

Heavy Quark Production, Propagation and Energy-Loss in Hot and Dense QCD Medium

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Content:

- Why Heavy Quark?
- Heavy Quark Production
- Heavy Quark Propagation
 - Collisional Energy Loss
 - Radiative Energy Loss
 - Heavy-flavoured hadron spectra (Jet Quenching)
- Conclusion

Why Heavy Quarks ?

Heavy Quarks \blacktriangleright Charm (c) & Bottom (b)

Charm: $M_c = 1.27_{-0.09}^{+0.07} \text{ GeV}$

Bottom: $M_b = 4.19_{-0.06}^{+0.18} \text{ GeV}$

Production time: $\tau_Q = 1/2M_Q \leq 0.1 \text{ fm}/c$

Thermalisation time (τ_{th}):

Successive equilibrium: $\tau_{th}^g < \tau_{th}^{u,d} < \tau_{th}^c < \tau_{th}^b < \tau_{th}^{life}$

[J. Alam, S. Raha & B. Sinha, PRL 73 (1994) 1895]

Therma. time for HQs $\tau_{th}^Q \sim \frac{M_Q}{T} \times \tau_{th}^{u,d}$ [G. Moore & D. Teaney, PRC 2005]

$\tau_{th}^{u,d} < \tau_{th}^Q$: no production at QGP and hadronic phase

Produced at very early time interactions in Hard Scattering of partons in Nucleons. Initial distribution of HQs \blacktriangleright frozen

Since g , u , d thermalise early and provide an expanding thermal background

Why Heavy Quarks ?

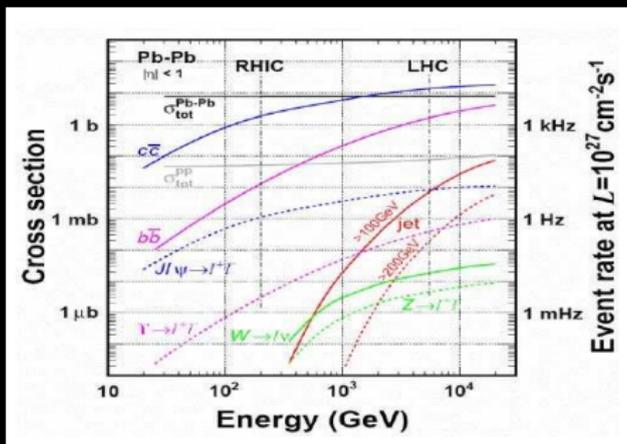
Heavy Quarks propagation:

- propagates in foreground with gluons & light quarks as an expanding thermal background for the non-equilibrated HQs ➤ expansion dynamics
- interacts with equilibrated degrees of freedom ➤ energy-loss of HQ
- requires dynamics ➤ final distrⁿ of HQs (Transport Eq.)

Uniqueness of Heavy Quarks:

- distinguishable down to lowest momenta ➤ medium coupling (Hydro)
- cleaner energy-loss probe ➤ reflected in leading particle p_{\perp} spectra, flow
- tests understanding of mass dependence
- Quarkonia: QGP thermometer

Heavy Quark Production in Heavy-Ion Collisions:



Heavy-Ion Coll.: **ALICE: 0809.1062[nucl-ex]**

Charm: **LHC ~ 10 × RHIC** Bottom: **LHC ~ 100 × RHIC**

Produced in pairs ($Q\bar{Q}$) at early times

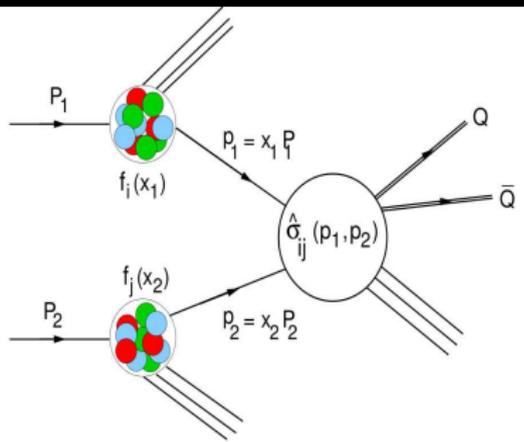
QCD provides a framework to compute production x-section and distribution

HQs Production and Initial Distribution

Factorisations allows that an observable can be expressed as a convolution of short distance (hard scattering of partons) and the long distance contribution describing the initial hadrons

$$\frac{d\sigma}{dy_1 dy_2 dp_\perp} = x_1 x_2 p_\perp \sum_{ij} f_i^{(1)}(x_1, \mu_F^2) f_j^{(2)}(x_2, \mu_F^2) \hat{\sigma}_{ij}(x_i, x_j, \alpha_s(\mu_F^2), \mu_F^2)$$

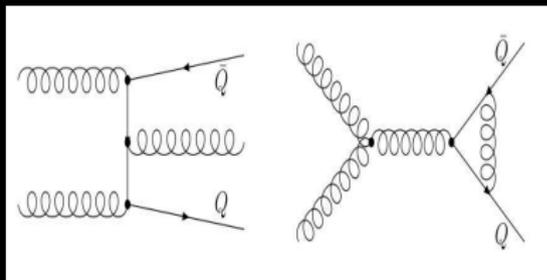
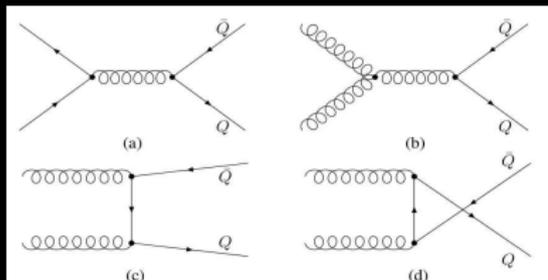
- i, j are partons in hadrons
- σ_{ij} are partonic x-sections \blacktriangleright pQCD
- f_i are parton distrⁿ fn. (PDFs) in hadron which are nonperturbative
- μ_F is the factorization scale between hard processes and the non-perturbative PDFs
- μ_F dependence should cancel in order by order in pQCD calculation



HQs Production and Initial Distribution

Factorisations:

- The short distance contribution describing the matrix elements of hard processes are free from mass singularities and calculable in pQCD
- $gg \rightarrow Q\bar{Q}$; $q\bar{q} \rightarrow Q\bar{Q}$; $gg \rightarrow Q\bar{Q}g$; $q\bar{q} \rightarrow Q\bar{Q}g$ [Leading Order (LO), NLO & FONLL]



- PDFs obey DGLAP evolution equation \rightarrow knowing PDFs at one scale can be calculated at any other scale
- In elementary [pp and $p\bar{p}$] collisions an observable is well controlled through factorization

Heavy-Ion Collisions & Uncertainties

- Does factorisation work in $A+A$? May or may not! But assumed!
- PDFs in nuclei (nPDFs) are different from those in hadrons: geometry, wavefunction are different, nPDFs require appropriate modifications
- Several nPDFs that parametrise the nuclear dependence of incoming parton distribution functions
- “uncertainty in initial state interactions”
- The fragmentation function, used to obtain p_{\perp} spectra, should also be affected by the presence of hot and dense QCD medium!
- “uncertainty in final state interactions”

Heavy Quark Propagation and Final distribution

Produced at early time ($\tau_Q < \tau_{th}$); No production at later time

- ➡ Total no. of HQ gets frozen very early in the history of collisions
- ➡ Immediately upon their production they will propagate through QGP
- ➡ One is left with the task of determining the HQ distribution
- ➡ Details of the distribution may reflect the characteristics and development of QGP

Fokker-Planck Equation

- ▶ Dynamics of a heavy quark propagation as a foreground particle in a QGP can be studied using the simplified Boltzmann approach when the background bulk medium is in thermal equilibrium

Extensive Body of Literatures

- ▶ Chakraborty & Syam, Lett. Nuvo Cim.'84
- ▶ Svetitsky, PRD'88
- ▶ Alam, Raha & Sinha, PRL'94
- ▶ Mustafa, Pal & Srivastava, PRC'97
- ▶ Mustafa PRC'05
- ▶ Moore & Teany PRC'05; Rapp & Hees'05

Fokker-Planck Equation

📌 Fokker-Planck Eq. ➤ Simplified Boltzmann Eq.

- 👉 Boltzmann Eq ➤ No external force and isotropic in space
- 👉 Soft Scatt. ➤ Taylor expan. of Coll. Integral ➤ Landau Eq.
- 👉 Landau Eq. ➤ an integro-diff^l Eq. involving transport coeffs. ➤ describes collision processes of two particles
- 👉 Landau Eq. ➤ FP Eq. when distribution of the background particles (QGP) is thermal whereas foreground particle (HQs) is non-thermal

Standard Dynamics of Heavy Quarks in the QGP

c,b quarks



Brownian Motion?

Fokker-Planck approach

$$\frac{df_{c,b}}{dt} = \gamma \frac{\partial(pf_{c,b})}{\partial p} + D \frac{\partial^2 f_{c,b}}{\partial p^2} \quad T \ll m_Q$$

From scattering matrix $|M|^2$

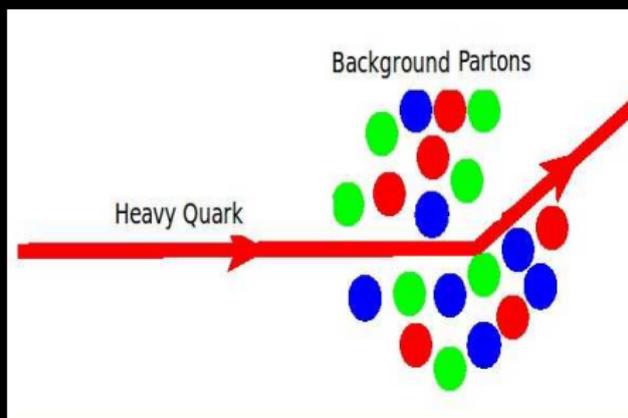
$$\gamma_P = \int d^3k |M(k,p)|^2 P$$

$$D = \frac{1}{2} \int d^3k |M(k,p)|^2 P^2 \quad \bullet \text{ from some theory...}$$

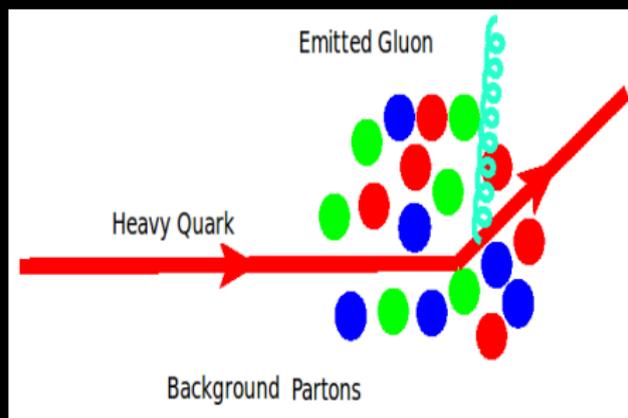
Propagation of Heavy Quark in QGP

Energy-Loss

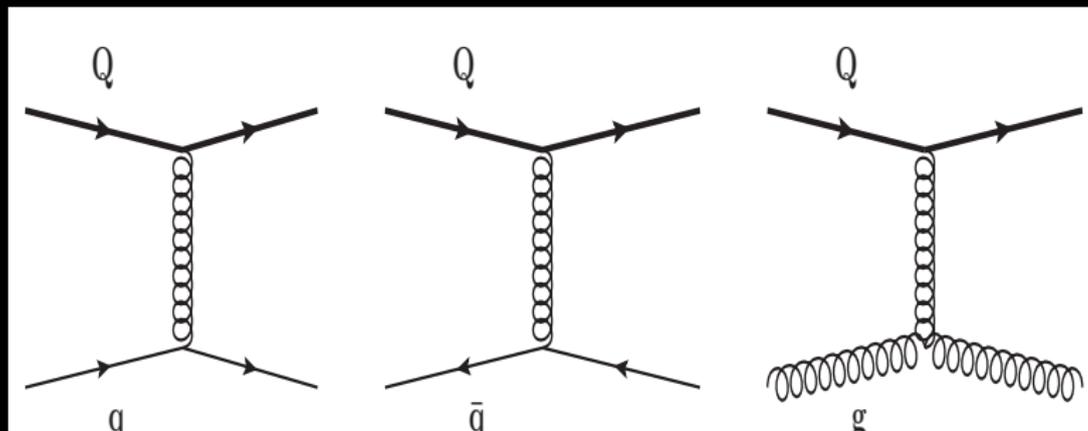
Collisional Loss



Radiative Loss

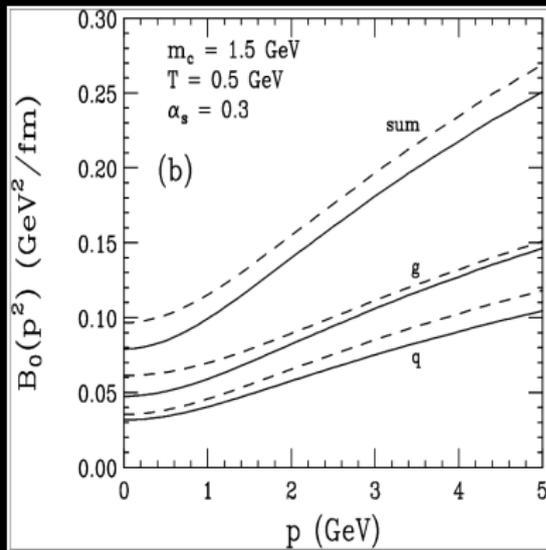
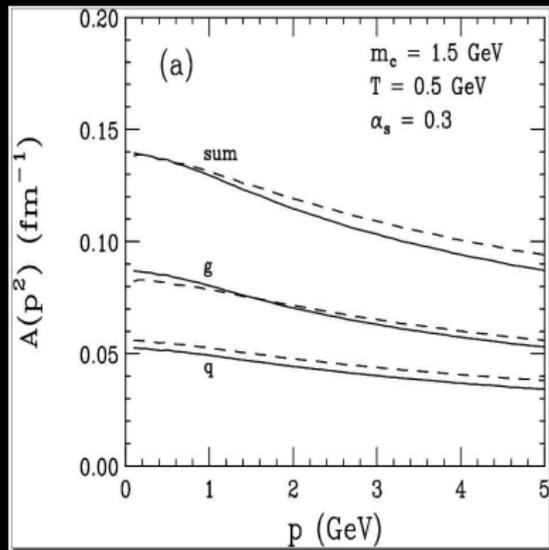


Collisional Processes

 Feynman Diagram

Heavy Quark Transport Coefficients from FP Equation

■ Drag $\mathcal{A}(p)$ and Diffusions $\mathcal{B}_0(p)$



■ Mustafa, Pal & Srivastava, PRC'97

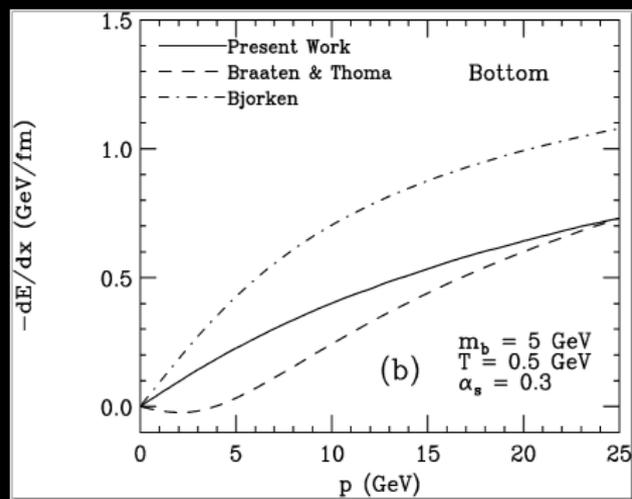
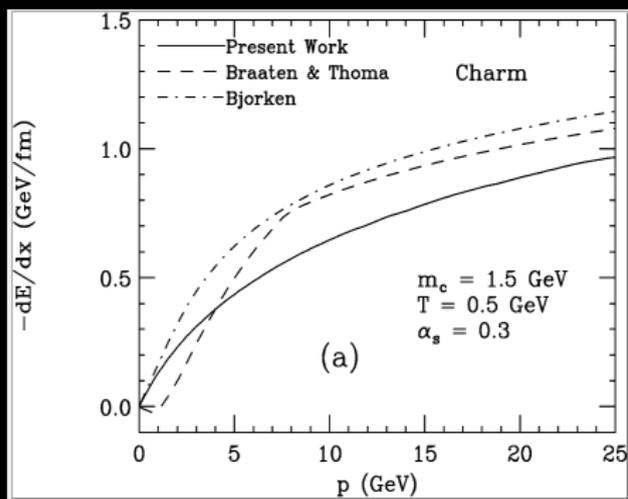
■ Mustafa PRC'05

Differential Collisional Energy Loss

 Drag Coeff.: $\mathcal{A}(p) = -\frac{1}{p} \frac{dE}{dx}$

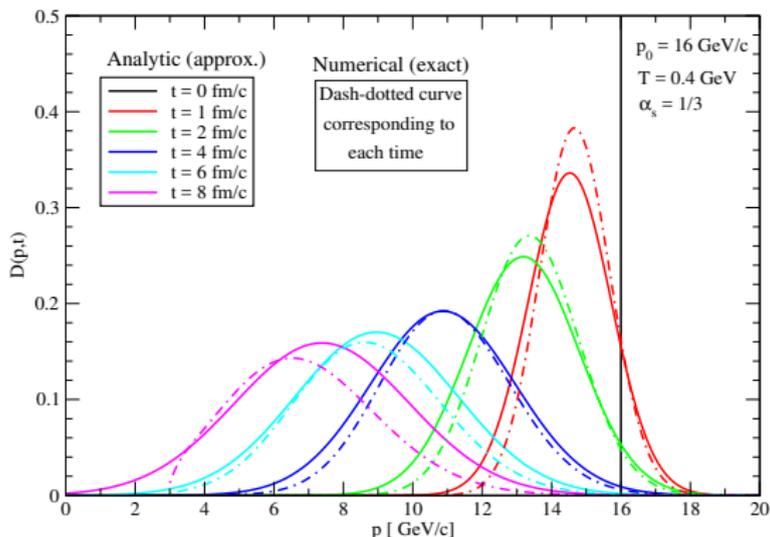
 Differential Collisional Energy-Loss: $\frac{dE}{dx}$

[Mustafa, Pal, Srivastava, PRC57 (1998) 889]



HQ Energy-Loss Distribution [Mustafa, PRC72 (2005) 014905]

Collisional Energy-Loss ➤ Transport-coeffs. ➤ Fokker-Planck Eq. ➤ Energy-Loss Distribution



Peak Shifts to lower p

Drag Force on mean p

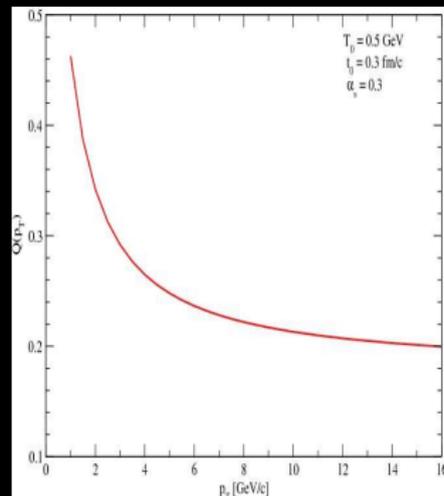
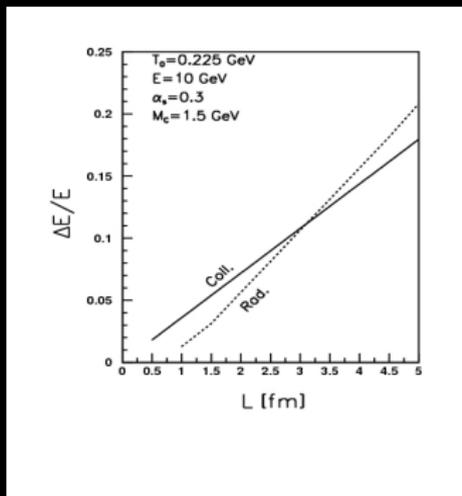
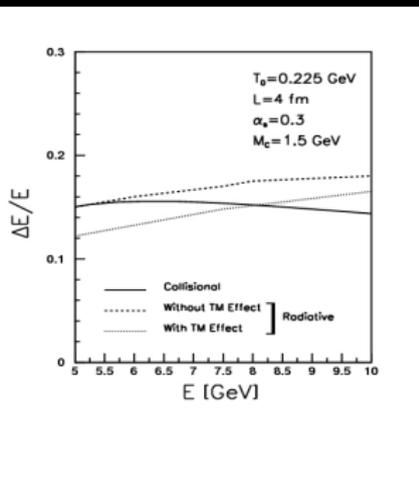
Peak broadens

Diffusion in p

Momentum Dispersion

Substantial Collisional E-Loss

 **Mustafa, PRC72 (2005) 014905**



 Coll. E-Loss is important

 Predicted R_{AA} but no data then

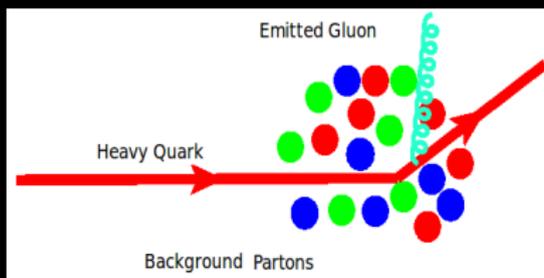
 For light quarks R_{AA} :

 **Mustafa et al., PRL100 (2008) 072301; Acta Phys.Hung. A22 (2005)**

Radiative Energy Loss

HQ encounters inelastic scattering as it traverses a medium

☞ Emits a gluon



☞ Differential Loss: dE/dx

☞ Very simple but first estimation:

☞ **Mustafa, Pal, Srivastava, Thoma, PLB 428 (1998) 234**

☞ Improvements in formalism

Radiative Energy Loss

Significant Points:

- ☞ The nature of the medium through which the energetic parton propagates ➤ thermally equilibrated perturbative medium with a collection of static scatt. centres with specified density
- ☞ Kinematic approximations for interaction between medium partons (thermal background) and the projectile parton that is propagating in the medium
- ☞ The virtuality and branching/splitting of hard parton to reduce its off-shellness ➤ multiple splitting may occur in the medium ➤ multigluon final state should include interference of emitted gluons

Radiative Energy Loss

Kinematic Approximations:

-  **Eikonal-I:** leading parton energy ($E = p_z, p_\perp = 0$) $E \gg q_\perp$, the transverse mom. of the exchanged gluons \blacktriangleright doesn't give sufficient transverse kick to deflect the parton from straightline trajectory
-  **Eikonal-II:** leading parton energy ($E = p_z, p_\perp = 0$) $E \gg k_\perp$, the transverse mom. of the emitted gluons \blacktriangleright doesn't get enough transverse kick from emitted gluons too
-  **Soft gluon emission:** the energy of the emitted gluon, $\omega \ll E \blacktriangleright$
 $x = \frac{\omega}{E} \sim 0$
-  **Small angle or colinear emission:** the energy of the emitted gluons, $\omega \gg k_\perp$, its transverse momentum. In broader sense $E \gg \omega \gg k_\perp$, which includes Eikonal-II

Suppression due to Mass and Dead Cone

General notion: heavy quark radiates less than light quark

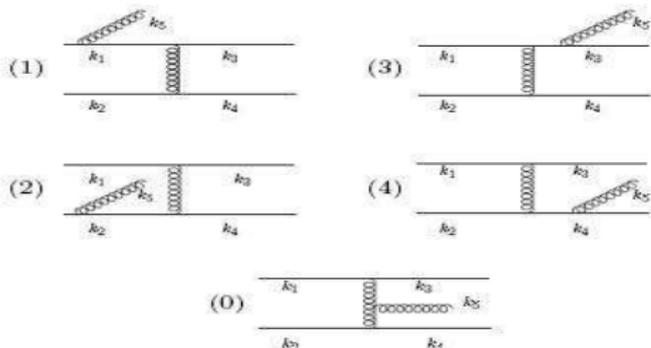
$qq' \rightarrow qq'$	$ \mathcal{M}_{qq' \rightarrow qq'} ^2 = \frac{8}{9} g^4 \frac{s^2}{t^2}$
$Qq \rightarrow Qq$	$ \mathcal{M}_{Qq \rightarrow Qq} ^2 = \frac{8}{9} g^4 \frac{s^2}{t^2} \left(1 - \frac{M^2}{s}\right)^2$
$qq' \rightarrow qq'g$	$ \mathcal{M}_{qq' \rightarrow qq'g} ^2 = 12g^2 \frac{8}{9} g^4 \frac{s^2}{t^2} \frac{1}{k_{\perp}^2}$
$Qq \rightarrow Qqg$	$ \mathcal{M}_{Qq \rightarrow Qqg} ^2 = 12g^2 \frac{8}{9} g^4 \frac{s^2}{t^2} \frac{1}{k_{\perp}^2}$?

2001 Dokshitzer and Kharzeev proposed (Phys. Lett. B 519, 199 (2001)) “dead cone” effect => heavy quark **small energy loss**.

$$\left(1 + \frac{\theta_0^2}{\theta^2}\right)^{-2} \quad \theta_0 = M/E$$

Generalised Dead Cone

RHIC data (PHENIX) $\rightarrow R_{AA}^{HQ} \sim R_{AA}^{LQ} \rightarrow$ (Heavy quark puzzle)



Hierarchy employed in this study

$$\sqrt{s}, E \gg \sqrt{|t|} \sim q_{\perp} \gg \omega > k_{\perp} \gg m_D$$

Mass Range $0 < M/E < 1$

Emission angle Range $-\pi < \theta < +\pi$

$$\begin{aligned}
 |\mathcal{M}_{Qq \rightarrow Qqg}|^2 &= 12g^2 |\mathcal{M}_{Qq \rightarrow Qq}|^2 \frac{1}{k_{\perp}^2} \left(1 + \frac{M^2}{s \tan^2(\frac{\theta}{2})} \right)^{-2} \\
 &= 12g^2 |\mathcal{M}_{Qq \rightarrow Qq}|^2 \frac{1}{k_{\perp}^2} \left(1 + \frac{M^2}{s} e^{2\eta} \right)^{-2}
 \end{aligned}$$

$$D = \left(1 + \frac{M^2}{s \tan^2(\frac{\theta}{2})} \right)^{-2}$$

..... Dead Cone Factor

Abir, Greiner,
Martinez,
Mustafa,
Uphoff, PRD85
(2012) 054012


 $Qq \rightarrow Qqg$ and $Q\bar{q} \rightarrow Q\bar{q}g$; Replace $Q \rightarrow \bar{Q}$

$$\left. \frac{dn_g}{d\eta dk_{\perp}^2} \right|_{Qq \rightarrow Qqg} = \frac{C_A \alpha_s}{\pi} \frac{1}{k_{\perp}^2} \mathcal{D}$$


 $Qg \rightarrow Qgg$ and Replace $Q \rightarrow \bar{Q}$

$$\left. \frac{dn_g}{d\eta dk_{\perp}^2} \right|_{Qg \rightarrow Qgg} = \frac{C_A}{C_F} \left. \frac{dn_g}{d\eta dk_{\perp}^2} \right|_{Qq \rightarrow Qqg}; \quad \frac{C_A}{C_F} = \frac{9}{4}$$

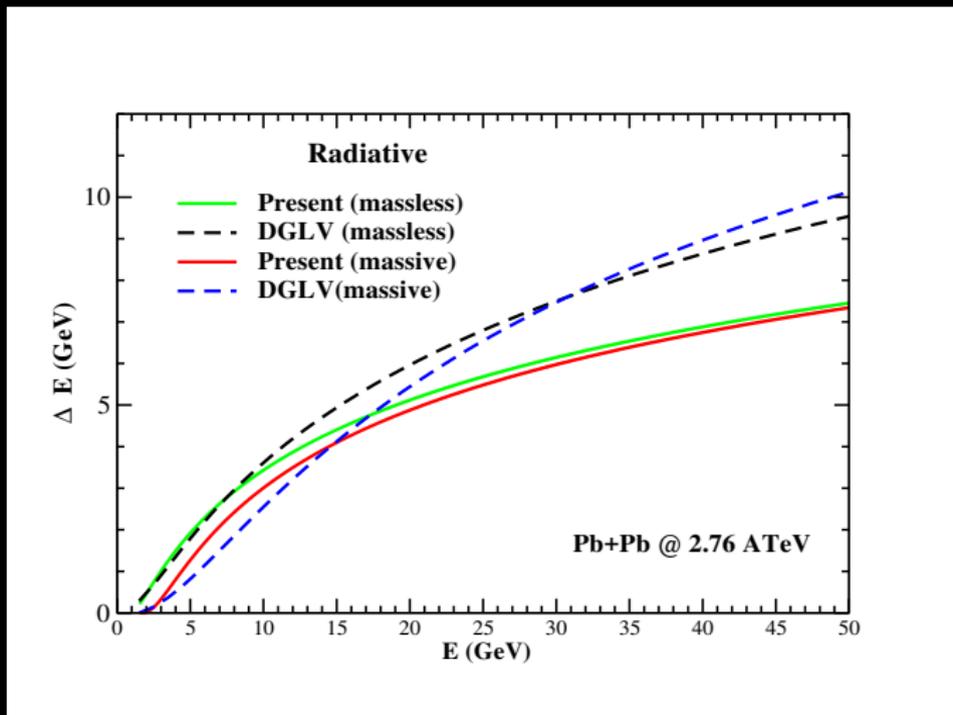
 Differential Radiative E-Loss

$$-\frac{dE}{dx} = \frac{\langle \omega \rangle}{\langle \lambda \rangle}$$

$\langle \omega \rangle$ = Mean energy of the emitted gluon

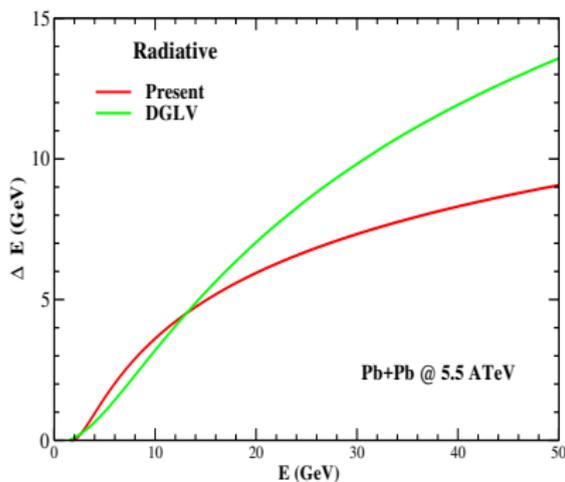
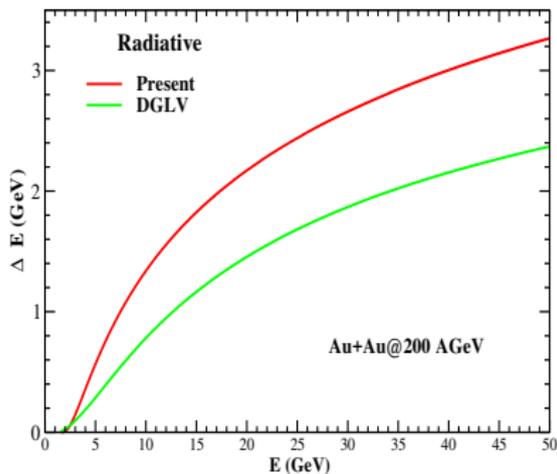
$\langle \lambda \rangle$ = Mean free path of the heavy quark

Radiative E-Loss [Abir,Jamil,Mustafa,Sivastava, PLB715 (2012) 183]

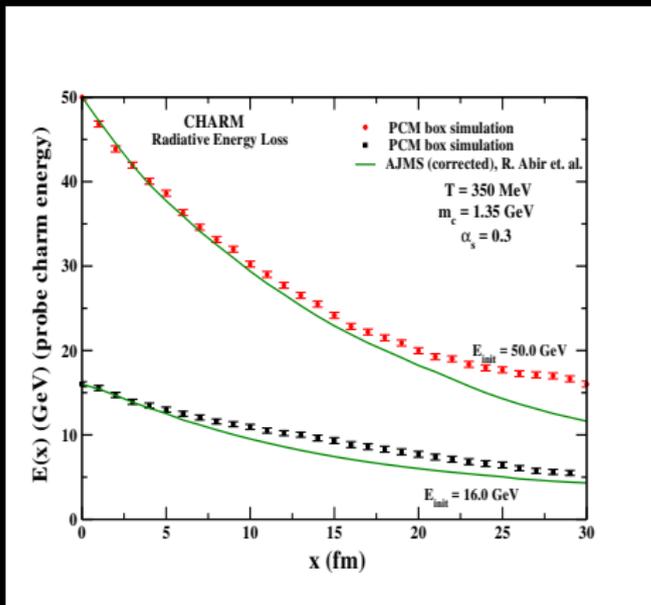


DGLV: PRL85 (2000); NPA784 (2007); NPA783 (2000); NPA733 (2004)

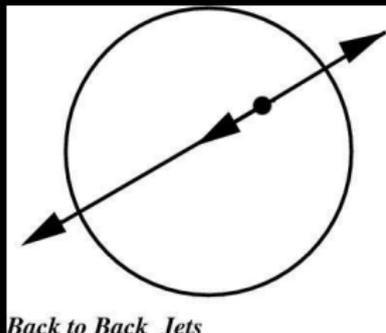
Radiative E-Loss [Abir,Jamil,Mustafa,Sivastava, PLB715 (2012) 183]



Comparison with Parton Cascade Model [Younus et al, Phys.Rev. C91 (2015) 024912]



Jet Quenching

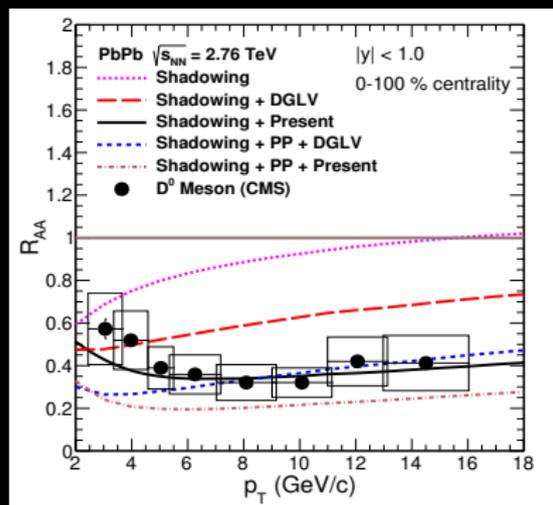
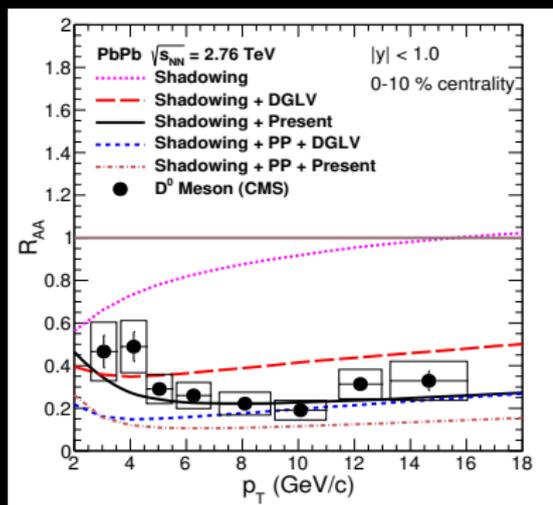


- Jets are produced back to back
 - ☞ Awayside will pass through medium
 - ☞ Interact with the medium
 - ☞ Lose energy and quenched
 - ☞ Results in suppression of hadronic yields in AA than NN

■ Nuclear Suppression Factor:
$$R_{AA} = \frac{(\text{Yield})^{AA}}{N_{\text{coll}} (\text{Yield})^{NN}}$$

- Quenching depends on amount of energy-loss suffered in the medium

D-Meson @ LHC 2.76A TeV

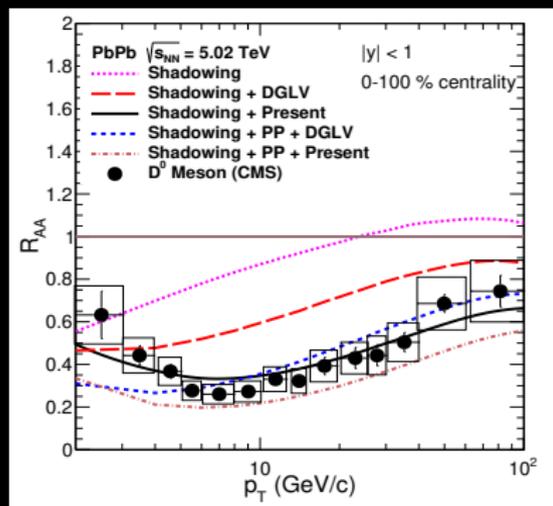
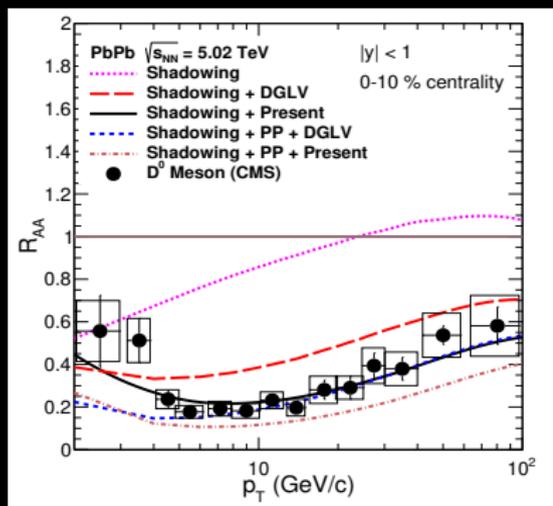


DATA: CMS Collaboration , CMS-PAS-HIN-16-011.

Theory: Abir, Jamil, Mustafa, Srivastava, PLB715 (2012) 183

Saraswat, Shukla, Singh, NPA961(2017)169

D-Meson @ LHC 5.02ATeV



☛ Data: CMS Collaboration, CMS-PAS-HIN-16-001.

☛ Theory: Abir, Jamil, Mustafa, Srivastava, PLB715 (2012) 183

☛ Saraswat, Shukla, Singh, NPA961(2017)169

Collisional Energy Gain

- It is assumed that the energy lost by the particle per unit time is small compared to the energy of the particle itself so that the change in the velocity of the particle during the motion may be neglected, i.e, the particle moves in a straight line trajectory.
- The energy loss of a particle is determined by the work of the retarding forces acting on the particle in the plasma from the chromo-electric field generated by the particle itself while moving:

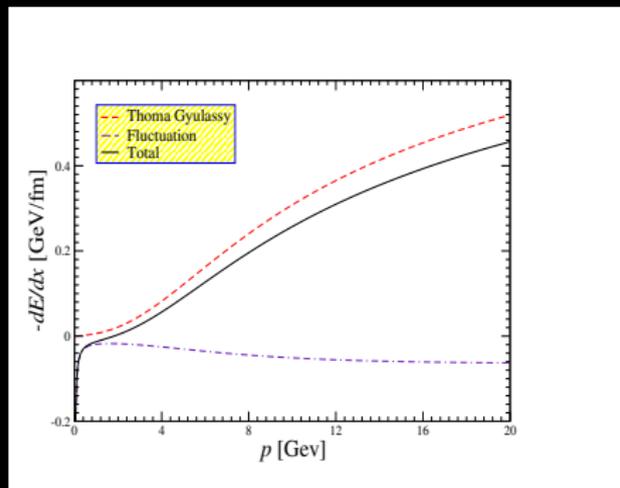
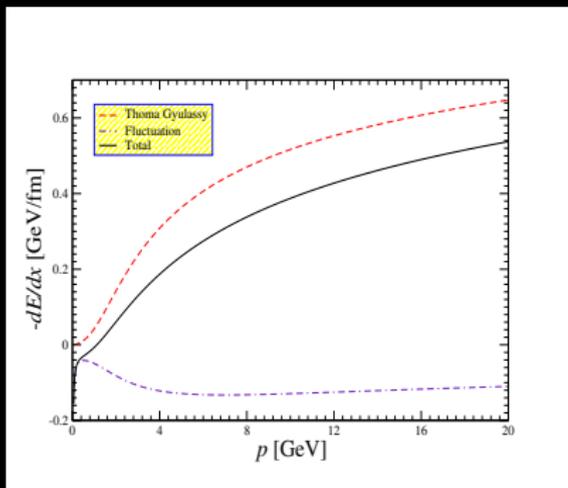
$$\left. \frac{dE}{dt} \right|_{\text{coll}} = Q^a \vec{v} \cdot \vec{\mathcal{E}}^a \Big|_{\vec{r}=\vec{v}t}$$

- The collisional energy loss does not take into account the field fluctuation in the plasma and the particle recoil in collisions.
- One need to take in to account fluctuation of chromo-electromagnetic fields

Collisional Energy Gain

Fluctuation of chromo-electromagnetic fields leads to energy gain

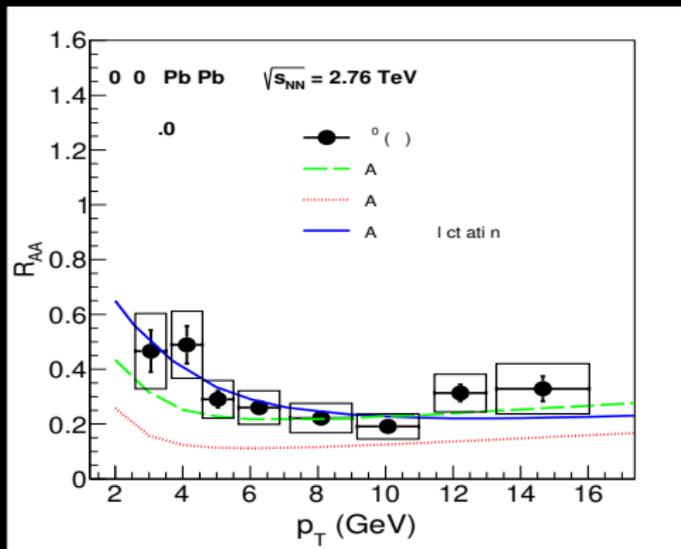
$$\left. \frac{dE}{dt} \right|_{\text{fluc}} = \left\langle Q^a \vec{v}(t) \cdot \vec{\mathcal{E}}^a(\vec{r}(t), t) \right\rangle$$



Chakraborty, Mustafa, Thoma, PRC75 (2007) 064908

Collisional Energy Gain

 Nuclear Modification factor (R_{AA}) for D-Meson



 DATA:CMS Collaboration , CMS-PAS-HIN-16-011

 A Ikbal, Z Ahamed, M G Mustafa, Phys .Rev. C98 (2018) 034915

Conclusion:

Discussed why are heavy quarks important

Heavy quark production in high energy HIC: X-section in pQCD

Collisional E-Loss & E-Gain

Relation to Transport Coeffs: Drag and Diffusion

Coll. E-Loss Distr. & **importance of Coll. E-Loss**

Radiative E-Loss

Discussed in general various constraints in Rad. E-Loss Model

Generalised Dead Cone and radiative E-Loss

Light and heavy quark lose energy in a similar fashion

Heavy quark nuclear suppression R_{AA} ; Jet quenching

D-Meson @LHC for 2.76ATeV in Pb+Pb

Collaborators:

- Raktim Abir (AMU, Aligarh)
- Purnendu Chakraborty (Basirhat College, W. Bengal)
- Umme Jamil (D.R. College, Golaghata, Assam)
- Dipali Pal (USA)
- Asik Ikbal (VECC, Kolkata)

- D. K. Srivastava (VECC, Kolkata)
- Zubayer Ahmed (VECC, Kolkata)

- Markus H. Thoma (Giessen Univ., Germany)
- Carsten Greiner (Goethe Univ. Frankfurt, Germany)
- Jan Uphoff (Goethe Univ., Frankfurt, Germany)
- M. Martinez (FIAS, Frankfurt, Germany)

THANK YOU