

# Next-to-leading order QCD calculation of jet production in pp collision

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Tuebingen, Germany

## Plan of Talk

- large  $p_T$  inclusive jet : calculation at NLO
- Analytic calculation at NLO : formalism
- Identified hadron in a jet
- Numerical results
- Conclusion

# Single Inclusive Jets in Hadronic Collision

Large  $p_T$  inclusive jet in pp collision : cross section can be written in a factorized form

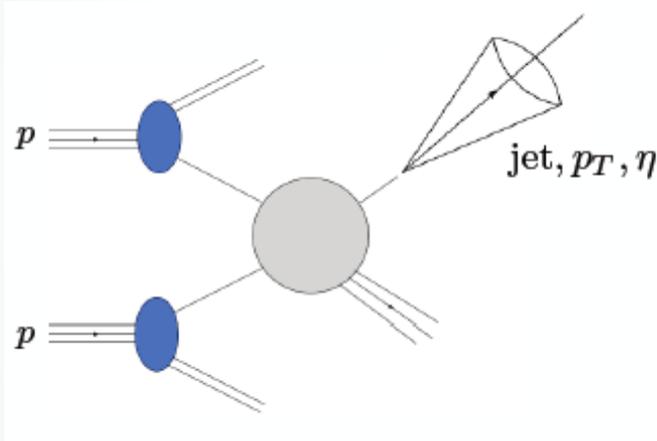


Figure by F. Ringer

Ellis, Kunszt, Soper (1990);

Aversa, Greco, Chiappetta, Guillet (1990)

Jaeger, Stratmann, Vogelsang (2004)

Log R resummation for semi-inclusive jets using SCET : Kang, Ringer, Vitev (2016)

Many more.....

# Single inclusive large $p_T$ jets in pp collision

Jets: Important tools at colliders for example RHIC and LHC

Single inclusive jet in polarized pp collision : probe for spin dependent parton distributions , in particular gluon distribution

No unique way to define a jet : depends on jet algorithm

Two main classes of jet algorithms (i) successive recombination , (ii) cone

# Jet Algorithms

(i) Successive recombination algorithm ( $k_T$ , anti  $k_T$ ,  
cambridge-aachen algorithms)

Ellis and Soper , PRD (1993); Catani *et al* , Nucl. Phys. B (1993);  
Dokshizer *et al*, JHEP (1997); Cacciari, Salam and Soyez, JHEP  
(2008)

(ii) Cone algorithm

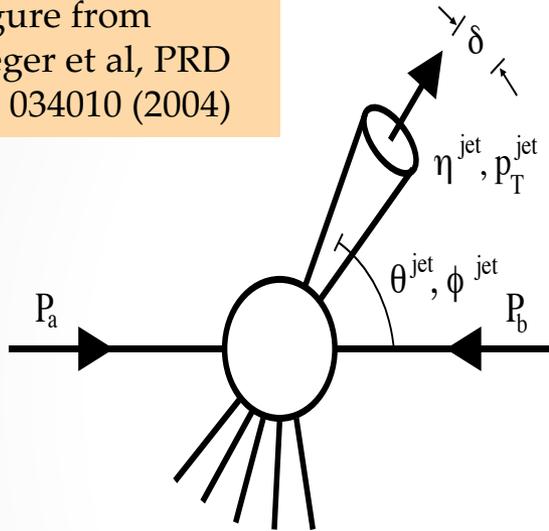
For a summary, see Blazey *et al* , “Proceedings of QCD and  
Weak Boson Physics “ (2005)

Problem with cone algorithm : midpoint cone and iterative  
cone algorithms are IR unsafe, becomes an issue at NNLO

Solution : seedless IR safe cone ; IR safe

# Cone Algorithm

Figure from  
Jaeger et al, PRD  
70, 034010 (2004)



Single inclusive jet production

$$pp \rightarrow \text{jet} X$$

Cone algorithm : all particles  $j$  that satisfy the following form a jet

$$R_{jJ}^2 = (\eta_j - \eta_J)^2 + (\phi_j - \phi_J)^2 \leq R^2$$

$R$  : jet cone aperture

$$\delta = \frac{R}{\cosh(\eta_J)} \quad \eta = -\log[\tan(\theta/2)]$$

Jet 4-momentum forms the axis of the cone; usually defined as the sum of 4-momenta of the particles forming the jet

# $k_T$ Type Algorithm

For each pair of objects (initially particles)  $j,k$ ; one defines a “distance”

$$d_{jk} \equiv \min(k_{T_j}^{2p}, k_{T_k}^{2p}) \frac{R_{jk}^2}{R^2},$$

Where  $p$  is a parameter that specifies the algorithm,  $k_{T_j}$  is the transverse momentum of particle  $j$  with respect to the beam direction,  $R$  is a parameter called the jet radius

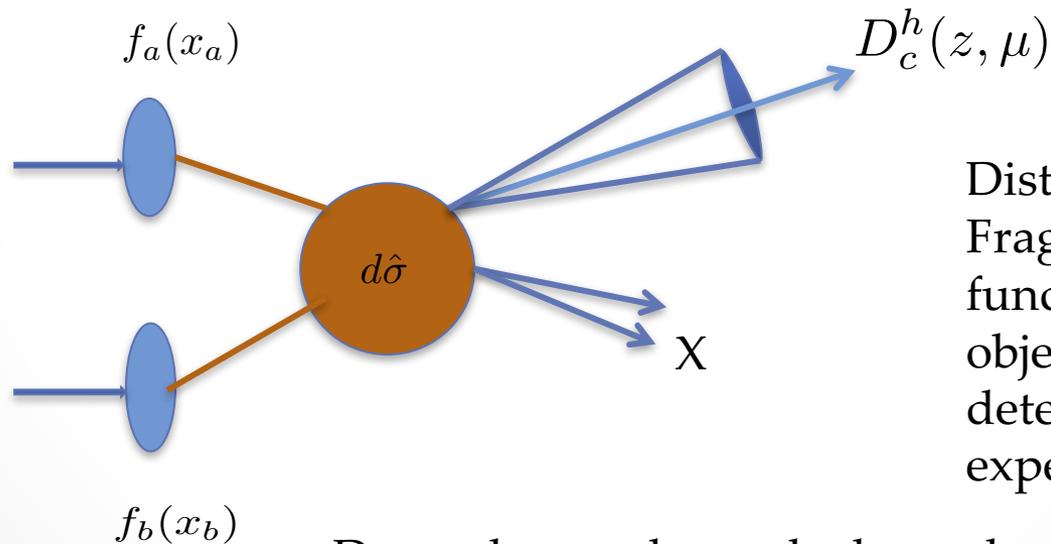
$$R_{jk}^2 = (\eta_j - \eta_k)^2 + (\phi_j - \phi_k)^2$$

Beam distance is defined as  $d_{jB} = k_{T_j}^{2p}$

Algorithm identifies the smallest of  $d_{jk}$  and  $d_{jB}$ . If it is a beam distance the object is defined as a jet and is removed from the list of objects. If it is  $d_{jk}$ , the algorithm merges  $j$  and  $k$  together into a single one. Procedure repeated until no object is left.

# High Energy pp Collision

Cross section for high  $p_T$  jet/hadron production can be factorized in terms of parton distribution and fragmentation functions and the hard scattering cross section



Distribution and Fragmentation function : non-pert object. To be determined from experimental data

Depend on scale : scale dependence of cross section decreases when higher order corrections are included

# Method to calculate single inclusive jet cross section in at NLO

$$pp \rightarrow jet X$$

Analytic approach

Aversa *et al*, Nucl. Phys. B (1989) ; Jaeger, Stratmann, Vogelsang, PRD 70, 034010 (2004); AM, Vogelsang, PRD 86, 094009 (2012)

Narrow jet approximation (NJA) ; analytic method

Expand partonic cross section in jet parameter R

$$A \text{Log} R + B + O(R^2)$$

Keep only A and B

Advantage : collinear singularities cancel analytically; code faster

Works when R is not too large; roughly  $0.2 < R < 0.7$

Start from single inclusive hadron production cross section at NLO

$$pp \rightarrow hX$$

$h$  : hadron with large transverse momentum

Difference with jet cross section : phase space of all final state partons except the observed one are integrated : leads to final state collinear singularities that are absorbed in the parton to hadron fragmentation function

Jet cross section : defined within the jet algorithm. An observable.

All final state singularities must cancel

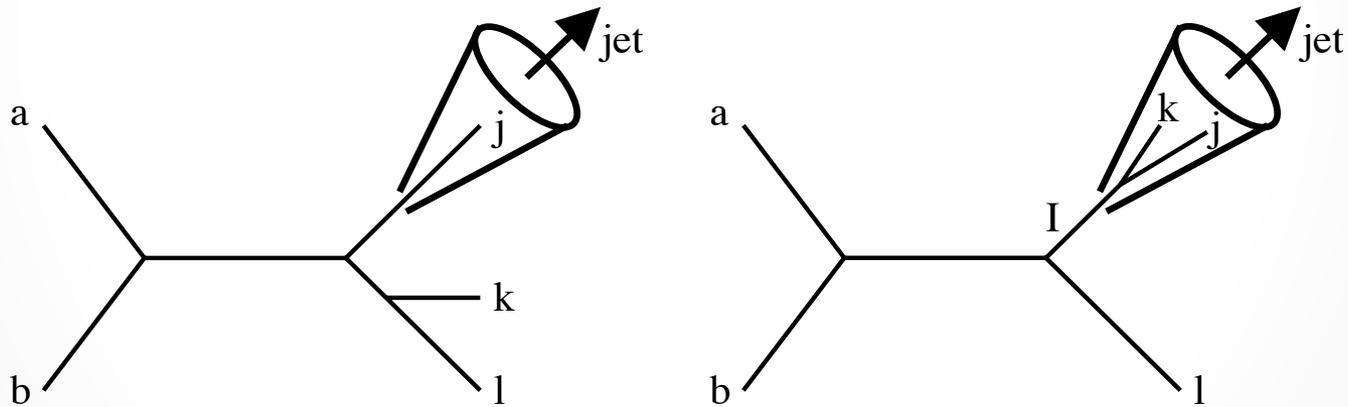
$$d\hat{\sigma}_{ab \rightarrow cX} \quad \swarrow \quad \searrow \quad pp \rightarrow hX$$

↘ Partonic cross section at NLO ↗

Imagine a jet cone around the observed parton  $j$

Single parton inclusive cross section : contains configurations when another parton  $k$  is there within this cone

For jets at NLO, both partons  $j$  and  $k$  that are in the cone and form the jet.



Jaeger, Stratmann, Vogelsang, PRD 70, 034010 (2004)

# Calculation of Jet Cross Section at NLO

$$\begin{aligned}d\hat{\sigma}_{ab \rightarrow \text{jet} X} = & [d\hat{\sigma}_c - d\hat{\sigma}_{c(d)} - d\hat{\sigma}_{c(e)}] \\ & + [d\hat{\sigma}_d - d\hat{\sigma}_{d(c)} - d\hat{\sigma}_{d(e)}] \\ & + [d\hat{\sigma}_e - d\hat{\sigma}_{e(c)} - d\hat{\sigma}_{e(d)}] \\ & + d\hat{\sigma}_{cd} + d\hat{\sigma}_{ce} + d\hat{\sigma}_{de}.\end{aligned}$$

$a, b$  .....quark, antiquark or gluon : sum over all subprocesses

$d\hat{\sigma}_j$  Single parton inclusive cross section with virtual corrections

$d\hat{\sigma}_{j(k)}$  Parton  $j$  observed but parton  $k$  also in the cone

$d\hat{\sigma}_{jk}$  Parton  $j$  and  $k$  inside the cone and form the jet

# Dependence on Jet Algorithm

Single parton inclusive cross section contains collinear singularities that are subtracted in the  $\overline{MS}$  scheme

$d\hat{\sigma}_{j(k)} + d\hat{\sigma}_{k(j)} - d\hat{\sigma}_{jk}$  Also contains collinear singularities, must match those in  $\frac{d\hat{\sigma}_j + d\hat{\sigma}_k}{2}$

Similar subtraction needs to be done so that cross section is finite

$d\hat{\sigma}_{jk}$  Introduces dependence on algorithm

AM and Vogelsang, PRD 86, 094009 (2012)

Technique remains the same both for polarized and unpolarized cross sections

# Single inclusive jet production cross section

$$\frac{d\sigma^{H_1 H_2 \rightarrow \text{jet} X}}{dp_T^{\text{jet}} d\eta^{\text{jet}}} = \frac{2p_T^{\text{jet}}}{S} \sum_{abc} \int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a^{H_1}(x_a, \mu_F) \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b^{H_2}(x_b, \mu_F) \times$$

$$\int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^c(\hat{s}, \hat{p}_T, \hat{\eta}, \mu_F, \mu'_F, \mu_R)}{v dv dw} \mathcal{J}_c \left( z_c, \frac{\mathcal{R} p_T^{\text{jet}}}{\mu'_F}, \mu_R \right),$$

$\mathcal{J}_{q/g}$  Jet function, calculated perturbatively

$$\hat{p}_T = p_T^{\text{jet}} / z_c \quad v \equiv 1 - \frac{\hat{p}_T e^{-\hat{\eta}}}{\sqrt{\hat{s}}}, \quad w \equiv \frac{\hat{p}_T^2}{\hat{s} v (1 - v)}.$$

In the hard scattering, a parton  $c$  “fragments” into the observed jet via the jet function ,

# Scale dependence

$\mu_R$  : Renormalization scale ,  $\mu_F$  : Initial state factorization scale,  $\mu_F'$  final state factorization scale

$\mu_F'$  : present in single inclusive partonic cross section  $d\hat{\sigma}_{ab}^c$

$\mu_F'$  dependence gets canceled in the cross section

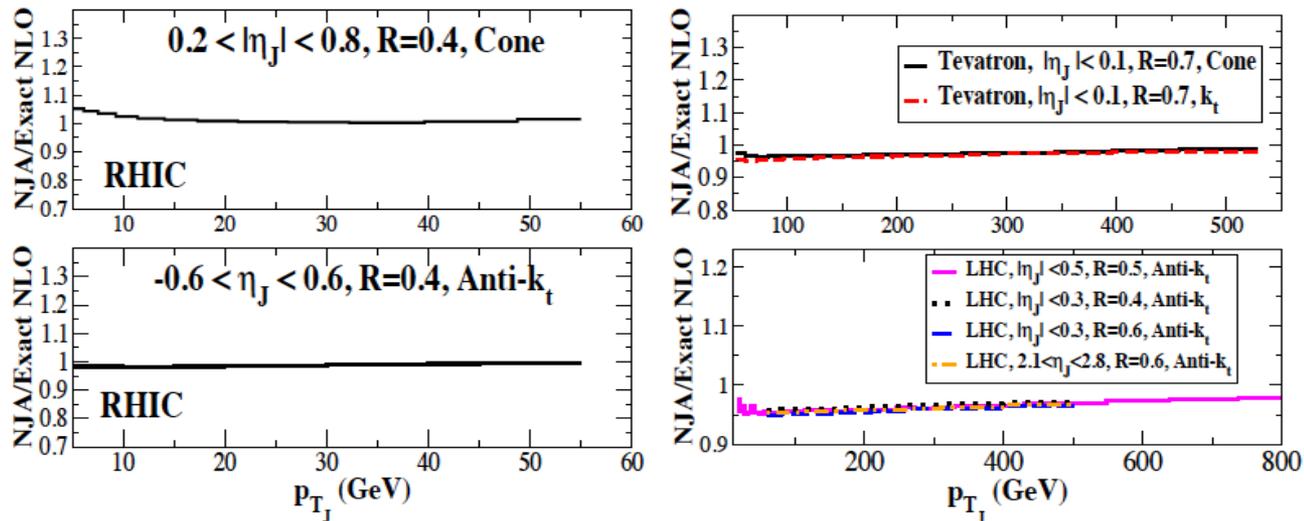
$z_c$  : fraction of parton momentum forming the jet

One can go directly from single inclusive hadron production cross section to jet cross section by replacing the fragmentation function by a jet function

Jet function depends on jet algorithm

# NJA vs exact NLO

## NJA vs. exact NLO



Upper left: Ratio of single-inclusive jet cross sections at RHIC for the cone algorithm, as computed within the NJA and with *fastNLO*. Lower left: Same for the jet cross sections for the  $k_t$ -type algorithms. Here, the exact NLO calculation was performed with the *FastJet* code. Right: Similar comparisons for Tevatron (upper,  $\sqrt{S} = 1960$  GeV) and LHC (lower,  $\sqrt{S} = 7$  TeV) energies. The exact NLO results for Tevatron and for LHC with  $R = 0.5$  were obtained from *fastNLO*, the others from *FastJet*

# Identified hadron inside a jet

Identified hadron in a jet : momentum correlation of hadron and jet : same-side observable

New probe of the fragmentation functions as a function of  $z$

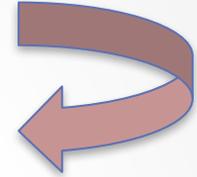
Hadrons in a jet observables were studied in  $e^+e^-$  collision

Procura and Waalewijn, PRD( 2011), Jain, Procura, Waalewijn, JHEP (2011)

For pp scattering, these have been studied in MC approach

F Arleo et al , J. High. En. Phys. 04 (2014) 147 Monte Carlo

Usually fragmentation functions are extracted from the measurements of  $e^+e^- \rightarrow hX$  or  $ep \rightarrow hX$



Gluon fragmentation : only at higher order in QCD

$$pp \rightarrow hX$$

Involves a convolution that forms an integration over  $z$  : information on  $D(z)$  smeared out

$$pp \rightarrow jet hX$$

Cross section differential in  $z_h$  : directly probes the fragmentation function at  $z=z_h$

New information on fragmentation functions, particularly gluon frag. functions which are not well known

Identified hadrons : may provide new information on the structure of jets and hadronization

# Hadron Produced Inside Jets

$$pp \rightarrow jet hX$$

Cross section specified by  $p_T^{jet}, \eta^{jet}, z_h = \frac{p_T}{p_T^{jet}}$

$z_h$ : fraction of jet transverse momentum carried by hadron

Factorized form of the cross section at NLO :

$$\frac{d\sigma_{H_1 H_2 \rightarrow (jet h) X}}{dp_T^{jet} d\eta^{jet} dz_h} = \frac{2p_T^{jet}}{S} \sum_{a,b,c} \int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a^{H_1}(x_a, \mu_F) \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b^{H_2}(x_b, \mu_F) \times$$

$$\int_{z_h}^1 \frac{dz_p}{z_p} \frac{d\hat{\sigma}_{ab}^{(jet,c)}(\hat{s}, p_T^{jet}, \hat{\eta}, \mu_F, \mu'_F, \mu_R, \mathcal{R}, z_p)}{v dv dw dz_p} D_c^h\left(\frac{z_h}{z_p}, \mu'_F\right),$$

Partonic cross section for producing a final state jet  $d\hat{\sigma}^{jet c}$  (subject to a jet algorithm), inside of which the parton  $c$  fragments to a hadron

# Hadron Produced Inside Jets

$$p_T^c = z_p p_T^{jet}$$

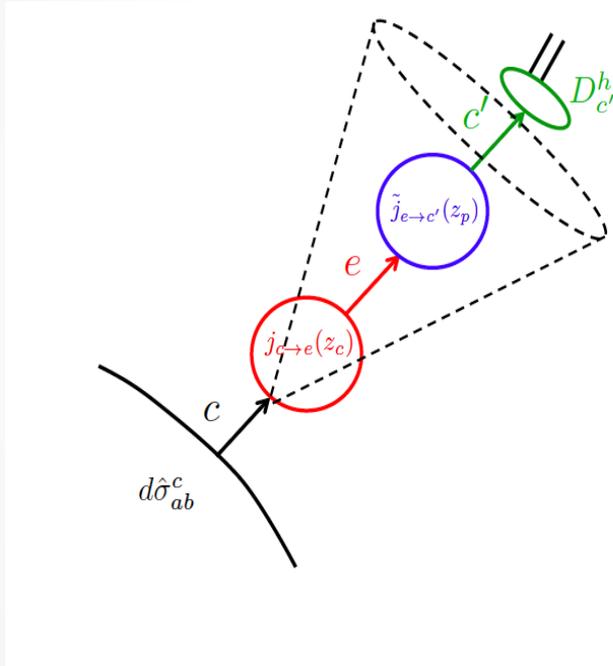
At LO, only one parton forms the jet, and this also fragments into the observed hadron

NLO : consider the same combination as for the single inclusive jet production but do not sum over the parton  $c$  : because we need different fragmentation functions

$$d\hat{\sigma}_{j(k)} + d\hat{\sigma}_{k(j)} - d\hat{\sigma}_{jk} \quad \text{No sum over parton } j \text{ ( } c \text{ )}$$

Double poles cancel, single poles are factorized in the fragmentation functions for parton  $c$

# Hadrons inside Jets



In NJA, production of a jet with an observed hadron factorizes into a production cross section of parton  $c$ , a jet function

$\dot{j}_{c \rightarrow e}$  that describes formation of a “jet”

consisting of parton  $e$  that has taken a fraction  $z_c$  of the momentum of  $c$

$\tilde{j}_{e \rightarrow c'}$  partonic fragmentation of parton  $e$  to parton  $c'$   
 $c'$  then fragments into a hadron

# Identified Hadron Inside Jet at NLO

$$\frac{d\sigma^{H_1 H_2 \rightarrow (\text{jet } h) X}}{dp_T^{\text{jet}} d\eta^{\text{jet}} dz_h} = \frac{2p_T^{\text{jet}}}{S} \sum_{a,b,c} \int_{x_a^{\text{min}}}^1 \frac{dx_a}{x_a} f_a^{H_1}(x_a, \mu_F) \int_{x_b^{\text{min}}}^1 \frac{dx_b}{x_b} f_b^{H_2}(x_b, \mu_F) \times$$

$$\int_{z_c^{\text{min}}}^1 \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^c(\hat{s}, \hat{p}_T, \hat{\eta}, \mu_F, \mu'_F, \mu_R)}{vdv dw} \sum_e j_{c \rightarrow e} \left( z_c, \frac{\mathcal{R} p_T^{\text{jet}}}{\mu'_F}, \mu_R \right) \times$$

$$\sum_{c'} \int_{z_h}^1 \frac{dz_p}{z_p} \tilde{j}_{e \rightarrow c'} \left( z_p, \frac{\mathcal{R} p_T^{\text{jet}}}{\mu''_F}, \mu_R \right) D_{c'}^h \left( \frac{z_h}{z_p}, \mu''_F \right).$$

Jet Functions : calculated analytically  $\hat{p}_T = p_T^{\text{jet}} / z_c$

Calculation based on soft collinear effective theory (SCET)

Chien, Kang, Ringer, Vitev, Xing; JHEP 1605, 125 (2016)

$\mu_F''$  : factorization scale in the fragmentation function

# A Few Jet Functions

$$j_{q \rightarrow q}(z, \lambda, \mu_R) \equiv \delta(1-z) - \frac{\alpha_s(\mu_R)}{2\pi} \left[ 2C_F(1+z^2) \left( \frac{\log(1-z)}{1-z} \right)_+ + P_{qq}(z) \log(\lambda^2) \delta(1-z) I_q^{\text{algo}} + C_F(1-z) \right]$$

$$\tilde{j}_{q \rightarrow g}(z, \lambda, \mu_R) \equiv -\frac{\alpha_s(\mu_R)}{2\pi} [P_{gq}(z) \log(\lambda^2(1-z)^2) + C_F z] ,$$

$$\lambda = \mathcal{R} p_T^{\text{jet}} / \mu'_F \quad \kappa = \mathcal{R} p_T^{\text{jet}} / \mu''_F \quad p_T^c = z_p p_T^{\text{jet}}$$

$$\tilde{\tilde{j}}_{q \rightarrow q}(z_p, \kappa, \mu_R) \equiv \delta(1-z_p) + \frac{\alpha_s(\mu_R)}{2\pi} \left[ 2C_F(1+z_p^2) \left( \frac{\log(1-z_p)}{1-z_p} \right)_+ + P_{qq}(z_p) \log(\kappa^2) \right. \\ \left. + C_F(1-z_p) + \mathcal{I}_{qq}^{\text{algo}}(z_p) + \delta(1-z_p) I_q^{\text{algo}} \right]$$

$$\tilde{\tilde{j}}_{q \rightarrow g}(z_p, \kappa, \mu_R) \equiv \frac{\alpha_s(\mu_R)}{2\pi} [P_{gq}(z_p) \log(\kappa^2(1-z_p)^2) + C_F z_p + \mathcal{I}_{gq}^{\text{algo}}(z_p)] ,$$

Kaufmann, AM, Vogelsang, PRD 92, 054015 (2015)

# Jet+hadron Production

If we drop the terms with sum over the parton  $c'$ , and perform a sum over the parton  $e$ , we would get the single inclusive jet cross section

Terms with sum over  $c'$  thus describes the production of an identified hadron in the jet

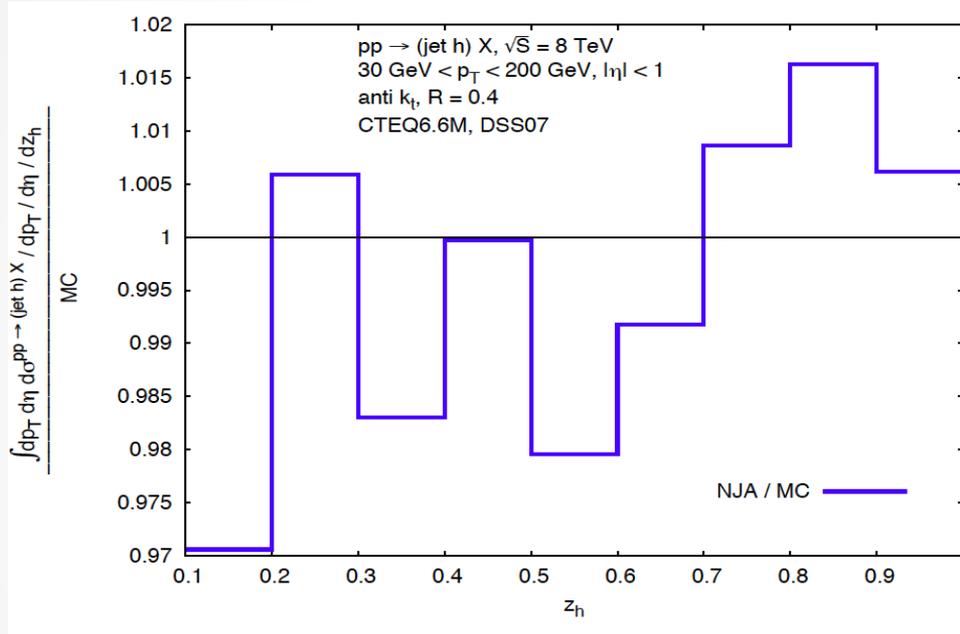
The two jet functions may be combined to give

$$\mathcal{K}_{c \rightarrow c'}(z, z_p; \lambda, \kappa, \mu_R) = \sum_e j_{c \rightarrow e}(z, \lambda, \mu_R) \tilde{j}_{e \rightarrow c'}(z_p, \kappa, \mu_R) ,$$

SCET calculation agrees with our result

Chien, Kang, Ringer, Vitev, Xing; JHEP 1605, 125 (2016)

# Numerical Result



Our results(NJA)  
compared with MC  
(F. Arleo et al, JHEP  
04 (2014), 147)

LHC, CM energy  
8 TeV, anti- $k_t$ ,  
R=0.4

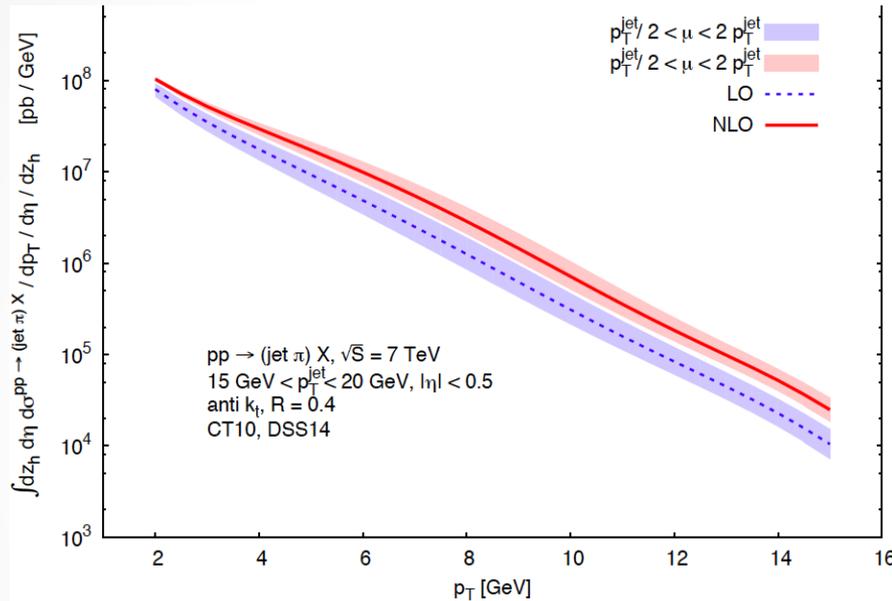
All scales equal to  $p_T$

Kaufmann, AM, Vogelsang, PRD 92, 054015 (2015)

Agreement very good, deviation within 3 %

Definition of  $z_h$  differ by terms  $O(R^2)$ , in the limit  $z_h$  close to 1, two definitions are equivalent. Agreement is also better

# Numerical Results (ALICE)



Fragmentation into charged pions

CT10 pdf (Lai et al, PRD 82, 074024 (2010)) , Frag fn DSS14 (De Florian et al , PRD 91 (2015) 014035 )

Kaufmann, AM, Vogelsang, PRD 92, 054015 (2015)

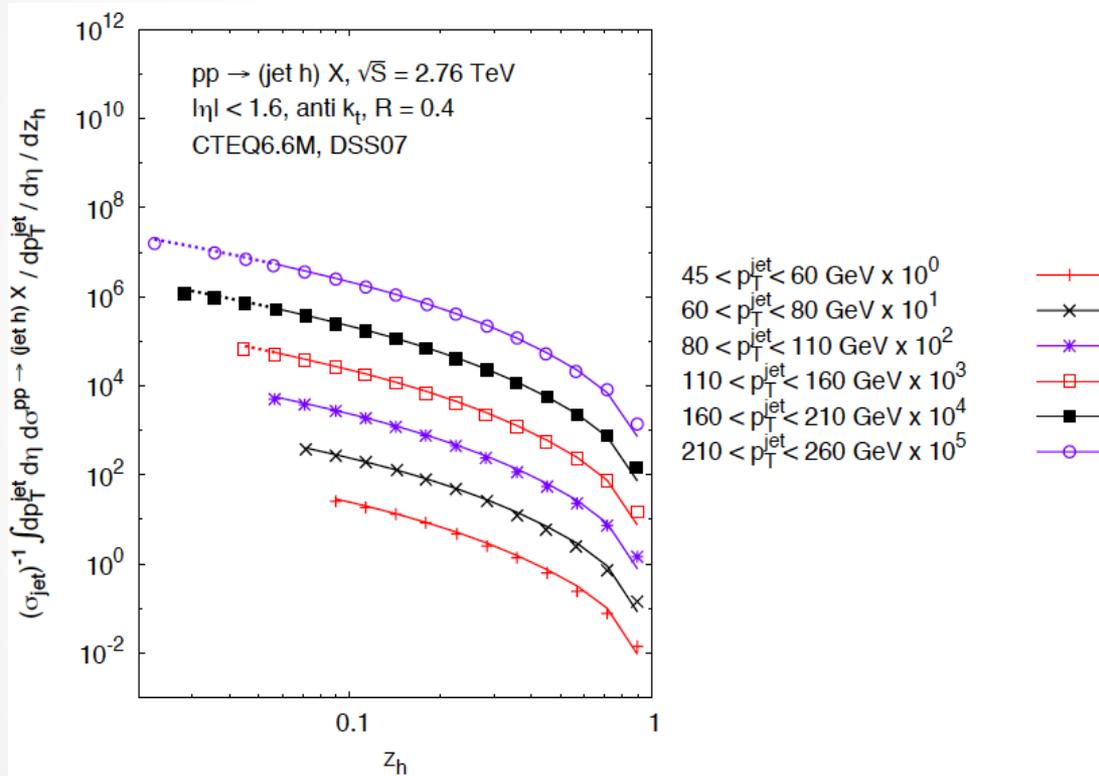
LO and NLO results  
for ALICE kinematics

$$pp \rightarrow jet \pi X$$

Factorization and  
renormalization  
scales are set  
equal, bands show  
scale dependence

Anti-kt, R=0.4

# Numerical Results (ATLAS)



Results compared with data from ATLAS (charged hadron in leading jet )

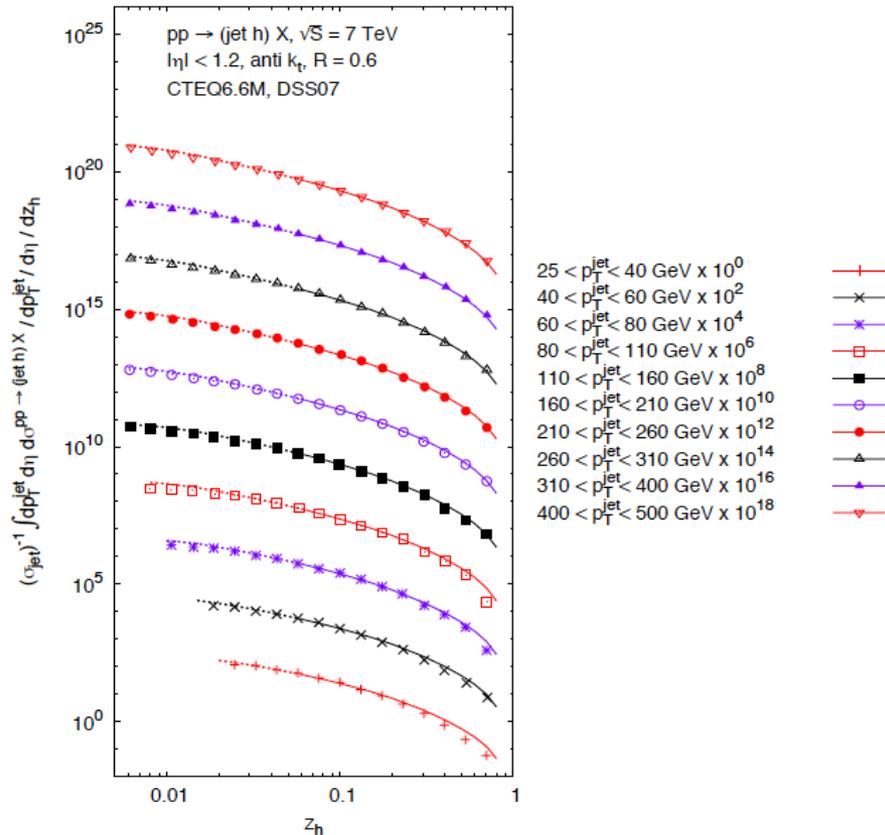
Cross section normalized to total jet rate

Directly maps out fragmentation functions

Overall good agreement, even down to low  $z_h$

Kaufmann, AM, Vogelsang, PRD 92 (2015) 054015  
 ATLAS-CONF-2015-022, ATLAS-COM-CONF-2015-027  
 (Preliminary)

# Numerical Results (ATLAS)



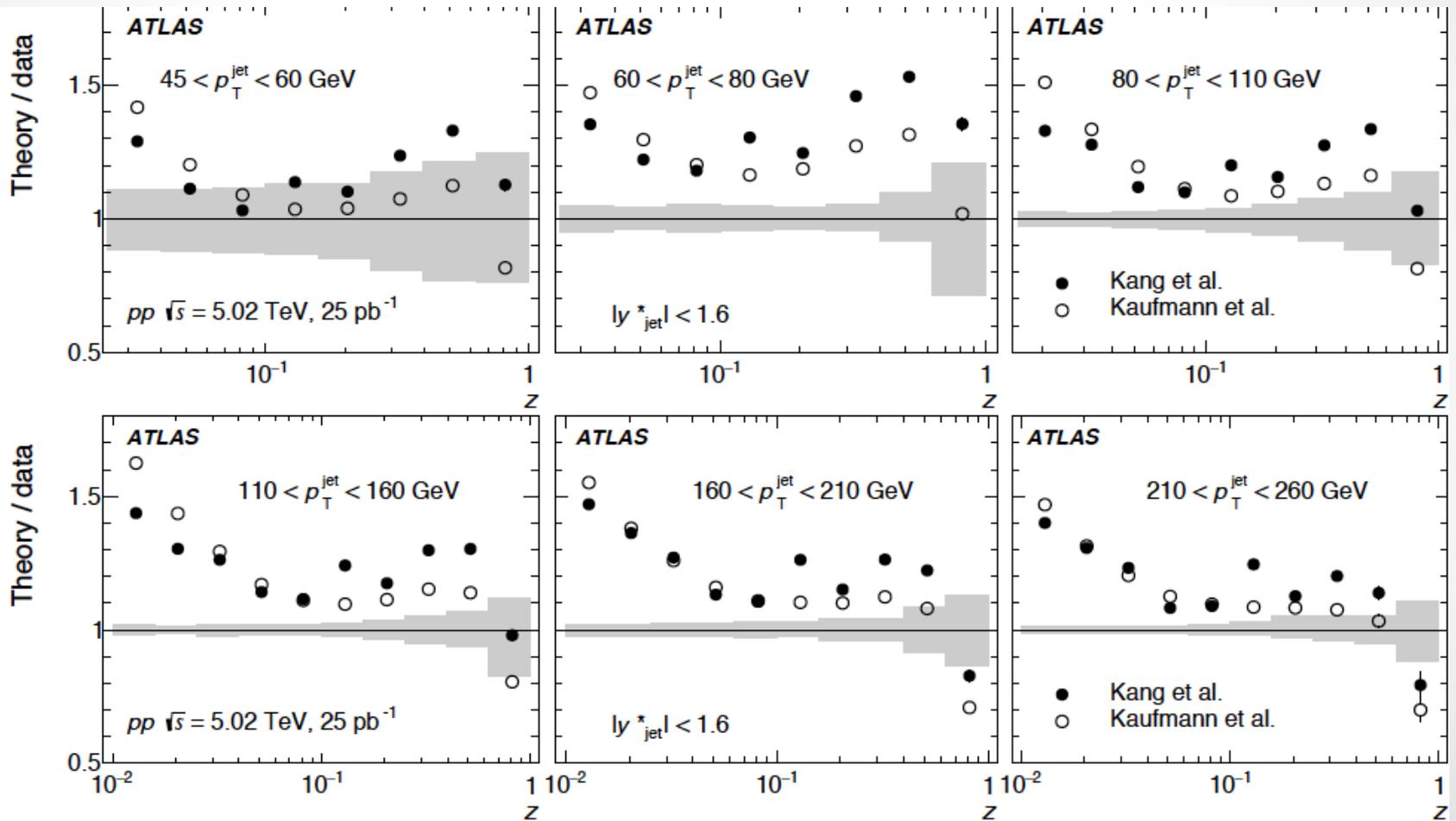
ATLAS charged hadron in leading jet data

EPJC 71, 1795 (2011)

DSS07 fragmentation function for the charged hadrons : extrapolated

Kaufmann, AM, Vogelsang, PRD 92, 054015 (2015)

# ATLAS results



# Conclusion

Analytic calculation of high  $p_T$  jet production cross section in pp collision : narrow jet approximation works for RHIC and LHC energies

NLO calculation of identified hadrons/photons inside a jet : fragmentation function as a function of  $z$

Analytic approach in NJA : good agreement with full MC, results confirmed in SCET

Photon fragmentation function in photon + jet : large background from pion decay

Resummation contributions, higher order dependence on  $R$  ...