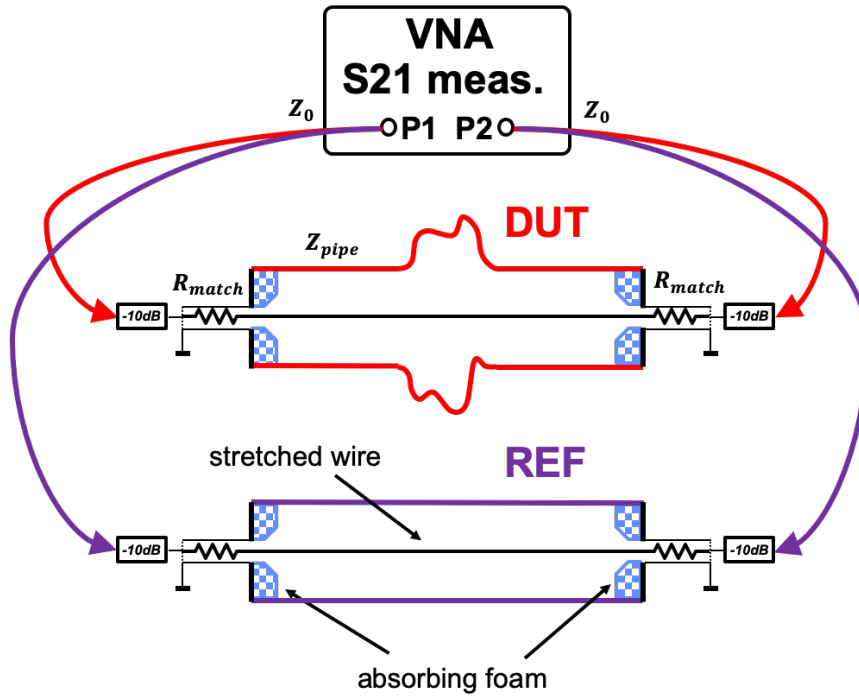


Figure 2. Measurement setup utilizing a stretched-wire to evaluate the longitudinal coupling impedance.

Among the variety of coupling impedance measurement techniques, the stretched-wire method can be kept rather simple to evaluate the longitudinal coupling impedance Z_{\parallel} , see also Fig. 2. An important parameter is the normalized electrical length Θ of the DUT (device under test) structure to be analyzed:

$$\Theta = 2\pi \frac{L}{\lambda}$$

With L being the physical length of the DUT, i.e. typically the part that has discontinuities and therefore non-TEM fields, and $\lambda = c_0/f$ the free-space wavelength, as we are in vacuum or air.

Different equations have been developed to extract Z_{\parallel} from the measured S_{21} :

Lumped impedance formula

$$Z_{\parallel} = 2Z_{pipe} \frac{1 - S_{21}}{S_{21}}$$

is valid for short DUTs, where $\Theta \leq 1$ and $L < D_{pipe}$ is smaller than the beam pipe diameter D_{pipe} . It may give unphysical results for long structures.

Log formula

$$Z_{\parallel} = -2Z_{pipe} \ln S_{21}$$

is versatile and give good results for distributed impedances. It may give less accurate results for lumped impedances.

Improved log formula

$$Z_{\parallel} = -2Z_{pipe} \ln S_{21} \left(1 + j \frac{\ln S_{21}}{2\Theta} \right)$$

is preferred over the “standard” log formula for high frequencies, $\Theta > 1$, and long components. However, it has limits for $\Theta \leq 1$, e.g. notice for a frequency-independent impedance the imaginary part $\ln S_{21}/(2\Theta)$ diverges, which is unphysical.

The transmission coefficient S_{21} has to be evaluated by referencing that of the DUT, $S_{21,DUT}$ to the measurement of a “reference” line $S_{21,REF}$ of same electrical length Θ :

$$S_{21} = \frac{S_{21,DUT}}{S_{21,REF}}$$

For a beam pipe with circular cross-section, the stretched-wire setup is basically a coaxial transmission line with air-dielectric, its impedance Z_{pipe} is calculated with the well-known formula:

$$Z_{pipe} = \frac{\eta_0}{2\pi\sqrt{\epsilon_r}} \ln \frac{D}{d} \cong 60 \ln \frac{D_{pipe}}{d_{wire}}$$

For non-circular beam pipe cross-sections Z_{pipe} needs to be approximated by an electrostatic calculation or by measurement.

For most cases, unfortunately $Z_{pipe} \neq Z_0 = 50 \Omega$. Among the many options for an impedance matching network we here chose for various practical reasons an abrupt beam-pipe to coaxial-cable transition with a simple series matching resistor $R_{match} = Z_{pipe} - Z_0$ to terminate the beam pipe. This termination is improved for high frequencies by adding microwave absorbing foam at the abrupt transition between the beam pipe of impedance Z_{pipe} and the matching resistor, and is part of the impedance matching network.

In this configuration, the VNA is not impedance-matched to the DUT or REF line, which would require a 2-resistor circuit. This however would complicate the practical realization, and is mostly unnecessary, as the reflected power at the DUT or REF line ports is absorbed at the ports of the VNA $Z_{VNA} = 50 \Omega$. Still, adding 10 dB attenuators at the matching resistors help to keep those reflections better under control.

Stretched-wire Measurements

Equipment: VNA (Keysight E5071C or equivalent), Ecal calibration kit, coaxial cables, 10 dB attenuators, stretched-wire impedance test setup.

A pair of simple metal-cylinders can be inserted to modify the test setup between DUT and REF line. Take S_{21} measurements using an appropriate frequency range of the VNA for different depth of the cylinders (but please, don't let them touch the wire!), and compare your results with the different formulas for the longitudinal coupling impedance.

Measurements with Network Analyzer test stand 3

Suggested topics:

- Measurements of the characteristic cavity features (Smith Chart analysis).
- Cavity perturbation measurements (bead pull).
- Perturbation measurements using rectangular wave-guides

Equipment:

VNA (Keysight E5071C), N-type calibration kit, three cavities: two pillbox cavities and one coaxial cavity. The first pillbox cavity (2.7.001) has a diameter of 30 cm. The second one (G.2.7.001) has a diameter of 31.5 cm, and its length is variable. The coaxial resonator (G.2.7.002) is short-circuited at one end, while the capacitive loaded open end is variable at the location of the capacitor plates. This resonator was built to determine the dielectric losses of isolators. You will need in addition a set of different inductive and capacitive couplers and two ~2m long N-type cables.

Pillbox measurements

- ◆ PRESET the instrument. Set the frequency to between 500 MHz and 1.2 GHz. (There is no resonance below 500 MHz and there is no time available here to look for higher modes above 1.2 GHz.)
- ◆ Set the number of points to 1601 (to have enough data points at a high Q-resonance).
- ◆ Choose two small inductive probes with N-connectors, look at how they are constructed.
- ◆ Activate two traces **Display, Number of Traces, 2**
- ◆ Define S_{11} and S_{21} measurements (menu **Measure**)
- ◆ Perform extended response calibration SOL on port 1 and Through for port 2
- ◆ Set the reference position of both channels to 9 vertical units.
- ◆ Connect the cables to the two installed inductive or capacitive probes and determine the frequency of all resonances you can find at S_{21} in the selected frequency range. Display only Trace 2.
- ◆ Calculate the length (actual length for pillbox 2) using the given mode patterns in your handouts and determine the type of the modes. Compare with results obtained using Figs. 4 and 5.
- ◆ Set the marker on the peak of the E_{010} mode (TM_{010}) and adjust it to the centre of the screen (MARKER => CENTER). For pillbox 2 (with variable length), the E_{010} mode is that which does not change its frequency if the length of the cavity is altered.
- ◆ Reduce the SCALE/DIV stepwise to 0.5 dB/DIV and the SPAN to fill the screen with the resonance curve. You may use also **Autoscale** function in the menu.
- ◆ Now use the width function to automatically get the -3 dB points. (**Marker search, Bandwidth ON**, width value -3 dB. You measure the nearly unloaded Q if both of the used probes are so small [noise level] that you get the result only by setting the following values: output power to 20 dBm, IF bandwidth to about 300 Hz, and the averaging on. It is also useful to insert a low-noise amplifier in front of port 2.

- ◆ Write down the centre frequency, 3 dB bandwidth and the measured Q.
- ◆ Now try to reach a critical coupling with a bigger loop or a capacitive pick-up. Use channel 1 with increased bandwidth (as the new loop may shift the resonance frequency). Go to 10 dB/DIV and display S_{11} in the formats LOG MAG and Smith Chart. Adjust a circle that goes through point 1 (50Ω). Turn the circle with the function PHASE OFFSET to the left side of the screen until it is symmetric to the locus of real impedance (de-tuned short position). Measure the loaded and unloaded Q using the markers. The points where the real and imaginary parts are equal give the bandwidth for the unloaded Q. You can find these points in the de-tuned short position looking at the real and imaginary parts of the marker. The loaded Q can be found at the crossing point of the circle with ± 45 degree lines starting at the zero point. This can be easily done on paper but not on the analyzer screen. It helps to know that the loaded points are always (not only at critical coupling) the highest and the lowest points of the circle, if this is brought to the de-tuned short position. For critical coupling, you will find the points 10Ω (or 0.2 when normalized to 50Ω) for the real part and $\pm 20 \Omega$ for the imaginary part.
- ◆ But there is an even easier method for reading out the loaded and unloaded Q, if you turn the circle with the phase offset (do not use the electrical delay as it would deform the circle) to the detuned open position. For critical coupling the circle lies directly on the circle of the Smith Chart, where the (normalized) real part is 1. Therefore, you find the points of the unloaded Q at the crossing to the lines where the (normalized) imaginary part is 1 too ($X = R$) and the loaded Q at the crossing to the lines where the (normalized) imaginary part is 2 ($X = R + 1$). You find more precise description in printouts: “RF Engineering Basic Concepts: The Smith Chart”, p. 15.
- ◆ Make sure that the de-tuned short and de-tuned open positions are well adjusted by checking for symmetry of the maxima of the imaginary parts using appropriate marker functions.
- ◆ Determine the loaded Q also by the -3 dB points of S_{11} for the critical coupling in the format LOG MAG.
- ◆ Determine the loaded Q in transmission by the 3 dB points of S_{21} with one probe in critical coupling and the other very small.
- ◆ Move your coupling loop to over-critical and under-critical coupling and calculate the coupling coefficient from the formula $k = 1/(2/D - 1)$. D is the diameter of the circle; its unit is the radius of the Smith Chart. If $D = 1$, then the coupling is 1 and you have critical coupling with the center frequency point at the center of the Smith Chart. Weakly coupled resonators have small Q-circles, strongly coupled have large ones.

Characterization and Measurements of a simple Pill-Box Cavity

ABSTRACT

This document presents an analysis of a cylindrical cavity, so-called “pill-box”, starting from analytical approximations and basic theory, followed by actual measurements and set-ups to obtain important parameters, such as the cut-off frequencies of the modes, the quality factor and the R/Q figure. Finally, a simulation of the cavity in CST Studio is performed, to compare the results from analytical theory and measurements, and evaluating the precision of the measurements.

Analytical calculations and theory

For the analytical analysis, it is necessary to obtain the dimensions of the cavity, as they define the eigen-modes that propagate, and other important figures of merit. These dimensions, along with an idealized sketch of the cavity structure are shown in Fig. 1 and summarized in Table I.

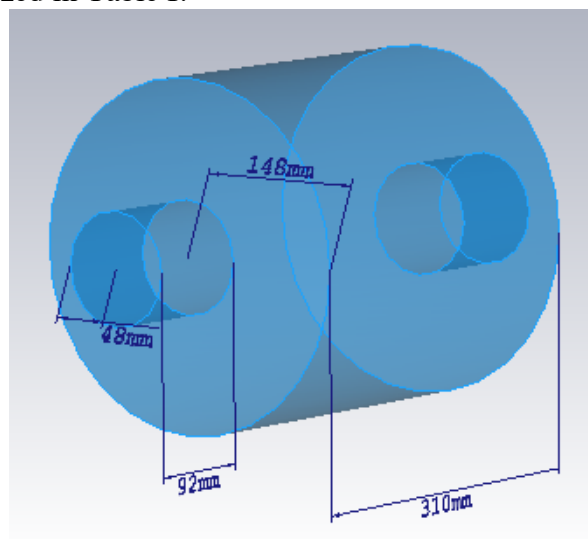


Figure 1. Drawing of the cavity with its dimensions.

Radius (a)	148 mm
Longitude (h)	310 mm
Port radius (a')	48 mm
Port longitude (h')	92 mm

Table I. Dimensions of the cylindrical cavity.

Applying a modal expansion to the boundary problem, the transverse magnetic modes propagate at cut-off frequencies given by eq. (1), while the frequencies of the transverse electric modes follow eq. (2).

$$f_{TM_{nm}l} = \frac{c}{2\pi} \sqrt{\left(\frac{p_{nm}}{a}\right)^2 + \left(\frac{\pi l}{h}\right)^2} \quad (1)$$

$$f_{TE_{nm}} = \frac{c}{2\pi} \sqrt{\left(\frac{p'_{nm}}{a}\right)^2 + \left(\frac{\pi l}{h}\right)^2} \quad (2)$$

where p_{nm} and p'_{nm} are the m roots of Bessel function $J_n(x)$ and the derivative Bessel function $J'_n(x)$, respectively. Table II shows the first two roots of these functions, which are of interest for our cavity, in our case these two lowest modes are the TE_{11} and the TM_{01} . Theoretically, their cut-off frequencies are 766 MHz and 776 MHz, but the exact values may vary due to inaccuracies of the dimensions and geometric variations (like the beam ports).

m	Bessel Function		Derivative Bessel Function	
	$J_0(x)$	$J_1(x)$	$J'_0(x)$	$J'_1(x)$
1	2.4048	3.8317	3.8317	1.8412
2	5.5201	7.0156	7.0157	5.3314

Table II. Roots of Bessel Functions.

The mode that is of interest is the TM_{01} mode, as it is the one which has a E-field along the z-axis that allows to accelerate the charged particles of the beam.

Another important parameter which characterizes the “quality” of the eigenmode, and can be used to compare cavities of same geometry but made out of different materials, is the Q-factor, or quality factor. It is defined by the losses of the cavity compared to the stored energy, and can be calculated by measuring the bandwidth relative to its central frequency, as shown in eq. (3), where f_0 is the resonant frequency of the eigenmode, and Δf is its -3 dB bandwidth. The Q-factor depends on the materials of the cavity walls (conductive losses), and on losses due to fillings or open boundaries (dielectric and radiation losses), and may be impacted by an external network, like in our case, where a measurement setup is attached. The *unloaded* Q , Q_0 , considers only out of the intrinsic losses, while the *loaded* Q , Q_L , takes into account also the external loading effects, and it is the one that is actually measured. They are related as shown in eq. (4), where Q_{ext} is the Q factor due to the external network only.

$$Q = \omega_0 \frac{\text{average energy stored}}{\text{energy loss/second}} = \frac{\omega_0 U}{P_d} = \frac{f_0}{\Delta f} \quad (3)$$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} \quad (4)$$

For a cylindrical cavity, Q_0 can be analytically calculated following eq. (5) using the skin depth, δ , defined in eq. (6), where ω_0 is the angular resonant frequency, σ is the conductivity and μ is the magnetic permeability. For the cavity under test, considering a conductivity of stainless steel of $1.3 \cdot 10^6$ S/m and a permeability of $4\pi \cdot 10^{-7}$ H/m (for air and vacuum), the analytical Q_0 factor is equal to **6321.8**.

$$Q_0 = \frac{a}{\delta} \left[1 + \frac{a}{h}\right]^{-1} \quad (5)$$

$$\delta = \sqrt{\frac{2}{\omega_0 \sigma \mu}} \quad (6)$$

The next important figure of merit is the R/Q . It describes how efficient the cavity transfers its stored energy to the beam [3], and it is defined as ratio between the

shunt impedance, R , and the Q_0 factor, as defined in eq. (7), being V_{acc} the accelerating voltage and U the stored energy.

$$\frac{R}{Q} = \frac{V_{acc}^2/P_d}{2\omega_0 U/P_d} = \frac{V_{acc}^2}{2\omega_0 U} \quad (7)$$

Analytically, for cylindrical cavities and the mode TM_{010} , the value of R/Q can be determined using the formula in eq. (8), where η is the characteristic impedance of air ($120\pi \Omega$), and χ_{01} is the first root of the Bessel function of 0th order. For the given dimensions of our cavity $R/Q = 10.4 \Omega$.

$$\frac{R}{Q} = \frac{4\eta}{\chi_{01}^3 \pi J_1^2(\chi_{01})} \frac{\sin^2\left(\frac{\chi_{01} h}{2a}\right)}{\frac{h}{a}} = 128 \cdot \frac{\sin^2\left(1.2024 \frac{h}{a}\right)}{\frac{h}{a}} \quad (8)$$

A summary of the discussed parameter values for our cavity structure is shown in Table III. It has to be noted that these values are a first approach, they depend on the exact geometry of the cavity and vary even for small deviations of the assumed dimensions.

Parameter	Value
f_{TE111}	766 MHz
f_{TM010}	776 MHz
Q_0	6321.8
R/Q	10.4 Ω

Table III. Analytical values of the most important cavity parameters.

Measurements with the Vector Network Analyser

To measure the aforementioned parameters and figures of merit, a vector network analyser (VNA) is used. A VNA allows the user to measure the S parameters of a device under test (DUT). It is of critical importance to perform a calibration for the parameter set which best suits the measurement problem.

The frequency range for the VNA measurement has to be set though the *Stimulus* panel (see Fig. 2), using the start and stop frequencies, or by setting a center frequency and a frequency span. Additionally, a sufficient number of points should be specified (Sweep Range \rightarrow Number of Points) to obtain better results. Once these parameters are given appropriately, a VNA calibration can has to be performed. For this purpose, the cables that are going to be used have to be connected to the electronic calibration device (see Fig. 3). Then press Cal \rightarrow E-Cal and select the number of ports required, using the correct assignment of VNA and E-Cal ports.



Figure 2. Button panel of the VNA.



Figure 3. Electronic calibration device (left) and two probes of the VNA (right).

Following the calibration, the DUT can be attached to the coaxial cables, and one or more S-parameter measurements (S_{11} , S_{21} ,..., under the *Meas* button) can be selected. The measurement format can be selected under the button *Format*, these include the logarithmic magnitude, phase, Smith chart, and real and imaginary parts, among others.

With the *Scale* button the size of the grid and amplitude range can be chosen. Also, a phase shift or electrical delay can be taken into account (useful when in Phase or Smith chart format). The *Display* button allows the user to select traces and the number of displays in the screen.

Another helpful feature is the *Marker* and *Marker Search*. With the first one several markers can be added to the display, while the second locates the markers in certain points of the signal (minimum, maximum) and allows tracking of points.

Lastly, it is possible to save the traces in the standard format *SnP* (S1P for one port and S2P for two ports measurements) with the button *Save/Recall* to post processing.

Measurement of the cavity eigen-modes

In order to measure the first modes of the cavity, and in particular to locate the TM_{010} mode, a magnetic loop is attached to one of the measurement ports of the cavity (see Fig. 4). Setting a start frequency of 760 MHz and a stop frequency of 780 MHz, the S_{11} measurement obtained is shown in Fig. 5.



Figure 4. Different magnetic (three on the left) and electric probes.

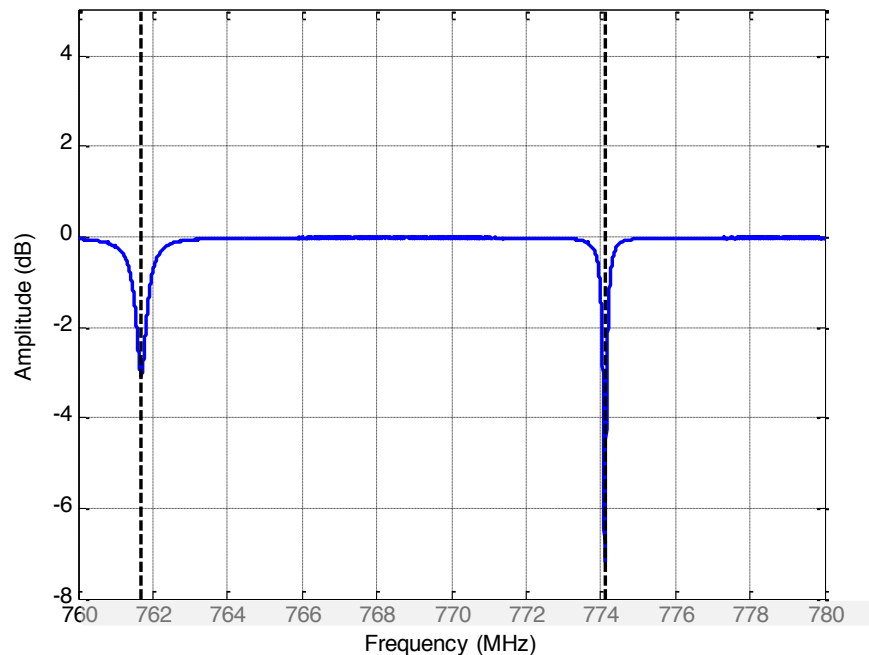


Figure 5. S_{11} measurement of the cavity.

The cut-off frequencies of these modes are close to the analytical calculations, but a further investigation needs to be performed to determine which mode corresponds

to each resonance. There are two simple ways of doing this: analyzing the reflected signal (S_{11}), or the transmitted signal (S_{21}).

a) Reflected signal

If an electric probe is introduced longitudinally in one of the beam ports along the symmetry axis of the cavity, the S_{11} measurement of the first resonance stays unaltered while the second one shifts slightly in frequency. This means, the second resonance couples to the electric probe, thus it has a longitudinal electric field component (TM mode), while the first one doesn't (TE mode).

b) Transmitted signal

On the other hand, a measurement with two magnetic probes can be performed to measure the transmitted signal. In this case, as Fig. 6 shows, the first mode disappears, while the 2nd TM mode preserves as its magnetic field couples to the two loop probes.

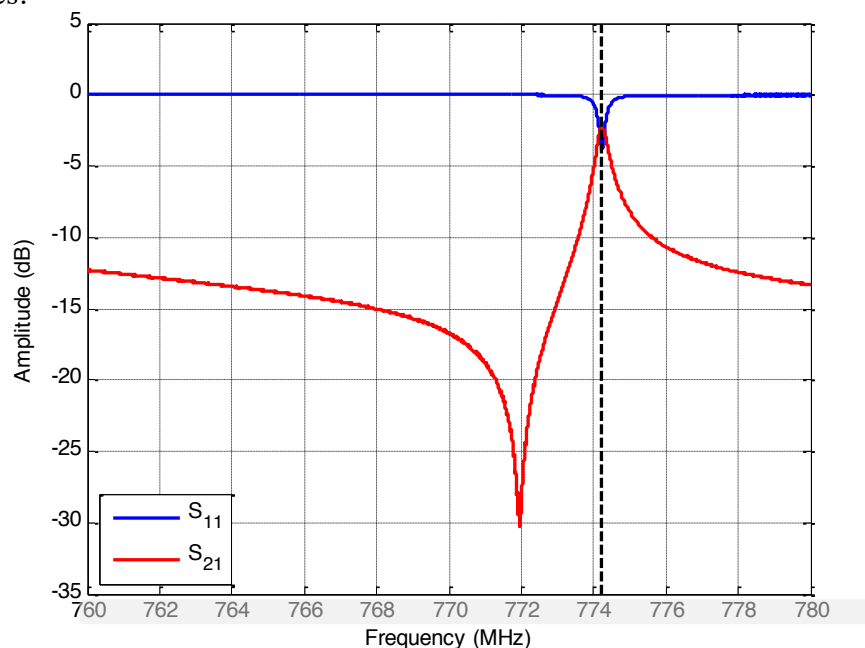


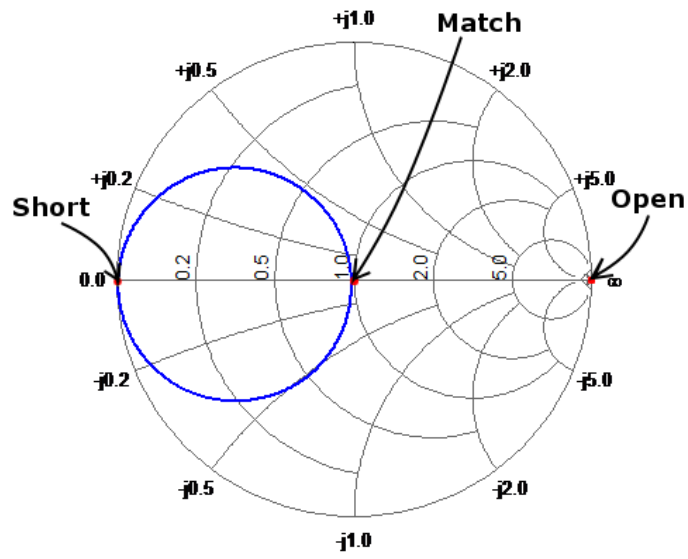
Figure 6. S_{11} and S_{21} measurement with two probes.

Using the Markers of the VNA or processing the traces, the frequencies of the cavity's modes can be determined. For the first mode, the TE_{111} , the cut-off frequency is 761.7 MHz and the frequency of TM_{010} is 774.1 MHz.

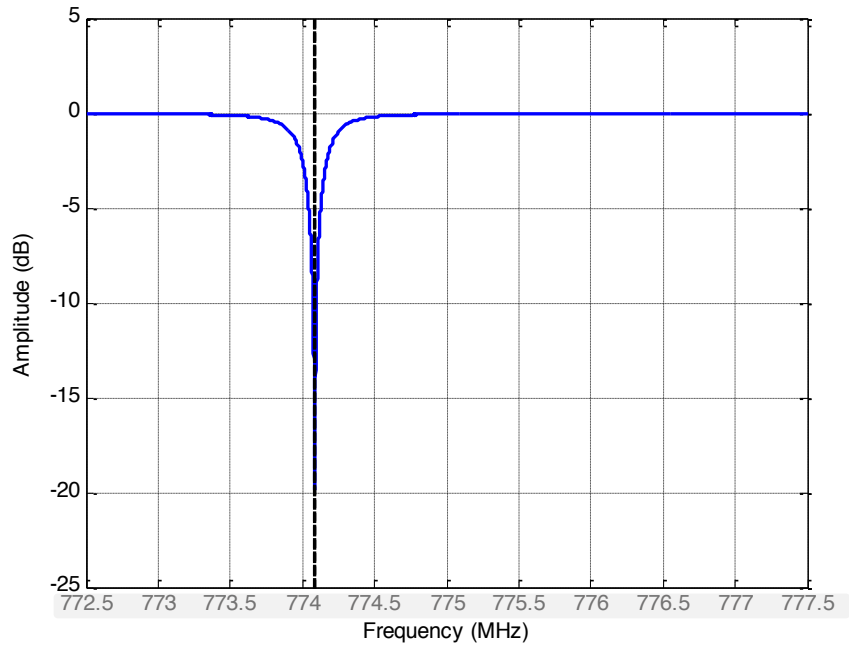
Measurement of the Q factor

Hereafter, only the mode TM_{010} is considered. To obtain the value of the unloaded Q_0 , it is necessary to take the effects of the measuring network into account, namely Q_{ext} . A solution to this problem is to reach *critical coupling*. In this state, probe and cavity are matched, and the power transfer is maximum. Additionally, the value of Q_{ext} is the same as the Q_0 . From the eq. (4) can be concluded that now the loaded Q_L (the one that is measured) is twice the unloaded Q_0 .

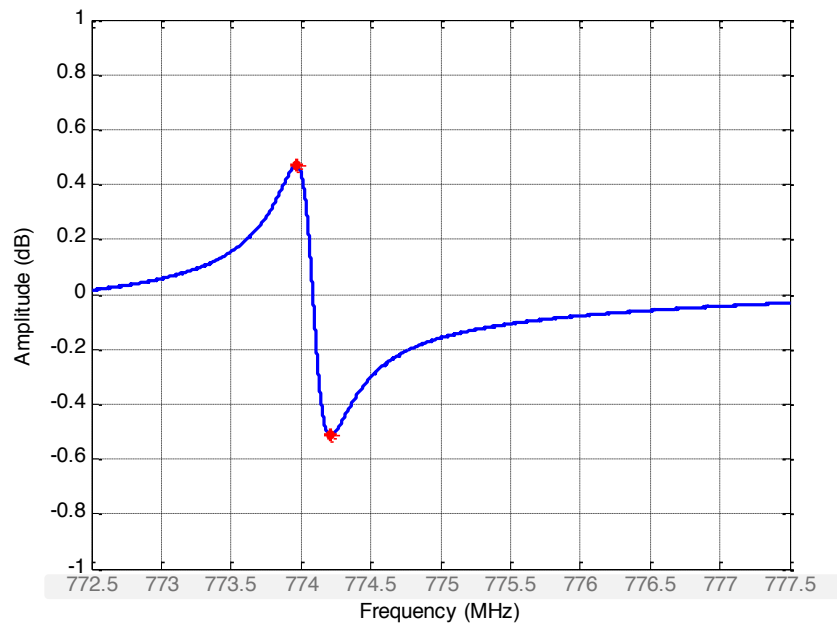
A critical coupling measurement can be easily performed using the Smith chart as display format. There are three important points (Fig. 7(a)) in the chart: the **short** (where the equivalent circuit is a short and reflexion coefficient equals -1), the **open** (where the equivalent circuit is an open and reflexion coefficient equals 1), and the **match** position (where the equivalent circuit is a matching load and the reflexion coefficient equals 0).



(a)



(b)



(c)

Figure 7. S_{11} of critical coupling in the Smith chart (a), logarithmic magnitude (b) and imaginary part (c).

This is related to the equivalent circuit that is considered for the resonant cavity: parallel or series (Fig. 8). In our case, the parallel circuit is used, and therefore, at direct current (frequency zero) the circuit acts like a short (due to the inductance). Then, to correctly display the critical coupling on the VNA, is necessary to add a phase shift until the trace is in the detuned short position to compensate the uncalibrated portion of the loop antenna. By rotation of the magnetic probe the coupling changes, and the match point for critical coupling at the resonant frequency can be tuned, as shown in Fig. 7 (a).

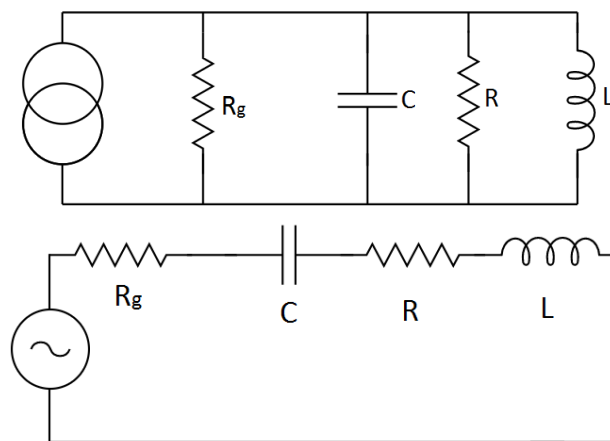


Figure 8. Equivalent parallel circuit (top) and series circuit (bottom).

Once in critical coupling, the resonance frequency can be obtained, as shown in Fig. 7 (b). The bandwidth can be calculated by taking the frequencies crossing the 0.5 circle in the Smith chart, at the points with -3 dB amplitude in the logarithmic format or at the maximum and minimum of the imaginary part (Fig. 7 (c)).

After the characterization of the resonance frequency and the bandwidth, the loaded Q can be calculated, and thus, the unloaded Q is twice that value, which is close to the analytical calculation. The results are shown in Table IV.

f_0	774.085 MHz
f_1	773.964 MHz
f_2	774.214 MHz
BW	250.5 kHz
Q_L	3090.2
Q_0	6180.3

Table IV. Analytical values of the most important parameters.

Measurement of the R/Q by pulling a perturbation bead

As already mentioned, the R/Q parameter defines how much accelerating voltage can be achieved, and depends only from the geometry of the cavity. This voltage is given by the integrated electric field, eq. (9), here the E_z -field component along the z symmetry axis of the cavity.

$$V_{acc} = \left| \int E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz \right| \quad (9)$$

To measure the electric field along the propagation axis, z , a *perturbation method* is used. When a small object is introduced in a cavity, the resonance frequency varies and the electric field can be calculated as is described from *Slater's* theorem in eq. (10):

$$\frac{f - f_0}{f_0} = \frac{1}{U} \left(-K_1 \varepsilon_0 \widehat{E}_{\parallel}^2 - K_2 \varepsilon_0 \widehat{E}_{\perp}^2 - K_3 \mu_0 \widehat{H}_{\parallel}^2 - K_4 \mu_0 \widehat{H}_{\perp}^2 \right) \quad (10)$$

$$E(z) = \sqrt{\frac{f(z) - f_0}{f_0} \cdot \frac{-1}{K_1 \varepsilon_0}} \quad (11)$$

Where K_1 , K_2 , K_3 and K_4 are coefficients proportional to the electric and magnetic polarizability of the object. For our analysis, an elongated object is preferred, such that the dominant coupling field the longitudinal electric field component. For that reason, K_1 is larger than the other coefficients, which can be neglected, see eq. (11). The coefficient of interest depends on the geometry of the perturbing object, in this case a needle, and can be evaluated following eq. (12), where the electric field is normalized by the root of the stored energy, U .

$$K_1 = \frac{\pi}{3} l^3 \left[\sinh^{-1} \left(\frac{2}{3\pi} \frac{l}{a} \right) \right]^{-1} \quad (12)$$

It should be noted that here l refers to half the length of the object! The needle used for our measurements had a length of 10 mm ($l = 5$ mm) and a radius $a = 0.59$ mm. With these dimensions follows: $K_1 = 9.65 \cdot 10^{-8}$.

To measure the frequency shift, the object is pulled through the cavity by utilizing a thin, non-metallic wire. The set-up schematics is shown in Fig. 9, and the practical set-up in Fig. 10. Additionally, metal flanges are used to close in the cavity ports, which present a better-defined boundary.

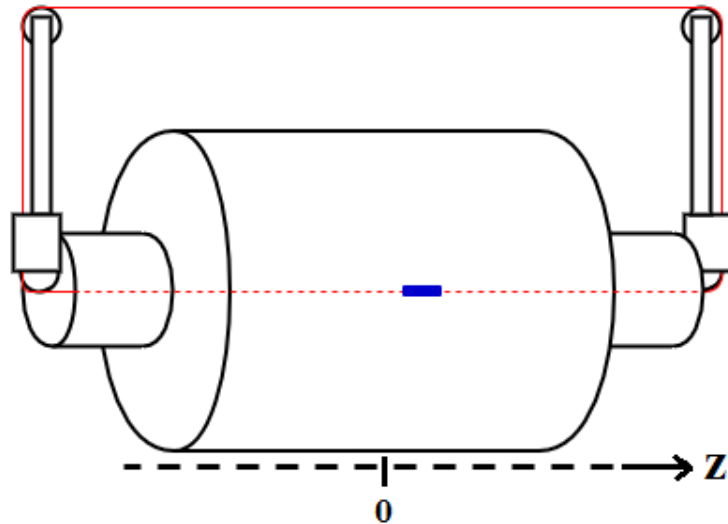


Figure 9. Set-up of the measurement, with the wire in red and the needle in blue.

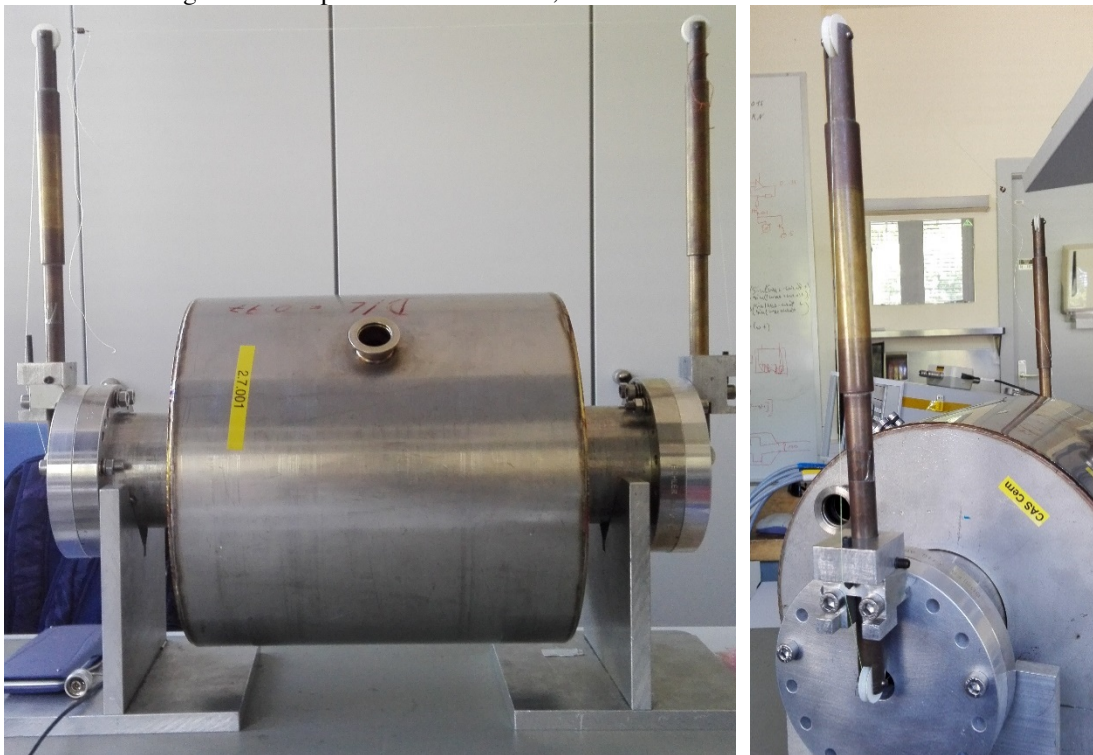


Figure 10. Real set-up of the measurement. In the right, detail of the structure used to move the wire.

There are two methods to measure the frequency shift:
Direct, or using the phase shift.

a) Direct measurement using a S11 reflection coefficient

Using one magnetic probe, first the resonant frequency has to be measured without the perturbing object. Then, the object is pulled through the cavity, and the frequency shift is monitored, e.g. in steps of 1 cm. Because of the symmetry of the cavity, it is sufficient to probe only half of the cavity. The results are shown in Fig. 11. The integration of the electrical field follows eq. (9), a value of 15.5 for R/Q is obtained.

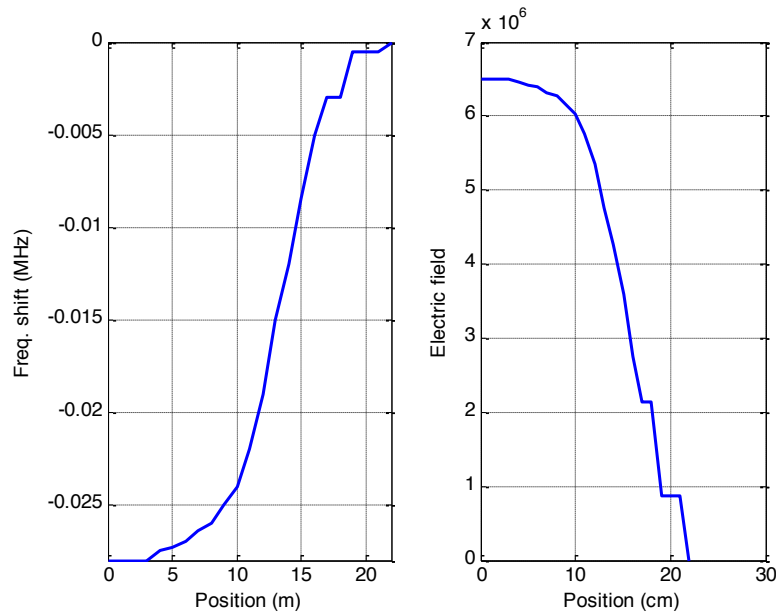


Figure 11. Measured frequency shift (left) and calculated electric field (right).

b) Phase shift measurement based on the S21 transmission coefficient

The second method is indirect, as the frequency shift is obtained through the phase of a transmitted signal at the resonance frequency. Therefore, two magnetic probes are used. It is of utmost importance to precisely excite the resonance frequency, as all following measurement steps depend on it. Additionally, both probes should be tuned for a very weak coupling, around -0.5 dB peak amplitude in S_{11} and S_{22} .

Once the exact value of the TM_{010} resonance frequency is determined, the center frequency of the VNA is set to f_0 , and a zero-span frequency (0 Hz) is configured to excite and analyze only at that particular frequency. With the needle out of the cavity, the reference phase of S_{21} is measured and, if necessary, a phase shift is added to set it to 0 degrees. Similar to the direct phase measurement method, the needle is pulled through the cavity, now monitoring the phase in steps of 1 cm. The results are shown in Fig. 12, in this case an R/Q of 16.5 is obtained, similar to the previous method.

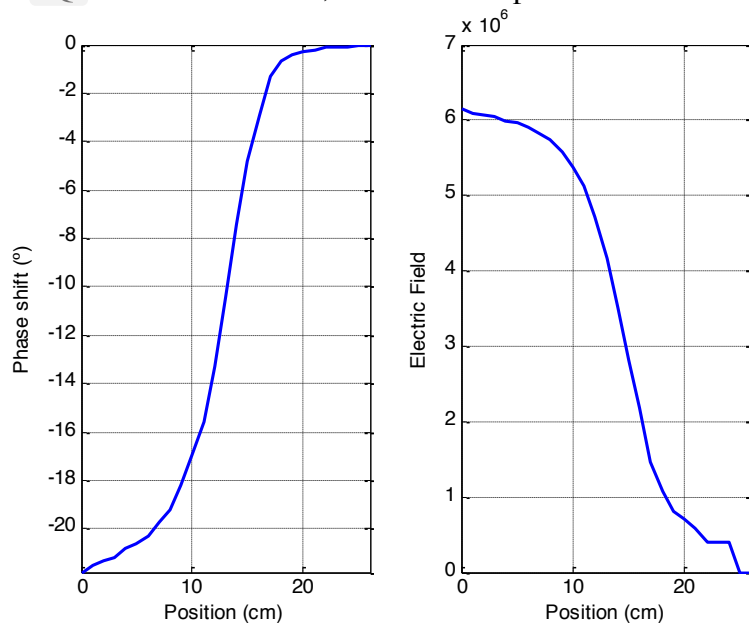


Figure 12. Measured phase shift (left) and calculated electric field (right).

It has been observed that phase drifts over time, most likely due to temperature variations, will alter the measurement result. To minimize this influence, the bead-pull phase measurement has to be performed quickly. A simple help consists in the attachment of a ruler, such that the centimetre steps can be determined faster, as shown in Fig. 13.



Figure 13. Ruler attached to the set-up.

Mode charts for cylindrical resonators

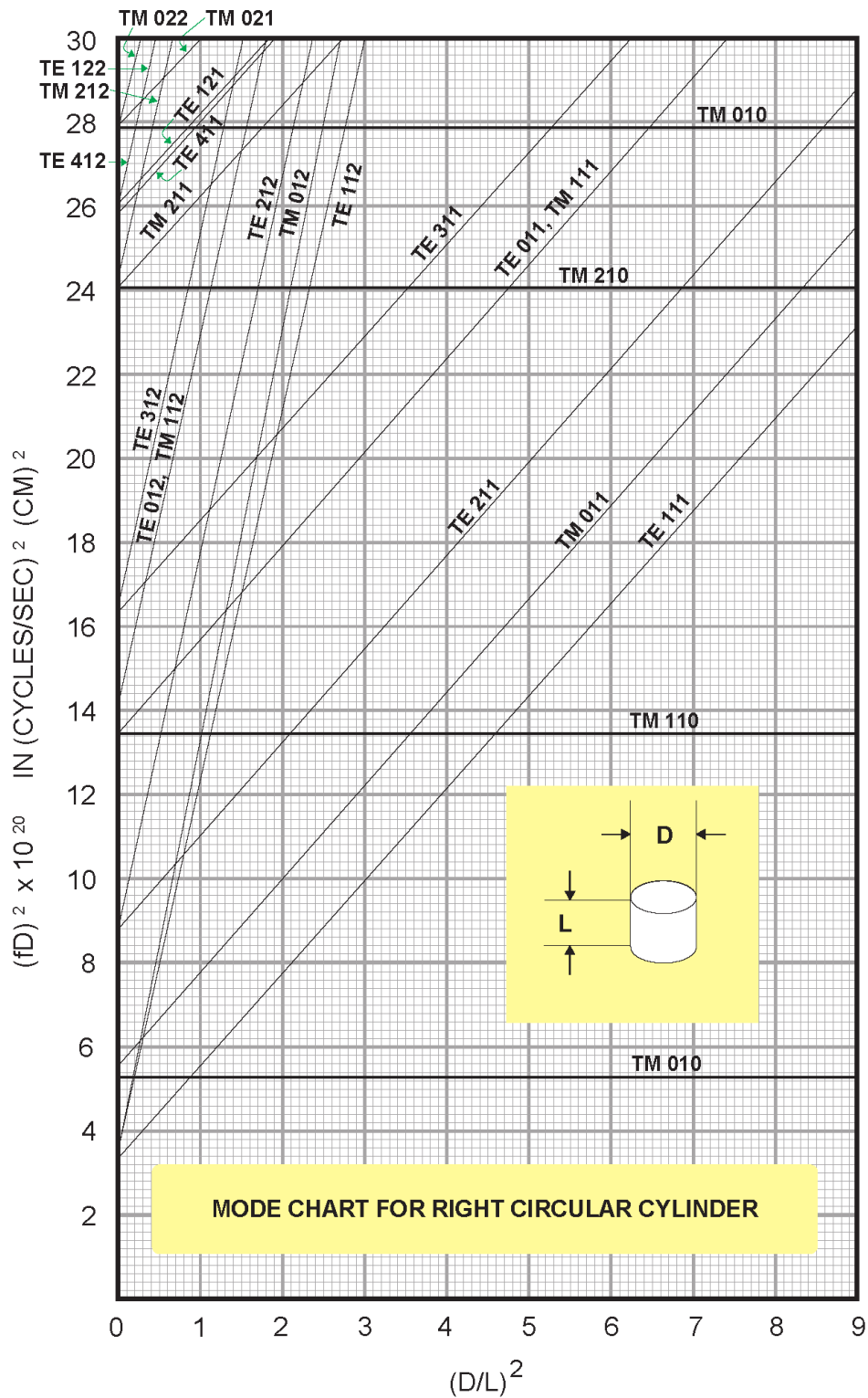


Fig. 4: Mode chart for a pillbox-type cavity
(reprinted from *Microwave Engineer's Handbook, Vol. 1*).

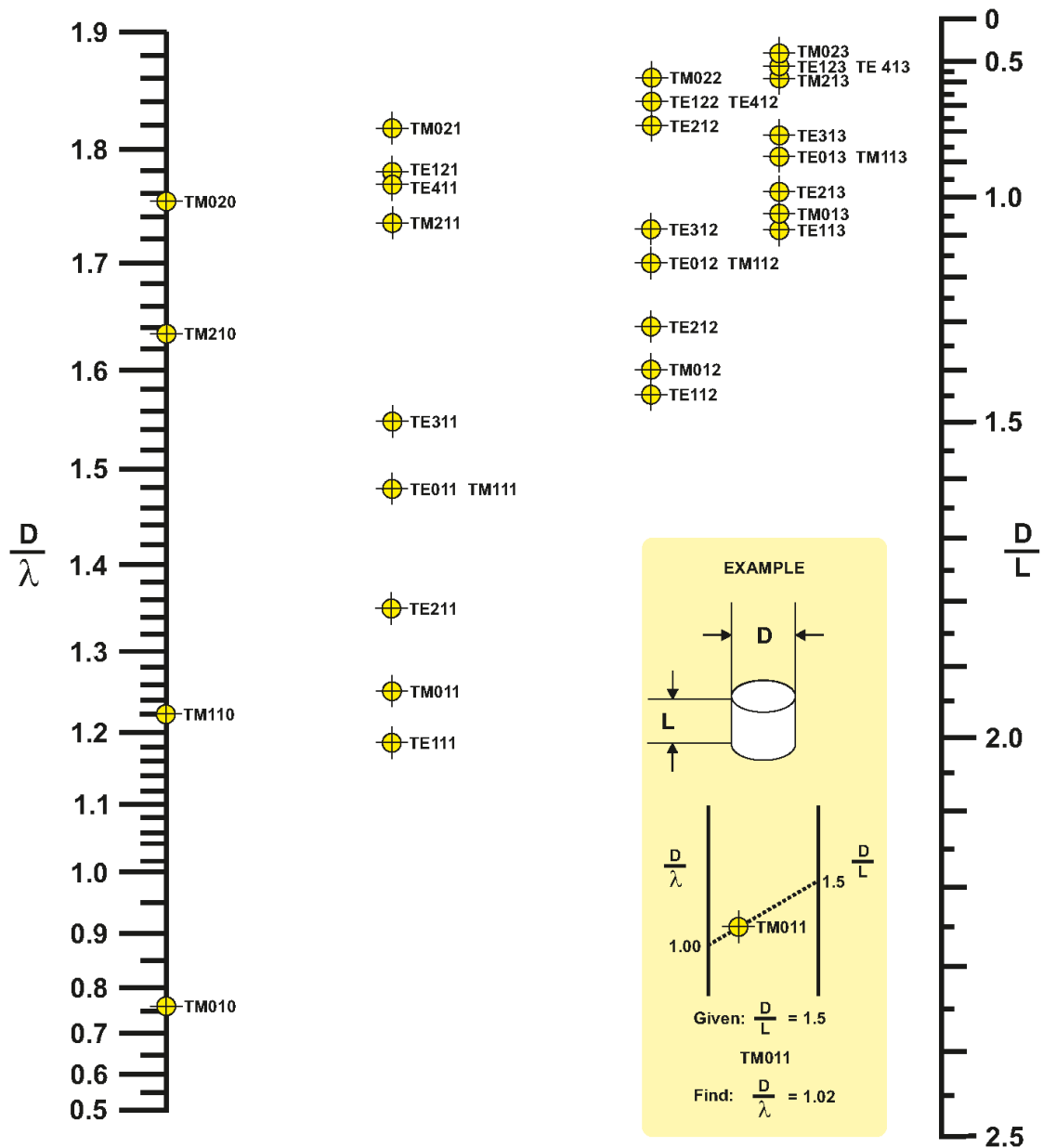
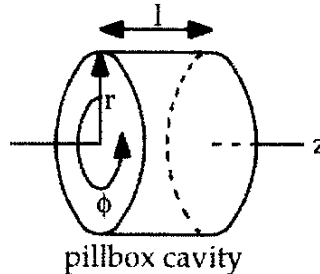


Fig. 5: Mode lattice for cylindrical resonators (reprinted from R.N. Bracewell, 'Charts for Resonant Frequencies of Cavities', Proc. IRE, August 1947).

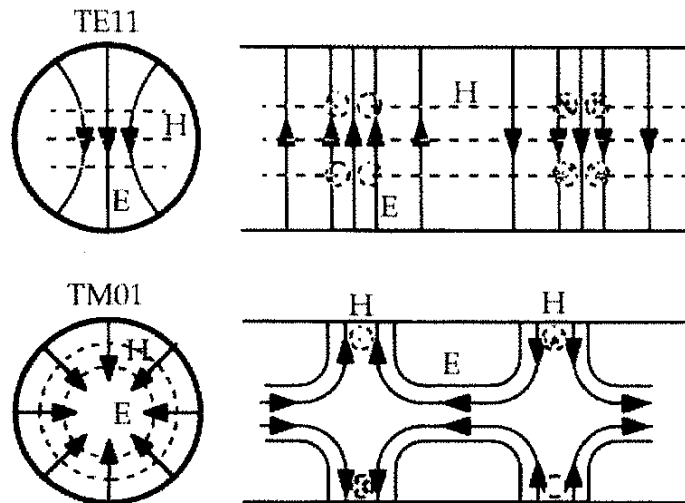
Additional background material: The script of the USPAS Microwave Measurements Laboratory by *J. Byrd* and *R. Rimmer*, reprinted below:

"Pillbox" cavity modes:

The "pillbox" is a simple closed shape for which analytical solutions can be derived for the field and current distributions of the resonant modes. Such a shape could in fact be used as an accelerating structure, however more efficient shapes are usually used in practice. Study of the modes of the pillbox is instructive however and provides much of the nomenclature that is used to describe modes in other axis-symmetric structures.

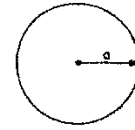
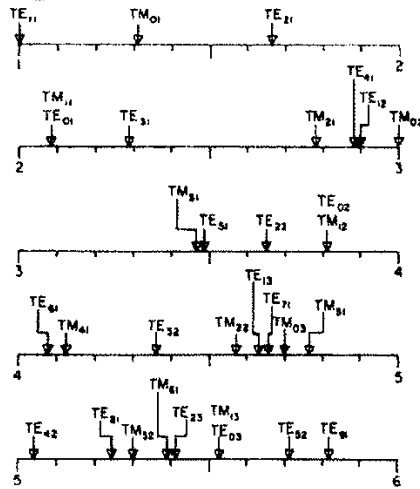


As presented above in the transmission-line analogy, cavity modes can be thought of as resonances between two short circuit planes in a waveguide. In the case of the pillbox this is a length of circular waveguide with a short-circuit boundary condition at each end, so the solutions are standing-waves of the TE and TM circular waveguide modes with an integer number of half-wavelengths between the end-plates. The boundary conditions also allow for TM modes with zero variation in the z axis, which are of particular interest for accelerator cavities. The waveguide modes (TE/ M_{mn}) are denoted by two subscripts, the first is the number of full periods in ϕ and the second is the number of radial zeros in the field. For cavity modes a third subscript is added which is the number of half-period variations in the z direction.



First two modes in circular waveguide

The following chart shows the cut-off frequencies for modes in circular waveguide, normalized to that of the lowest mode (TE₁₁).

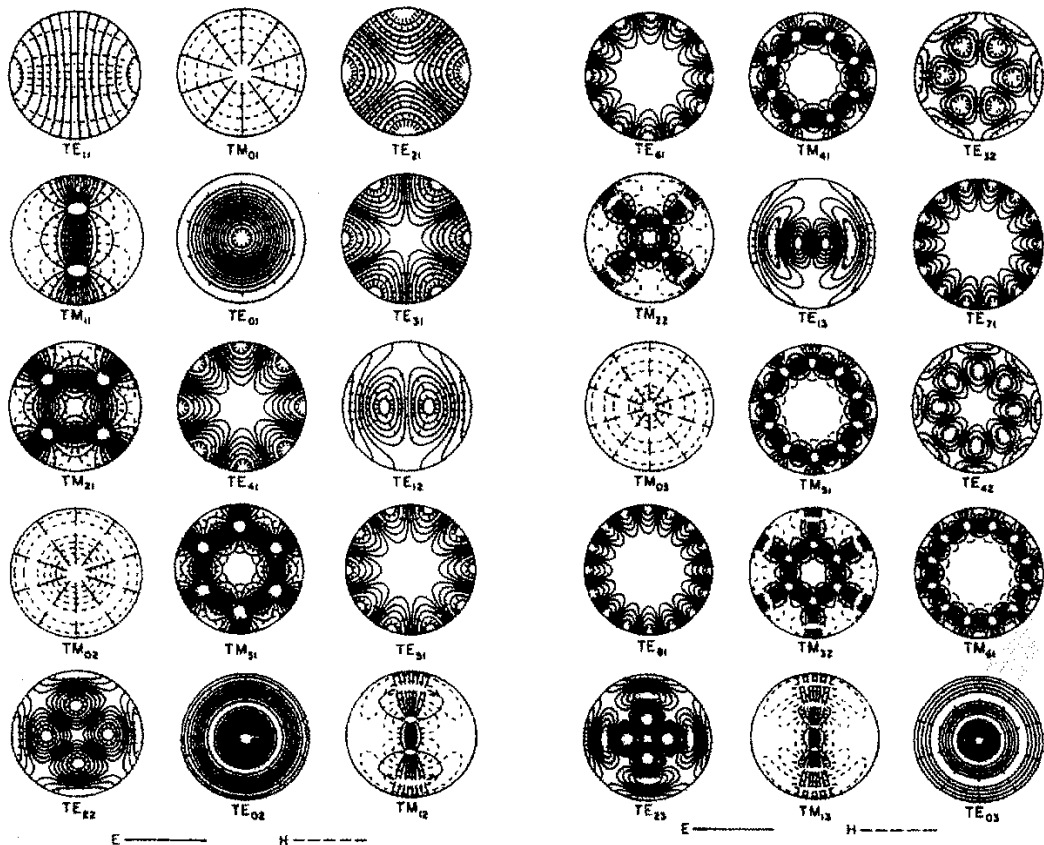


$$kc_{TE11} = \frac{1.841}{a}$$

$$\lambda c_{TE11} = 3.41a$$

$$f_{cTE11} = \frac{0.293}{a\sqrt{\mu\epsilon}}$$

The figures below show plots of the E and H fields for the first thirty modes [Lee et al., IEEE Trans. MTT, vol. MTT-33, No. 3, March 1985, p 274].



Only those modes with a component of electric field in the direction of motion of the particle can interact with the beam (Panofsky-Wenzel). For the pillbox this means only the TM modes are of interest. The transverse variations of the longitudinal field are solutions of Maxwell's equations within a circular boundary condition and are Bessel functions of the first kind.

$$E_z(r, \phi) = E_0 J_m(k_{mn} r) \cos m\phi$$

where: J_m are the first order Bessel functions
 $k_{mn} = x_{mn}/r$ is the transverse wave number
 x_{mn} are the roots of the Bessel functions J_m

For modes with $E_z(z) = \text{constant}$ ($k_z = 0$), $\omega = ck_{mn}$

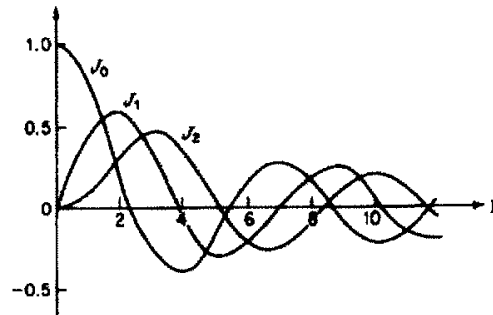
For modes with $E_z(z) \propto \cos(k_z z)$, where k_z is the axial wave number:

$$k_0^2 = k_{mn}^2 + k_z^2 \quad \text{or} \quad \omega_0 = c\sqrt{k_{mn}^2 + k_z^2}$$

For TM_{mnz} modes the fields are thus:

$$E_z(r, z, t, \phi) = E_0 J_m\left(\frac{x_{mn} r}{a}\right) e^{j\omega t} \cos(m\phi) \cos(k_z z)$$

$$H_\phi(r, z, t, \phi) = H_0 J_m'\left(\frac{x_{mn} r}{a}\right) e^{j\omega t} \cos(m\phi) \cos(k_z z)$$



Low-order Bessel functions of the first kind

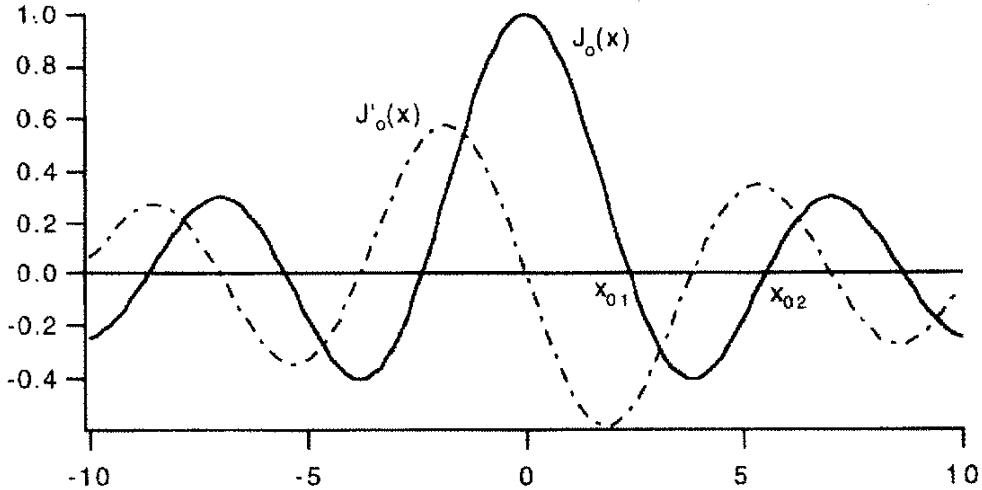
ZEROS AND ASSOCIATED VALUES OF BESSEL FUNCTIONS AND THEIR DERIVATIVES

n	$j_{0,n}$	$J_0'(j_{0,n})$	$j_{1,n}$	$J_1'(j_{1,n})$	$j_{2,n}$	$J_2'(j_{2,n})$
1	2.40482 55577	-0.51914 74973	3.83171	-0.40276	5.13562	-0.33967
2	5.52007 81103	+0.34026 48065	7.01559	+0.30012	8.41724	+0.27138
3	8.65372 79129	-0.27145 22999	10.17347	-0.24970	11.61984	-0.23248
4	11.79153 44391	+0.23245 98314	13.32369	+0.21836	14.79595	+0.20654
5	14.93091 77086	-0.20654 64331	16.47063	-0.19647	17.95982	-0.18773

n	$j_{3,n}$	$J_3'(j_{3,n})$	$j_{4,n}$	$J_4'(j_{4,n})$	$j_{5,n}$	$J_5'(j_{5,n})$
1	6.38016	-0.29827	7.58834	-0.26836	8.77148	-0.24543
2	9.76102	+0.24942	11.06471	+0.23188	12.33860	+0.21743
3	13.01520	-0.21828	14.37254	-0.20636	15.70017	-0.19615
4	16.22347	+0.19644	17.61597	+0.18766	18.98013	+0.17993
5	19.40942	-0.18005	20.82693	-0.17323	22.21780	-0.16712

Monopole modes (m=0):

Modes which have no azimuthal variation are labelled "monopole" modes and TM modes of this type have longitudinal electric field on axis and thus can interact strongly with the beam. The radial distribution of E_z follows J_0 , where the zeros satisfy the boundary condition that $E_z = 0$ at the conducting wall at radius a . Similarly H_ϕ and E_r (if present) follow J'_0 and are zero in the center and have a finite value at the wall.



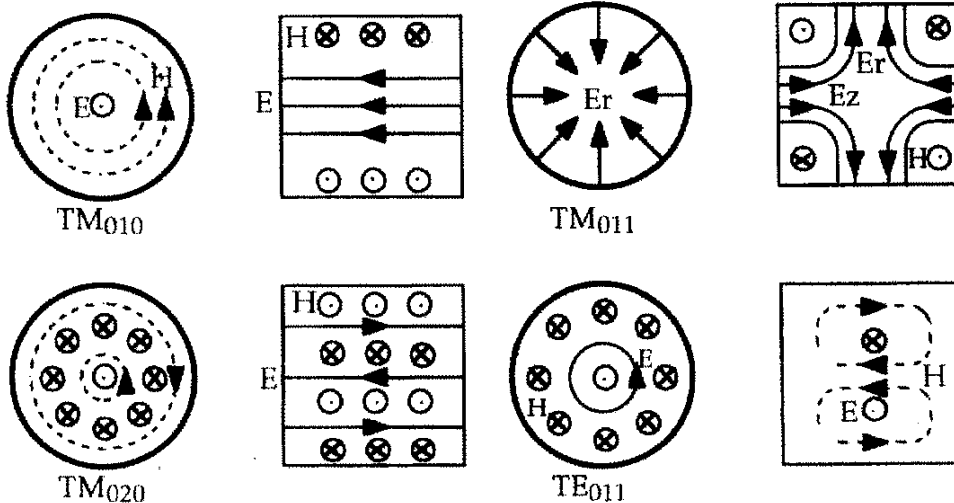
For TM_{0ni} modes:

$$E_z = E_0 J_0(k_{0n}r) \cos(k_z z) \text{ where } k_{0n} = x_{0n}/a \text{ and } k_z = i\pi/\text{length} \ (i \geq 0)$$

$$H_\phi = H_{\phi 0} J'_0(k_{0n}r) \cos(k_z z) \quad x_{01} = 2.405$$

$$E_r = E_{r0} J'_0(k_{0n}r) \sin(k_z z) \quad x_{02} = 5.520$$

$$x_{03} = 8.654$$



Dipole modes (m=1):

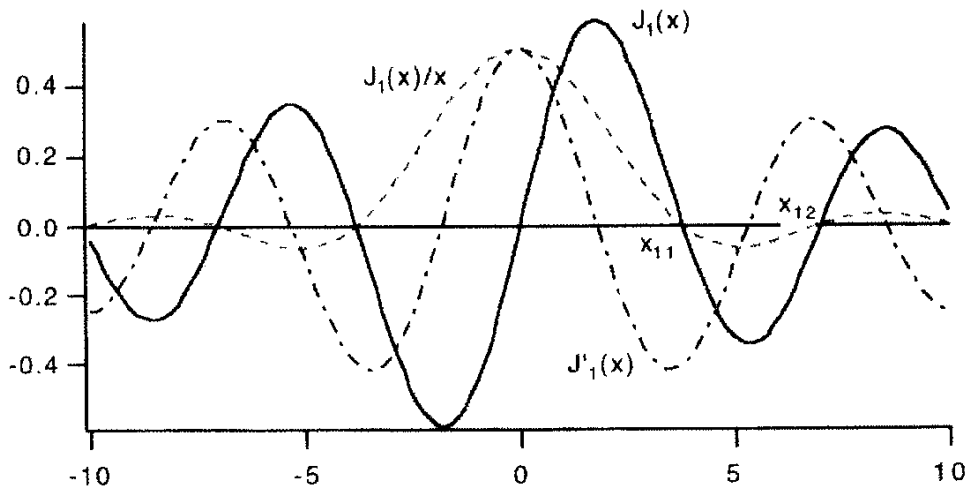
Dipole modes have one full period of variation around the azimuth. For TM modes this means there is no longitudinal field on axis and that the field strength grows linearly with radius close to the center, with opposite sign either side of the axis. This transverse gradient to the longitudinal field gives rise to a transverse voltage kick which is proportional to the beam current and the beam offset. This can be expressed through a **transverse impedance** Z_{\perp} :

$$Z_{\perp}[\Omega m^{-1}] = j \frac{-V_x}{I_b x_0}$$

where $I_b(0)x_0$ is the dipole moment of the beam. It can be shown that Z_{\perp} is related to $Z_{||}$ by

$$Z_{\perp}[\Omega m^{-1}] = \frac{Z_{||}(r)}{kr^2}$$

where $Z_{||}(r)$ is the longitudinal impedance evaluated at radius r



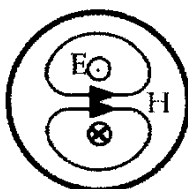
For TM_{1n} modes:

$$E_z = E_0 J_1(k_{1n} r) \cos(\phi) \cos(k_z z) \text{ where } k_{1n} = x_{1n}/a \text{ and } k_z = i\pi/\text{length} \text{ (} i \geq 0 \text{)}$$

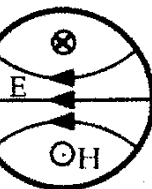
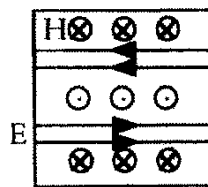
$$H_{\phi} = H_{\phi 0} J_1(k_{1n} r) \cos(\phi) \cos(k_z z) \quad x_{11} = 3.383171$$

$$|H_r| = H_{r0} \frac{J_1'(k_{1n} r)}{r} \sin(\phi) \cos(k_z z) \quad x_{12} = 7.01559$$

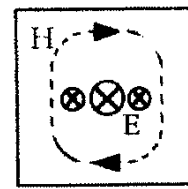
$$x_{13} = 10.17347$$



TM₁₁₀



TE₁₁₁

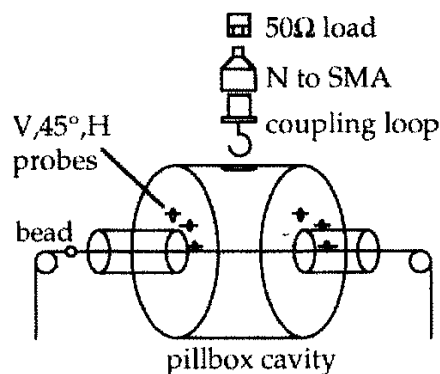


Introduction

The experiment is designed to introduce the student to the concept of cavity modes by studying the mode spectrum of a simple model cavity using a network analyzer (NWA). The first few modes are identified by frequency and their Q 's measured by a transmission method. For the TM_{010} mode, which would be the accelerating mode in a real cavity, the coupling through a drive loop is determined by means of a reflection measurement and the loaded and unloaded Q 's are calculated. The loading of the higher-order modes (HOMs) is also observed and their Q reduction is measured. Finally The longitudinal field profile of the TM_{010} mode is measured using a perturbation method and the shunt impedance is calculated.

Equipment:

Aluminum model pillbox cavity
Network Analyzer + calibration kit
2 cables
2 N-type to SMA adapters
2 50Ω SMA loads
Coupling loop
E-Field probe
1 SMA F-F connector (for Thru calibration)
Bead-pull apparatus



Experiment:

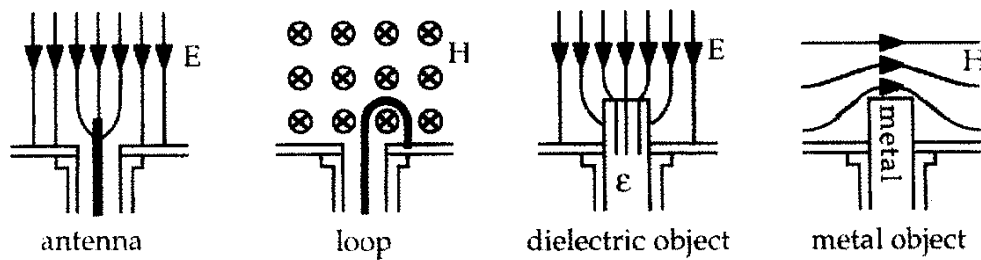
Field mapping using bead pull

- Remove the coupler and leave the NWA connected for transmission measurement. Assemble the bead-pull apparatus around the cavity with the thread running along the central axis and the bead just outside the beam-pipe. Set the frequency span so that the peak of the TM_{010} mode is on the right of the display and moves to the left of the display as the bead is pulled to the center of the cavity. This will give acceptable resolution of the frequency shift without having to adjust the display during the measurement. Measure the frequency and Q_0 with the bead outside the cavity and then proceed to pull the bead through the cavity and make frequency measurements at 1 cm intervals. Tabulate and plot $\delta f/f$ vs axial position and calculate the beam impedance $Z_{||}$. It is most efficient to use a spreadsheet to calculate the impedance.
- If time permits, repeat for the dipole TM_{110} mode with the alumina bead offset close to the wall of the beam pipe at a known radius r . Calculate $Z_{||}(r)$ and Z_{\perp} .

Background material

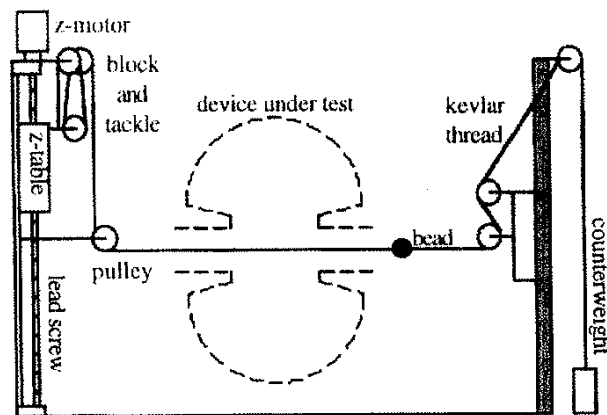
Field measurement:

Information about the field distribution and mode orientation can be obtained by observing the coupling to E and H field components at various places in the cavity. This can be done using E-field antennas or H-field loops or by introducing perturbing objects of dielectric, ferrite or metal.



Introduction of a dielectric object in a region of electric field produces a negative shift in the resonant frequency while introducing a metal object into a region of magnetic field causes a positive frequency shift. If both fields are present when a metal object is inserted the resulting frequency shift will depend on the relative strengths of the E and H fields.

Small objects pulled through the cavity on a string can be used to map the field distributions of the modes and determine the beam impedances.



schematic of a motorized bead-puller apparatus

Perturbation measurement:

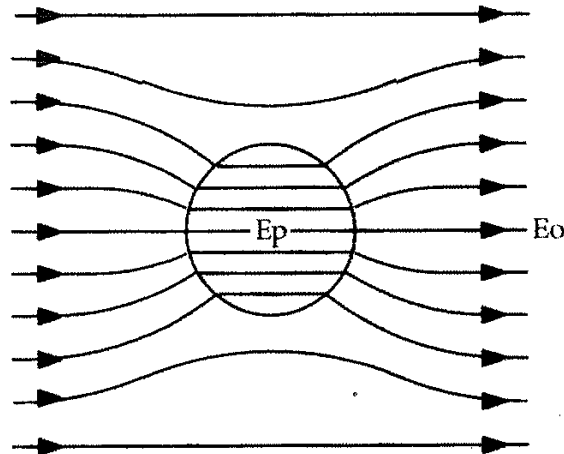
It has been shown (by Slater and others), that the change in resonant frequency upon introducing an object into the cavity field is proportional to the relative change in stored energy:

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US/CERN/Japan RF School, Sept. 1996

J. Byrd and R.A. Rimmer

$$\frac{\Delta\omega}{\omega} = \frac{\Delta U_E - \Delta U_M}{U}$$



$$E_p = \frac{3E_0}{\epsilon_r + 2}$$

perturbation of a uniform E-field by a dielectric bead

For the case of a small non-conducting sphere, radius r , where the unperturbed field may be considered uniform over a region larger than the bead, it can be shown that:

$$\frac{\Delta\omega}{\omega} = \frac{\Delta U}{U} = -\frac{\pi r^3}{U} \left[\epsilon_0 \frac{\epsilon_r - 1}{\epsilon_r + 2} E_0^2 + \mu_0 \frac{\mu_r - 1}{\mu_r + 2} H_0^2 \right]$$

and since $U = PQ/\omega$

$$\frac{\Delta\omega}{\omega} = \frac{\Delta U}{U} = -\frac{\omega \pi r^3}{PQ} \left[\epsilon_0 \frac{\epsilon_r - 1}{\epsilon_r + 2} E_0^2 + \mu_0 \frac{\mu_r - 1}{\mu_r + 2} H_0^2 \right]$$

so to calculate the absolute fields the Q and the input power must be known, however to get R/Q from the longitudinal field distribution these are not required.

Cases of special interest:

For a dielectric bead ($\mu_r = 1$) the expression reduces to:

$$\frac{\Delta\omega}{\omega} = -\frac{\pi r^3}{U} \left[\epsilon_0 \frac{\epsilon_r - 1}{\epsilon_r + 2} E_0^2 \right]$$

For a metal bead ($\epsilon_r \rightarrow \infty, \mu_r \rightarrow 0$):

$$\frac{\Delta\omega}{\omega} = -\frac{\pi r^3}{U} \left[\epsilon_0 E_0^2 - \frac{\mu_0}{2} H_0^2 \right]$$

A metallic bead can be used to measure the electric field if the magnetic field is known to be zero (e.g.: on axis of a monopole mode), and gives a larger

frequency shift than common dielectric materials such as Teflon ($\epsilon_r = 2.08$) or Alumina ($\epsilon_r = 9.3$). Shaped beads such as needles or disks can be used to enhance the perturbation and give directional selectivity. The enhancement or “form factor” can be calculated for ellipsoids or calibrated in a known field.

Calculation of R, R/Q:

By mapping the longitudinal distribution of E_z and integrating, the cavity shunt impedance can be determined

$$RT^2 = \frac{(VT)^2}{2P} = \frac{[\int E_z(z)e^{j\omega z/v} dz]^2}{2P}$$

where v is the velocity of the particles (usually = c), while

$$E^2 = -\frac{\Delta\omega PQ(\epsilon_r+2)}{\omega^2\pi r^3\epsilon_0(\epsilon_r-1)}$$

If the cavity is symmetric in z and $t=0$ at $z=0$ in the center, the impedance can be written

$$\frac{RT^2}{Q} = \frac{1}{2\omega\pi r^3\epsilon_0} \frac{\epsilon_r+2}{\epsilon_r-1} \left[\int dz \sqrt{\frac{\Delta\omega}{\omega}}(z) (\cos kz) \right]^2$$

where Q is the measured quality factor, r is the bead radius, ϵ_r is the relative dielectric constant and k is the wave number ($=\omega/c$).

Values of $\Delta f/f$ can be measured at discrete intervals and the function

$$\sqrt{\frac{\Delta\omega}{\omega}}(z) (\cos kz)$$

can be tabulated, integrated numerically and multiplied by the constants to obtain RT^2/Q . If Q is measured at the same time then the beam impedance $Z_{||} = RT^2$ can be calculated. This process is often automated, using a computer to move a motorized bead positioning apparatus, take frequency data from the network analyzer and calculate the integrals. For modes with weak fields where the frequency perturbation may be hard to measure it may be advantageous to measure the phase shift with the source fixed at the unperturbed resonant frequency. This is a more sensitive measurement and the phase data can be used directly to calculate RT^2 , eliminating the need to measure the Q .

Measurements with Network Analyzer test stand 4

Suggested topics:

- Measurements of the cavity driven by an amplifier (critical coupling, transfer from input coupler to cavity probes, tuning of resonance frequency).
- Direct RF feedback (decrease and increase of cavity impedance).
- Transfer function of periodic filters.
- Cavity with 1-turn delay feedback (effect on cavity impedance at specific frequencies).

Equipment:

VNA (Keysight E5071C), N-type calibration kit, coaxial cavity, amplifiers, power splitters/combiners, trombones, optical fiber delay line, oscilloscope and pulse generator. In addition you will need a number of cables of different length, as well as RF adaptors between BNC, SMC and N. The resonance of the coaxial cavity is about 57 MHz, which is achieved by a strong capacitive loading. The cavity is equipped with an input coupling loop, two probes and a tuning small tuning plunger. The latter allows to fine tune the resonance frequency. Figure 1 shows the inside of the coaxial cavity.

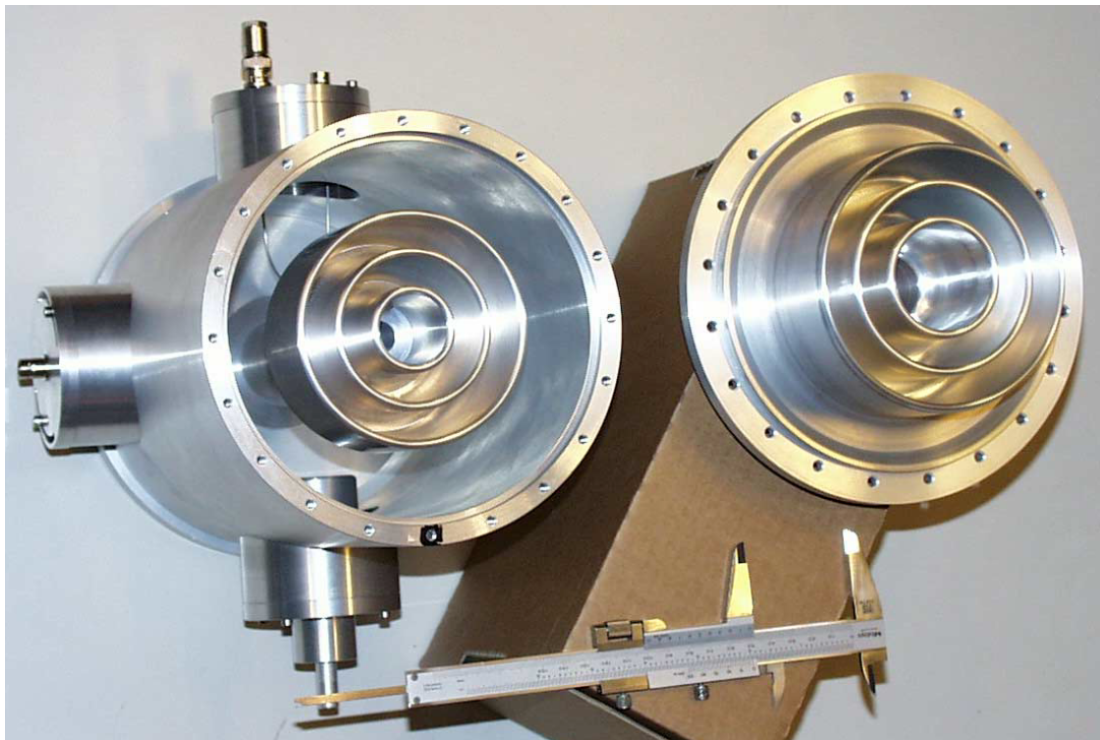


Figure 1. Inside view of the coaxial cavity.

Measurements of cavity parameters

- ◆ **Preset** the instrument. Start with a large frequency range (e.g. **center frequency**: 50 MHz and **span**: 100 MHz).
- ◆ You can either measure S_{11} at the input coupler port (port with two BNC connectors) or S_{21} from the input coupler to one of the probe ports, as well as between both probe ports. The coupling loop is connected between the central pins of both BNC connectors. One of the connectors must therefore be short-circuited to achieve the expected coupling. The probe ports are small loops. All ports are close to the short-circuit side of the cavity, hence in the area of large magnetic field (inductive coupling).
- ◆ Once the fundamental resonance of the cavity is found, narrow down the **span** to 1 MHz and set the **center frequency** to the resonance of the cavity.
- ◆ Optionally, you can now perform a calibration of port 1 of the VNA, including the cable to connect to the device under test, and measure the impedance (S_{11}) of the cavity at the input coupler. Observe the impedance in various formats, e.g. logarithmic amplitude or Smith chart. How well does the cavity impedance correspond to 50Ω at its resonance? If needed, adjust the coupling.

Cavity with direct feedback

- ◆ **Preset** the instrument. Select the measurement of S_{21} and set a **center frequency** of about 57 MHz a **span** of 1 MHz. Reduce the **source power** to -30 dBm, as well as the **resolution bandwidth** below 3 kHz. How can a smaller resolution bandwidth improve the quality of the measurement?
- ◆ Calibrate the network analyzer for S_{21} transmission measurements (**through**).
- ◆ Connect the output port of the VNA (port 1) to the input coupler of the cavity and the input port (port 2) to one of the cavity probes. Observe the cavity transfer function in transmission and set the **center frequency** such that it is displayed symmetrically. Alternatively you can also move the tuning plunger of the cavity to fine tune the resonance frequency.
- ◆ Now insert a power combiner (one input terminated with 50Ω) and amplifiers with a gain of about 30 to 50 dB into the signal path connected to the input coupler of the cavity. This gain can be achieved with two Minicircuits ZFL-500-BNC+ amplifiers in series with a 6 dB attenuator between them. The attenuator is required to not exceed the maximum input power of +5 dBm (**damage limit!**). The measured transfer function should be shifted in amplitude, but its shape should remain unchanged. Why is it important that the amplifiers are chosen such that the shape of the resonance curve is not altered?
- ◆ Now close the feedback loop around the cavity by feeding back the signal from the second cavity probe of the cavity into the second input of the power combiner. How does this affect the resonance curve of the cavity?
- ◆ Change the length of the feedback cable and install a trombone to fine tune the length more comfortably.
- ◆ Try to minimize the transfer function of the cavity. Compare the case without and with the feedback around the amplifier. A typical comparison of the resonance curve with and without the direct feedback is shown in Figure 2.

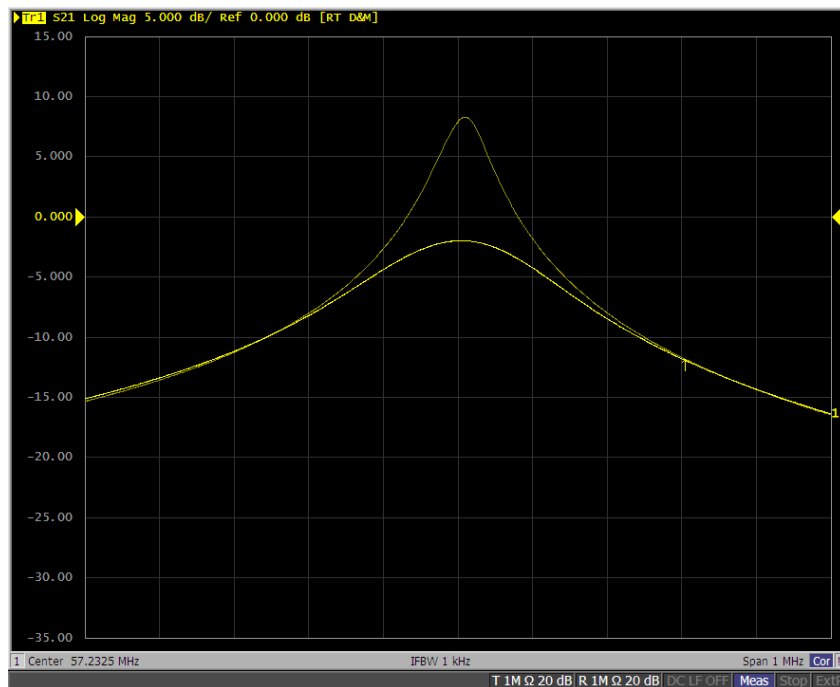


Figure 2. Resonance curve of the coaxial cavity with and without feedback.

- ◆ Optionally, change the gain of the feedback loop by installing additional attenuators or an amplifier in the feedback branch.
- ◆ Alternatively, you can also excite the cavity like the beam would do. Try to use a short wire as an antenna connected to the cable from the excitation port (port 1 in case of an S_{21} measurement) of the VNA and measure the transfer function. Hold the wire acting as an antenna into the gap region of the cavity. Again compare the case with and without the direct RF feedback. When correctly set-up in terms of gain and phase, the direct feedback reduces the cavity impedance seen by the beam, while keeping the impedance seen by the driving RF amplifier unchanged.
- ◆ Starting from a cavity impedance as low as possible, change the length of the feedback cable by half a wavelength (by inserting ~ 9 ns of cable delay). The apparent quality factor of the cavity should now increase significantly, also compared to the case without the feedback cable. Connect an oscilloscope to one of the cavity probes. What do you observe? If nothing happens you can increase the gain of the amplifier chain.
- ◆ Optionally, try perform a time domain measurement, exciting the cavity with a pulse generator simulating a bunch in a particle accelerator. Again the cavity should be excited at the gap with a small antenna. Connect an oscilloscope to a cavity probe to observe the transient response.

Comb filter

- ◆ Construct the Comb filter according to the diagram Figure 3, using to power splitters/combiners and the optical delay line followed by an amplifier and a 10 dB attenuator.

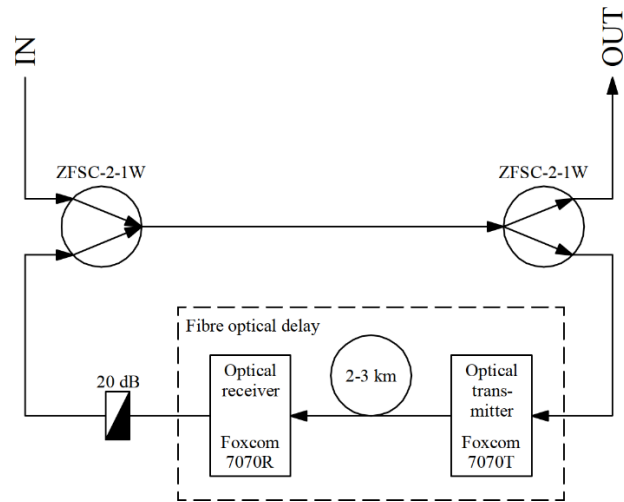


Figure 3. Comb filter set-up

- ◆ The comb filter will work best in terms of amplitude difference between pass- and stop-bands when both signals to the power combiner have equal amplitude. The coarse adjustment can be performed by measuring both signals with an oscilloscope.
- ◆ **Preset** the instrument. Select the measurement S_{21} and set a **center frequency** of 57 MHz with a **span** of 1 MHz. Reduce the **source power** to -20 dBm, as well as the **resolution bandwidth** to 3 kHz. How can a smaller resolution bandwidth improve the quality of the measurement?
- ◆ Calibrate the network analyzer for S_{21} transmission measurements.
- ◆ Measure the transfer function of the comb filter in amplitude and phase. Check the frequency spacing of two adjacent passband. Which component of the filter defines this spacing?
- ◆ Fine tune the gain of the delayed branch of the filter (trimmers on the optical modules) to achieve maximum amplitude difference between pass- and stop-band (Figure 4).

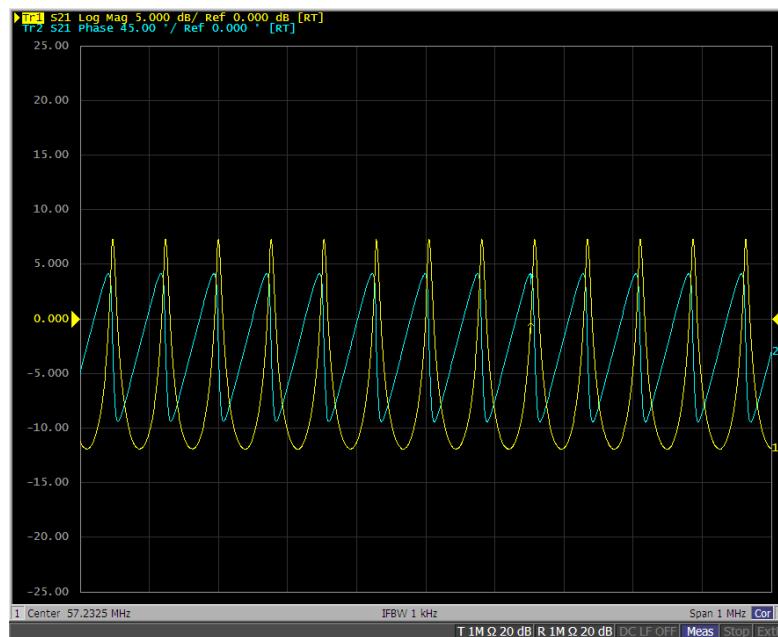


Figure 4. Typical comb filter transfer function in amplitude (yellow) and phase (blue).

1-turn delay feedback

- ◆ Rebuild the configuration for the measurement of the transfer function with direct feedback. Excite the cavity VNA through the power combiner.
- ◆ Replace the simple cable of the feedback branch (from cavity probe to power combiner) by the comb filter.
- ◆ Adjust the delay (cable delay and trombone) to achieve sharp notches in the transfer function of the cavity. In a real accelerator these notches are located exactly at the revolution frequency harmonics of the circulating beam (Figure 5).

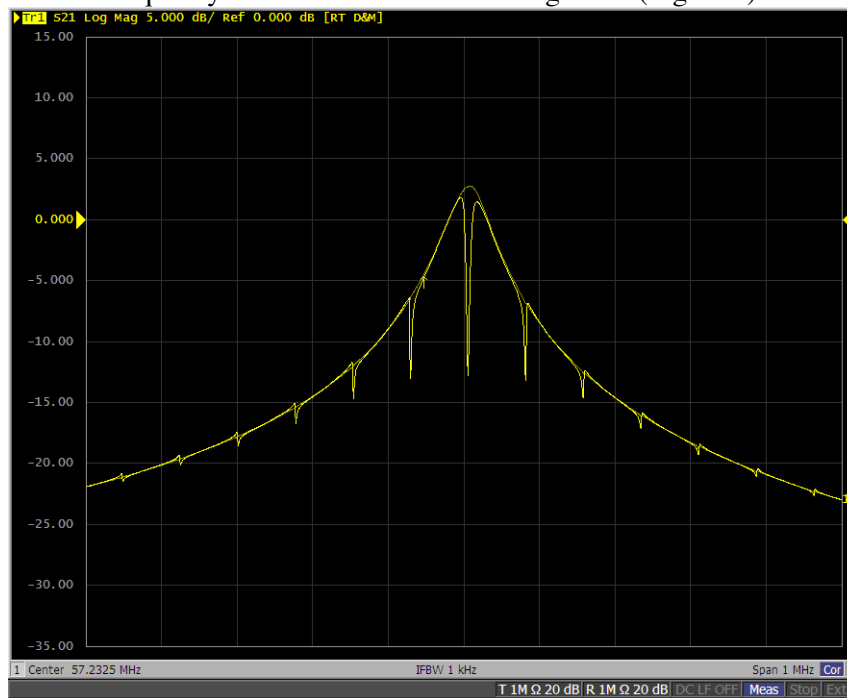
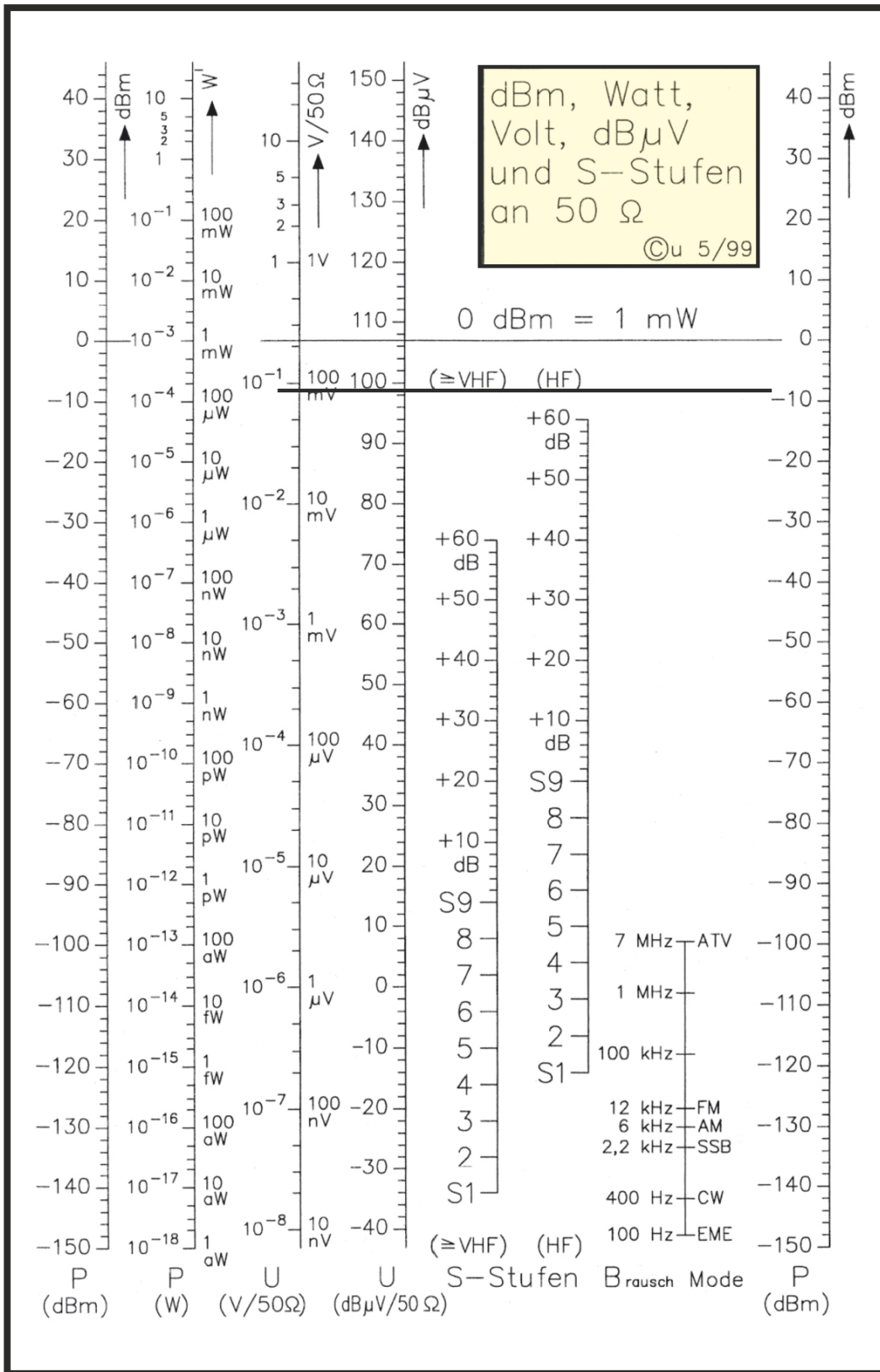


Figure 5. Resonance curve with and without comb filter feedback.

- ◆ Compare the transfer function with and without the comb filter. What happens at the frequencies between the notches?
- ◆ Optionally try again to excite the cavity with the pulse generator. To simulate the periodic spectrum of the beam, the pulse repetition rate of the pulse generator represents the revolution frequency of the circulating beam. It must match the frequency spacing of the passbands of the comb filter.

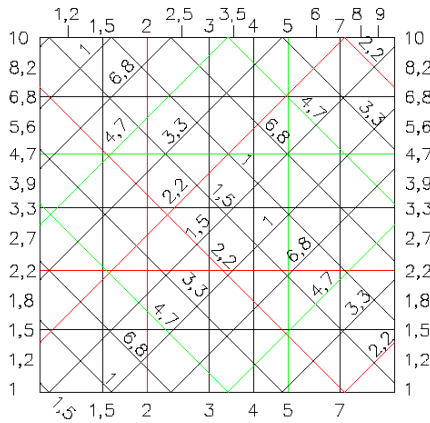
Appendix – useful charts



DECIBELS vs. VOLTAGE AND POWER

The Decibel Chart below indicates dB for any ratio of voltage or power up to 100 dB. For voltage ratios greater than 10 (or power ratios greater than 100) the ratio can be broken down into two products, the dB found for each separately, the two results then added. For example: To convert a voltage ratio of 200:1 to dB: 200:1 VR equals the product of 100:1 and 2:1. 100:1 equals 40 dB; 2:1 equals 6 dB. Therefore, 200:1 VR equals 40dB + 6 dB or 46 dB.

Voltage Ratio	Power Ratio	-db +	Voltage Ratio	Power Ratio	Voltage Ratio	Power Ratio	-db +	Voltage Ratio	Power Ratio	Voltage Ratio	Power Ratio	-db +	Voltage Ratio	Power Ratio
1.0000	1.0000	0	1.0000	1.0000	.4467	.1995	7.0	2.239	5.012	.1995	.03981	14.0	5.012	25.12
.9886	.9772	.1	1.012	1.023	.4416	.1950	7.1	2.265	5.129	.1972	.03890	14.1	5.070	25.70
.9772	.9550	.2	1.023	1.047	.4365	.1905	7.2	2.265	5.248	.1950	.03802	14.2	5.129	26.30
.9661	.9333	.3	1.035	1.072	.4311	.1862	7.3	2.317	5.370	.1928	.03715	14.3	5.188	26.92
.9550	.9120	.4	1.047	1.096	.4266	.1820	7.4	2.344	5.495	.1905	.03631	14.4	5.248	27.54
.9441	.8913	.5	1.059	1.122	.4217	.1778	7.5	2.371	5.623	.1884	.03548	14.5	5.309	28.18
.9333	.8710	.6	1.072	1.148	.4169	.1738	7.6	2.399	5.754	.1862	.03467	14.6	5.370	28.84
.9226	.8511	.7	1.084	1.175	.4121	.1698	7.7	2.427	5.888	.1841	.03388	14.7	5.433	29.51
.9120	.8318	.8	1.096	1.202	.4074	.1660	7.8	2.455	6.026	.1820	.03311	14.8	5.495	30.20
.9016	.8128	.9	1.109	1.230	.4027	.1622	7.9	2.483	6.166	.1799	.03236	14.9	5.559	30.90
.8913	.7943	1.0	1.122	1.259	.3981	.1585	8.0	2.512	6.310	.1778	.03162	15.0	5.623	31.62
.8810	.7762	1.1	1.135	1.288	.3936	.1549	8.1	2.541	6.457	.1758	.03090	15.1	5.689	32.36
.8710	.7586	1.2	1.148	1.318	.3890	.1514	8.2	2.541	6.607	.1738	.03020	15.2	5.754	33.11
.8610	.7413	1.3	1.161	1.349	.3846	.1473	8.3	2.600	6.761	.1718	.02951	15.3	5.821	33.88
.8511	.7244	1.4	1.175	1.380	.3802	.1445	8.4	2.630	6.918	.1698	.02884	15.4	5.888	34.67
.8414	.7079	1.5	1.189	1.413	.3758	.1413	8.5	2.661	7.079	.1679	.02818	15.5	5.957	35.48
.8318	.6918	1.6	1.202	1.445	.3715	.1380	8.6	2.692	7.244	.1660	.02754	15.6	6.026	36.31
.8222	.6761	1.7	1.216	1.479	.3673	.1349	8.7	2.723	7.413	.1641	.02692	15.7	6.095	37.15
.8128	.6607	1.8	1.230	1.514	.3631	.1318	8.8	2.754	7.586	.1622	.02630	15.8	6.166	38.02
.8035	.6457	1.9	1.245	1.549	.3589	.1288	8.9	2.786	7.762	.1603	.02570	15.9	6.237	38.90
.7943	.6310	2.0	1.259	1.585	.3548	.1259	9.0	2.818	7.943	.1585	.02512	16.0	6.310	39.81
.7852	.6166	2.1	1.274	1.622	.3508	.1230	9.1	2.851	8.128	.1567	.02455	16.1	6.383	40.74
.7762	.6026	2.2	1.288	1.660	.3467	.1202	9.2	2.884	8.318	.1549	.02399	16.2	6.457	41.69
.7674	.5888	2.3	1.303	1.698	.3428	.1175	9.3	2.917	8.511	.1531	.02344	16.3	6.531	42.66
.7586	.5754	2.4	1.318	1.738	.3388	.1148	9.4	2.951	8.710	.1514	.02291	16.4	6.607	43.65
.7499	.5623	2.5	1.334	1.778	.3350	.1122	9.5	2.985	8.913	.1496	.02239	16.5	6.683	44.67
.7413	.5495	2.6	1.349	1.820	.3311	.1096	9.6	3.020	9.120	.1479	.02188	16.6	6.761	45.71
.7328	.5370	2.7	1.365	1.862	.3273	.1072	9.7	3.055	9.333	.1462	.02138	16.7	6.839	46.77
.7244	.5248	2.8	1.380	1.905	.3236	.1047	9.8	3.090	9.550	.1445	.02089	16.8	6.918	47.86
.7161	.5129	2.9	1.396	1.950	.3199	.1023	9.9	3.126	9.772	.1429	.02042	16.9	6.998	48.98
.7079	.5012	3.0	1.413	1.995	.3162	.1000	10.0	3.162	10.000	.1413	.01995	17.0	7.079	50.12
.6998	.4898	3.1	1.429	2.042	.3126	.09772	10.1	3.199	10.23	.1396	.01950	17.1	7.161	51.29
.6918	.4786	3.2	1.445	2.089	.3090	.09550	10.2	3.236	10.47	.1380	.01905	17.2	7.244	52.48
.6839	.4677	3.3	1.462	2.138	.3055	.09333	10.3	3.273	10.72	.1365	.01862	17.3	7.328	53.70
.6761	.4571	3.4	1.479	2.188	.3020	.09120	10.4	3.311	10.96	.1349	.01820	17.4	7.413	54.95
.6683	.4467	3.5	1.496	2.239	.2985	.08913	10.5	3.350	11.22	.1334	.01778	17.5	7.499	56.23
.6607	.4365	3.6	1.514	2.291	.2951	.08710	10.6	3.388	11.48	.1318	.01738	17.6	7.586	57.54
.6531	.4266	3.7	1.531	2.344	.2917	.08511	10.7	3.428	11.75	.1303	.01698	17.7	7.674	58.88
.6457	.4169	3.8	1.549	2.399	.2884	.08318	10.8	3.467	12.02	.1288	.01660	17.8	7.762	60.26
.6383	.4074	3.9	1.567	2.455	.2851	.08128	10.9	3.508	12.30	.1274	.01622	17.9	7.852	61.66
.6310	.3981	4.0	1.585	2.512	.2818	.07943	11.0	3.548	12.59	.1259	.01585	18.0	7.943	63.10
.6237	.3890	4.1	1.603	2.570	.2786	.07762	11.1	3.589	12.88	.1245	.01549	18.1	8.035	64.57
.6166	.3802	4.2	1.622	2.630	.2754	.07586	11.2	3.631	13.18	.1230	.01514	18.2	8.128	66.07
.6095	.3715	4.3	1.641	2.692	.2723	.07413	11.3	3.673	13.49	.1216	.01479	18.3	8.222	67.61
.6026	.3631	4.4	1.660	2.754	.2692	.07244	11.4	3.715	13.80	.1202	.01445	18.4	8.318	69.18
.5957	.3548	4.5	1.679	2.818	.2661	.07079	11.5	3.758	14.13	.1189	.01413	18.5	8.414	70.79
.5888	.3467	4.6	1.698	2.884	.2630	.06918	11.6	3.802	14.45	.1175	.01380	18.6	8.511	72.44
.5821	.3388	4.7	1.718	2.951	.2600	.06761	11.7	3.846	14.79	.1161	.01349	18.7	8.610	74.16
.5754	.3311	4.8	1.738	3.020	.2570	.06607	11.8	3.890	15.14	.1148	.01318	18.8	8.710	75.86
.5689	.3236	4.9	1.758	3.090	.2541	.06457	11.9	3.936	15.49	.1135	.01288	18.9	8.811	77.62
.5623	.3162	5.0	1.778	3.162	.2512	.06310	12.0	3.981	15.85	.1122	.01259	19.0	8.913	79.43
.5559	.3090	5.1	1.799	3.236	.2483	.06166	12.1	4.027	16.22	.1109	.01230	19.1	9.016	81.28
.5495	.3020	5.2	1.820	3.311	.2455	.06026	12.2	4.074	16.60	.1096	.01202	19.2	9.120	83.18
.5433	.2951	5.3	1.841	3.388	.2427	.05888	12.3	4.121	16.98	.1084	.01175	19.3	9.226	85.11
.5370	.2884	5.4	1.862	3.467	.2399	.05754	12.4	4.169	17.38	.1072	.01148	19.4	9.333	87.10
.5309	.2818	5.5	1.884	3.548	.2371	.05623	12.5	4.217	17.78	.1059	.01122	19.5	9.441	89.13
.5248	.2754	5.6	1.905	3.631	.2344	.05495	12.6	4.266	18.20	.1047	.01096	19.6	9.550	91.20
.5188	.2692	5.7	1.928	3.715	.2317	.05370	12.7	4.315	18.62	.1035	.01072	19.7	9.661	93.33
.5129	.2630	5.8	1.950	3.802	.2291	.05248	12.8	4.365	19.05	.1023	.01047	19.8	9.772	95.50
.5070	.2570	5.9	1.972	3.890	.2265	.05129	12.9	4.416	19.50	.1012	.01023	19.9	9.886	97.72
.5012	.2512	6.0	1.995	3.981	.2239	.05012	13.0	4.467	319.95	.1000	.01000	20.0	10.000	100.00
.4955	.2455	6.1	2.018	4.074	.2213	.04898	13.1	4.519	20.42		10 ⁻³	30		10 ³
.4898	.2399	6.2	2.042	4.169	.2188	.04786	13.2	4.571	20.89		10 ⁻⁴	40	10 ²	10 ⁴
.4842	.2344	6.3	2.065	4.266	.2163	.04677	13.3	4.624	21.38		10 ⁻⁵	50		10 ⁵
.4786	.2291	6.4	2.089	4.365	.2138	.04571	13.4	4.677	21.88		10 ⁻⁶	60		10 ⁶
.4732	.2239	6.5	2.113	4.467	.2113	.04467	13.5	4.732	22.39		10 ⁻⁷	70	10 ³	10 ⁷
.4677	.2188	6.6	2.138	4.571	.2089	.04365	13.6	4.788	22.91		10 ⁻⁸	80		10 ⁸
.4624	.2138	6.7	2.163	4.677	.2065	.04266	13.7	4.842	23.44		10 ⁻⁹	90	10 ⁴	10 ⁹
.4571	.2089	6.8	2.188	4.786	.2042	.04169	13.8	4.898	23.99		10 ⁻¹⁰	100		10 ¹⁰
.4519	.2042	6.9	2.213	4.898	.2018	.04074	13.9	4.955	24.55					



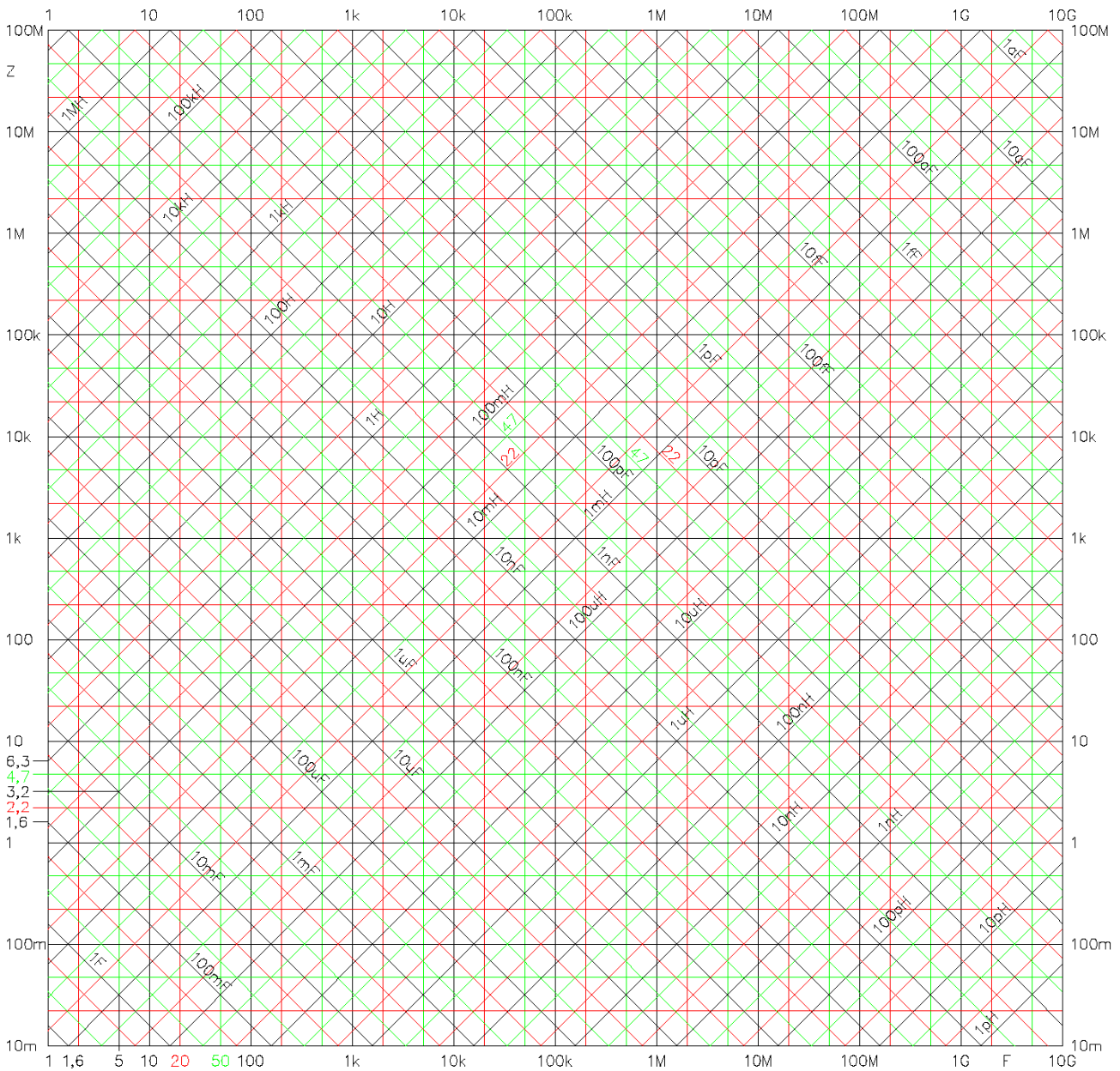
HF-Tapete: Nomogramm, das die Impedanz von Kondensator und Induktivität in Abhängigkeit von der Frequenz darstellt.

X-Achse: Frequenz (1 – 2 – 5)
 Y-Achse: Absolutwert der Impedanz (1 – 2,2 – 4,7)
 Diagonalen: Kapazität/Induktivität (1 – 2,2 – 4,7)

In der einzelnen Masche (links) kann der Wert genauer bestimmt werden.

Beispiel: 4,7nF und 47uH kreuzen sich bei ca. 330kHz (Resonanzfrequenz des Schwingkreises). Beide Bauteile haben dabei eine Impedanz von $\pm j \cdot 100 \text{ Ohm}$.

2002-03-27 Oliver Betz (www.oliverbetz.de)



NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES

