Systematic uncertainties in atmospheric neutrino calculations

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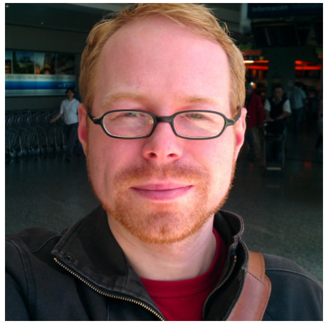
Origin of the series of models, methods and tools



Hans
Dembinski

Anatoli
Fedynitch

Ralph
Engel

Thomas K.
Gaisser

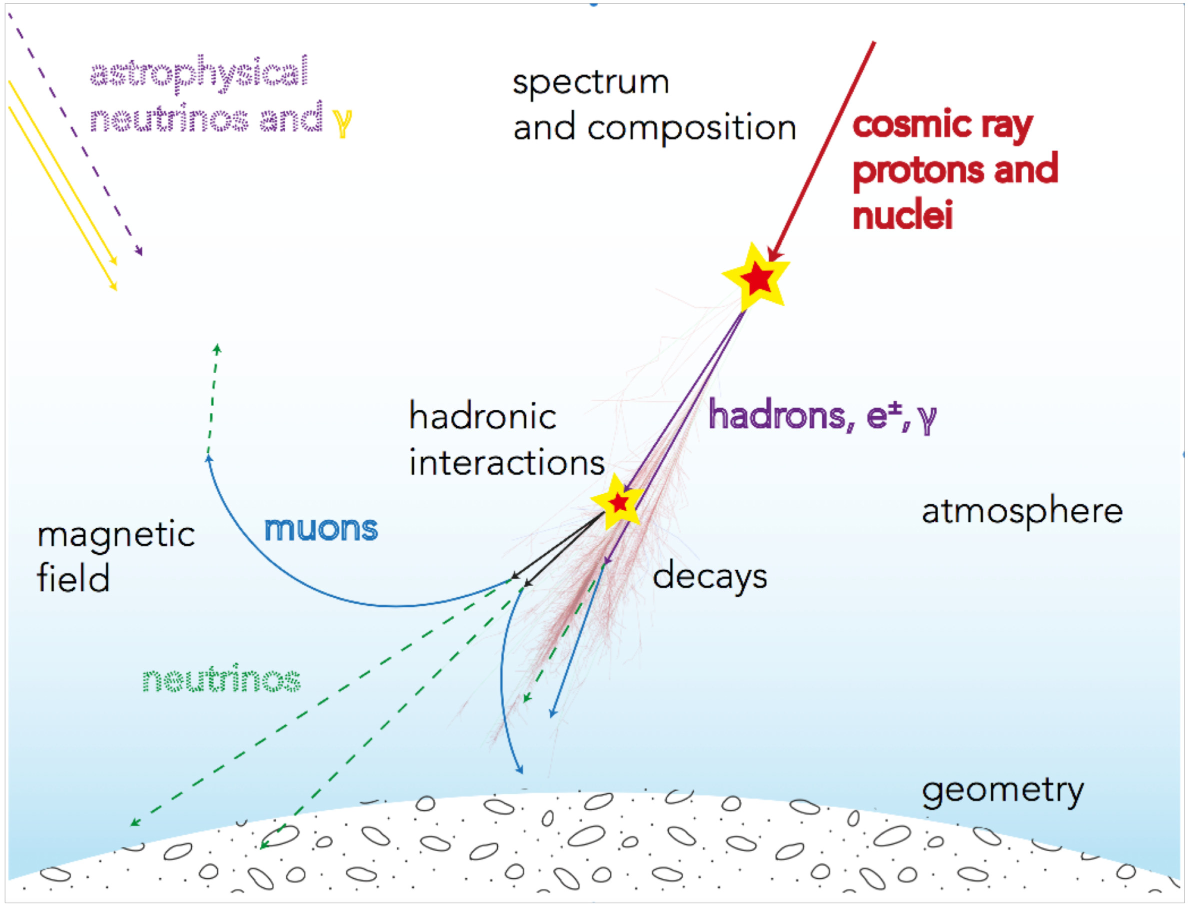
Stephan
Meighen-Berger

Felix
Riehn

Todor
Stanev

Atmospheric neutrinos

Ingredients for high-precision atmospheric neutrino flux calculation



- For high precision calculations all phenomena need accurate modeling
- Uncertain "ingredients":
  - Cosmic ray spectrum and composition
  - Hadronic interactions
  - Atmosphere (dynamic, depends on use case)
  - (Rare) decays
  - Geometry, magnetic fields, solar modulation
- No clear prescription how to handle uncertainties.
- Energy range MeV – EeV!

Transport equations (hadronic cascade equations)

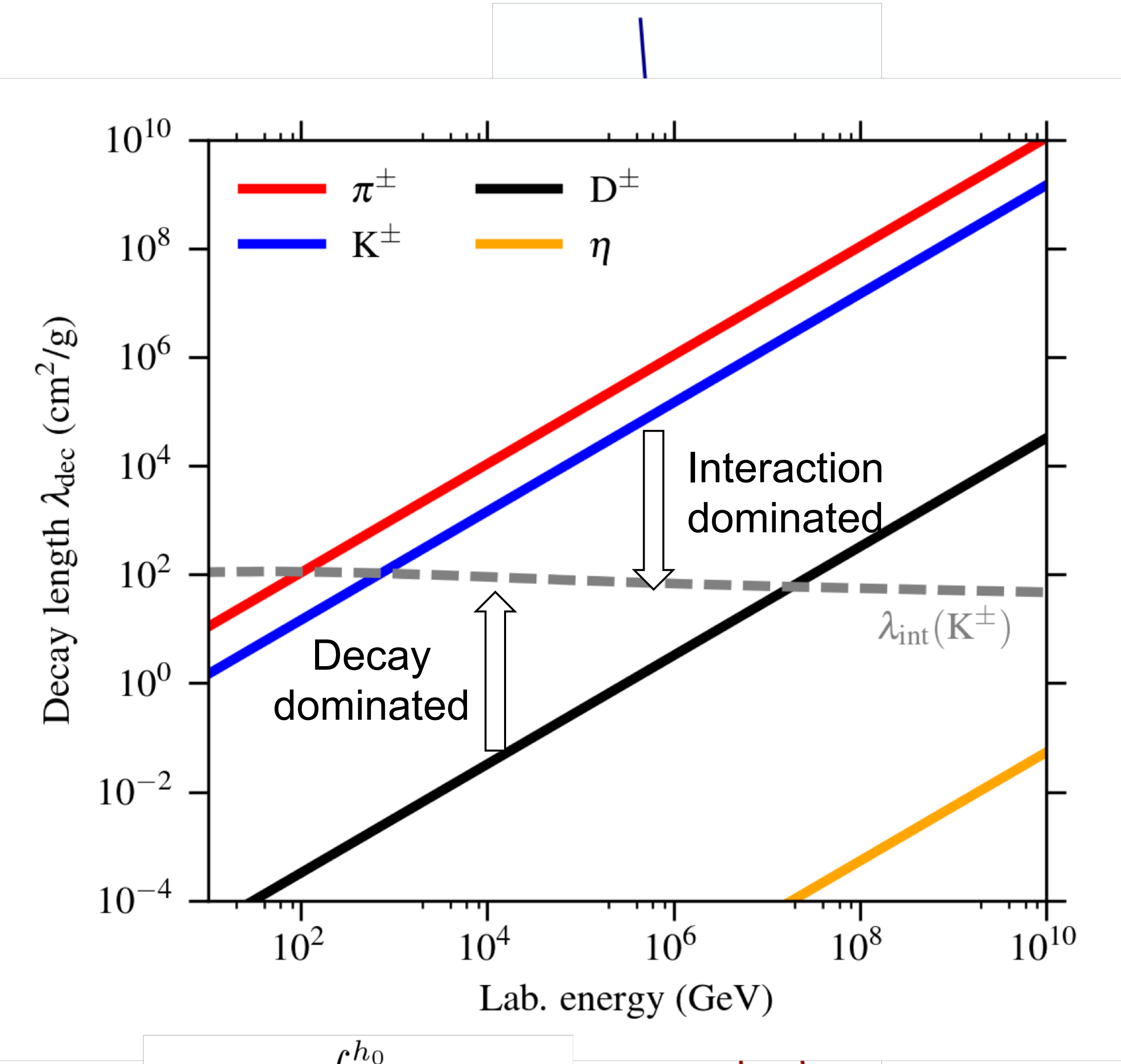
System of coupled non-linear PDE for each particle species h :

$$
\begin{aligned}
\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = &-\frac{\Phi_h(E,X)}{\lambda_{\mathrm{int},h}(E)} \quad \text{Interactions with air} \\
&-\frac{\Phi_h(E,X)}{\lambda_{\mathrm{dec},h}(E,X)} \quad \text{Decays} \\
&-\frac{\partial}{\partial E}\left(\mu(E)\Phi_h(E,X)\right) \quad \text{Continuous losses} \\
&+\sum_k \int_E^\infty \mathrm{d}E_k \, \frac{\mathrm{d}N_{k(E_k)\to h(E)}}{\mathrm{d}E} \frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{int},k}(E_k)} \\
&+\sum_k \int_E^\infty \mathrm{d}E_k \, \frac{\mathrm{d}N^{\mathrm{dec}}_{k(E_k)\to h(E)}}{\mathrm{d}E} \frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{dec},k}(E_k,X)}
\end{aligned}
$$

cosmic ray physics

atmospheric physics

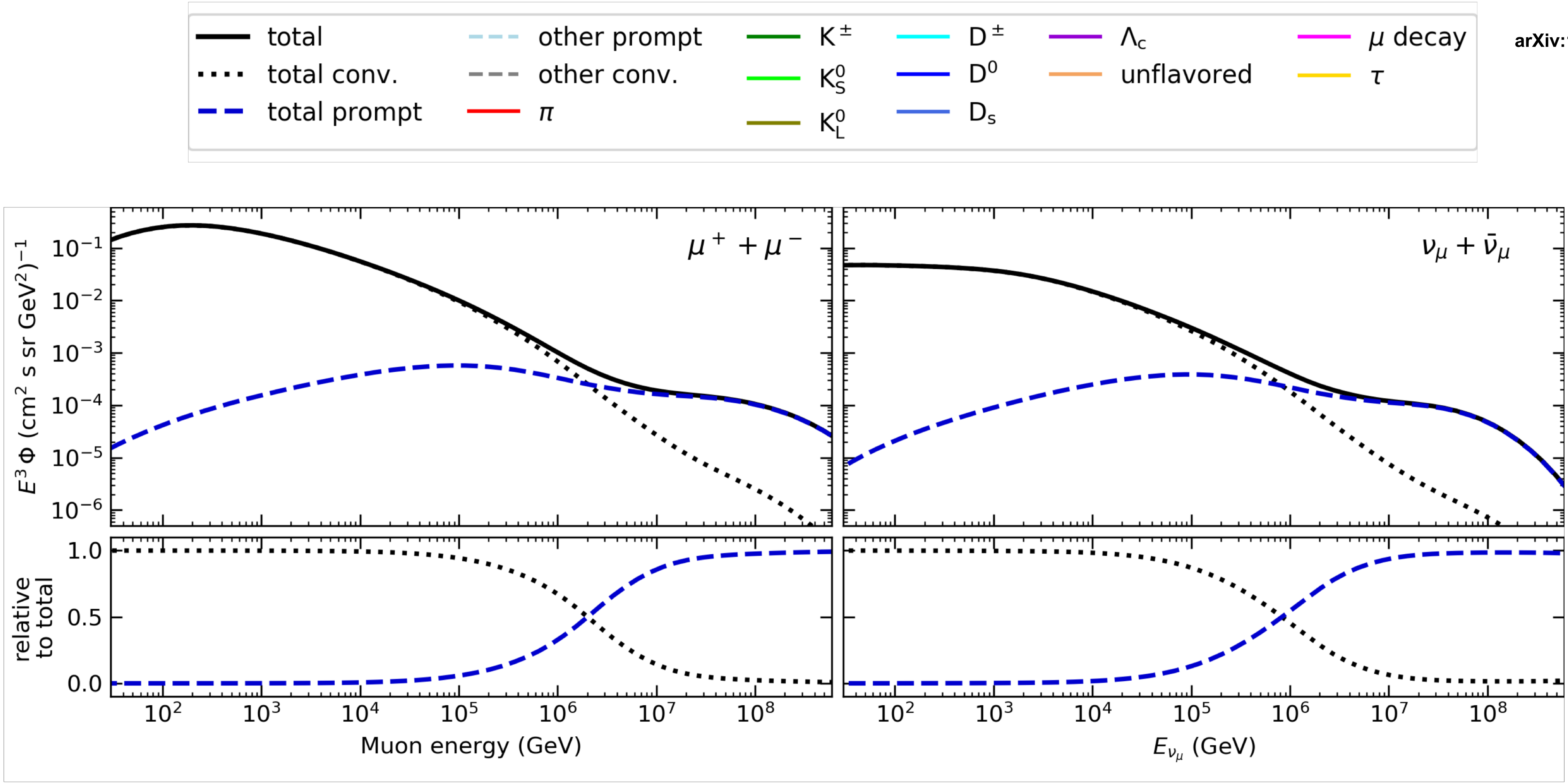
particle physics



$$X(h_0) = \int_0^{h_0} \mathrm{d}\ell \; \rho_{\mathrm{air}}(\ell)$$

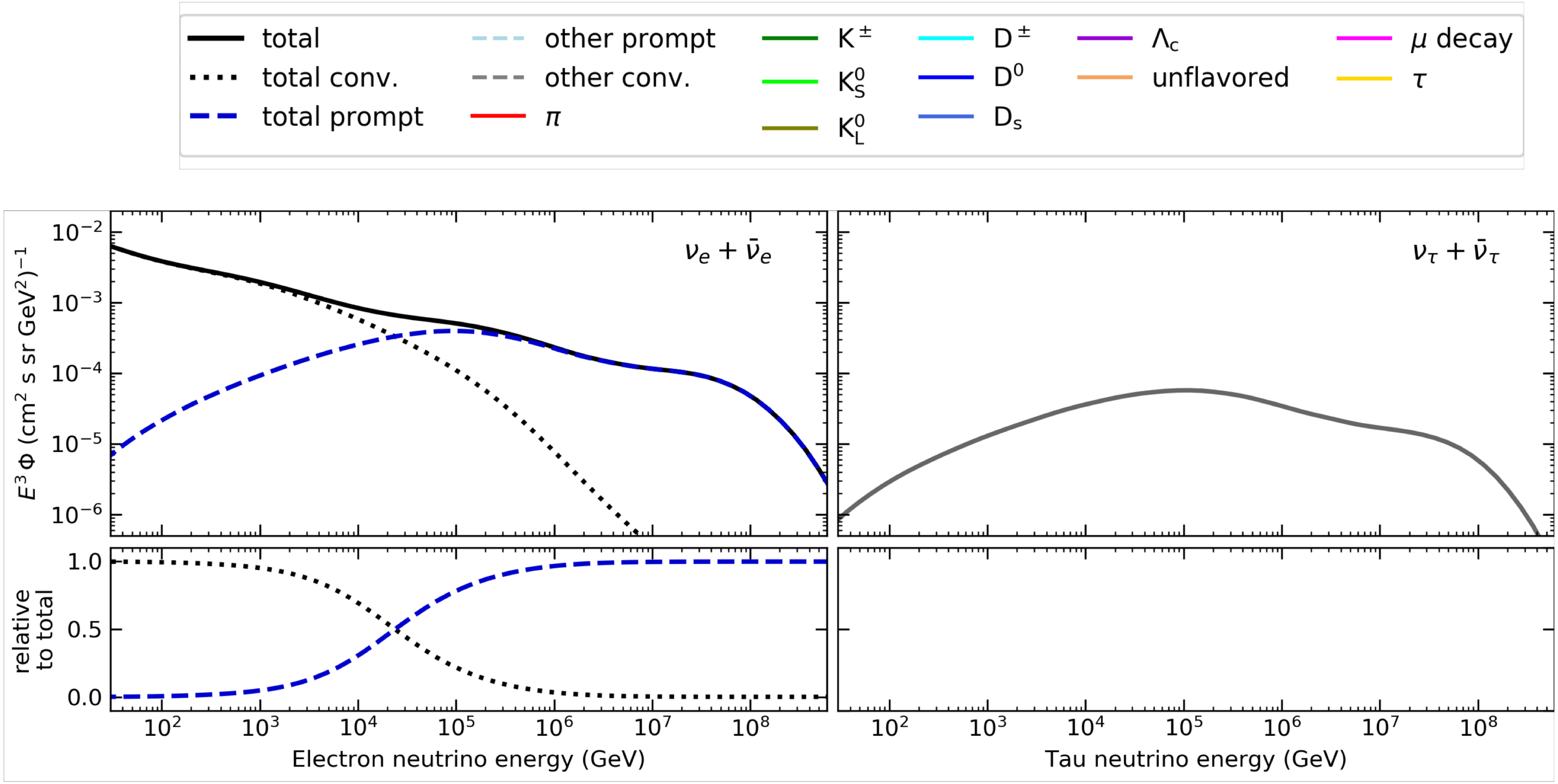
Hadrons contributing to muonic leptons

arXiv:1806.04140

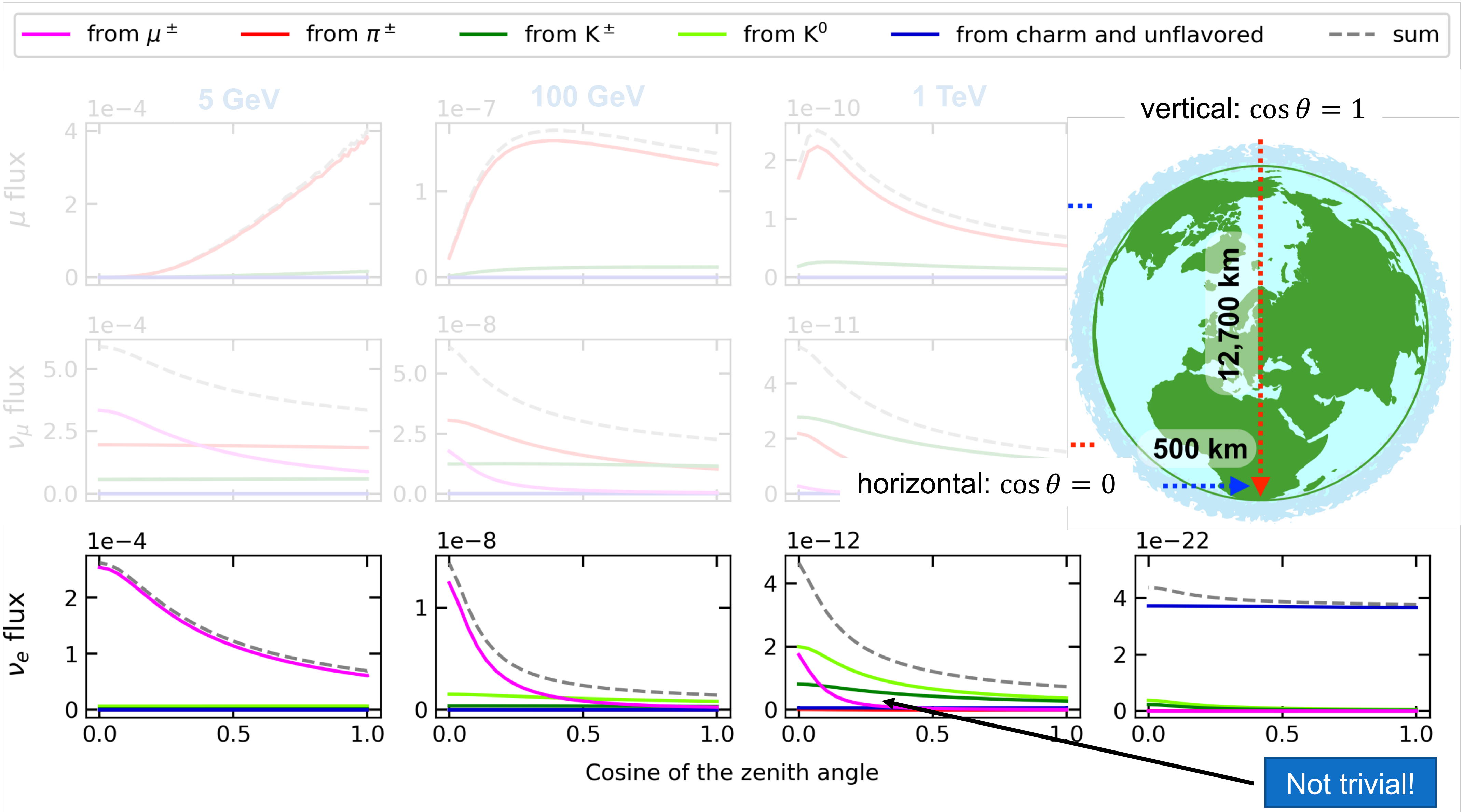


Hadrons contributing to electron and tau neutrinos

arXiv:1806.04140



Different hadronic components shape the zenith distribution



AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, 1806.04140 (2018)

MCEq: Matrix Cascade Equations

A. Fedynitch, R. Engel, T. K. Gaisser, F. Riehn and S. Todor
PoS ICRC 2015, 1129 (2015), EPJ Web Conf. 99, 08001 (2015)
and EPJ Web Conf. 116, 11010 (2016)

$$\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = -\frac{\Phi_h(E,X)}{\lambda_{\mathrm{int},h}(E)} - \frac{\Phi_h(E,X)}{\lambda_{\mathrm{dec},h}(E,X)} - \frac{\partial}{\partial E}(\mu(E)\Phi_h(E,X)) + \sum_\ell \int_E^\infty \mathrm{d}E_\ell \frac{\mathrm{d}N_{\ell(E_\ell)\to h(E)}}{\mathrm{d}E}\frac{\Phi_\ell(E_\ell,X)}{\lambda_{\mathrm{int},l}(E_\ell)} + \sum_\ell \int_E^\infty \mathrm{d}E_\ell \frac{\mathrm{d}N^{\mathrm{dec}}_{\ell(E_\ell)\to h(E)}}{\mathrm{d}E}\frac{\Phi_\ell(E_\ell,X)}{\lambda_{\mathrm{dec},l}(E_\ell,X)}$$

$$\frac{\mathrm{d}\Phi^h_{E_i}}{\mathrm{d}X} = -\frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{int},E_i}} - \frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{dec},E_i}(X)} - \vec{\nabla}_i(\mu^h_{E_i}\Phi^h_{E_i}) + \sum_{E_k\geq E_i}^{E_N}\sum_\ell \frac{c_{\ell(E_k)\to h(E_i)}}{\lambda^\ell_{\mathrm{int},E_k}}\Phi^\ell_{E_k} + \sum_{E_k\geq E_i}^{E_N}\sum_\ell \frac{\mathrm{d}_{\ell(E_k)\to h(E_i)}}{\lambda^\ell_{\mathrm{dec},E_k}(X)}\Phi^\ell_{E_k}$$

State (or flux) vector

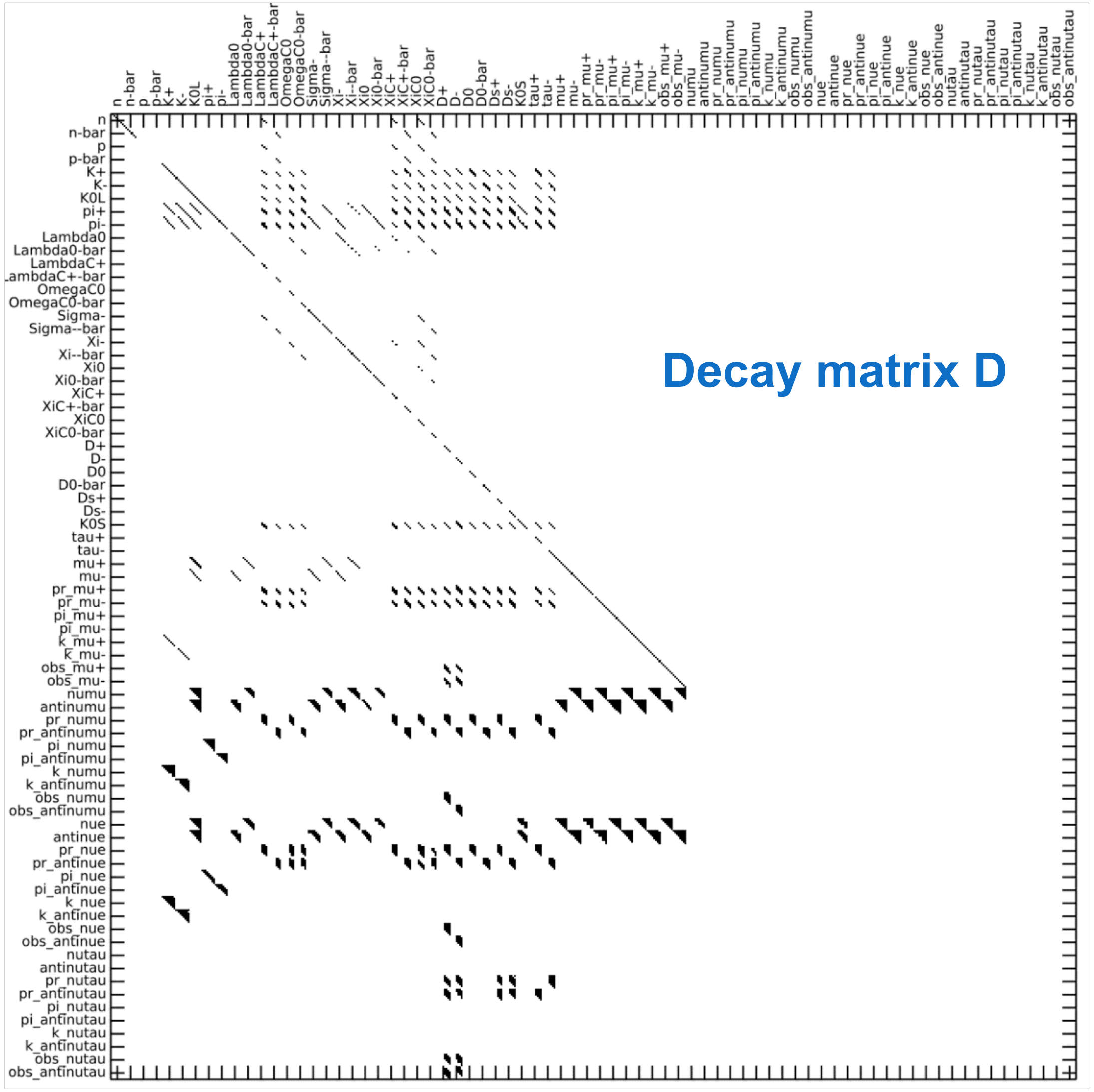
$$\vec{\Phi} = \left(\vec{\Phi}^{\mathrm{p}} \quad \vec{\Phi}^{\mathrm{n}} \quad \vec{\Phi}^{\pi^+} \quad \cdots \quad \vec{\Phi}^{\bar{\nu}_\mu} \quad \cdots\right)^T$$

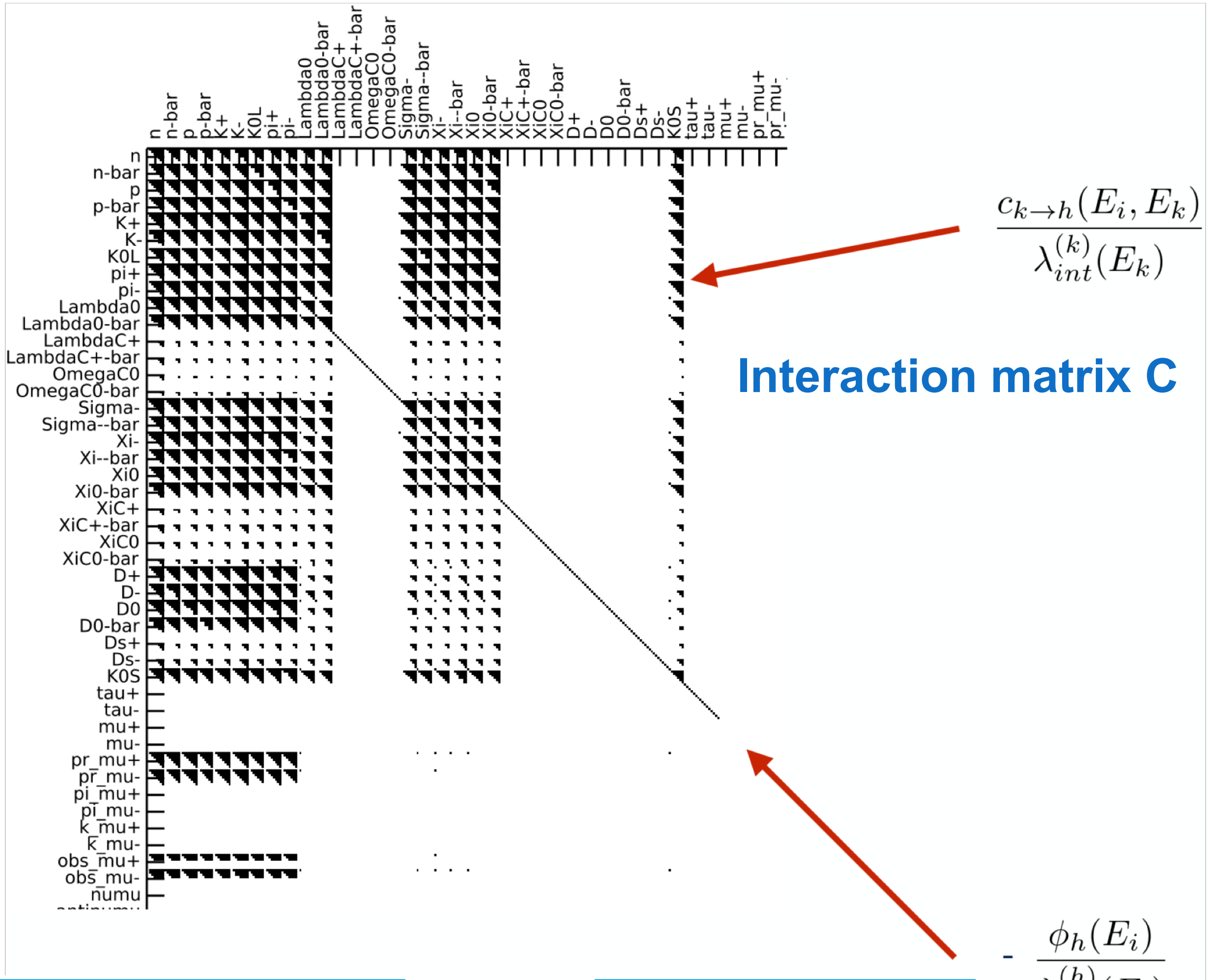
$$\vec{\Phi}^{\mathrm{p}} = \left(\Phi^{\mathrm{p}}_{E_0} \quad \Phi^{\mathrm{p}}_{E_1} \quad \cdots \quad \Phi^{\mathrm{p}}_{E_N}\right)^T$$

"Matrix form"

$$\frac{\mathrm{d}}{\mathrm{d}X}\vec{\Phi} = -\vec{\nabla}_E(\mathrm{diag}(\vec{\mu})\vec{\Phi}) + (-\mathbf{1}+\mathbf{C})\mathbf{\Lambda}_{\mathrm{int}}\vec{\Phi} + \frac{1}{\rho(X)}(-\mathbf{1}+\mathbf{D})\mathbf{\Lambda}_{\mathrm{dec}}\vec{\Phi}$$

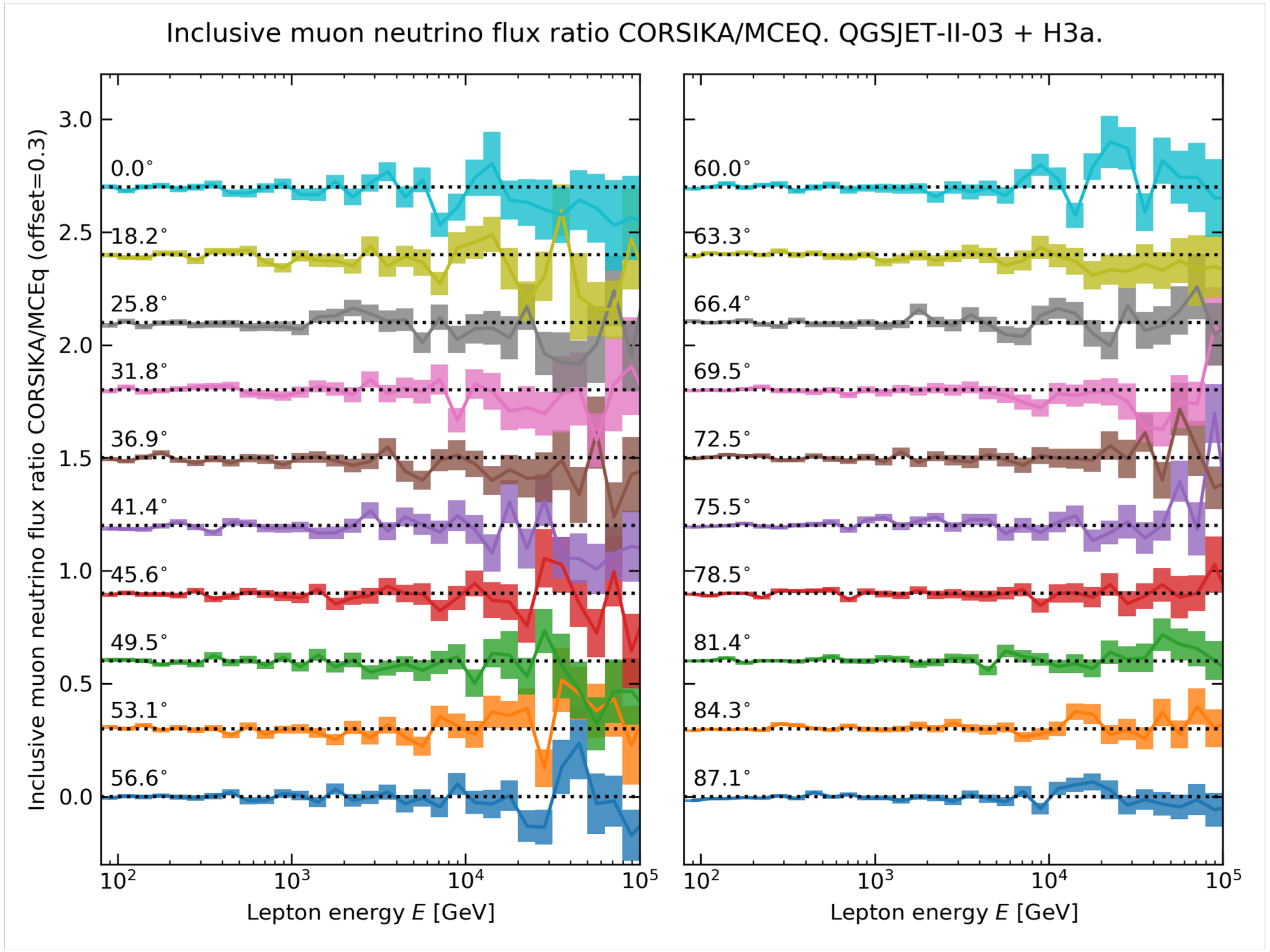
Sparse matrix structure

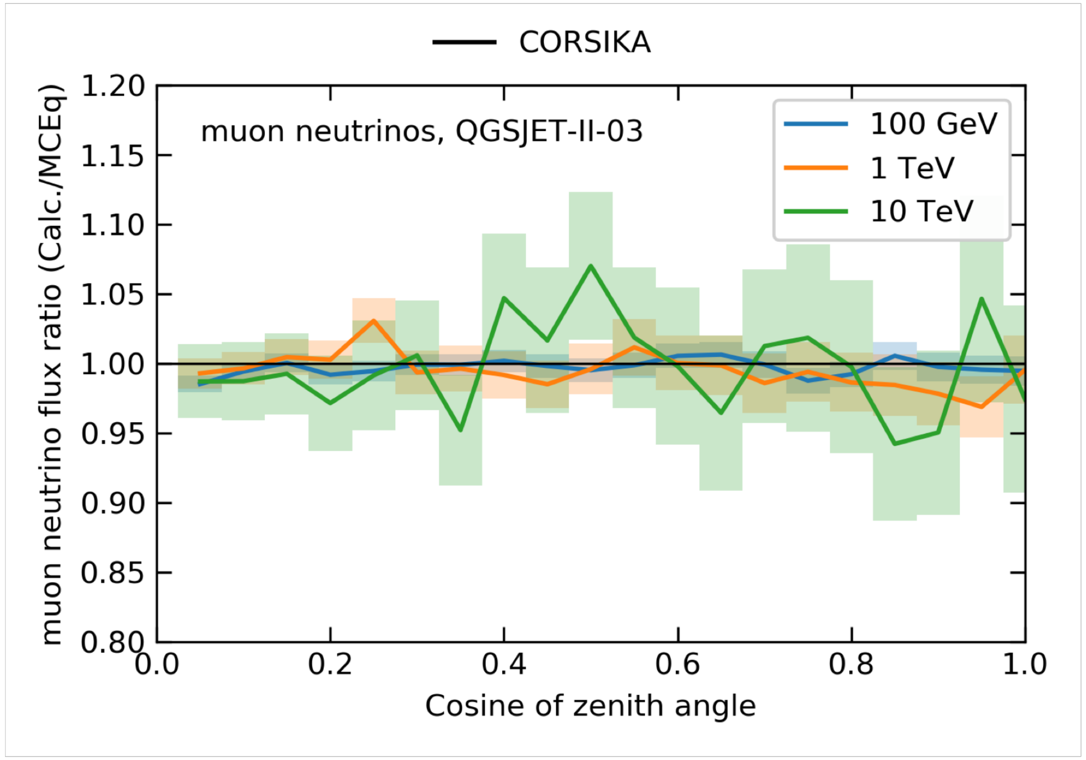




matrices are sparse → high performance

MCEq vs (thinned) CORSIKA calculation in 1D





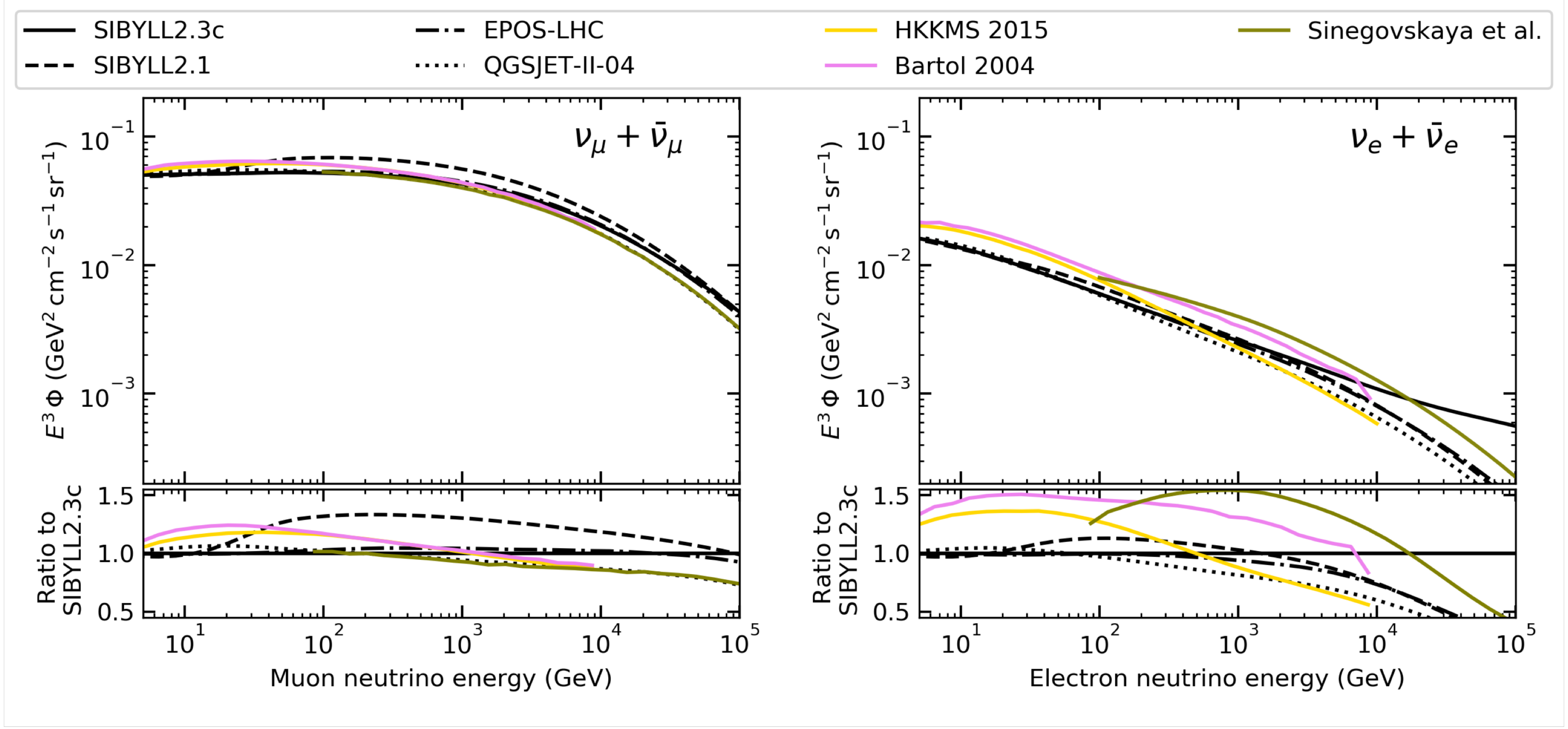
> MIT licensed @

https://github.com/afedynitch/MCEq

CORSIKA: A. Fedynitch, J. Becker Tjus and P. Desiati, PRD 2012
MCEq: Code paper, AF, R. Engel, in prep. for submission

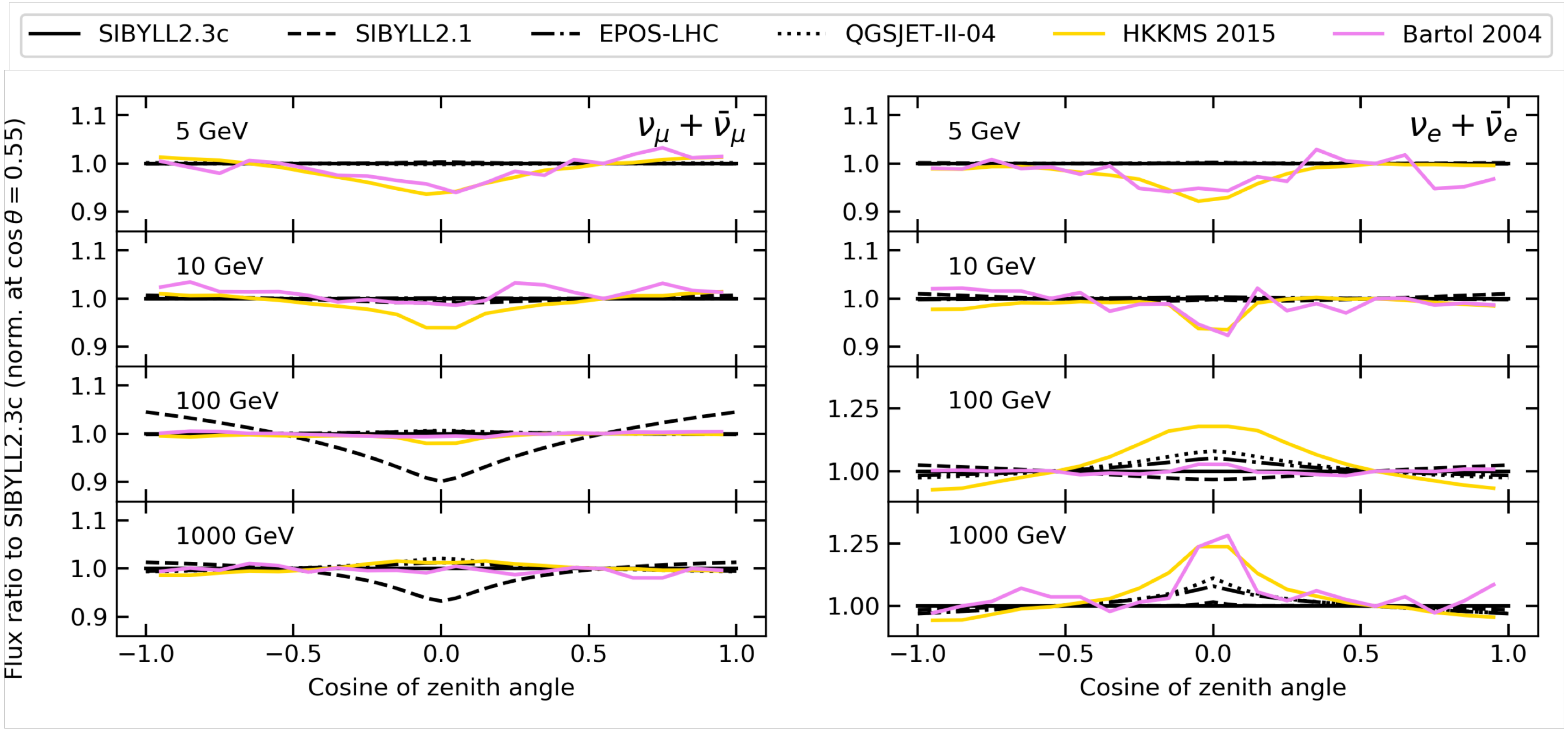
MCEq vs. traditional calculations

HKKMS: M. Honda et al., PRD 92 (2015)
Bartol: G. Barr et al., PRD 70 (2004)
Sinegovskaya et al. PRD 91 (2015)
MCEq: AF, R. Engel in prep.



- Old 2002 (GH) primary model for HKKMS and Bartol, H3a for the rest
- Data can not discriminate between calculations
- Shown are zenith and azimuth averages

Hadronic model dependence of zenith distributions



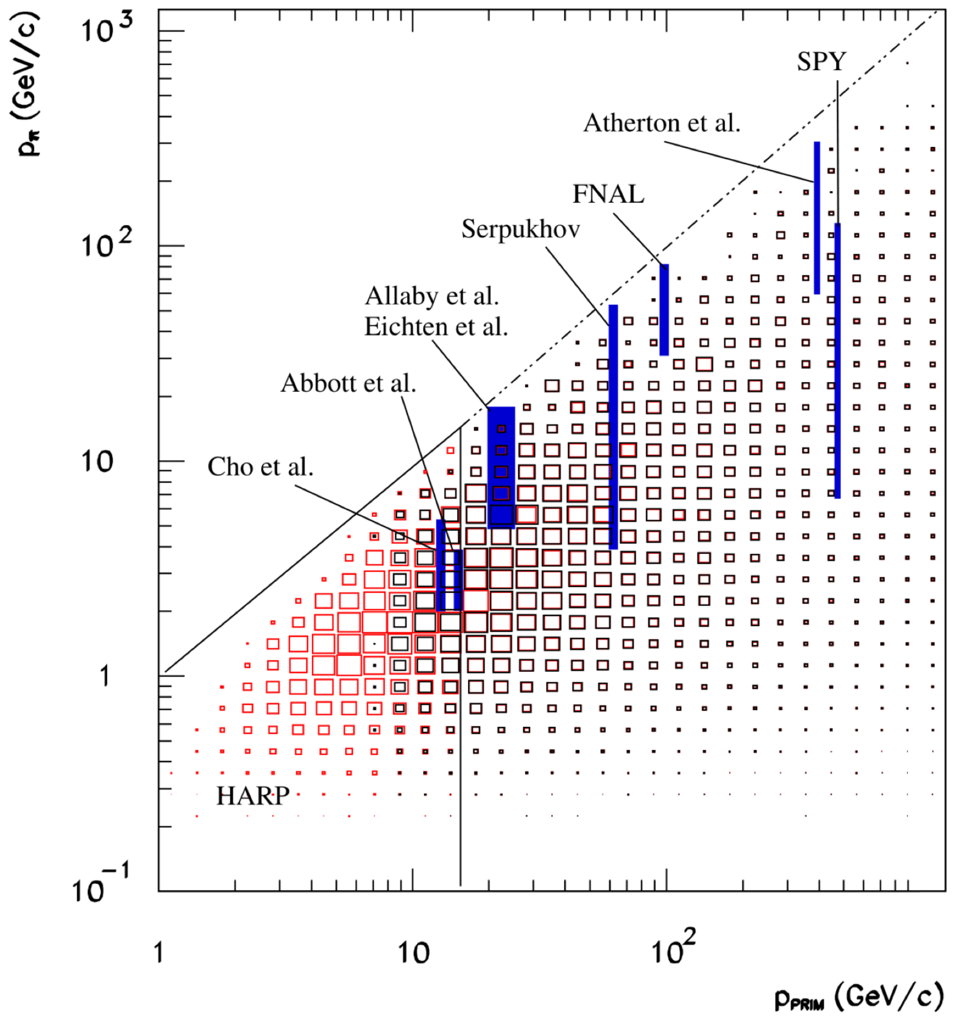
- Good agreement above tens of GeV for muon neutrinos
- Some tension between calculations at the horizon in electron neutrinos
- Affected by K/Pi, K^{+-}/K^0_L ratios

AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, 1806.04140 (2018)

Hadronic uncertainties: re-spin of Barr et al. approach

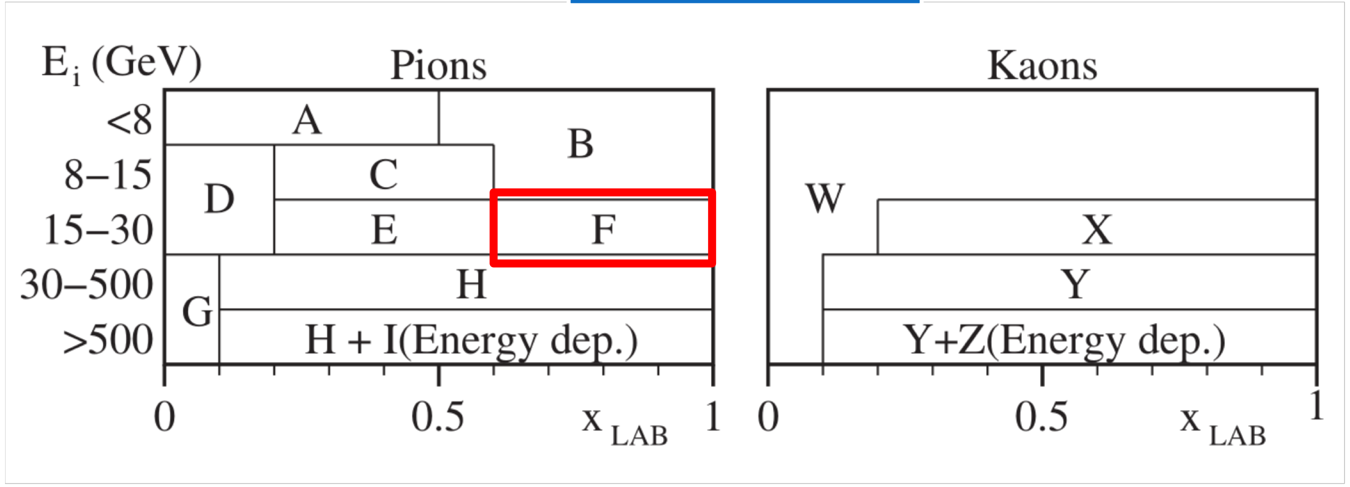
- "*Uncertainties in atmospheric neutrino fluxes*", G. D. Barr, S. Robbins, T. K. Gaisser, and T. Stanev, Phys. Rev. D 74, 094009 (2006) (extensive discussion also in Sanuki et al. PRD 75 (2007)
- Cut phase-space in regions/slices in E_{lab} and x_{lab} and **assign** uncertainty to each slice (uncorrelated)
- Uncertainty assigned by hand and not derived from data. Assignment based on availability of data, not how well the model [TARGET2.1] describes it
- Many "free" parameters with unclear correlations

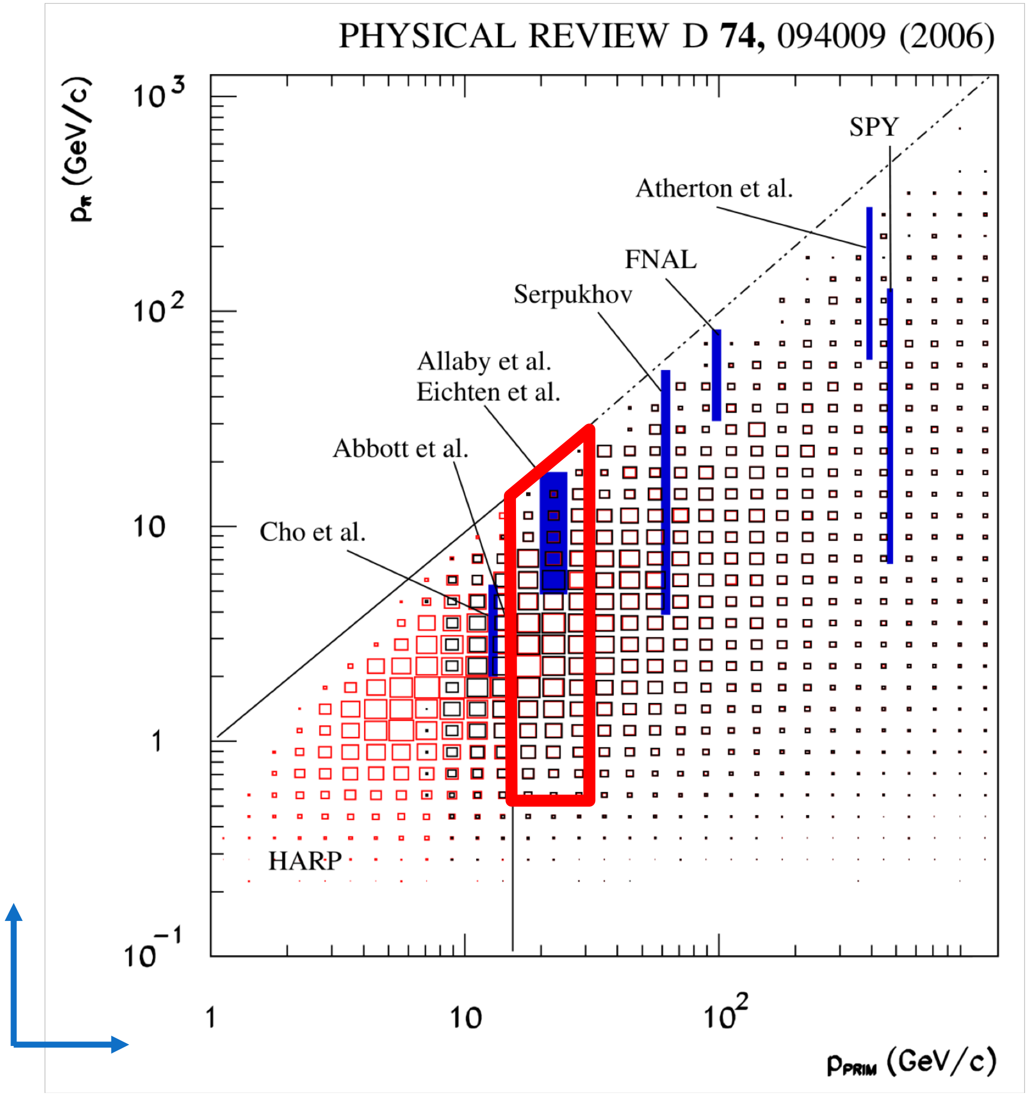
PHYSICAL REVIEW D **74,** 094009 (2006)



Phase space regions

"Barr regions"

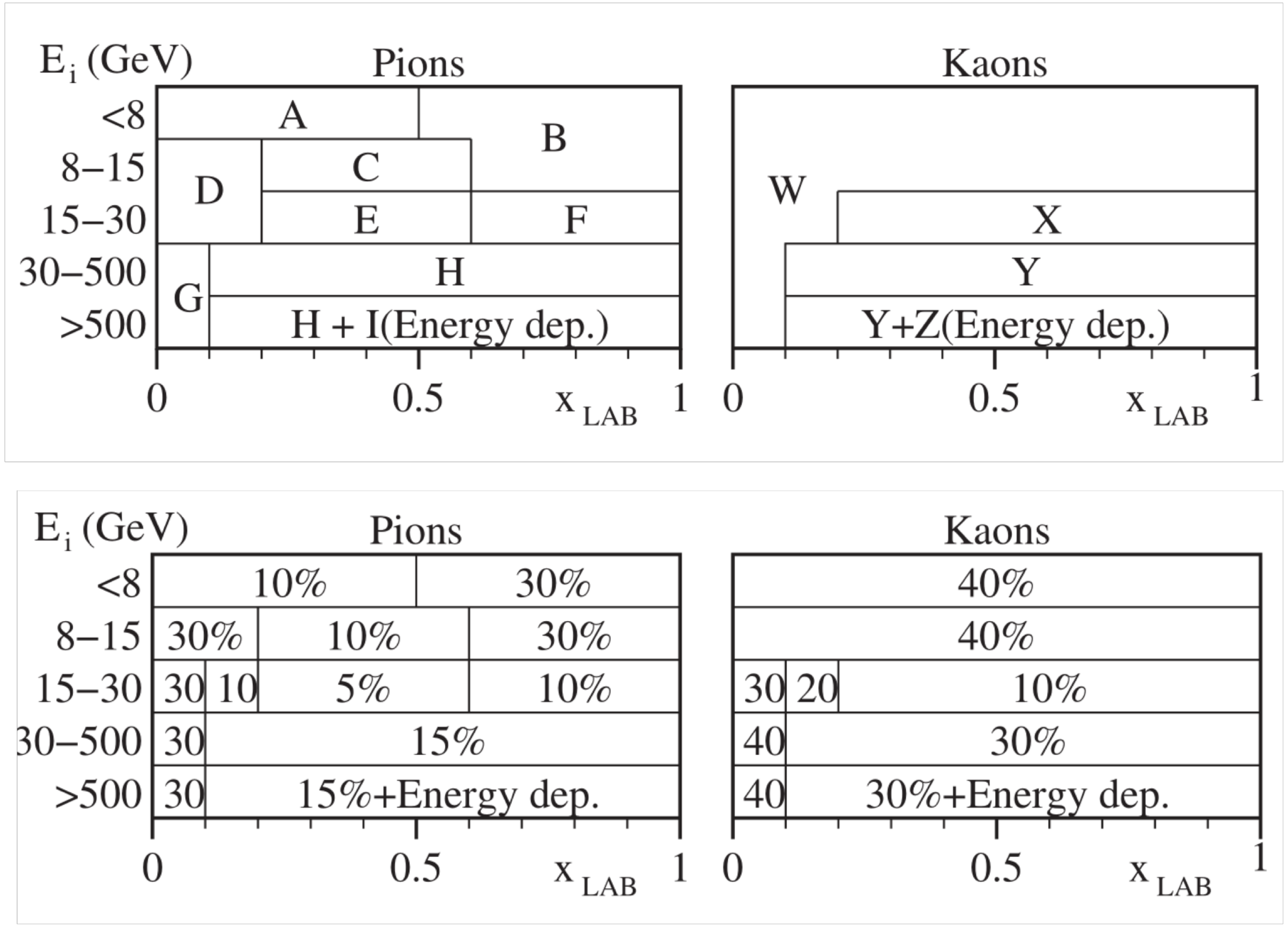




Same axes as in the MCEq matrices

MCEq-based implementation

“Barr regions”



- Compute partial derivatives wrt. phase-space regions (Taylor expansion), i.e. $\frac{\partial \Phi_\nu}{\partial \mathrm{W}}$
- No correlations between phase-space regions (as in Barr et al.) or add. correlations

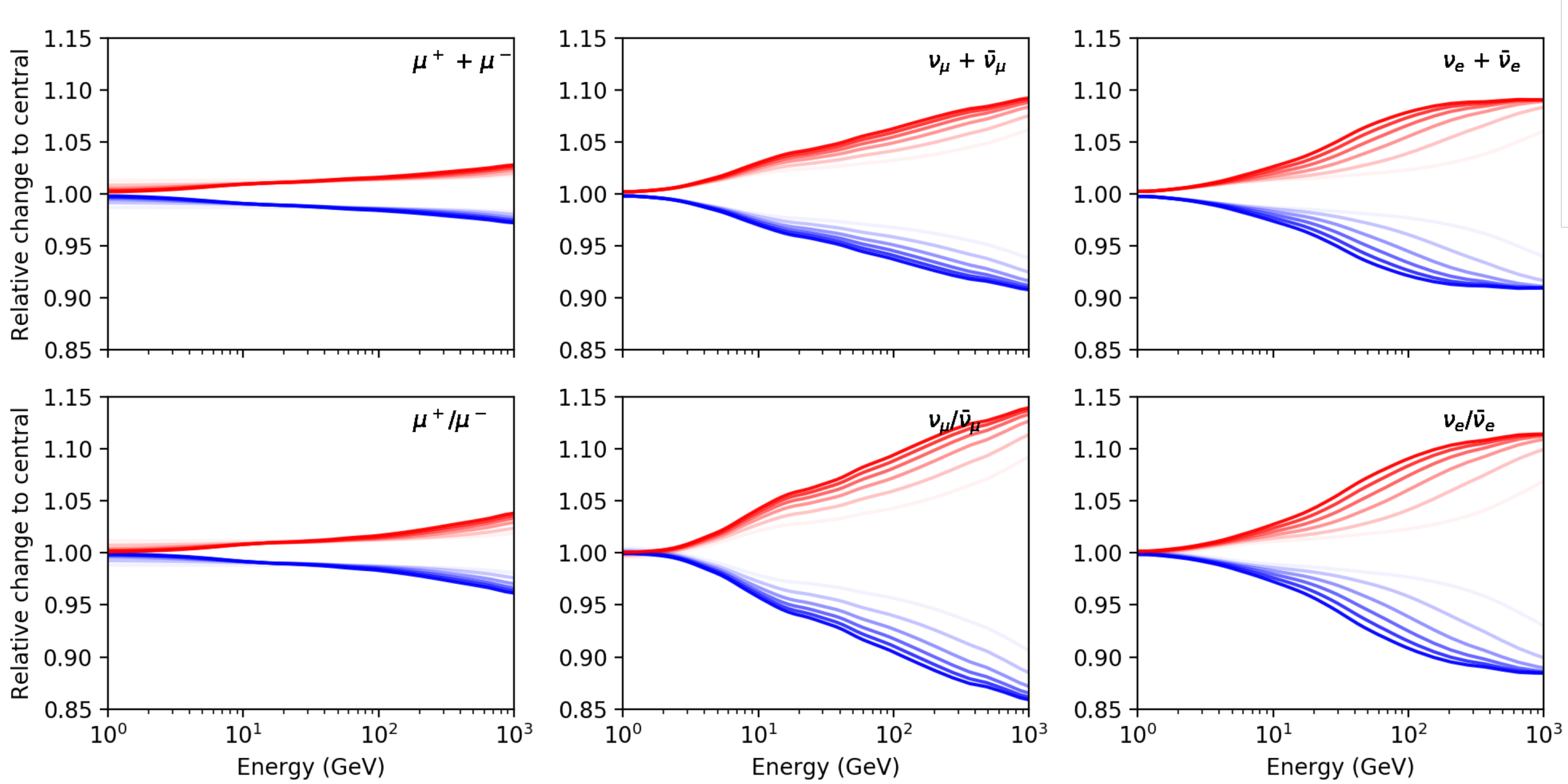
Elements of Jacobian (numerical)

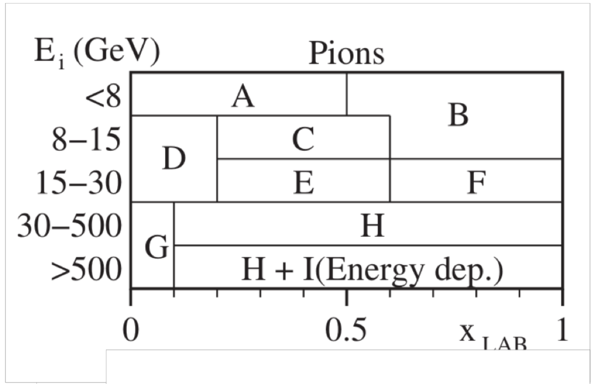
$$J_{E_i j} = \frac{\partial \Phi_\nu(E_i)}{\partial p} = \frac{\Phi_\nu(\delta p_j +) - \Phi_\nu(\delta p_j -)}{2\delta p_j}$$

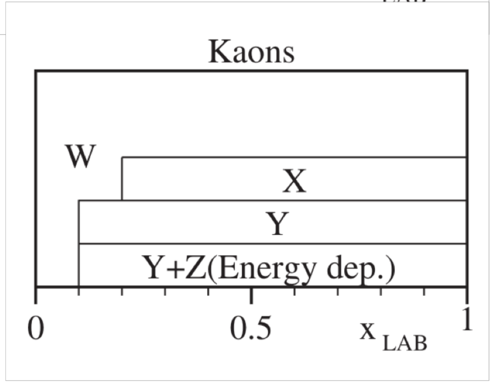
Error propagation

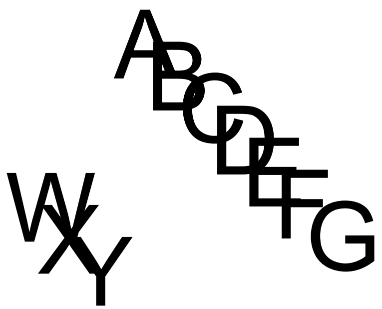
$$\mathrm{cov}[\Phi_\nu(E_i), \Phi_\nu(E_j)] = \sum_{mn} J_{E_i m} J_{E_j n} \mathrm{cov}[p_m, p_l]$$

… impact on flux

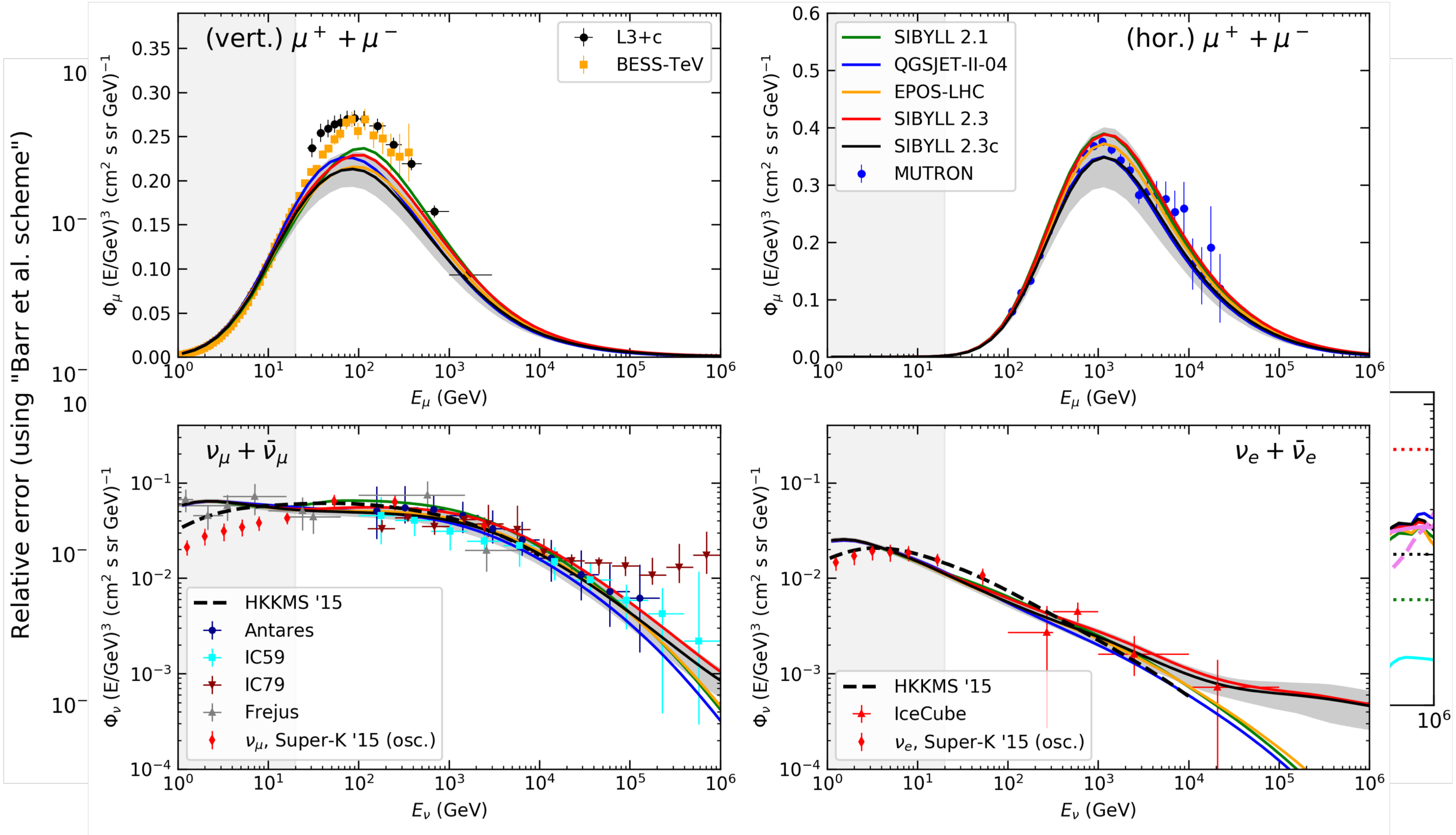




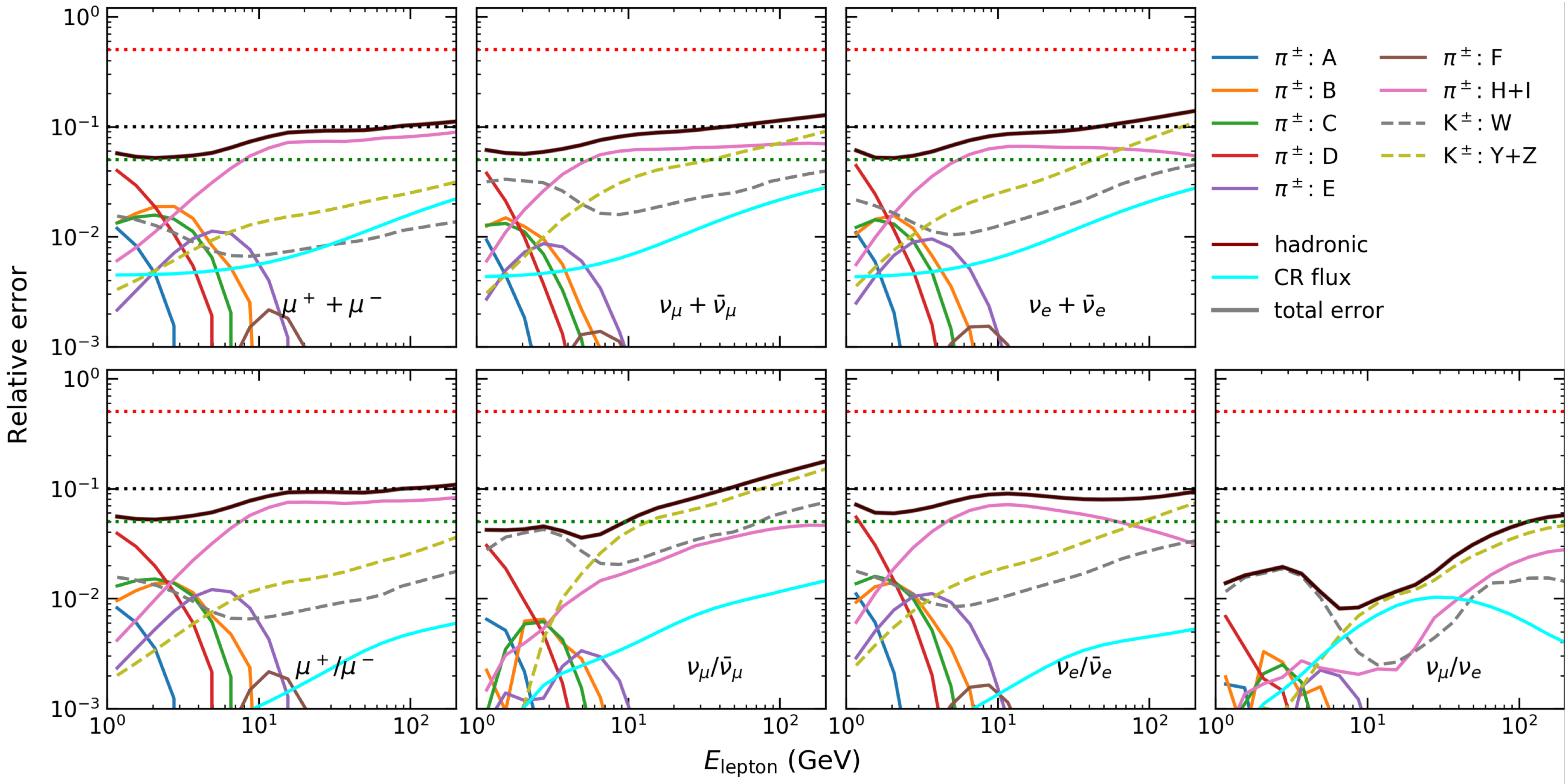




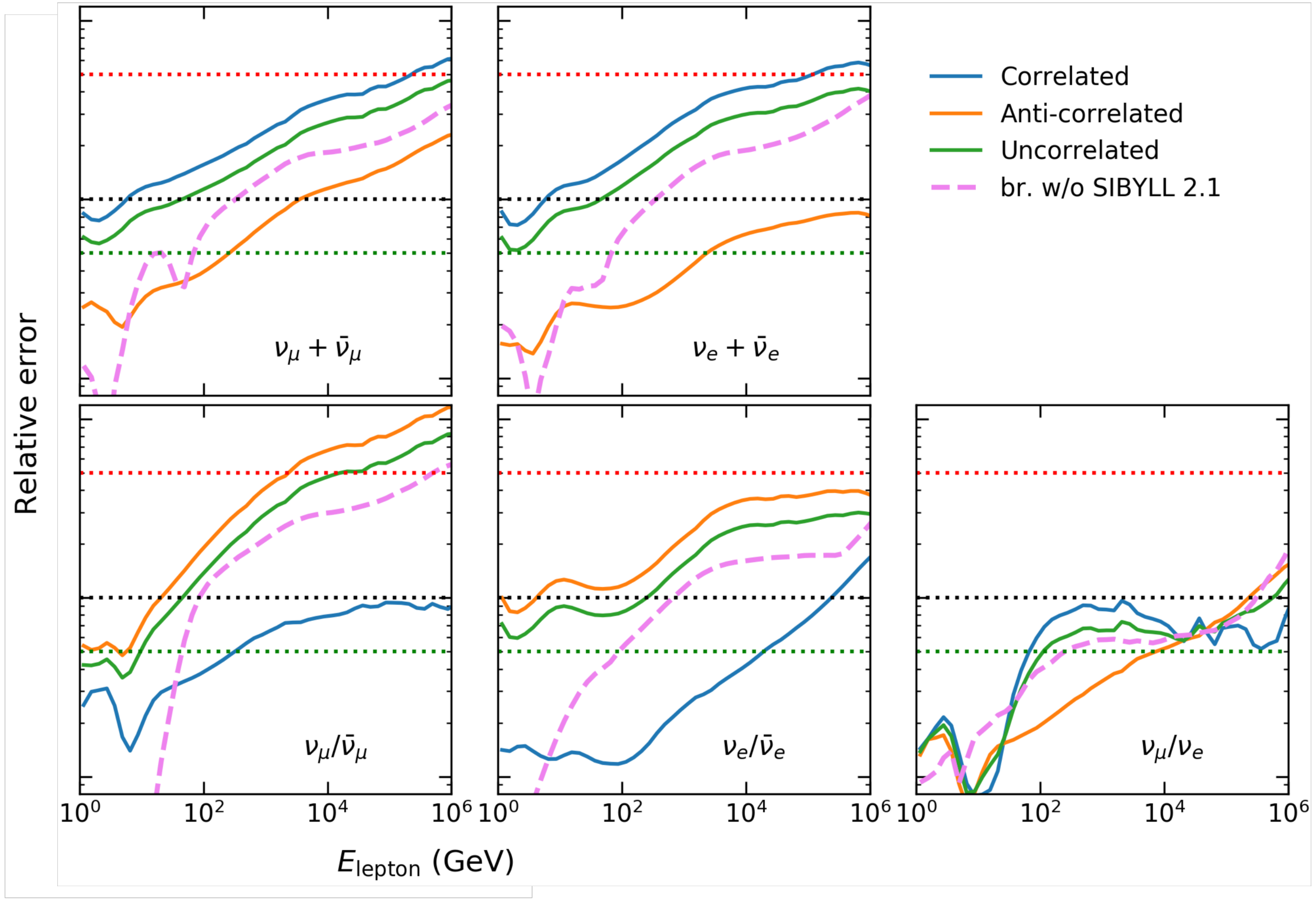
Computation of error bands through error propagation



Contribution of individual “Barr groups”



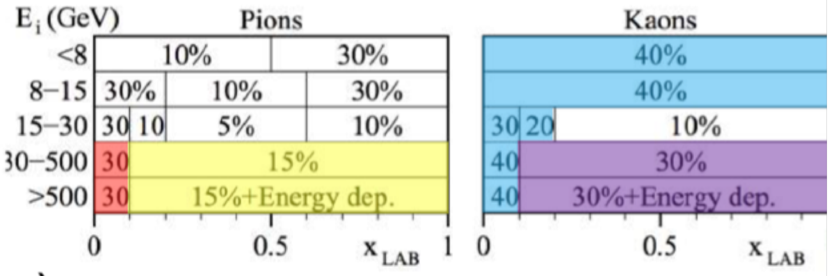
Correlations between phase-space patches unclear



Examples	For one "Barr" - parameters
symmetric	$p\pi^+\uparrow$ $n\pi^+\uparrow$ $p\pi^-\uparrow$ $n\pi^-\uparrow$
asymmetric	$p\pi^+\uparrow$ $n\pi^+\uparrow$ $p\pi^-\downarrow$ $n\pi^-\downarrow$
uncorrelated	$p\pi^+\uparrow$ $n\pi^+0$ $p\pi^-0$ $n\pi^-0$

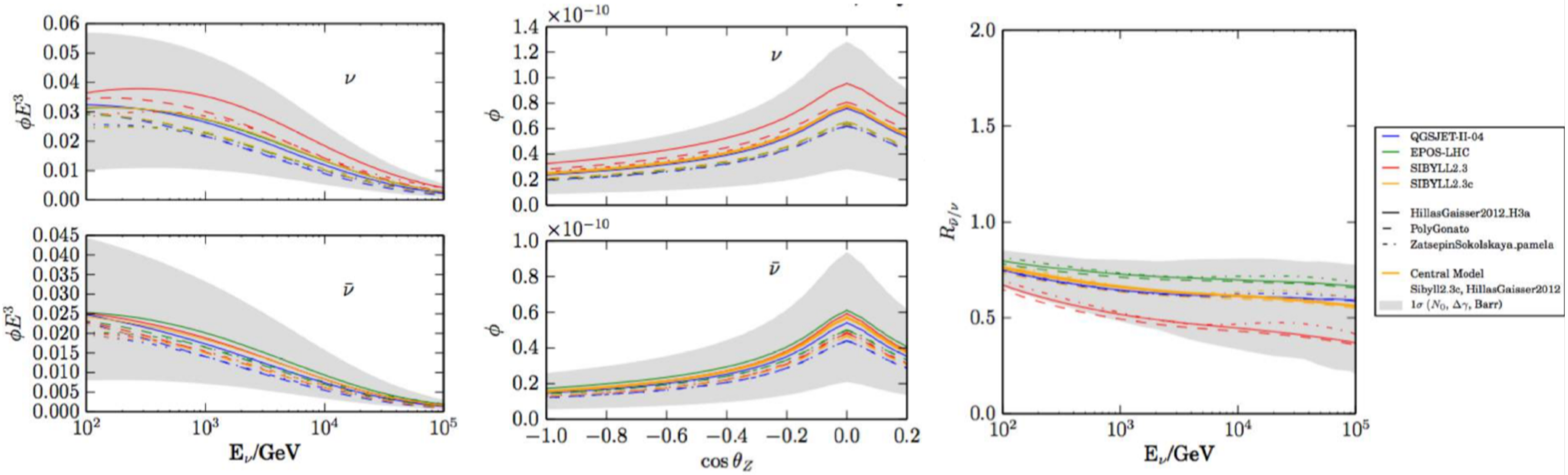
- The production of charged secondaries is physically not independent
- It is very difficult to extract this information from hadronic interaction models directly

Flux systematics with MCEq



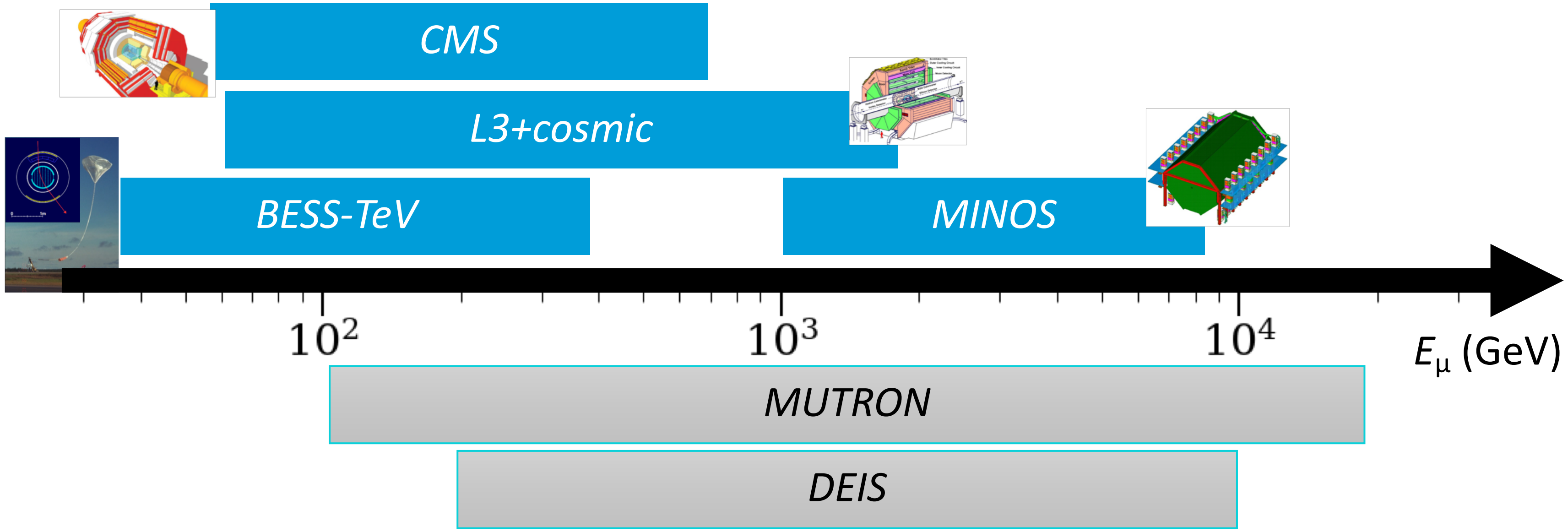
- Implementation in analyses:
  - Treat **“Barr blocks”** as nuisance parameters (Gaussian priors)
    - Also **normalisation** and **spectral index**
  - Re-calculate flux using MCEq for each case and re-weight simulation accordingly
  - Fit to find best fit physics and nuisance parameters to data

Future is now ☺



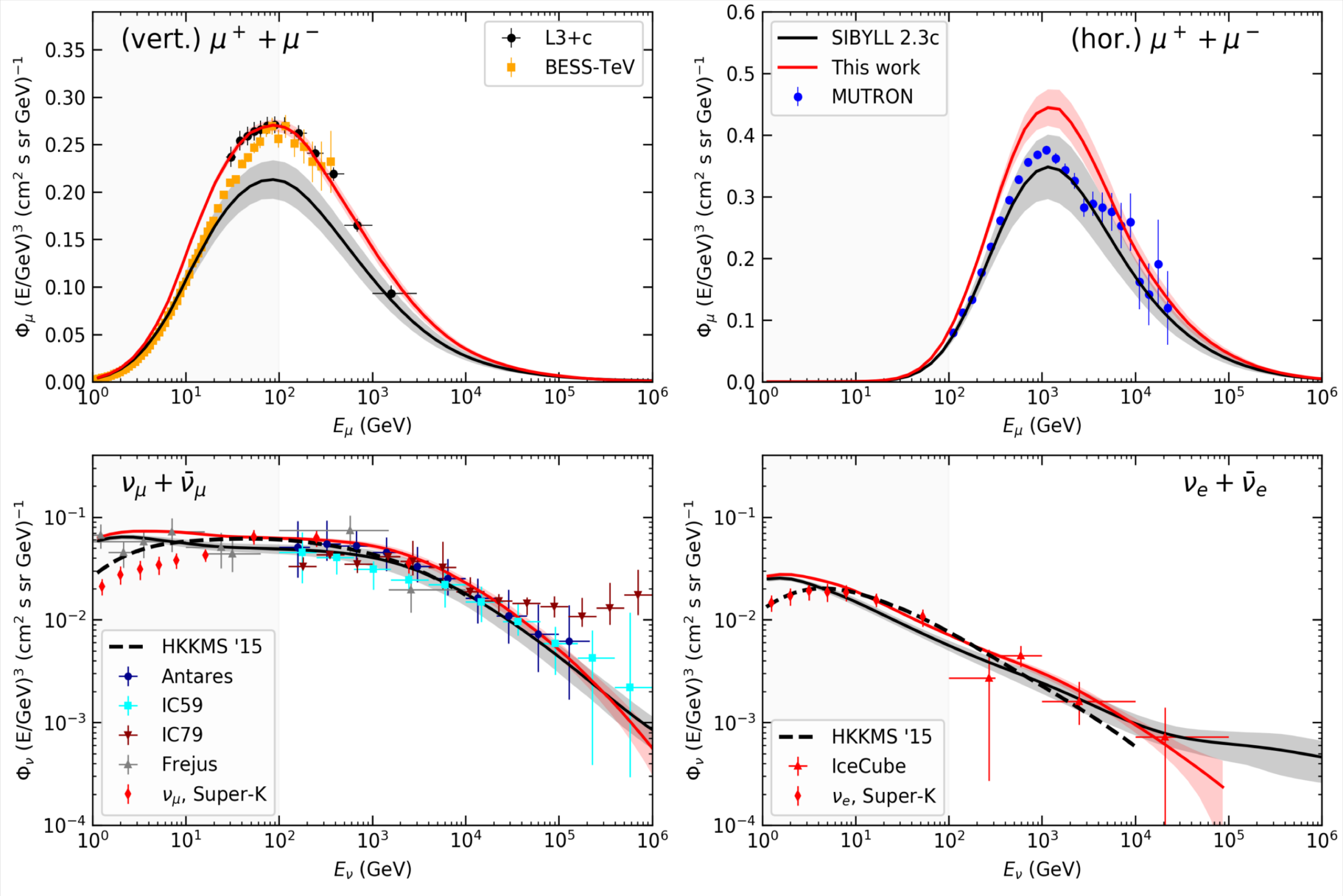
Calibration of ν uncertainties with μ data

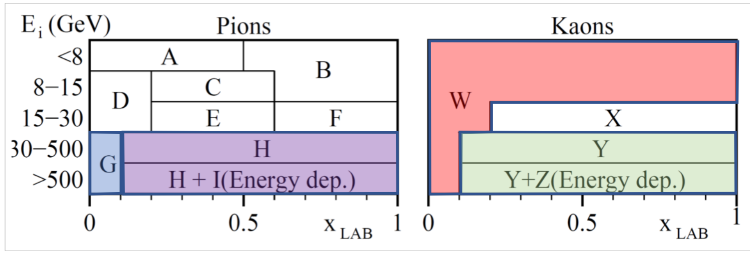
Cosmic ray flux: Global Spline Fit [7] + Atmospheric conditions + Hadronic interactions: SYBILL 2.3c = Flux of muons and neutrinos

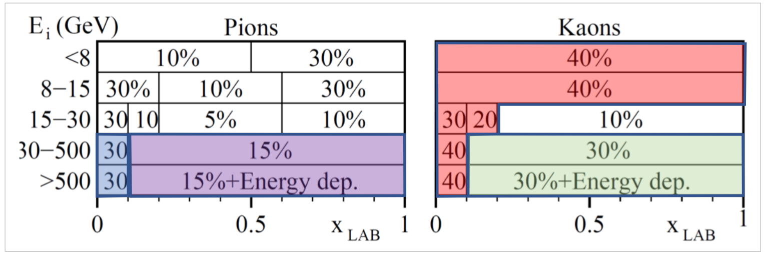


J.P. Yanez Garza and AF, Neutrino 2018

Results of the fit on fluxes

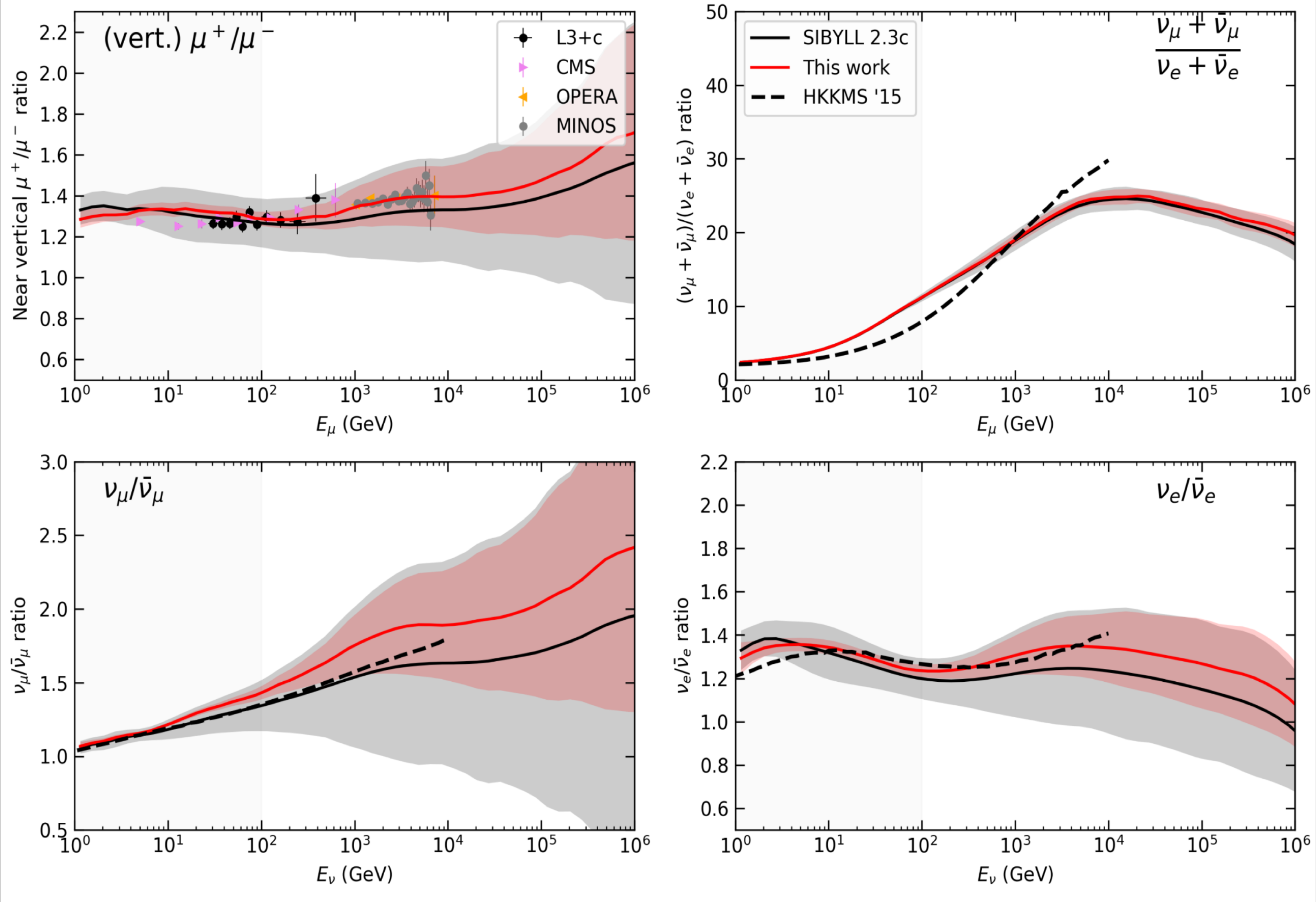


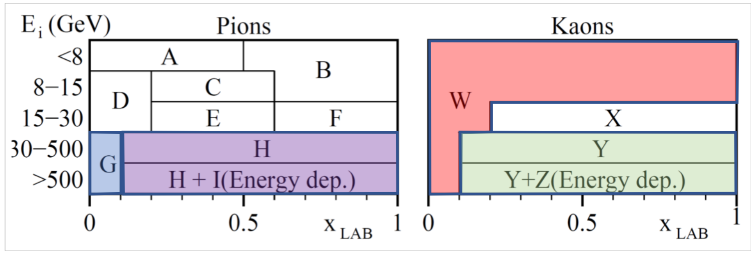


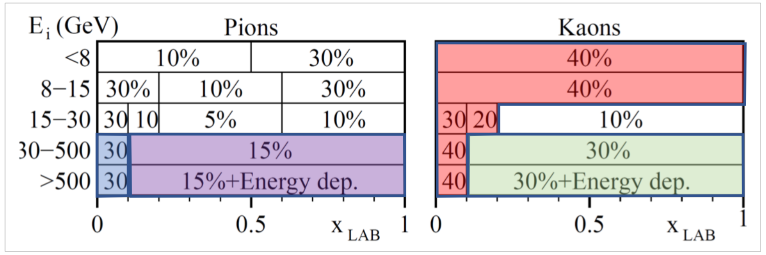


Name	value, error
π^+: G	0.13±0.10
π^+: H	0.30±0.03
K^+: W	0.14±0.08
K^+: Y	0.47±0.07
π^-: G	0.44±0.08
π^-: H	0.16±0.04
K^-: W	0.20±0.10
K^-: Y	0.11±0.07

Results of the fit







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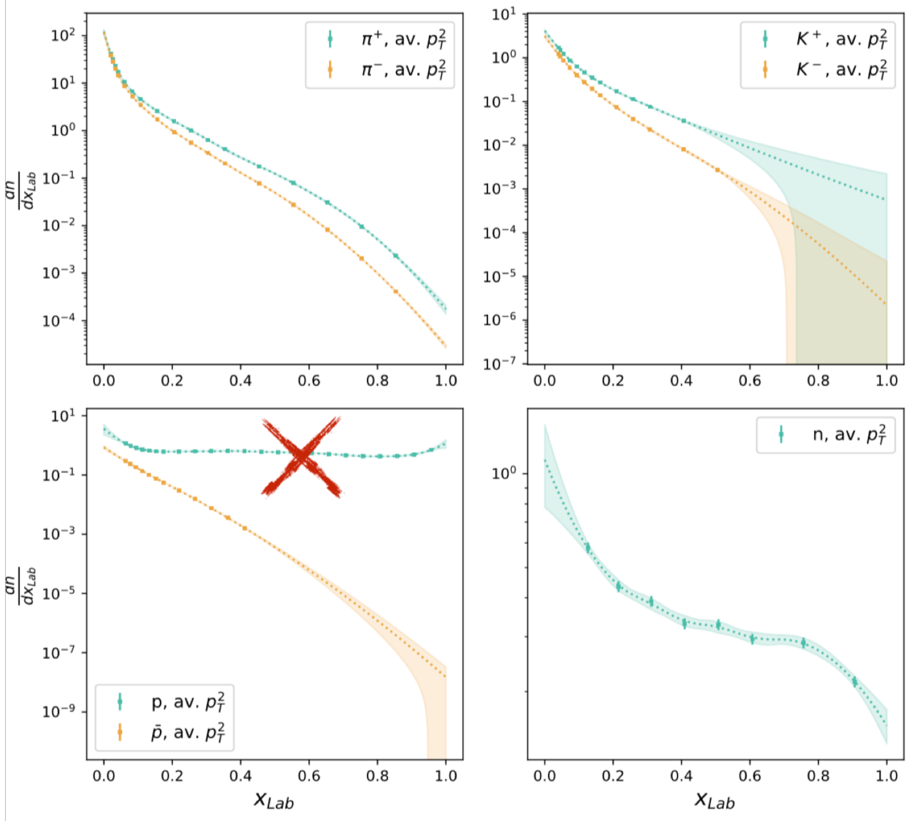
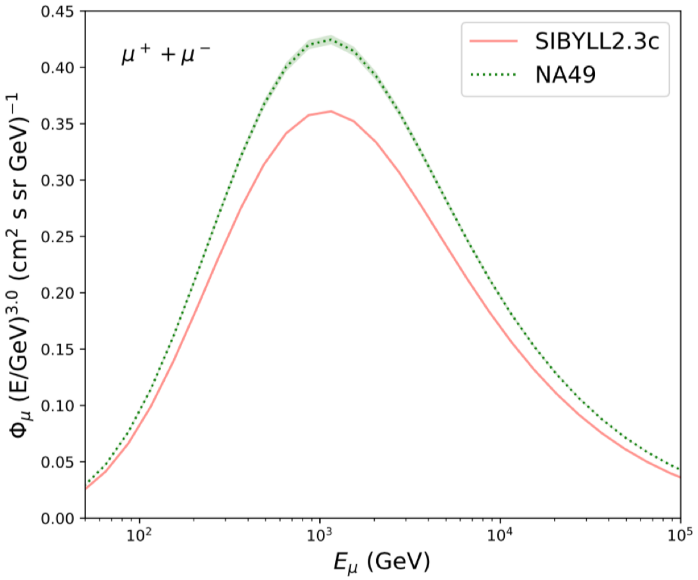
Alternative approach, data-driven inclusive interaction model

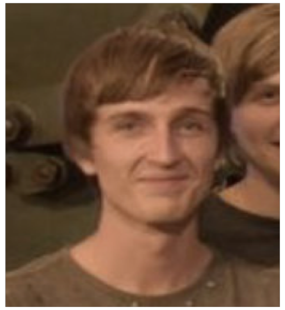
NA49 pp data (158GeV)



Experiment	Interaction	E_p [GeV]	yields
NA49	pp	158	$\pi^{\pm}, K^{\pm}, \overset{(-)}{p}, n$
NA49	pC	158	$\pi^{\pm}, \overset{(-)}{p}, n$
NA61/SHINE	pC	31	$\pi^{\pm}, K^{\pm}, K^0_S, \Lambda$
NA61/SHINE	pp	20, 31, 40, 80, 158	$\pi^{\pm}, K^{\pm}, \overset{(-)}{p}$
NA61/SHINE	$\pi^- C$	158, 350	ρ^0, ω, K^{*0}
NA61/SHINE (upcoming)	$\pi^- C$	158, 350	$\pi^{\pm}, K^{\pm}, \overset{(-)}{p}$



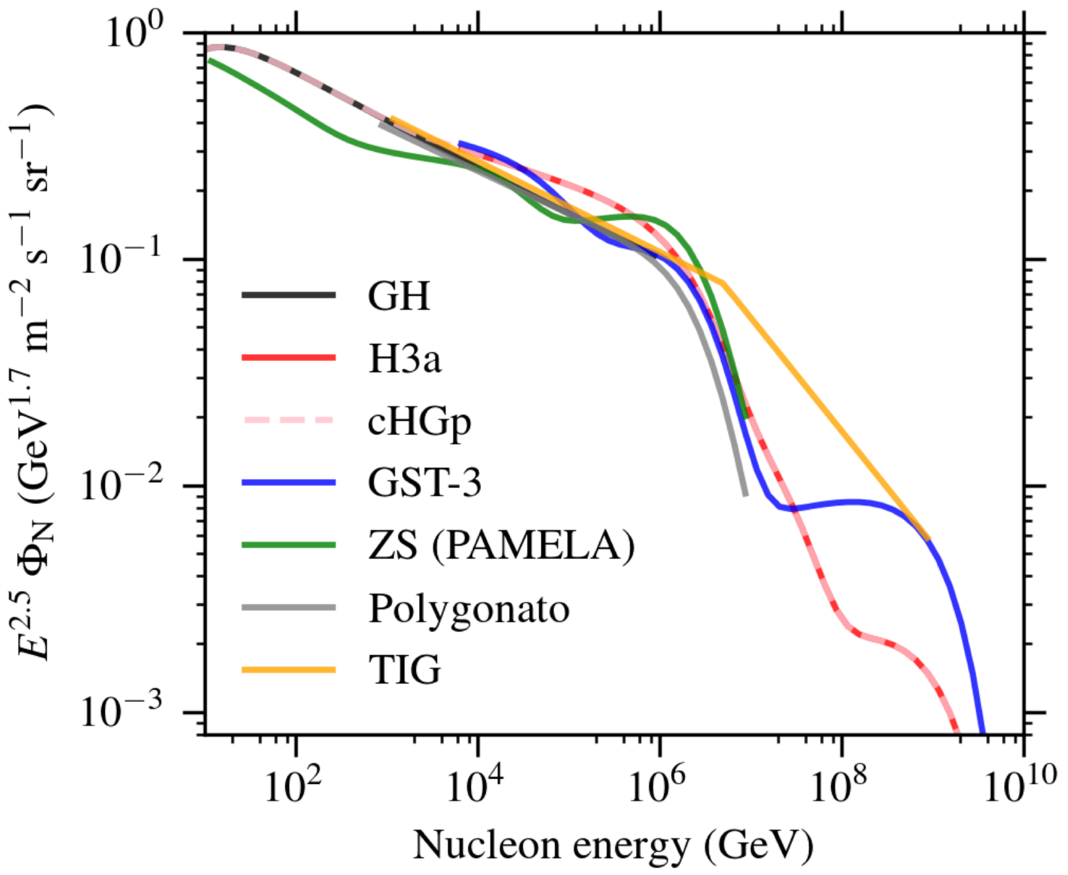
p+p → part+X



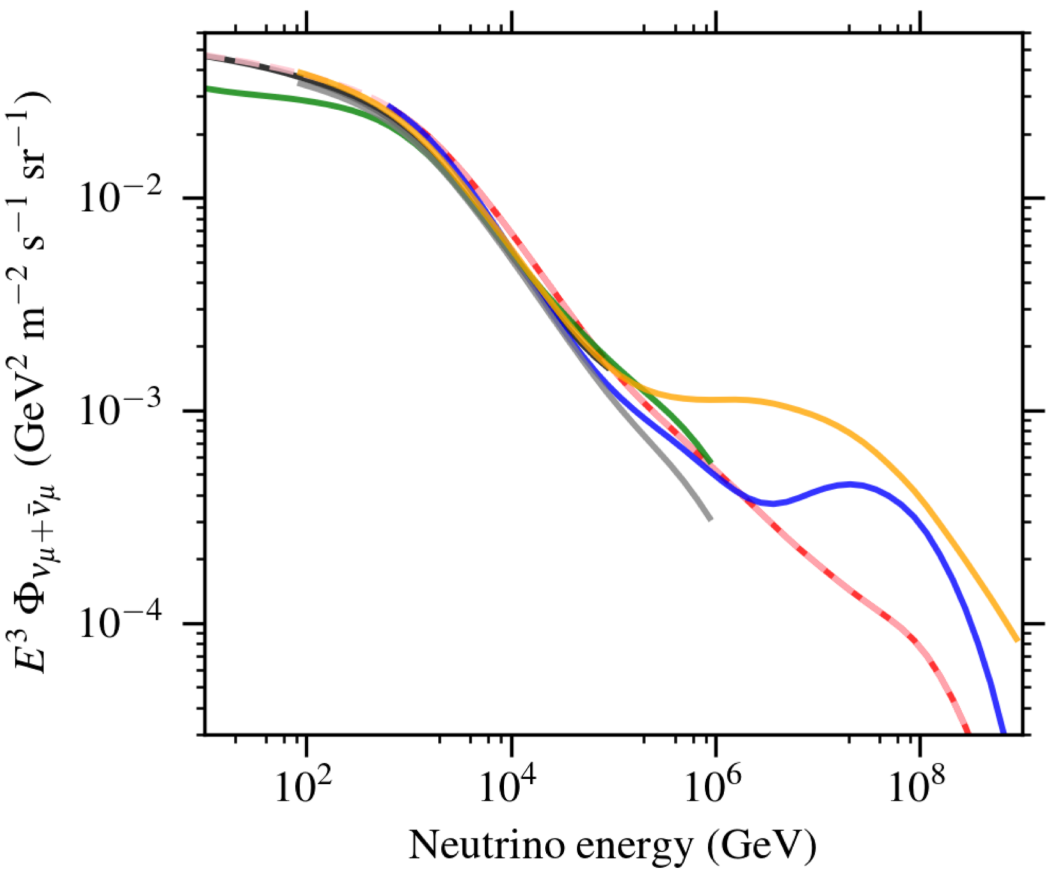
Matthias Huber

Cosmic ray flux uncertainties – ‘bracketing’ overestimates

Cosmic ray nucleon flux



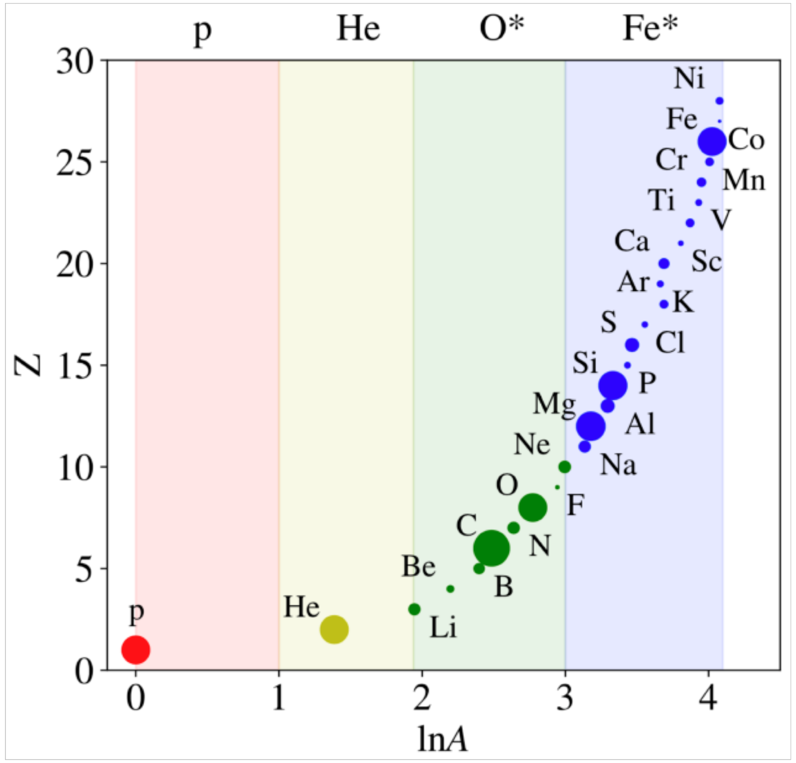
Atmospheric neutrino flux

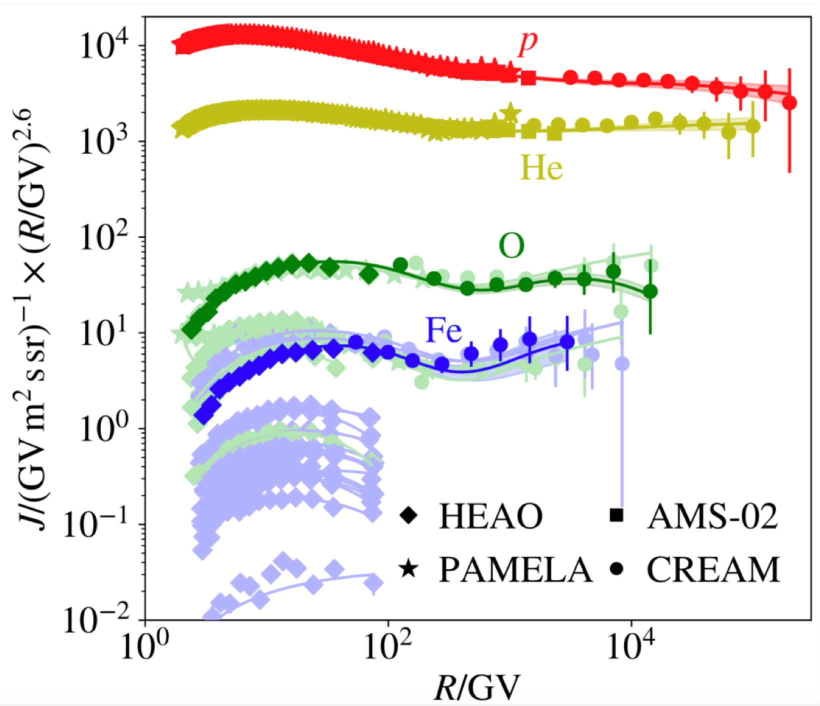


Global Spline Fit – fit to direct & indirect observations

H. Dembinski, AF, T. Gaisser
PoS(ICRC2017)533

- Fit **four** independent mass groups, which cover equal ranges in lnA: proton (p), helium (He), oxygen group (O*), and iron group (Fe*)
- Assumption: this holds **at all energies**
- One leading element *L* per group described by smooth spline curve
- Other elements *j* in a group kept in constant ratio: $J_j(R)/J_L(R) = const.$

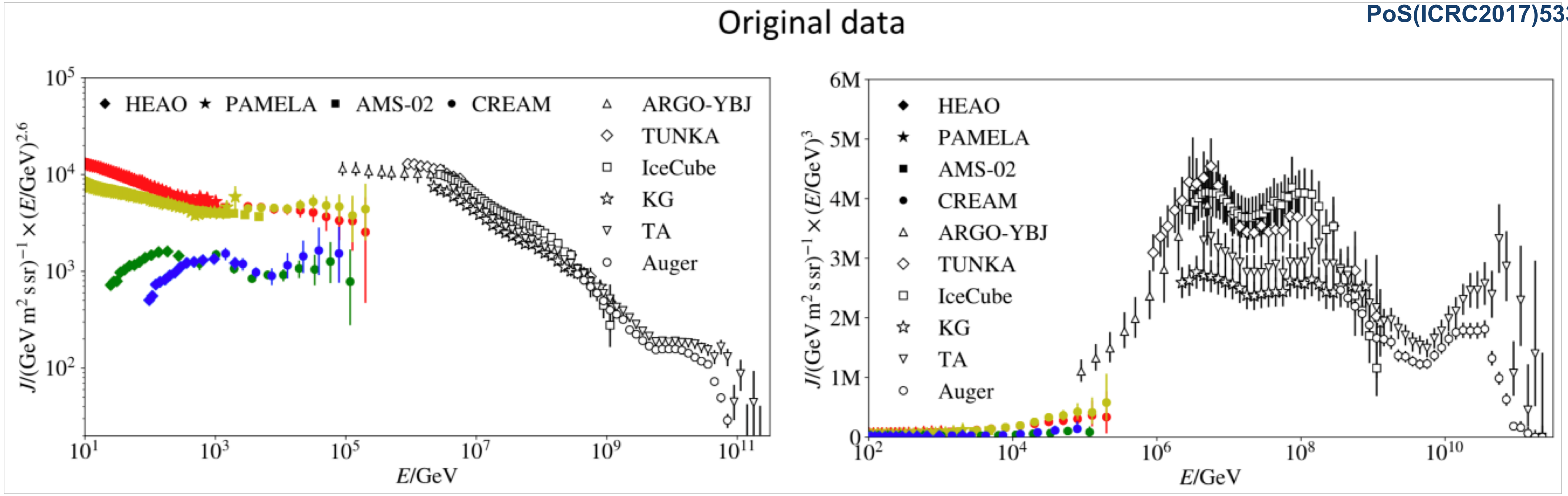




Mass sensitivity of air-shower experiments is ~ lnA

Handling energy-scale uncertainty

H. Dembinski, AF, T. Gaisser
PoS(ICRC2017)533



- The determination of **energy scale in air-shower experiments is uncertain**
- This is caused by inconsistencies of **hadronic interaction models**
- Fit adjusts energy scales **within systematic uncertainties** of the experiment

$$\tilde{J}(\tilde{E}) = J(E)\frac{\mathrm{d}E}{\mathrm{d}\tilde{E}} = J\left(\frac{\tilde{E}}{1+z_E}\right)\frac{1}{1+z_E}$$

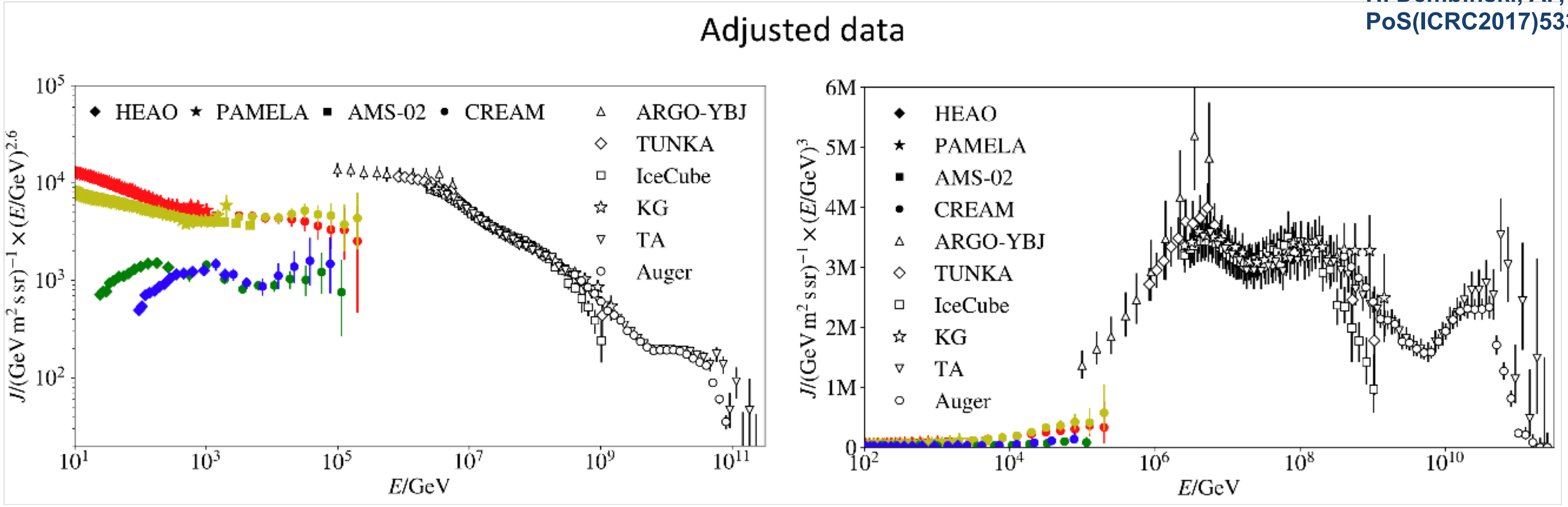
Flux distortion caused by energy-scale offset z_E

$$S = \sum_i z_i^2 + \sum_j \left(\frac{z_{Ej}}{(\sigma[E]/E)_j}\right)^2$$

Flux residuals Energy-scale offset residuals

Handling energy-scale uncertainty

H. Dembinski, AF, T. Gaisser
PoS(ICRC2017)533



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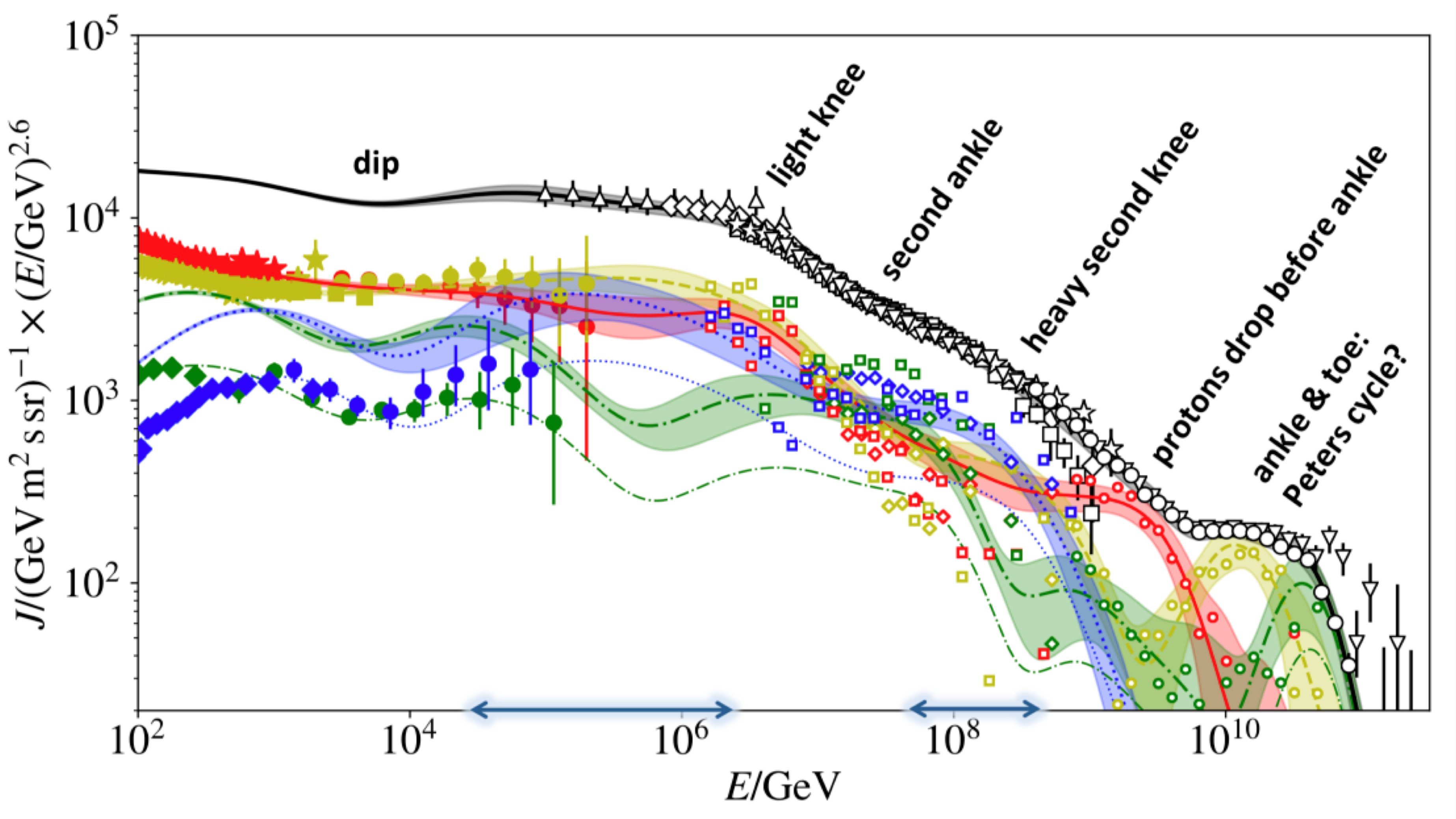
Flux residuals Energy-scale offset residuals

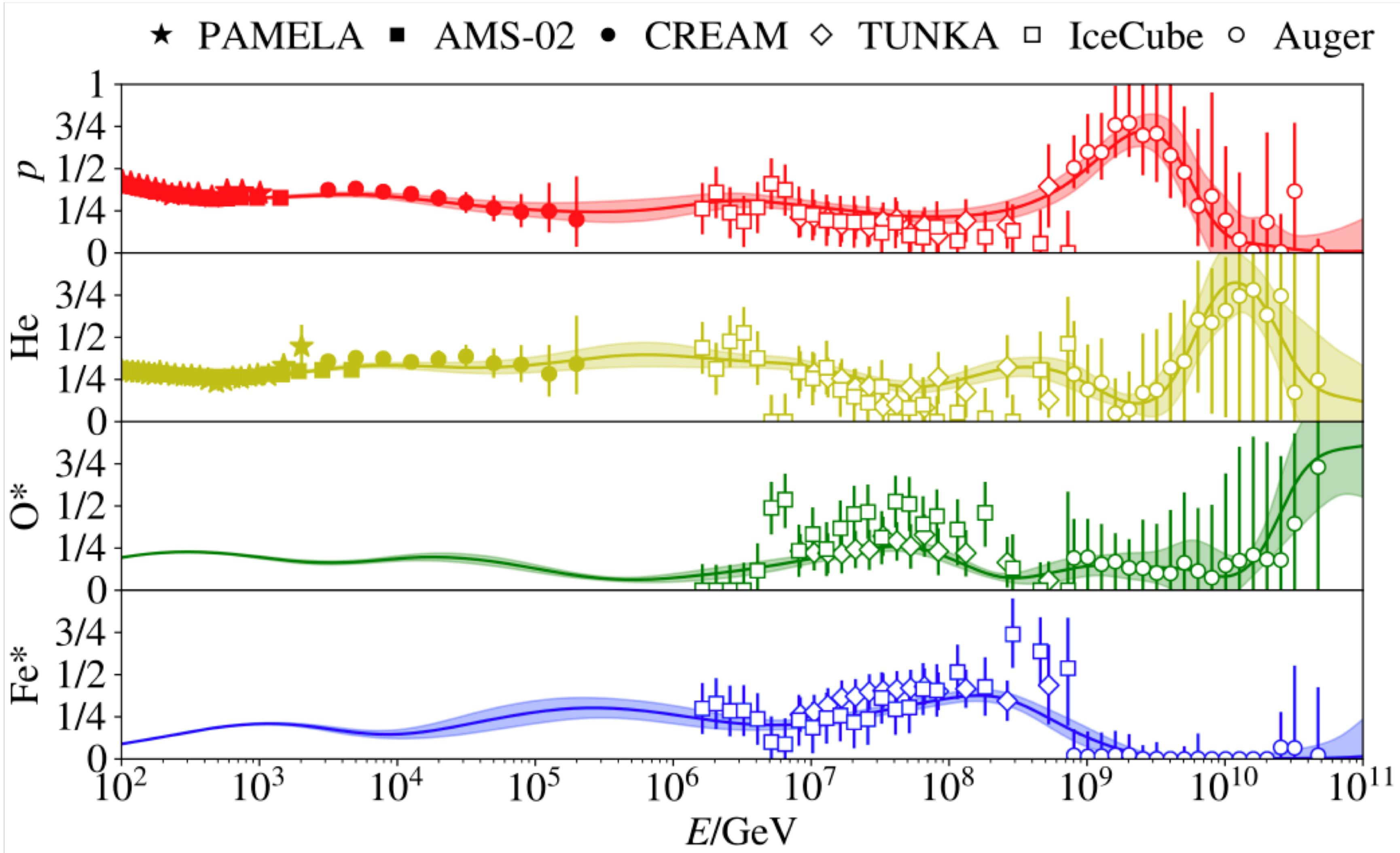
The Global Spline Fit



More composition data needed

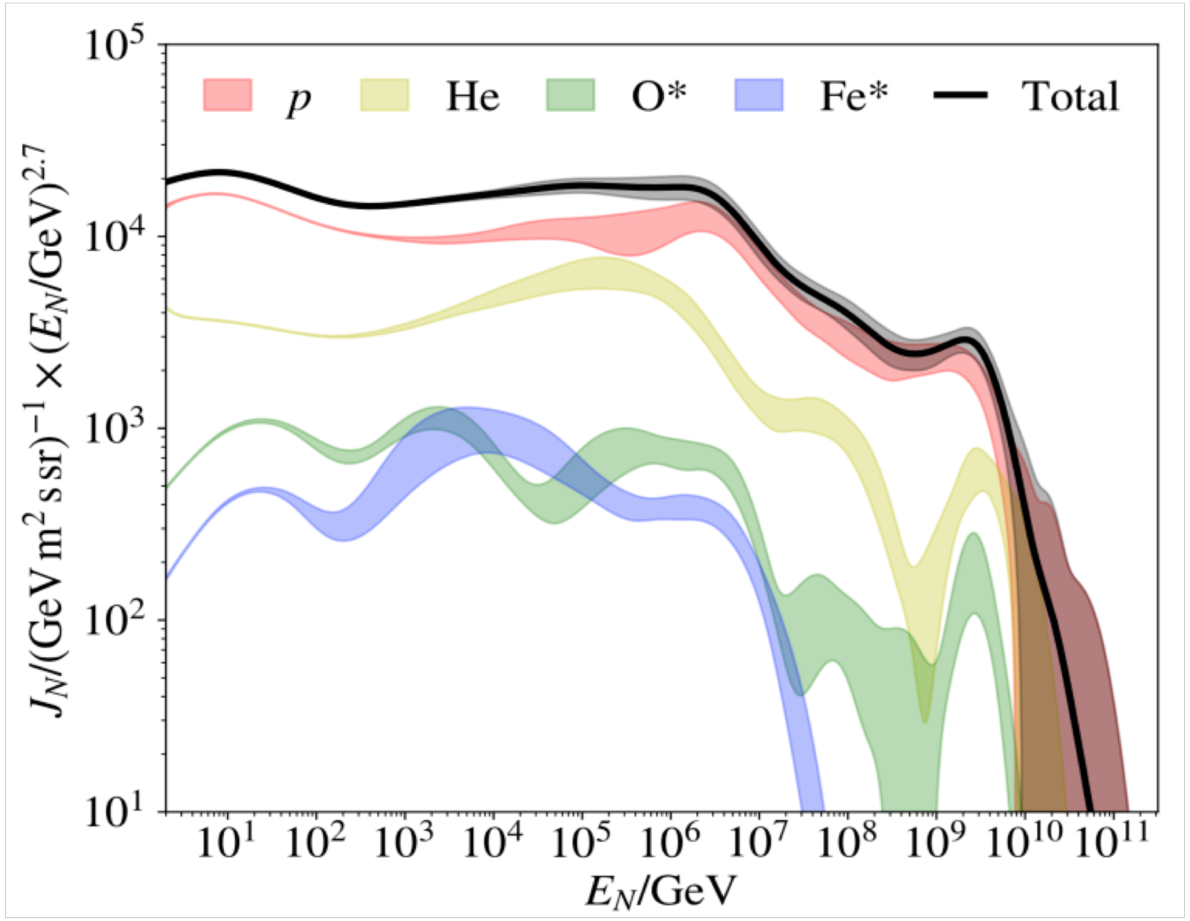
Fitted composition data

4-mass group experiments

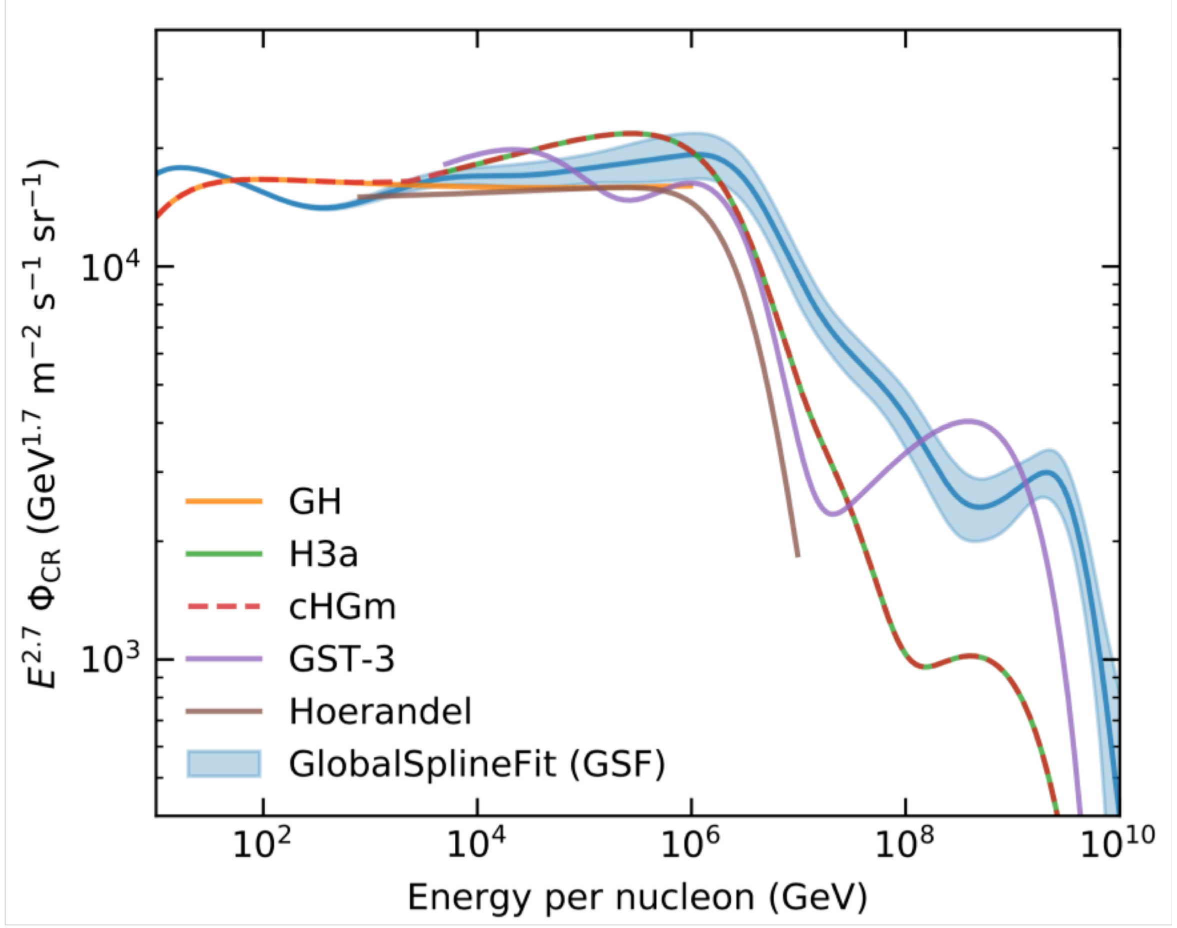


Derived result: nucleon flux

AF et al, PoS(ICRC2017)1019

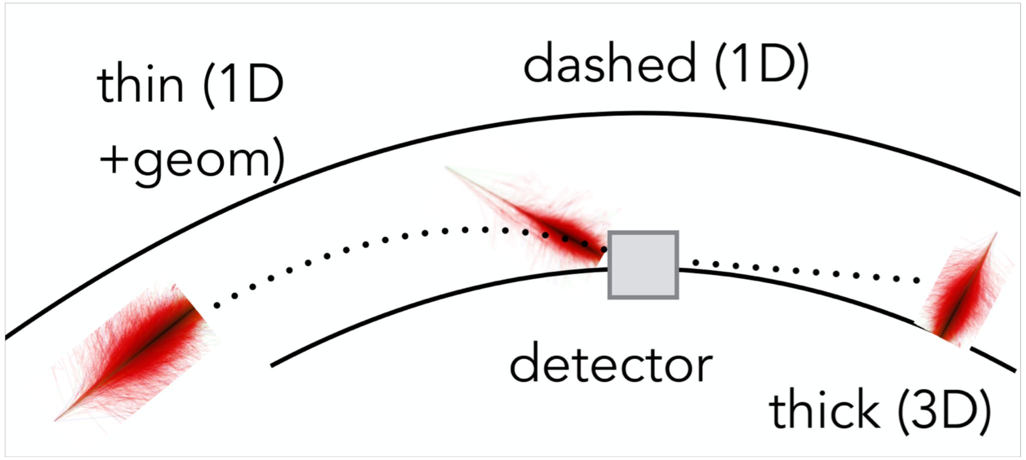


Dominated by proton flux. Details of sub-leading elements not important.



Harder spectrum at the knee due to lighter composition as assumed by 3-population models

Extension of these methods below few GeV – 3D vs 1D



A subset of 3D calculations

[1] G. Barr, P. Lipari, S. Robbins, and T. Stanev, International Cosmic Ray Conference 3, 1411 (2003).

[2] M. Honda, T. Kajita, K. Kasahara, and S. Midorikawa, Phys. Rev. D 83, (2011).

[3] M. Honda, T. Kajita, K. Kasahara, S. Midorikawa, and T. Sanuki, Phys. Rev. D 75, (2007).

[4] [1] G. Battistoni, A. Ferrari, P. Lipari, T. Montaruli, P. R. Sala, and T. Rancati, Astroparticle Physics **12**, 315 (1999).

[5] J. Wentz, I. M. Brancus, A. Bercuci, D. Heck, J. Oehlschläger, H. Rebel, and B. Vulpescu, Phys. Rev. D 67, 073020 (2003).

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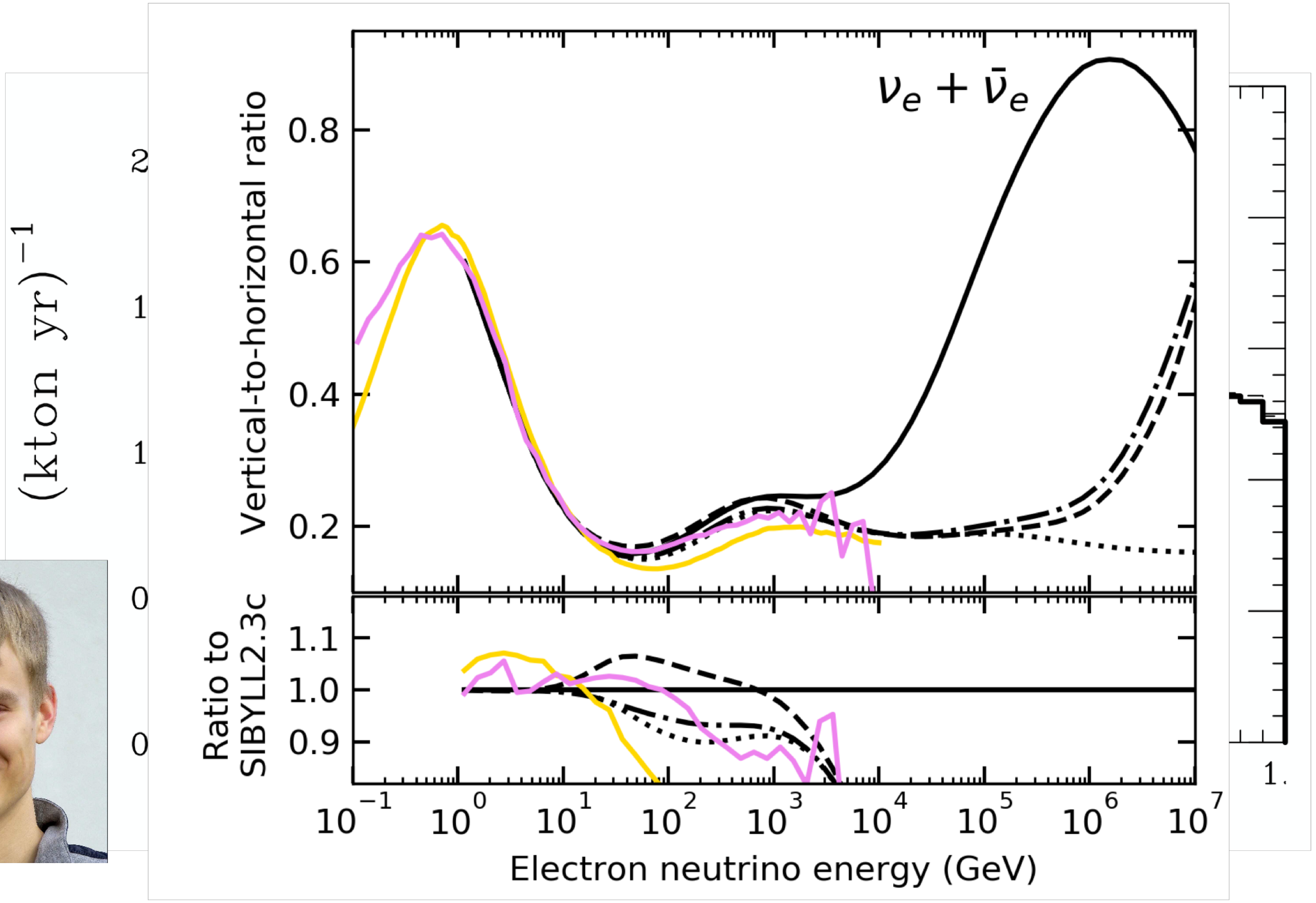

3D needed < ~ 5 - 10 GeV

Conclusions and future path

- Current atmospheric neutrino detectors cover **9 orders of magnitude** in energy (MeV-PeV) → **challenge** for modeling!
- **Unsolved problems remain**, in particular hadronic interactions, but data-driven techniques can improve the precision as in the HKKM calculations
- **High-precision** (and high-performance) calculations available through MCEq that well match full Monte Carlo
- This allows for **new approaches towards flux systematics** in data analysis, replacing effective parameters with physical parameters

Atmospheric flux		
ν flux template	discrete (7)	
ν / $\bar{\nu}$ ratio	continuous	0.025
π / K ratio	continuous	0.1
Normalization	continuous	none[1]
Cosmic ray spectral index	continuous	0.05
Atmospheric temperature	continuous	model tuned